DISSERTATIO PHYSICO-MATHEMATICA,

THEORIAM MOTUS BILANCIS, A DUABUS HORIZONTALITER SOLLICITATÆ VIRIBUS,

SISTENS.

Quam,

Venia Ampl. Facult. Philos. Aboëns.

PUBLICO EXAMINI OFFERUNT

JOHANNES FREDRICUS AHLSTEDT,

Mathes. puræ Docens,

&

JOHANNES LÖNNMARK,

Stipend. Archenholtz. Satac.

In Auditorio Physico die XVII Dec. MDCCCVI.

h. a. m. s.

ABOÆ, Typis FRENCKELLIANIS.

4.

Effectum gravitatis corporum terrestrium in se invicem, si vel quantulo distant intervallo, ratione effectus illius gravitatis, qua terram petunt, fere evanescere sensusque effugere notum est. Illum tamen ope bilancis exigui ponderis, non tantum observari, verum etiam ad numerum absolutum reduci posse, cura inprimis *Cel.* HENR. CAVENDISH, compertum esse testatur *Cel.* GILBERT in *Annalen der Physik 2. B. 1. St. 1799.* Huic consilio, bilanx levissima *AB Fig. 1.* circiter 6. ped. paris. longa, e ligno fabricata, e medio *C* horizontaliter filo tenuissimo metallico in *D* suspensa, & globos plumbeos duorum pollicum parisin. in extremitatibus suis portans instruitur, totumque receptaculo ligneo, quo impetus aëris abstineatur, obducitur.

In plano bilancis horizontali & in peripheria, quam describit bilanx, duæ Massæ ex. gr. plumbeæ, sphæricæ, æquales, diam. 8. poll. *E.* & *F*, ad ad vicinitatem globorum *A* & *B*, e partibus bilancis oppositis, admoveantur, quo facto bilanx, e pristino suo statu quietis *AB* turbata, massas petit plumbeas, motu quidem valde tardo, attamen tanto, ut vi inertiæ suæ oscillationes perficiat juxta puncta, in quibus vis attractionis Massarum *E* & *F*, vi torsionis fili metallici *CD*, huic contrariæ, æquipolleat. — Quod inventum, uti latissimum tam Physicæ quam Chemiæ allaturum videbatur usum, ita merito attentionem excitavit eruditorum sollertiorem. Sed quoniam Theoria motus hujus bilancis, monente *Cel.* GILBERT (*), nondum veris ex fontibus hausta est, nostras quoque vires qualescumque in illa evolvenda periclitari voluimus.

(*) *In Scediasmate subsequente l. c. p.* 68. Was sich bei der vortrefflichen Abhandlung Cavendish's vielleicht noch wünschen liesse, ist eine etwas andere Berechnungs methode. Der Arm wird bei Cavendish's Versuchen nicht wie der Pendel von einer, sondern von zwei beschleunigenden Kräften getrieben: der Kraft des Drahts, die dem Windungswinkel proportional ist, und der Anziehung der Bleimassen, die in umgekehrten Verhältnisse der Quadrate der Entfernungen wächst. Die Theorie der Bewegung eines Pendels auf der zwei so einfache beschleunigende Kräfte wirken, kann schwerlich so schwierig seyn, dass sie sich nicht in geschmeidigen Formeln unmittelbar sollte darstellen lassen - - - Es thut mir leid, das meine Zeit es mir nicht erlaubt, jetzt eine solche Theorie zu versuchen, und nach ihr Cavendish's Rechnungen zu wiederholen.

Duabus ſollicitatur bilanx viribus: Altera, attractione Masſarum plumbearum *E* & *F*, quam, cum ſphæricæ ſint formæ, directam masſarum, (et forſan, pro data materia & temperatura, coëfficientis inſimul cujusdam conſtantis.) inverſamque quadratorum diſtantiarum centrorum ſequi ſupponere licebit rationem; altera, torſione fili metallici, quæ, ſecundum perplurima experimenta *Cel.* COULOMB, pro eadem longitudine & crasſitie fili, rationem angulorum torſionis quam accuratisſime ſervat.

Sit ſtatus bilancis, a gravitate ſola in terram ſollicitatæ, quieſcentis *GC*, (*Fig.* 2.) & admoveatur masſa plumbea *F* ita, ut centrum ejus in plano bilancis horizontali eandem ac centrum globi *G* deſcribat peripheriam circuli. Vi attractiva hujus masſæ ſollicitata, bilanx, niſi reſiſteret vis torſionis fili, motu accelerato masſam *F* attingeret. In puncto autem quodam *A*, vis torſionis usque adeo accreſcere poteſt, ut vim masſæ *F* plene tollerę valeat. Ob velocitatem vero, quam ſub motu ſuo e *G* ad *A* obtinuerit bilanx, in *A* quieſcere non poteſt, verum tanquam iners ultro illud punctum usque ad *D* vacillare cogitur, inde vero rurſus revertitur ad *B*, & ſic oſcillationes circa punctum *A* perficeret innumeras, niſi reſiſtentia aëris & elaſticitas fili imperfecta eas continuo minuerent, donec tandem in *A* quietem recuperaverit bilanx.

Sit

Sit angulus $ACF = 2a$, $ACG = 2e$, $ACB = 2\varphi$, $ACD = 2f$, situs bilancis oscillantis in linea CP, & dicatur $ACP = 2x$; sit porro Massa F, (& quæ forsan reliqua insimul fuerit caussa coëfficiens,) $= m$, & coëfficiens, qua vis torsionis fili metallici multiplicabitur $= b$, erit $\frac{m}{FP^2}$ = vi, qua sollicitatur bilanx ad F in directione PF, & $b.\ PG$ = vi, qua resistit filum in directione tangentis PM. Resolvatur vis $\frac{m}{FP^2}$ in vires laterales directiones tangentis PT, & radii bilancis $PC\ (= r)$ sequentes. Hinc, cum sit $PCF = 2\,(a + x)$, erit dimidium ejus $PCQ = a + x$, & $(Rad =)\ 1 : Sin\,(a + x) :: r : PQ = r\,Sin\,(a + x)$, unde $PF = 2\,r\,Sin\,(a + x)$. Ob æqualitatem autem angulorum PCQ, FPT, et PQC, PTF, erit $1 : Cos\,(a + x) ::$ *vis in directione* PF $\left(= \frac{m}{4\,r^2\,Sin\,(a + x)^2}\right)$: *vim in directione* $PT = \frac{m\,Cos\,(a + x)}{4\,r^2\,Sin\,(a + x)^2}$, qua bilanx ab A ad B mota retardatur. Reliqua vis in directione normali PN, ab opposita parte bilancis tollitur. Vis vero torsionis, cum angulorum torsionis fili sequatur rationem, erit in eodem puncto $P = 2\,br\,(e - x)$, motumque bilancis ab A ad B accelerabit; quare vis residua, qua ad punctum B tendit bilanx, erit

$$2\,br\,(e - x) - \frac{m\,Cos\,(a + x)}{4\,r^2\,Sin\,(a + x)^2}.$$

Ducto

Ducto autem differentiali arcus $AP = 2\,r\,dx$ in vim inventam, habebitur differentiale altitudinis v velocitati in P debitæ, (*Elem. Mechan.*), h. e.

$$dv = 4\,br^2\,(e-x)\,dx - \frac{2\,mr\,Cos\,(a+x)\,dx}{4\,r^2\,Sin\,(a+x)^2},$$

quo integrato enascitur

$$v = 2\,br^2\,(2e-x)\,x + \frac{m}{2\,r\,Sin\,(a+x)} + Const.$$

In puncto vero B, unde revertitur bilanx, esse debet $v = 0$, quare, facto $x = \varphi$, erit

$$Const = -\,2\,b\,r^2(2\,e-\varphi)\,\varphi - \frac{m}{2\,r\,Sin\,(a+\varphi)},$$ &

integrale correctum:

$$v = \frac{m}{2r}\left(\frac{1}{Sin\,(a+x)} - \frac{1}{Sin\,(a+\varphi)}\right) - 2br^2\,(2e-\varphi-x)(\varphi-x).$$

Temporis autem, arcui AP describendo impensi $= t$, fluxio, (denotante g altitudinem, unde gravia in superficie telluris primo minuto secundo delabantur,) erit $= \frac{d.\,AP}{2\sqrt{gv}}$, (*Elem. Mechan.*), sive insertis valoribus

$$dt = r\,dx : \sqrt{\left(g\left(\frac{m}{2r}\left(\frac{1}{Sin\,(a+x)} - \frac{1}{Sin\,(a+\varphi)}\right), - 2\,b\,r^2\,(2e-\varphi-x)(\varphi-x)\right)\right)}.$$

Hæc formula, cum non niſi per approximationem integrari poſſit, in ſeriem convergentem reſolvetur; quapropter exiſtente $Sin\ (a + x) = Sin\ a\ Cos\ x + Cos a\ Sin\ x$, $Sin\ x = x - \frac{x^3}{6} + \dots$ & $Cos\ x = 1 - \frac{x^2}{2} + \dots$, erit, ob parvitatem arcus x, quam proxime $Sin\ (a + x) = Sin\ a \times \left(1 + \frac{x\ Cos a}{Sin\ a} - \frac{x^2}{2} - \frac{x^3\ Cos a}{6\ Sin\ a}\right)$ & hinc $\frac{1}{Sin\ (a + x)} = \frac{1}{Sin\ a} \left(1 - \frac{x\ Cos a}{Sin\ a} + \frac{x^2}{2} \cdot \frac{1 + Cos a^2}{Sin\ a^2} - \frac{x^3}{6} \cdot \frac{(5 + Cos a^2)\ Cos a}{Sin\ a^3}\right)$, quo adhibito valore habebitur

$$\sqrt{g\left(\frac{m}{2r}\left(\frac{1}{Sin\ a + x)} - \frac{1}{Sin\ (a + \varphi)}\right) - 2br^2(2e - \varphi - x)(\varphi - x)\right)}$$

$$= \sqrt{\left(g\left(\frac{m}{2r\ Sin\ a} - \frac{m}{2r\ Sin\ (a + \varphi)} - 2br^2(2e - \varphi)\varphi + \left(4br^2 e - \frac{m\ Cos a}{2r\ Sin\ a^2}\right)x - \left(2br^2 - \frac{m(1 + Cos a^2)}{4r\ Sin\ a^3}\right)x^2 - \frac{m(5 + Cos a^2)\ Cos a}{12\ r\ Sin\ a^4}\ x^3\right)\right)}.$$

Fiat

$$A = \frac{m}{2r\ Sin\ a} - \frac{m}{2r\ Sin\ (a + \varphi)} - 2br^2\ (2e - \varphi)\varphi,$$

$$B = 4br^2\ e - \frac{m\ Cos a}{2\ r\ Sin\ a^2},$$

$$C = 2br^2 - \frac{m(1 + Cos a^2)}{4\ r\ Sin\ a^3}\ \&$$

$$D = \frac{m(5 + Cos a^2)\ Cos a}{12\ r\ Sin\ a^4}.$$

Vi vero tangentiali in puncto A, quæ coëfficiente $B:2r$ exprimitur, existente $= o$, erit quoque $B = o$. Hinc

$$dt = r\,dx : \sqrt{g\,(A - Cx^2 - Dx^3)} = \frac{r}{\sqrt{Ag}}\left(dx + \frac{C}{2A}.x^2dx + \frac{D}{2A}.x^3dx + \ldots\right),$$

cujus formulæ integrale

$$t = \frac{r}{\sqrt{Ag}}\left(x + \frac{C}{6A}.x^3 + \frac{D}{8A}x^4 + \ldots\right),$$

cum posito $x = o$, euanescat, nulla eget correctione. Totum ergo tempus, quo bilanx ab A ad B pervenit, posito $x = \phi$, emergit $=$

$$\frac{r\phi}{\sqrt{Ag}}\left(1 + \frac{C}{6A}\phi^2 + \frac{D}{8A}.\ \phi^3 + \ldots\right).$$

Quod si bilanx in altera parte arcus oscillationis suæ AD in situ quocumque Cp versaretur, et arcus ACp fuerit $= 2z$, simili ac supra evincitur calculo, esse vim attractionis massæ F in directione tangentis $pt = \frac{m\,Cos\,(a-z)}{4r^2\,Sin\,(a-z)^2}$, (sublato, ut ante, effectu vis normalis in directione pn per oppositam bilancis partem), & vim torsionis fili in contraria directione pm agens $= 2br\,(e + z)$. Mutatis porro in antecedentibus x in $-z$ & ϕ in $-f$, invenitur altitudo velocitati in p debita $= \frac{m}{2r\,Sin\,(a-z)}$

$- 2br^2$

$-2br^2(2e+z)z - \frac{m}{2r\,Sin(a-f)} + 2br^2(2e+f)f$,
& tempus arcui *AD* defcribendo impenfum $=$
$\frac{rf}{\sqrt{\alpha g}}\left(1 + \frac{C}{6\alpha}f^2 - \frac{D}{8\alpha}f^3 + \ldots\right)$, exiftente
$\alpha = \frac{m}{2r\,Sin\,a} - \frac{m}{2r\,Sin(a-f)} + 2br^2(2e+f)f$. Sed quoniam α eandem ac *A* altitudinem velocitati in puncto *A* debitam indicat, ipfi *A* æqualis esfe debet.

Ex his ergo colligitur, esfe tempus unius ofcillationis *BD*

$$I.)\; T = \frac{r(\varphi+f)}{\sqrt{Ag}}\left(1 + \frac{C}{6A}(\varphi^2 - \varphi f + f^2) + \frac{D}{8A}(\varphi^3 - \varphi^2 f + \varphi f^2 - f^3) + \ldots\right).$$

Comparatis præterea valoribus *A* & α, reperitur

$$II.)\; \varphi - f = 2e - \frac{m}{4br^3(\varphi+f)}\left(\frac{1}{Sin(a-f)} - \frac{1}{Sin(a+\varphi)}\right).$$

Valor denique ipfius *B* evanefcentis præbet

$$III.)\; e = \frac{m\,Cof\,a}{8br^3\,Sin\,a^2}$$

Cognofcentur porro ex obfervationibus

$IV.)\; 2(e+f) = n$
$V.)\; 2(\varphi+f) = p$ &
$VI.)\; 2(a+e) = q$.

Po-

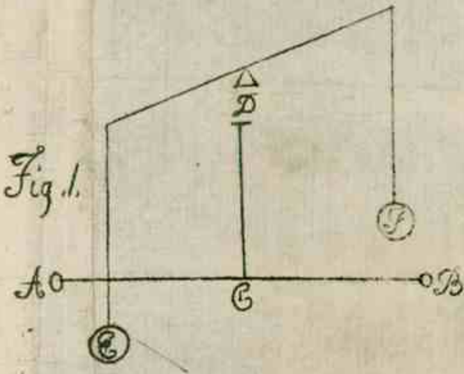

Fig. 1.

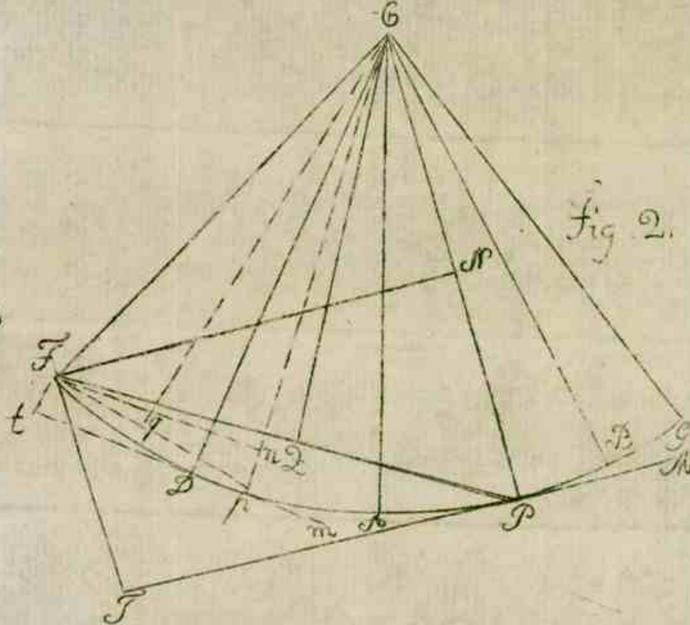

Fig. 2.

Poſtremo, ſi, ſublatis masſis ſollicitantibus, bilanx ex A libere oſcillare inceperit, ejusque ſitus fuerit in CP, erit, dicto $GCP = 2y$, vis torſionis fili in puncto P verſus G ſollicitans $= -2bry$, qua in $2rdy\ (= d.\ GCP)$ ducta, invenitur altitudo velocitati in P debita $= Conſt - \int 4br^2\, y\, dy = 2br^2(e^2 - y^2)$, integrali ita correcto, ut, facto $y = e$, evaneſcat. Hinc tempus deſcribendo arcui GP impenſum $=$

$$\int \frac{2r\,dy}{2\sqrt{g.\,2br^2(e^2-y^2)}} = \frac{1}{\sqrt{2bg}} \int \frac{dy}{\sqrt{e^2-y^2}}$$

$$= \frac{1}{\sqrt{2bg}}\ \text{Arc Sin}\ \frac{y}{e},$$

unde, facto $y = e$, exſurgit tempus, quo arcum AG abſolverit bilanx

$$VII.)\ \theta = \frac{\pi}{2\sqrt{2gb}}.$$

Ex his VII. æquationibus incognitæ m, b, φ, f, a, e & g inveniri posſunt.