DISSERTATIO PHYSICO-MATHEMATICA,

PHÆNOMENA LUMINIS, VIRIBUS ATTRACTIVIS & REPULSIVIS CORPORUM SUBJACERE & EX HIS DERIVARI POSSE,

STATUENS;

CUJUS PARTEM TERTIAM,

CONSENTIENTE AMPLISS. ORDINE PHILOSOPH. IN IMPERIALI ACADEMIA ABOËNSI,

PRÆSIDE

Mag. JOH. FREDR. AHLSTEDT,
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ABOÆ, TYPIS FRENCKELLIANIS.

I.

In *Tomo primo, Propoſit. LXXIII. Theoremate XXXIII. Principiorum Philoſophiæ Naturalis*, hæc Ipſe adſert NEWTON: "*Superficies, ex quibus Solida componuntur, hic (ſcil. in Lege Attractionis exploranda) non ſunt purè Mathematicæ, ſed orbes adeo tenues, ut eorum crasſitudo inſtar nihili ſit; nimirum orbes evaneſcentes, ex quibus Sphæra ultimo conſtat, ubi orbium illorum numerus augetur & crasſitudo minuitur in infinitum. Similiter per puncta, ex quibus lineæ, ſuperficies & ſolida componi dicuntur, intelligendæ ſunt particulæ æquales magnitudinis contemnendæ*". *Vide Operis laudati, cura Le Seur & Jacquier Coloniæ Allobrogum Anno 1760 editi, Partem I. pag. 470.* Ex hoc asſerto, (quod quidem jam haud verum habere posſunt Mathematum cultores), in Propoſitione ſequente, hanc Legem Attractionis ſancivit: *Corpusculum extra ſphæram conſtitutum attrahi vi reciproce proportionali quadrato diſtantiæ ſuæ ab ipſius centro.* — Negandum minime eſt, vires, quibus corpora cœleſtia in ſe vicisſim agunt, huic eo magis appropinquare Theorematì, quo majore intervallo a ſe invicem disſita ſint, & quo minore gaudeant volumine. Verum, uti omne asſertum, quod ſpeciem tantummodo veritatis habet, ſed abſolute non valet, ad omnes caſus extendere haud liceat, ita quoque de hoc Theoremate non injuſte dicendum esſe putamus Manifeſtum enim eſt, hac lege asſumta, particula. ejusdem corporis, (ex. gr. Terræ), quo propiores

 ſints,

ſint, eo majori vi comprimi, donec in contactu infinita omnino actæ vi, in unicum redigantur punctum, necesſe erit; atque ſic omnis extenſio corporum deſtrueretur, nec, niſi puncta, infinitæ denſitatis, remanerent.

Inſiſtentes vero Illuſtrisſimi Viri veſtigiis nuper deſcriptis, & certo perſvaſi, ſolida neque ex ſuperficiebus, neque ex punctis unquam componi posſe, necesſarium omnino reputavimus, extenſionem corporum præcipue esſe conſulendam; atque adeo vim, qua afficiuntur particulæ, non ex ſuperficiebus Sphærarum, ſed ex ipſis ſupra dictis Orbibus, cujuscunque demum fuerint crasſitiei, (ſemper tamen asſignabilis), esſe derivandam.

Hisce exploratis, ſequentem pro attractione & repulſione, ſimul agentibus, Legem Naturæ compoſuimus:

$$P = \sqrt[3]{a^3 y^3 + A^3} - \sqrt[3]{r^3 y^3 + R^3},$$

denotantes: P vim aut attrahentem aut repellentem, y diſtantiam inter corpus agens & particulam, a, A conſtantes, e vi attractiva, denſitate, affinitate, cet. pendentes, & r, R itidem conſtantes, vim repulſivam & quæ ei fuerint communia exprimentes. Hujus autem Legis ortum paucis explicare primum juvat.

Sint

Sint M & N duæ Sphæræ concentricæ, quarum radii respondeant quantitatibus b & $b + c$ respective, erunt soliditates harum Sphærarum $\frac{4}{3}b^3\pi$ & $\frac{4}{3}(b + c)^3\pi$, (existente π = semiperipheriæ circuli, radio $= 1$, descripti), unde $N - M$ sive soliditas Sphæræ cavæ, quam in sequentibus cum Newtono *Orbem* appellare licebit, erit

$$\tfrac{4}{3}\pi\,((b + c)^3 - b^3).$$

Sint porro duæ aliæ Sphæræ Y & Z, radiis y & $y + z$ ex eodem ac primæ descriptæ centro, habebitur orbis $Z - Y$ soliditas

$$\tfrac{4}{3}\pi\,((z + y)^3 - y^3).$$

Si jam affectiones reliquas, a natura materiei pendentes, æquales in quacunque Sphæra assumere liceret, oriretur pro æqualibus Solidis, (uti Newton pro æqualibus superficiebus),

$$\tfrac{4}{3}\pi((y + z)^3 - y^3) = \tfrac{4}{3}\pi((b + c)^3 - b^3), \text{ sive}$$

$$(y + z)^3 - y^3 = (b + c)^3 - b^3;$$

sed quoniam vis, de qua sermonem facimus, in quavis Sphæra, quoad constantes has quantitates, inæqualis esse potest, ponamus coëfficientem hanc pro Sphæra Z esse $= 1$, pro $Y, = a^3$, pro $M, = m$ & pro $N, = n$, erit nova facies hujus æquationis

$$(y + z)^3 - a^3y^3 = n(b + c)^3 - mb^3,$$

 quæ,

quæ, facto $n(b+c)^3 - mb^3 = A^3$, abit in

$$(y+z)^3 - a^3y^3 = A^3, \text{ unde}$$

$$z = \sqrt[3]{a^3y^3 + A^3} - y,$$

formula, quantitatem vis attractivæ ſolius repræſentans.

Iisdem pro repulſione ſola computatis, facile apparebit, vim hanc exprimi posſe formula:

$$z' = \sqrt[3]{r^3y^3 + R^3} - y.$$

Differentia vero harum virium $z - z'$, vim ſive ſollicitantem ſive reagentem P conſtituit, quare erit:

$$P = \sqrt[3]{a^3y^3 + A^3} - \sqrt[3]{r^3y^3 + R^3}.$$

En ergo vim, quam nos, loco formulæ Newtonianæ $\frac{M}{y^2}$, (quæ componitur ex Maſſa attrahente, directe, & quadrato diſtantiæ, inverſe,) in ſequentibus adhibebimus.

Cum vero valor potentiæ P nullo pacto ab irrationalitate plene liberari posſit, per ſeries infinitas definiatur, oportet: & quidem ſecundum exponentes diſtantiæ y tam decreſcentes, quam creſcentes.

Ex

Ex tenore formulæ notisſimæ:

$$(1+Q)^n = 1 + nQ + \frac{n.\overline{n-1}}{2}Q^2 + \frac{n.\overline{n-1}.\overline{n-2}}{2.3}Q^3$$

$$+ \frac{n.\overline{n-1}.\overline{n-2}.\overline{n-3}}{2.3.4}Q^4 + \frac{n.\overline{n-1}.\overline{n-2}.\overline{n-3}.\overline{n-4}}{2.3.4.5}Q^5$$

+ &cra, oritur:

$$\sqrt[3]{a^3y^3+A^3} = ay\sqrt[3]{1+\frac{A^3}{a^3y^3}} = ay\Big(1+\tfrac{1}{3}.\frac{A^3}{a^3y^3} - \frac{1.2}{2.3^2}.\frac{A^6}{a^6y^6}$$

$$+ \frac{1.2.5}{2.3.3^3}.\frac{A^9}{a^9y^9} - \frac{1.2.5.8}{2.3.4.3^4}.\frac{A^{12}}{a^{12}y^{12}}$$

$$+ \frac{1.2.5.8.11}{2.3.4.5.3^5}.\frac{A^{15}}{a^{15}y^{15}} - \ldots\Big), \ \&$$

$$\sqrt[3]{r^3y^3+R^3} = ry\sqrt[3]{1+\frac{R^3}{r^3y^3}} = ry\Big(1+\tfrac{1}{3}.\frac{R^3}{r^3y^3} - \frac{1.2}{2.3^2}.\frac{R^6}{r^6y^6}$$

$$+ \frac{1.2.5}{2.3.3^3}.\frac{R^9}{r^9y^9} - \frac{1.2.5.8}{2.3.4.3^4}.\frac{R^{12}}{r^{12}y^{12}}$$

$$+ \frac{1.2.5.8.11}{2.3.4.5.3^5}.\frac{R^{15}}{r^{15}y^{15}} - \ldots\Big), \text{ unde}$$

$$P = (a-r)y + \tfrac{1}{3}.\Big(\frac{A^3}{a^2} - \frac{R^3}{r^2}\Big)\frac{1}{y^2} - \frac{1.2}{2.3^2}.\Big(\frac{A^6}{a^5} - \frac{R^6}{r^5}\Big)\frac{1}{y^5}$$

$$+ \frac{1.2.5}{2.3.3^3}.\Big(\frac{A^9}{a^8} - \frac{R^9}{r^8}\Big)\frac{1}{y^8} - \frac{1.2.5.8}{2.3.4.3^4}.\Big(\frac{A^{12}}{a^{11}} - \frac{R^{12}}{r^{11}}\Big)\frac{1}{y^{11}}$$

$$+ \frac{1.2.5.8.11}{2.3.4.5.3^5}.\Big(\frac{A^{15}}{a^{14}} - \frac{R^{15}}{r^{14}}\Big)\frac{1}{y^{14}} - \ldots \quad {}^{\circ}$$

° Asſumta $a = r$, habebitur valor potentiæ P ſingularis:

Hinc Integrale quantitatis $C \pm \int P dy$ invenitur esse:

$$C \pm \tfrac{1}{2}(a-r)y^2 \mp \frac{1}{3}\cdot\left(\frac{A^3}{a^2}-\frac{R^3}{r^2}\right)\frac{1}{y}$$
$$\pm \frac{1\cdot 2}{2\cdot 4\cdot 3^2}\cdot\left(\frac{A^6}{a^5}-\frac{R^6}{r^5}\right)\frac{1}{y^4}$$
$$\mp \frac{1\cdot 2\cdot 5}{2\cdot 3\cdot 7\cdot 3^3}\cdot\left(\frac{A^9}{a^8}-\frac{R^9}{r^8}\right)\frac{1}{y^7}$$
$$\pm \frac{1\cdot 2\cdot 5\cdot 8}{2\cdot 3\cdot 4\cdot 10\cdot 3^4}\cdot\left(\frac{A^{12}}{a^{11}}-\frac{R^{12}}{r^{11}}\right)\frac{1}{y^{10}}$$
$$\mp \frac{1\cdot 2\cdot 5\cdot 8\cdot 11}{2\cdot 3\cdot 4\cdot 5\cdot 13\cdot 3^5}\cdot\left(\frac{A^{15}}{a^{14}}-\frac{R^{15}}{r^{14}}\right)\frac{1}{y^{13}} \pm \ldots (\mathfrak{A}).$$

Quod si vero y adeo fuerit parva, ut series allata cre-

$$P = \tfrac{1}{3}\cdot\frac{A^3-R^3}{a^2}\cdot\frac{1}{y^2} - \frac{1\cdot 2}{2\cdot 3^2}\cdot\frac{A^6-R^6}{a^5}\cdot\frac{1}{y^5}$$
$$+ \frac{1\cdot 2\cdot 5}{2\cdot 3\cdot 3^3}\cdot\frac{A^9-R^9}{a^8}\cdot\frac{1}{y^8} - \frac{1\cdot 2\cdot 5\cdot 8}{2\cdot 3\cdot 4\cdot 3^4}\cdot\frac{A^{12}-R^{12}}{a^{11}}\cdot\frac{1}{y^{11}}$$
$$+ \frac{1\cdot 2\cdot 5\cdot 8\cdot 11}{2\cdot 3\cdot 4\cdot 5\cdot 3^5}\cdot\frac{A^{15}-R^{15}}{a^{14}}\cdot\frac{1}{y^{14}} - \ldots$$

quæ series, quo majores fuerint a & y respectu differentiæ inter valores dignitatum ipsarum A & R in numeratore, eo vehementius convergit.

crescat, formula $P = \sqrt[3]{a^3 y^3 + A^3} - \sqrt[3]{r^3 y^3 + R^3}$ sequente modo evolvatur:

$$\sqrt[3]{a^3 y^3 + A^3} = A\sqrt[3]{1 + \frac{a^3 y^3}{A^3}} = A\Big(1 + \frac{1}{3} \cdot \frac{a^3 y^3}{A^3}$$

$$- \frac{1 \cdot 2}{2 \cdot 3^2} \cdot \frac{a^6 y^6}{A^6} + \frac{1 \cdot 2 \cdot 5}{2 \cdot 3 \cdot 3^3} \cdot \frac{a^9 y^9}{A^9}$$

$$- \frac{1 \cdot 2 \cdot 5 \cdot 8}{2 \cdot 3 \cdot 4 \cdot 3^4} \cdot \frac{a^{12} y^{12}}{A^{12}} + \frac{1 \cdot 2 \cdot 5 \cdot 8 \cdot 11}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 3^5} \cdot \frac{a^{15} y^{15}}{A^{15}} - \ldots\Big),$$

$$\sqrt[3]{r^3 y^3 + R^3} = R\sqrt[3]{1 + \frac{r^3 y^3}{R^3}} = R\Big(1 + \frac{1}{3} \cdot \frac{r^3 y^3}{R^3}$$

$$- \frac{1 \cdot 2}{2 \cdot 3^2} \cdot \frac{r^6 y^6}{R^6} + \frac{1 \cdot 2 \cdot 5}{2 \cdot 3 \cdot 3^3} \cdot \frac{r^9 y^9}{R^9}$$

$$- \frac{1 \cdot 2 \cdot 5 \cdot 8}{2 \cdot 3 \cdot 4 \cdot 3^4} \cdot \frac{r^{12} y^{12}}{R^{12}} + \frac{1 \cdot 2 \cdot 5 \cdot 8 \cdot 11}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 3^5} \cdot \frac{r^{15} y^{15}}{R^{15}} - \ldots\Big).$$

Quarum differentia præbet

$$P = A - R + \tfrac{1}{3} \cdot \Big(\frac{a^3}{A^2} - \frac{r^3}{R^2}\Big) y^3 - \frac{1 \cdot 2}{2 \cdot 3^2} \cdot \Big(\frac{a^6}{A^5} - \frac{r^6}{R^5}\Big) y^6$$

$$+ \frac{1 \cdot 2 \cdot 5}{2 \cdot 3 \cdot 3^3} \cdot \Big(\frac{a^9}{A^8} - \frac{r^9}{R^8}\Big) y^9 - \frac{1 \cdot 2 \cdot 5 \cdot 8}{2 \cdot 3 \cdot 4 \cdot 3^4} \cdot \Big(\frac{a^{12}}{A^{11}} - \frac{r^{12}}{R^{11}}\Big) y^{12}$$

$$+ \frac{1 \cdot 2 \cdot 5 \cdot 8 \cdot 11}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 3^5} \cdot \Big(\frac{a^{15}}{A^{14}} - \frac{r^{15}}{R^{14}}\Big) y^{15} - \ldots \quad \&$$

$C \pm \int P\, dy$

$$C \pm \int P dy = C \pm (A - R) y \pm \frac{1}{3 \cdot 4} \cdot \left(\frac{a^3}{A^2} - \frac{r^3}{R^2} \right) y^4$$

$$\mp \frac{1 \cdot 2}{2 \cdot 7 \cdot 3^2} \cdot \left(\frac{a^6}{A^5} - \frac{r^6}{R^5} \right) y^7 \pm \frac{1 \cdot 2 \cdot 5}{2 \cdot 3 \cdot 10 \cdot 3^3} \cdot \left(\frac{a^9}{A^8} - \frac{r^9}{R^8} \right) y^{10}$$

$$\mp \frac{1 \cdot 2 \cdot 5 \cdot 8}{2 \cdot 3 \cdot 4 \cdot 13 \cdot 3^4} \cdot \left(\frac{a^{12}}{A^{11}} - \frac{r^{12}}{R^{11}} \right) y^{13}$$

$$\pm \frac{1 \cdot 2 \cdot 5 \cdot 8 \cdot 11}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 16 \cdot 3^5} \cdot \left(\frac{a^{15}}{A^{14}} - \frac{r^{15}}{R^{14}} \right) y^{16} \mp \ldots \quad (\mathfrak{B}).$$

ex quibus pro ($\mathfrak{A}$) & ($\mathfrak{B}$) evolutis terminis, natura hujus functionis ſatis elucet.

Applicentur valores jam inventi quantitatis $C \pm \int P dy$, Formulis (A) pag. 5, & (B) pag. 6, hujus opusculi, quo pateat æquatio lineæ curvæ quam corpus deſcribit.

Quod ad priorem, ſive Integralis (A) valorem attinet, oritur hic ex formula

$$dx = \frac{dy}{\sqrt{\frac{D}{C \pm \int P dy} - 1}} = dy \left(\frac{C \pm \int P dy}{D - C \mp \int P dy} \right)^{\frac{1}{2}} =$$

dy =