DE
INVENIENDA LINEA CURVA,
QUÆ IN
CORPORE LIQUIDO MOTA MINIMAM PATITUR RESISTENTIAM.

DISSERTATIO,

Quam

Conſ. Ampliſſ. Facult. Philoſ. Aboënſ.

PRÆSIDE

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PRO GRADU PHILOSOPHICO

publice examinandam proponit

JACOBUS WEGELIUS,

Oſtrobotnienſis.

In Audit. Majori die 19 Maji MDCCCII,

Horis a. m. ſolitis.

ABOÆ, typis Frenckellianis.

18.

VIRO

ADMODUM REVERENDO atque PRÆCLARISSIMO

DOMINO MAGISTRO

ESAIÆ WEGELIO,

Præposito atque Pastori Ecclesiarum, quæ Deo in Wörå colliguntur, meritissimo.

PARENTI OPTIMO!

Plura & majora sunt, Parens Indulgentissime! beneficia paterna, quæ Tibi debeo, quam ut ea vel verbis persequi, multo minus persolvere unquam possim. Has igitur pagellas Tibi consecrans, ut eas, tanquam pignus summæ pietatis animique gratissimi, hilari benignoque, quo soles, vultu accipias, supplex oro atque obtestor, ad cineres permansurus

PARENTIS OPTIMI

filius obsequentissimus

JACOBUS WEGELIUS.

Si figura plana curvilinea, quæ diametro est prædita, & duas juxta hunc diametrum habet partes perfecte similes & æquales, in aqua secundum directionem diametri movetur, resistentiam patitur, illamque diversam pro diversis angulis incidentiæ, quibus curva aquæ impingit. Inveniri itaque forsitan posset talis curvatura figuræ, quæ minimam pateretur resistentiam. Ut vero hæc disquisitio succedat, lex, secundum quam minuitur resistentia aquæ pro diminutis angulis incidentiæ, cognoscatur necesse est. A priori quidem demonstrarunt plures Mathematici celeberrimi (*) resistentiam aquæ rationem sequi dupplicatam sinus anguli incidentiæ, & hanc legem ad inveniendam resistentiam in figuras planas & corpora solida applicarunt. Considerarunt autem aquam uti congeriem particularum minimarum solidarum, quæ

A nulla

(*) DE L'HOPITAL in *Mem. de l'Acad. Roy. des Sciences de Paris* 1699, *pag.* 147. JOH. BERNOULLI in *Essai d'une nouv. theorie de la manoeuvre des vaisseaux*, *Basle* 1714, *Cap. I*, § 1. BOUGUER in *Traité du navire*, *Paris* 1746, *L. III*, *Sect. I*, *Chap. II*, § 1. LEONH. EULER *in Scientia naval. Petrop.* 1749, *Part. I*, *Cap.* 5, § 474.

nulla vi inter se cohærerent, tacite simul supponentes, particulas aquæ, ne sequentium resistentiam mutarent, mox post impulsionem vel omnino evanescere, vel saltem ita reflecti, ut ceteris non occurrant (*). Cumque hæc hypothesis minime sit admittenda, docentibus experimentis, quæ in Gallia communi sollertia D'ALEMBERT, DE CONDORCET & BOSSUT (**), nec non in Anglia VINCE, instituerunt, resistentiam aquæ in motu obliquo ceteris paribus non diminui in ratione quadrati Sinus anguli incidentiæ; theoria illa resistentiæ vulgaris nullius in praxi est usus. Experimentis itaque invenienda erat nova lex resistentiæ, naturæ rei magis apta, taliaque anno 1794 instituit classis nostræ bellicæ Præfectus gener. FREDR. H. A CHAPMAN. Ex iis hanc pro determinanda resistentia aquæ deduxit regulam, ut facto angulo incidentiæ $= w$, & Sinu toto $= 1$, quantitas $Sin\ 45^\circ + Sin\, w^2 - \frac{1}{2\, Cos\, w}$, si w non excedit 45°, sed pro $w > 45^\circ$ quantitas $\frac{1}{2} + \frac{1}{2} Sin\, w \left(Sin\ 45^\circ - \frac{1}{2\, Sin\, w} \right)$ ducta in aream projectionis lineæ, cui resistit aqua, orthogonalis in illa linea, quæ directio-

(*) Cfr. *Essai d'une nouvelle theorie de la resistance des fluides, par* D'ALEMBERT, *Paris* 1752, *introd. pag* XXIII; SAM. VINCE in *Philos. transactions of the roy Soc of London* 1798, & in *Annalen der Physik, herausgeg. von* L. W. GILBERT, *Halle* 1800, 4 *B.* 1 *St. pag.* 34.

(**) Vide: *Nouvelles experiences sur la resistance des fluides, par* D'ALEMBERT; DE CONDORCET & BOSSUT; *Paris* 1777; *Chap. V, Sect. IV, pag.* 175.

rectioni motus normalis est, sumtæ, proportionalis sit resistentia (*). Observavit allatas formulas resistentiæ valere, si planum quoddam in situ vel horizontali vel verticali movetur.

Ad inveniendam figuram plani, quod minimam patiatur resistentiam, leges has Chapmanianas bona lectoris venia jam applicabimus. Cum autem minor sit resistentia pro minoribus angulis incidentiæ, per se patet, illam tantum in hac quæstione esse adhibendam regulam, quæ supponit $w > 45°$.

Sint (Fig. 1) AC & AB axes coordinatarum orthogonalium in plano, quod horizontaliter in directione axis AC movetur, & ipsæ coordinatæ $AP = x$ atque $PM = y$. Sumta $Pp = dx$, atque ductis pn & mn rectis PM & AP parallelis, nec non facto arcu $BM = s$; erit $Mm = -dy$, & $Mn = ds$. Cum sit mn motus directioni parallela, erit Mnm angulus incidentiæ aquæ in elementum curvæ Mn, adeoque ang. $Mnm = w$. In triangulo vero mMn est $ds : -dy :: 1 : Sin\, w$, & $ds : dx :: 1 : Cos\, w$, quare habetur $Sin\, w = -\frac{dy}{ds}$ & $Cos\, w = \frac{dx}{ds}$. His valoribus substitutis, factoque $Sin\, 45° = a$, erit quantitas resistentiæ in elementum ds, cujus projectio orthogonalis in ordinata y est $-dy$, hæc: $-\left(a + Sin\, w^2 - \frac{1}{2\, Cos\, w}\right) dy = \frac{ds\, dy}{2dx}$

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(*) Cfr. *Kongl. Vetensk. Acad. nya Handlingar, Tom. XVI, för år 1795, 2 qvart.*

$= \frac{dy^3}{ds^2} - ady$. Expressio autem hæc evanescit adeoque minima evadit facta $dy = 0$, quod indicat fieri y constantem, & quidem in casu minimæ resistentiæ $y = 0$; unde apparet planum quæsitum absolute minimæ resistentiæ non posse inveniri.

Felicius vero succedit disquisitio, si areæ plani moti simul ratio habeatur. Quæratur scilicet talis curva *BMCD*, quæ inter omnes curvas ejusdem constantis areæ puncta fixa *B* & *C* conjungentes, minimam patiatur resistentiam, quando in aqua secundum directionem *AC* movetur. Ante vero quam hanc disquisitionem suscipiamus, sequens præmittere debemus

LEMMA. *Inter omnes lineas curvas, ad eandem datam abscissam relatas, in quas idem valor expressionis indefinitæ W competit, determinare eam, in qua expressio alia V sit minima vel maxima.*

Solutio. Expressis quantitatibus W & V, quæ nullas alias variabiles nisi coordinatas orthogonales x & y cum earum fluxionibus continebunt, formula integrali indefinita, ponatur in illis, pro dx constante, $dy = pdx$, $dp = qdx$, $dq = rdx$, $dr = sdx$, &c; quo facto eæ in talem formam $\int Zdx$ reductæ habentur, ut sit $W = \int Zdx$ & $V = \int Z'dx$, existente Z, ut etiam Z', functione aliqua quantitatum x, y, p, q, r, &c. Erunt autem tum fluxiones dZ & dZ' hujus formæ: $dZ = Mdx + Ndy + Pdp + Qdq + Rdr + Sds +$ &c., & $dZ' = M'dx + N'dy + P'dp + Q'dq + R'dr + S'ds +$ &c., e quibus formentur quan-

quantitates $L = N - \frac{dP}{dx} + \frac{d^2 Q}{dx^2} - \frac{d^3 R}{dx^3} + \frac{d^4 S}{dx^4} - \&c.$, &

$L' = N' - \frac{dP'}{dx} + \frac{d^2 Q'}{dx^2} - \frac{d^3 R'}{dx^3} + \frac{d^4 S'}{dx^4} - \&c.$, determinabitque curvam quæſitam hæc æquatio: $\alpha L + \beta L' = 0$, denotantibus α & β quantitatibus aliquibus conſtantibus (*).

In disquiſitione noſtra præſenti communis illa quantitas eſt area $\int y dx$, quare ponenda eſt $W = \int y dx = \int Z dx$, ut habeatur $Z = y$, $dZ = dy$, adeoque $N = 1$, $P = 0$, &c. $= 0$, & $L = 1$. Cumque eſſet reſiſtentia in elementum curvæ $ds = \frac{ds dy}{2dx} - \frac{dy^3}{ds^2} - a dy$, in totam curvam s erit reſiſtentia $= \int \left(\frac{ds dy}{2dx} - \frac{dy^3}{ds^2} - a dy \right)$, quæ minima evadet. Facta itaque $dy = p dx$, erit $\int \left(\frac{ds dy}{2dx} - \frac{dy^3}{ds^2} - \alpha dy \right) = \int \left(\frac{1}{2} p \sqrt{1 + p^2} - \frac{p^3}{1+p^2} - ap \right) dx = V = \int Z' dx$, adeoque $Z' =$

(*) Vide hujus æquationis demonſtrationem, quam brevitatis cauſa omiſimus, in libro qui inſcribitur: *methodus inveniendi lineas curvas maximi minimive proprietate gaudentes*, *auctore* LEONH. EULERO. *Lauſannæ & Genevæ* 1744, 4. *Cap. V*, *Prop. IV*, *pag.* 184 *ſeqv.*

$\frac{1}{2} p \sqrt{1+p^2} - \frac{p^3}{1+p^2} - ap$, & $dZ' = \frac{(1+2p^2)dp}{2\sqrt{1+p^2}}$ $- \frac{(3+p^2)p^2 dp}{(1+p^2)^2} - adp$. Hinc videtur esse $N' = 0$; $P' = \frac{1+2p^2}{2\sqrt{1+p^2}} - \frac{(3+p^2)p^2}{(1+p^2)^2} - a$; $Q = 0$, &c. $= 0$, adeoque $L' = -\frac{dP'}{dx}$, & pro curva quæsita $\alpha - \frac{\beta dP'}{dx} = 0$, seu $\frac{\alpha}{\beta}. dx = dP'$, unde integrando obtinetur $\frac{\alpha}{\beta} . x = P' + C = \frac{1+2p^2}{2\sqrt{1+p^2}} - \frac{(3+p^2)p^2}{(1+p^2)^2} - a + C$. Si autem in B est $w = 0$, ob $Tg\ w = -p$, pro $x = 0$ fit $p = 0$, adeoque in hoc casu $0 = \frac{1}{2} - a + C$, seu $-a + C = -\frac{1}{2}$; qua facta correctione habetur $\frac{\alpha}{\beta} . x = \frac{1+2p^2}{2\sqrt{1+p^2}} - \frac{(3+p^2)p^2}{(1+p^2)^2} - \frac{1}{2}$.

Quum ulterius sit $dy = pdx$, habetur integrando $\frac{\alpha}{\beta}.y = \frac{\alpha}{\beta}.px - \frac{\alpha}{\beta}\int xdp = \frac{(1+2p^2)p}{2\sqrt{1+p^2}} - \frac{(3+p^2)p^3}{(1+p^2)^2}$ $- \frac{1}{2}p - \int\frac{(1+2p^2)dp}{2\sqrt{1+p^2}} + \int\frac{(3+p^2)p^2 dp}{(1+p^2)^2} + \frac{1}{2}p =$ $\frac{(1+2p^2)p}{2\sqrt{1+p^2}} - \frac{(3+p^2)p^3}{(1+p^2)^2} - \frac{1}{2}p\sqrt{1+p^2} + \frac{p^3}{1+p^2}$ $+ D =$

$+ D = \frac{p^3}{2\sqrt{1+p^2}} - \frac{2p^3}{(1+p^2)^2} + D$. Facta jam $AB = 1$, pro $y = 1$ evadit $p = 0$, adeoque $\frac{\alpha}{\beta} = D$, qua facta correctione prodit $\frac{\alpha}{\beta} \cdot y = \frac{p^3}{2\sqrt{1+p^2}} - \frac{2p^3}{(1+p^2)^2} + \frac{\alpha}{\beta}$.

Cum sit $p = - Tg\, w$, erit pro curva quæsita BMC

$$x = \frac{\beta}{\alpha} \left(2 Sin\, w^4 - 3 Sin\, w^2 - \tfrac{1}{2} Cos\, w + \frac{1}{Cos\, w} - \tfrac{1}{2}\right), \&$$

$$y = 1 + \frac{\beta}{\alpha}\left(2 Sin\, w^3\, Cos\, w - \frac{Sin\, w^3}{2 Cos\, w^2}\right).$$

Ordinata y decrescere debet crescente abscissa x; quare patet, quantitatem $\frac{\beta}{\alpha}$ esse debere negativam, & quidem talem, ut fieri queat $y = 0$. Hoc autem accidere debet quando quantitas $2\, Sin\, w^3\, Cos\, w - \frac{Sin\, w^3}{2\, Cos\, w^2}$ est maxima. Sumtis itaque ejus fluxionibus eruitur

$$6 Sin\, w^2\, Cos\, w^2\, dw - 2 Sin\, w^4\, dw - \frac{3 Sin\, w^2\, dw}{2\, Cos\, w} - \frac{Sin\, w^4\, dw}{Cos\, w^3} = 0,$$

unde invenitur $Cos\, w^5 - \frac{1}{4} Cos\, w^3 - \frac{1}{16} Cos\, w^2 - \frac{1}{8} = 0$. Hujus autem æquationis radix una realis est $Cos\, w - 0{,}7744648 = 0$, unde invenitur $w = 39^\circ\, 14'\, 36{,}5''$; quo valore adhibito evanescere debet y. Erit itaque

$0 = 1 + 0{,}1811 \cdot \frac{\beta}{\alpha}$, adeoque $\frac{\beta}{\alpha} = -5{,}52181$, nec non

$$x = 5{,}52181 \left(\tfrac{1}{2} - 2 \, Sin \, w^4 + 3 \, Sin \, w^2 - \frac{1}{Cos \, w} + \tfrac{1}{2} \, Cos \, w\right),$$

atque $y = 1 - 5{,}52181 \left(2 \, Sin \, w \; Cos \, w - \frac{Sin \, w^3}{2 \, Cos \, w^2}\right)$.

Harum æquationum ope per puncta construi potest curva quæsita; inveniuntur enim tales correspondentes x & y, quales sequens præbet tabula:

w	x	y
0	0,0000000	1,0000000
5°	0,0935980	0,9945307
10°	0,3623390	0,9579586
15°	0,7712539	0,8663584
20°	1,2657890	0,7104545
25°	1,7768865	0,4982182
30°	2,2270135	0,2646439
35°	2,5361734	0,0693548
39° 14′ 36,5″	2,6300600	0,0000000

Curvam hac constructione inventam attentius considerantes, invenimus arcum circuli alicujus in eam proxime incidere. Hujus vero circuli centrum & radius sequenti modo facile determinantur. Producta recta BA ad E, atque erecta huic normali EF, sit F centrum illud. Factis itaque $EF = a$, $AE = b$, & radio $= r$, erit ex natura circuli: $r^2 = (a + x)^2 + (b + y)^2$. Ut determinentur a, b & r, assumamus in tribus punctis hunc circulum in curvam incidere, pro quibus punctis sit $x = x_{,}$; $x_{,,}$; $x_{,,,}$;
& respondens - - - $y = y_{,}$; $y_{,,}$; $y_{,,,}$.

Si

Si horum valorum fit ſubſtitutio, tres habentur æquationes ſequentes:

$r^2 = (a + x_{,})^2 + (b + y_{,})^2$;

$r^2 = (a + x_{,,})^2 + (b + y_{,,})^2$;

$r^2 = (a + x_{,,,})^2 + (b + y_{,,,})^2$;

unde exterminando r obtinentur

$x^2_{,} - x^2_{,,} + y^2_{,} - y^2_{,,} + 2a(x_{,} - x_{,,}) + 2b(y_{,} - y_{,,}) = 0$,

$x^2_{,,} - x^2_{,,,} + y^2_{,,} - y^2_{,,,} + 2a(x_{,,} - x_{,,,}) + 2b(y_{,,} - y_{,,,}) = 0$:

Exterminando ulterius quantitatem b invenitur

$a = -\frac{1}{2}[(x^2_{,} + y^2_{,})(y_{,,} - y_{,,,}) - (x^2_{,,} + y^2_{,,})(y_{,} - y_{,,,})$
$+ (x^2_{,,,} + y^2_{,,,})(y_{,} - y_{,,})] : [x_{,}(y_{,,} - y_{,,,}) - x_{,,}(y_{,} - y_{,,,}) +$
$+ x_{,,,}(y_{,} - y_{,,})]$, qua cognita eruuntur

$b = -\frac{1}{2}.\frac{x^2_{,} - x^2_{,,} + y^2_{,} - y^2_{,,} + 2a(x_{,} - x_{,,})}{y_{,} - y_{,,}}$, atque

$r = \sqrt{(a + x_{,})^2 + (b + y_{,})^2}$.

Si jam hic circulus cum curva in punctis B, M & C, ubi eſt $w = 0$, $w = 20°$ & $w = 39° 14' 36,5''$ reſpective, coincidet, habentur e tabula computata valores ſequentes:

$x_{,} = 0$; $x_{,,} = 1,265789$; $x_{,,,} = 2,63006$;

$y_{,} = 1$; $y_{,,} = 0,7104545$; $y_{,,,} = 0$;

quibus adhibitis inveniuntur

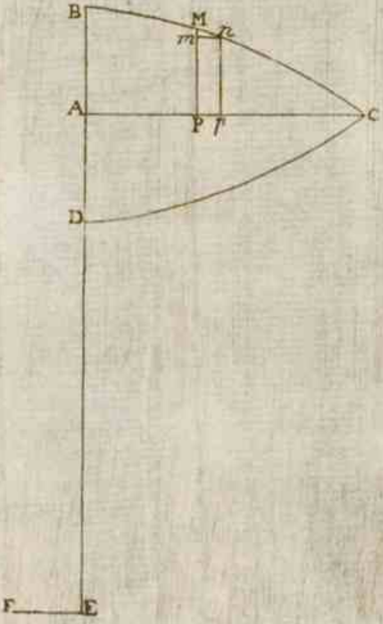
$a = 0,6012031$; $b = 4,539805$; $r^2 = 31,050074$.

His valoribus in æquatione: $r^2 = (a + x)^2 + (b + y)^2$ adhibi-

 biſtitis, & deſumta magnitudine abſciſſæ x e tabula antecedente, invenitur in circulo correſpondens y, ut etiam ejus differentia ab ordinata y in curva conſtructa, ut in ſequente videtur tabula:

x	y	Differentia
0,	1.	0.
0,0935980	0,9890	+ 0,0055
0,3623390	0,9485	+ 0,0094
0,7712539	0,8608	+ 0,0055
1,2657890	0,7104	0,
1,7768865	0,4995	— 0,0013
2,2270135	0,2614	+ 0,0032
2,5361734	0,0653	+ 0,0040
2,6300600	0,	0.

Cum hinc jam appareat, circulum inventum aut in curvam minimæ reſiſtentiæ incidere, aut ne centeſima quidem parte latitudinis AB ab illa diſtare; concludimus in praxi circulum ſine errore notabili adhiberi ſemper poſſe. Quo major ſumitur AB, eo quoque eſt major diſtantia inter circulum & curvam minimæ reſiſtentiæ. Attamen, ſi ad naves etiam majores, quarum maximam latitudinem $= 50$ ped. Svecan. aſſumimus, hæc curva applicatur, diſtantia maxima a circulo non eſt major, quam ut negligi queat. Inferendo enim exempli cauſa $1 : 0,0094 : AB = 25$ ped. : 0,235 ped., invenitur error 2,35 pollicum, ſi arcus circularis adhibetur.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