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Early Numeracy Development:

Identifying and Supporting Children at Risk for Mathematical Learning Difficulties

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Portrait photo: Anna Widlund

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Abstract

Early numeracy skills are important for later mathematical learning. Already in kindergarten and the early school years there are large individual differences in children's early numeracy skills. Early identification and support are argued to potentially decrease later mathematical learning difficulties. However, this puts a high demand on the tools for identifying and supporting children at risk for mathematical learning difficulties. This thesis, therefore, aims to contribute to the research on early numeracy skills development by focusing on identifying and supporting children at risk for mathematical learning difficulties in kindergarten and during the early school years. This aim has been achieved by investigating how early numeracy skills develop and are interrelated, examining the psychometric criteria of an early numeracy test, and investigate the effects of a computer-assisted intervention program on children's early numeracy skills.

This thesis is based on three substudies (Studies I, II and III). Study I investigated how early numeracy skills (symbolic and non-symbolic number knowledge, understanding mathematical relations, counting skills and basic skills in arithmetic) develop and are interrelated, in order to reach an understanding of which early numeracy skills at the beginning of the school year, predicted later early numeracy performance. Study II focused on assessment for identifying children at risk for mathematical learning difficulties by examining the psychometric criteria of the Early Numeracy test. The reliability (internal consistency) and validity (structural validity, known group validity, and cross-cultural validity) evidence were investigated. Study III examined the effects of supporting children at risk for mathematical learning difficulties using the evidence-based Number Race computer game in a computer-assisted intervention in an authentic school setting. All three substudies were based on data collected in kindergarten (n = 361), first grade (n = 321) and second grade (n = 457) during one school year.

The results from Study I indicated that counting skills in the beginning of first grade was an important predictor of later early numeracy skills while basic arithmetic skills emerged as a key predictor of later early numeracy skills in both first and second grades. Latent profile analysis identified three early numeracy profiles that differed in the early numeracy skills in the beginning of the school year: well-performing, average-performing, and at risk for mathematical learning. Low-performing children showed deficits in all early numeracy subskills and were unable to close the gap to their average- or well-performing peers during the school year. Study II supported the reliability and validity evidence of the Early Numeracy test, indicating that the test can fulfil its purpose to identify children at risk for mathematical learning difficulties in kindergarten, first grade and second grade. This test can also specifically describe children's performances, as it defines and interprets the various early numeracy subskills as different factors. Study III revealed no statistically significant effect of the Number Race-intervention on the low-performing children's early numeracy skills. At none of the three assessment time points did the low-performing groups differ significantly from the low-performing comparison group, which only received ordinary classroom instruction in mathematics during the intervention period. These findings highlighted the importance of developing targeted interventions, which will be easy to implement in the general classroom to enhance children's early numeracy skills.

Combining the findings of the three substudies ca provide additional empirical evidence for the model of core early numeracy skills: symbolic and non-symbolic number knowledge, understanding mathematical relations, counting skills and basic skills in arithmetic (Aunio & Räsänen, 2016). This model of core early numeracy skills can provide a working model for educational practice, in which assessment and instruction are brought closer together to improve educational practise and provide all children with foundational early numeracy skills.

Keywords: assessment, at risk, computer-assisted intervention, early numeracy development, evidence-based educational practice, low performance, mathematical learning difficulties, validation

Abstrakt

Tidiga grundläggande matematiska färdigheter är betydelsefulla för fortsatt lärande i matematik. Redan i förskola och nybörjarundervisning finns det stora individuella skillnader i barns matematiska färdigheter. Tidig identifiering och tidiga stödåtgärder har påvisats kunna förebygga och minska på fortsatta svårigheter i matematik. Syftet med denna avhandling är att bidra med kunskap om utvecklingen av barns tidiga grundläggande matematiska färdigheter genom att lyfta fram identifiering och stöd för barn i risk för matematiska inlärningssvårigheter i förskola och nybörjarundervisning. Syftet nås genom att studera hur tidiga grundläggande matematiska färdigheter samspelar och utvecklas, de psykometriska kriterierna för ett kartläggningsverktyg för identifiering av barn i behov av stöd samt effekten av ett datorbaserat interventionsprogram för att stärka barns tal- och antalsuppfattning.

Denna avhandling bygger på tre delstudier. Den första delstudien fokuserade på hur tidiga grundläggande matematiska färdigheter (tal- och antalsuppfattning, förståelse för matematiska samband, räknefärdigheter och aritmetiska basfärdigheter) samspelade och vilka delfärdigheter i början av årskurs ett och två förutspådde prestationerna i slutet av läsåret samt hur dessa färdigheter utvecklades för barn inom olika prestationsprofiler. Den andra delstudien fokuserade på identifiering av barn i risk för matematiska inlärningssvårigheter genom att studera de psykometriska egenskaperna för kartläggningsverktyget Lukimat (Early Numeracy test) ämnat för barn i förskola, årskurs 1 och årskurs 2. Kartläggningsverktygets psykometriska kriterier granskades genom att ta fasta på reliabiliteten (intern konsistens) och validiteten (strukturell validitet, grupptillhörighetsvaliditet och tvärkulturell validitet). Den tredje delstudien studerade effekten av en datorbaserad intervention med datorspelet ”Tal i farten” för lågpresterande barn i årskurs 1. Datorspelet ”Tal i farten” är utvecklat för att specifikt stärka barns tal- och antalsuppfattning. Delstudierna i denna avhandling baserar sig på datainsamling i finlandssvenska förskolor och skolor där totalt 361 barn i förskola, 321 barn i årskurs 1 och 457 barn i årskurs 2 deltog.

Resultaten från den första delstudien visade att räknefärdigheter och aritmetiska färdigheter var de starkaste prediktorerna för prestationerna i slutet av läsåret i årskurs 1, medan aritmetiska färdigheter utgjorde den starkaste prediktorn i årskurs 2. Med latenta profilanalyser identifierades i början av läsåret i årskurserna 1 och 2 tre prestationsgrupper: hög-, medel- och lågpresterande. Barnen som tillhörde den lågpresterande gruppen uppvisade bristande färdigheter inom alla fyra färdighetsområden (tal- och antalsuppfattning, förståelse för matematiska samband, räknefärdigheter och aritmetiska basfärdigheter) under hela läsåret. I båda årskurserna var skillnaderna mellan alla prestationsgrupperna signifikanta såväl i början

som i mitten och i slutet av läsåret, vilket indikerar att klyftan mellan de lågpresterande och de medelpresterande barnen inte minskade under läsårets gång. Resultaten från den andra delstudien ger belägg för att Lukimat-kartläggningsmaterialet kan ses som lämpligt för att identifiera barn i risk för matematiska inlärningssvårigheter. De psykometriska kriterierna uppfylls genom att den interna konsistensen (reliabiliteten) och strukturella validiteten är goda. Vidare visar grupptillhörighetsvaliditeten att kartläggningsmaterialet fungerar för såväl flickor som pojkar och yngre som äldre barn inom gruppen och den tvärkulturella validiteten visar att kartläggningsmaterialet fungerar oberoende skolspråk (svenska och finska). Kartläggningsmaterialets styrka är att det inte enbart identifierar de barn som behöver stöd utan också visar vilka delfärdigheter som stödet behöver fokuseras på. Resultaten från den första delstudien gav ytterligare belägg för kartläggningsmaterialets prediktiva validitet. Resultaten från den tredje delstudien visade inte på några signifikanta framsteg för de lågpresterande barnen i årskurs 1 som deltog i interventionen med datorspelet ”Tal i farten”. Dessa resultat belyser vikten av skräddarsydda interventioner som utgår från barnets individuella behov. Därmed behövs olika typer av forskningsbaserade interventionsprogram som stärker barns tidiga grundläggande matematiska färdigheter.

Sammantaget belyser resultaten från avhandlingens tre delstudier vikten av tidig och kontinuerlig kartläggning av tidiga grundläggande matematiska färdigheter samt skräddarsydda stödåtgärder för att på bästa sätt säkerställa att barn med risk för matematiska inlärningssvårigheter identifieras och får ändamålsenligt stöd. Kunskap om hur tidiga matematiska färdigheter samspelar och hur dessa färdigheter utvecklas borde ligga till grund för såväl undervisning som utveckling av forskningsbaserat material för kartläggning och intervention. Därtill ger resultaten empirisk grund för den pedagogiska modellen av Aunio och Räsänen (2016) med fyra grundläggande matematiska färdighetsområden: tal- och antalsuppfattning, förståelse för matematiska samband, räknefärdigheter och aritmetiska basfärdigheter. Modellen utgör en god pedagogisk modell att användas för att planera, förverkliga och utvärdera ändamålsenliga stödåtgärder. Då både kartläggning och stödåtgärder baseras på samma forskningsbaserade modell möjliggör det en närmare koppling mellan kartläggning och stödåtgärder, med målsättningen att säkerställa att alla barn får tillräckliga tidiga grundläggande matematiska färdigheter.

Nyckelord: datorstödd intervention, forskningsbaserad undervisning, kartläggning, lågpresterande, matematiska inlärningssvårigheter, tidiga grundläggande matematiska färdigheter, utveckling av matematiska färdigheter, validering

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Vaasa, March 2021

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List of Original Publications

This thesis is based on the following three publications, referred to as substudies I, II, and III:

Study I: Hellstrand, H., Korhonen, J., & Aunio, P. (manuscript revision). Development of early numeracy skills during first and second grade.

Study II: Hellstrand, H., Korhonen, J., Linnanmäki, K., Räsänen, P., & Aunio, P. (2020). Reliability and validity evidence of early numeracy test for identifying children at risk for mathematical learning difficulties. *International Journal of Educational Research, 102,* 101580. https://doi.org/10.1016/j.ijer.2020.101580

Study III: Hellstrand, H., Korhonen, J., Linnanmäki, K., & Aunio, P. (2020). The Number Race – computer-assisted intervention for mathematically low-performing first graders. *European Journal of Special Needs Education, 35(1)*, 85-99. https://doi.org/10.1080/13488678.2019.1615792

Author contribution

Heidi Hellstrand is the first author for all three manuscripts that are included in this doctoral thesis. Hellstrand is responsible for the data collection, all the data analysis, interpreting the results and written all the manuscripts under the supervision of Associate Professor Johan Korhonen, Docent Karin Linnanmäki, and Professor Pirjo Aunio.

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1. Introduction

Numeracy skills are highly important in modern everyday life, both for individuals and for society. Low numeracy skills can affect an individual's life in many ways: in daily activities (Geary, 2011; Jansen et al., 2016), vocational and educational choices (Jordan et al., 2015; Korhonen et al., 2014; Widlund et al., 2020), employment (Parsons & Bynner, 2005), socio-economic status (Estrada-Mejia et al., 2016; Ritchie & Bates, 2013) and well-being (Widlund et al., 2018). For society, low numeracy skills can even have long-term effects on the national economy (Hanushek & Woessmann, 2010). Early numeracy skills start to develop from birth and evolve throughout our lifetime. The early years are highly important and form a foundation for later mathematical learning, as early numeracy is proposed to be a precursor of later mathematical skills (Aunio & Niemivirta, 2010; Geary et al., 2018; Jordan et al., 2009; Zhang et al., 2020).

Already in kindergarten and the early school years, children show large individual differences in their early numeracy performance (Aunio & Niemivirta, 2010; Jordan et al., 2009). Some of the children are proficient and have grasped a wide range of early numeracy skills (e.g., number words, counting sequence, comparison of quantities and numbers, simple addition and subtraction), whereas others struggle in their development of early numeracy skills. Children who do not develop foundational early numeracy skills in their first school years are at risk for encountering mathematical difficulties (Clements & Sarama, 2014; Jordan et al., 2015). Previous studies also indicate that the gap between low-performing and average-performing children tends to persist and even grow wider across grades (Aunola et al., 2006; Geary, 2011). Luckily, adequate support at an early stage may prevent or reduce later mathematical learning difficulties (Fuchs et al., 2007; Zhang et al., 2020). In order to support children at risk for mathematical learning difficulties at an early stage, educators need to identify the children in need of support. To be able to identify and support children at risk for mathematical learning difficulties, educators need evidence-based tools as well as guidance to improve their knowledge regarding what to focus on in mathematics (Clements & Sarama, 2014; Desoete et al., 2009). The assessment for identifying children at risk for mathematical learning difficulties should not only differentiate the children that are low-performing, but it should also give more detailed information that can be used for planning and conducting the support (Aunio, 2019). Intensive and individualised support that focuses on enhancing specific subskills is suggested to be beneficial for children at risk for mathematical learning difficulties (Zhang et al., 2020). Computer games can be a good way to provide children with intensive and individualised support. Computer-assisted interventions (CAI) with adaptive computer games are suggested to be advantageous ways to support children at risk for mathematical learning difficulties, offering practice in a motivational and entertaining context, providing direct feedback and multiple representations, and enabling progress at the child's own pace (Fengfeng, 2008; Kroesbergen & Van Luit 2003). Additionally, the practical advantage of computer assisted interventions is that they are independent of the educator and the educator can be freed to support other children. However, findings from previous CAI studies have reported contradictory findings of the effects of enhancing low-performing children's early numeracy skills (Mononen et al., 2014). Altogether, this highlights the importance of early identification and support, which puts high demands on the tools that are used to identify and support children at risk for mathematical learning difficulties.

This thesis aims to contribute to the research on early numeracy skills development by focusing on identifying and supporting children at risk for mathematical learning difficulties in kindergarten and early school years with the aspiration to improve evidence-based educational practice. This thesis is based on three substudies (Studies I, II and III). Study I focuses on how early numeracy skills develop and are interrelated, in order to reach an understanding of which early numeracy skills predict later mathematical learning and are important to focus on when identifying and supporting children at risk for mathematical learning difficulties. Study II focuses on assessment for identifying children at risk for mathematical learning difficulties by examining the reliability (internal consistency) and validity (structural validity, known group validity, and cross-cultural validity) evidence of an early numeracy test. Study III focuses on examining the effects of supporting children at risk for mathematical learning difficulties using the evidence-based Number Race computer game in a computer-assisted intervention in an authentic school setting. In this thesis, "early numeracy" refers to the numeracy skills of children in kindergarten and early school years (ages 5-9). The term "kindergarten" is used for the one-year education that is provided in Finland to all children the year before they enter primary school. "Early school years" refers to first and second grade in compulsory formal education. "Evidence-based practice" refers to the principle that educational practice (e.g., decisions, activities, and tools) should be based on research findings from objective, valid and scientific studies (Gregory, 2015; Hempenstall, 2006).

This present thesis consists of two parts. The first part includes a theoretical introduction, the aims and research questions, methods, an overview of the original studies and a discussion. The second part of the thesis consists of an overview of the three original articles. On due day, two of them are published and one is submitted to an international peer-reviewed journal. The introduction section aims to theoretically frame the subject of the thesis. First, the introduction focuses on early numeracy skills by presenting an overview of theoretical models operationalising early numeracy skills, describing in more detail early numeracy skills based on the core factor model constructed by Aunio and Räsänen (2016), and then highlighting predictors and risk factors in children's early numeracy skill development. Second, the introduction focuses on identifying children at risk for mathematical learning difficulties by operationalising mathematical learning difficulties and giving an overview of criteria for evidence-based assessment for identifying children at risk for mathematical learning difficulties. Third, in the introduction section, supporting children at risk for mathematical learning difficulties is discussed from the view of computer-assisted interventions and a specific evidence-based program designed to improve young children's early numeracy skills, the Number Race game.

1.1 Early Numeracy Skills

Children develop their numeracy skills from the first years of life and when entering formal education they already have a wide range of knowledge and skills. Early numeracy skills have been found to be important for later mathematical learning (Aunio & Niemivirta, 2010; Geary et al., 2018; Jordan et al., 2009; Zhang et al., 2020). A number of frameworks have been constructed to highlight early numeracy skills that form the foundation for later mathematical learning and to describe the development of these early numeracy skills. The developmental models of early numeracy skills have set the base for research on how these skills develop and are interrelated in order to reach an understanding of which early numerical skills are most important to identify and support with the aim to prevent and minimise later mathematical learning difficulties.

1.1.1 Theoretical Frameworks of Early Numeracy Skills Development

Current frameworks of early numeracy skills development in early school years (Aunio & Räsänen, 2016; Fritz et al., 2013; Krajewski & Schneider, 2009; Sarama & Clements, 2009; Steffe, 1992; Wright et al., 2006) have focused on describing the essential skills involved in order to promote instruction in early mathematics. The mathematical development has been described in children aged four to eight, relying more on theoretical underpinnings than empirical data on children's development. All these frameworks (Fritz et al., 2013; Krajewski & Schneider, 2009; Sarama & Clements, 2009; Steffe, 1992; Wright et al., 2006) have emphasised early number knowledge, counting and basic arithmetical skills. The content in these frameworks taps similar early numeracy skills but the frameworks differ in extensity, aim, purpose and target group. An overview of these frameworks is presented in Table 1.

Table 1
Overview of Theoretical Frameworks of Early Numeracy Skills Development.

Author/authors	Main aim	Main components	Age/target group	Purpose	Focus group
Aunio & Räsänen, 2015	A core factor model of early numerical skills	Four subskills describing core numerical skills for young children 1. Symbolic and non-symbolic number sense 2. Understanding mathematical relations 3. Counting skills 4.Basic arithmetical skills	Children aged 5 to 8 years	A model for educators to help to work with young children who are at risk for learning difficulties	Average- and low-performing children
Fritz, Ehlert, & Balzer, 2013	A conceptual model of hierarchical mathematics competence development	A six-level model for describing, explaining and predicting children's development of key numerical concepts and arithmetic skills: 1. Count number 2. Mental number line 3. Cardinality and decomposability 4. Class inclusion and embeddedness 5. Relationality 6. Units in numbers	Children aged 4 to 8 years	The model can be used for both diagnostics and training	Average-performing children
Krajewski & Schneider, 2009	Developmental model describing the transition from a procedural to an increasingly conceptual understanding of number words	Early quantity-number competencies acquired via three levels: 1. Basic numerical skills 2. Quantity-number concept and linking number words with quantity 3. Number relationships and linking quantity relations with the number words	Focus on early quantity-number competencies, in early years up to school grade 4	Focus on the competencies that can be predictive of mathematical learning and difficulties in school	Average-performing children
Sarama & Clements, 2009	A model of early mathematics core skills, with focus on learning trajectories	Early mathematics concepts and skills categorised under four categories: 1. Quantity, number and subitising 2. Verbal and object counting 3. Comparing, ordering and estimating 4. Arithmetic: addition and subtraction	Focus on early mathematics for young children	Helping educators to understand and identify the children's developmental level and help children to learn better, both in classes and individually	Average-performing children
Steffe, 1992	A psychological model of the development of children's counting-based strategies	Three composite units categorised in two general categories (units-coordinating schemes and unit segmenting schemes): - Initial number sequence (INS) - Tacitly nested number sequence (TNS) - Explicitly nested number sequence (ENS)	Children in early school years (In the case studies, the three children were aged 8, 3rd grade)	Constructing models of children's individual construction of schemes of action and operation involving composite units, through individual learning situations	Average-performing children
Wright, Martland, & Stafford, 2006	The Learning Framework in Number (LFIN) in children's early numerical learning	Eleven aspects of early number, organised into four categories: 1. Early arithmetical strategies (6 levels), base-ten arithmetical strategies (3 levels) 2. Forward number word sequences and number word after (6 levels), backward number word sequences and number word before (6 levels), numeral identification (5 levels) 3. Structuring number 1 to 20 - Combining and partitioning - Spatial patterns and subitising - Temporal sequences - Finger patterns - Five-based strategies 4. Early multiplication and division (5 levels)	Children aged 5 to 8 years	Framework used as a basis for classroom teaching and assessment in early numbers, as well as for intervention for low-attaining children. Detailed information of a child's individual stage and levels	Average- and low-performing children

Note. Only the main reference for each model is mentioned in this table

The existing frameworks have focused on average-performing children (Fritz et al., 2013; Krajewski & Schneider, 2009; Sarama & Clements, 2009; Steffe, 1992) or on both average- and low-performing children (Aunio & Räsänen, 2016; Wright et al., 2006). The focus in all the frameworks is early numeracy, more precisely enumeration, number word sequence, number combinations and arithmetical principles. Numerical and arithmetical skills are by definition the base for diagnosing mathematical learning difficulties. Therefore, areas such as geometry, measurement and statistics are excluded in the frameworks, as they are not typically included in tasks used in researching mathematical learning difficulties (DSM-5; American Psychiatric Association, 2013; ICD-10; World Health Organization, 2016), even though these areas are emphasised in mathematical didactics and are therefore key areas in the national curricula and in mathematical assessments. Directly mastering of arithmetical skills (i.e., addition and subtraction) is only included in the framework of Sarama and Clements (2009).

The existing frameworks were originally constructed based on either theoretical or empirical findings or using both approaches. Aunio and Räsänen (2016) constructed their framework of four core numerical skills for learning mathematics based on findings from longitudinal studies and existing test batteries. Fritz and colleagues (2013) constructed a hierarchical framework of competencies grounded on earlier theoretical and empirical findings. The authors verified their framework with empirical data. The theoretical framework of Krajewski and Schneider (2009) focused on describing the transition from a procedural to an increasingly conceptual understanding of number words via three levels, based on theoretical knowledge of how the competencies develop. The framework of Sarama and Clements (2009) is based on theory and research on early childhood learning and teaching, with a focus on children's learning trajectories. Steffe's (1992) framework has a constructivist view on learning and is based on case studies of children's individual mathematical development trajectories. This framework was later tested with a larger sample of children (e.g., Biddlecomb & Carr, 2011). Steffe's framework has been a starting point for the framework that Wright, Martland, and Stafford (2006) constructed. The framework of Wright and colleagues is grounded in research in early number learning and was developed in a project (Count Me In, CMI), and was later used by educators and researchers (e.g., Wright, 1994), mostly for providing individualised educational support for low-performing children. The framework of Wright, Martland, and Stafford (2006) focuses on low-performing children but includes many levels and is therefore very extensive for educators to integrate in their everyday work with children in whole class setting.

Some of the frameworks of early numeracy skills development in early school years have been used for development of mathematical test batteries

(Aunio & Räsänen: Early Numeracy test; Fritz et al.: Marko-D test) and intervention and training programs (Aunio & Räsänen: ThinkMath; Fritz et al.: MARKO-T, an individual training program; Sarama & Clements: The Learning Trajectories Approach and the Building Blocks project; Wright et al.: Mathematics Recovery Program, an intervention program).

It is important for both theoretical and practical reasons to identify the foundational skills that would predict further success in acquiring mathematical skills (Lyons et al., 2014; Purpura & Lonigan, 2013), and to integrate and support these skills by new research findings (Mulligan & Vergnaud, 2006). Even though many of the current theoretical frameworks have been constructed with an educational approach and without being curriculum-based, a substantial number are either too extensive or too detailed to be easily understandable and implemented in educational practise, in order to provide all children with fundamental early numeracy skills and to identify and support those children who struggle in their early numeracy development. The model of Aunio and Räsänen (2016) highlights predictors and risk factors in children's early numeracy skill development and was constructed explicitly to be a working model for educators to use in order to implement evidence-based practice in kindergarten and early school years.

1.1.2 A Model of Four Core Early Numeracy Skills

Aunio and Räsänen's (2016) theoretical model of four core numeracy skills for learning mathematics in children aged five to eight was constructed based on the findings from developmental research on mathematical skills. Based on a literature review with longitudinal studies on early mathematical development, Aunio and Räsänen categorised numerical skills and tested the framework against the contents of internationally used test batteries (Utrecht Test of Early Numeracy: Van Luit et al., 1994; Number Knowledge Test: Griffin, 2003; Early Numeracy Test: Wright et al., 2006; Test of Early Mathematics Ability, TEMA-3: Ginsburg & Baroody, 2003). The criteria for the selected tests were that they were published with norms, they were widely used by educators and researchers in the field, they focused on multiple mathematical skills relevant for this age group, and they were curriculum independent. One key point for the framework of core numerical skills by Aunio and Räsänen (2016) was that the model should emphasise early numeracy skills based on research of mathematical development instead of the content in curriculums.

Based on their analysis, Aunio and Räsänen (2016) categorised early numeracy skills into four main groups of factors that are fundamental to the development of mathematical skills: (1) symbolic and non-symbolic number sense, (2) understanding mathematical relations, (3) counting skills, and (4) basic skills in arithmetic. The first group, symbolic and non-symbolic number sense, consists of the earliest and most profound number competencies and is operationalised by subitising, approximate representation of magnitudes, magnitude comparison and pattern recognition. It is assumed that children are born with the capacity to discriminate quantities (Dehaene, 1997; Wynn, 1998) and already at early ages make approximate evaluations of magnitudes or symbols representing magnitudes (Jordan & Levine, 2009). The concept of number sense has various definitions, ranging from broader to narrower. In the narrower definition number sense is restricted to those numerical abilities that have been identified both for animals and humans from infants to adults (e.g., Dehaene, 1997/2011). The broader definition of number sense can also include operating with and understanding number symbols. Aunio and Räsänen used a narrower definition of the concept number sense, but in order to clarify the distinction between the symbolic and the non-symbolic skills, they extended the concept to symbolic and non-symbolic number sense. A differentiation between symbolic and non-symbolic number sense is supported in recent research findings, as symbolic magnitude processing is suggested to be a stronger predictor for later mathematical learning than non-symbolic magnitude processing (De Smedt et al., 2013; Schneider et al., 2016; Xenidou-Dervou et al., 2013; Vanbinst et al., 2015; Vogel et al., 2017). Non-symbolic number sense refers to the earliest preverbal numeracy skills, the ability to process and discriminate numerical magnitudes, whereas symbolic number sense refers to symbolic numerical magnitude processing and operating with number symbols and number words (Lyons et al., 2018; Vanbinst et al., 2016). Learning to connect quantities to numerals and a good understanding of numerical meaning helps children to develop more advanced counting strategies, which in turn might lead to a faster reliance on arithmetic combinations (facts) (Chu et al., 2018; Vanbinst et al., 2016) and more sophisticated word problem-solving skills (Thevenot & Barrouillet, 2015).

This second group, understanding mathematical relations, refers to understanding the quantitative and non-quantitative relationships between the elements in a task. This group includes understanding of early mathematical–logical principles, basic arithmetic principles, mathematical operational symbols, and place-value and base-10 systems. Mathematical–logical principles are operationalised by comparison, classification, seriation and one-to-one correspondence, as well as cardinality and ordinality of numbers. An understanding of these concepts and principles has been pointed out in early research on children's mathematical development to form a base in understanding and doing early mathematics (Dowker, 2005; Gelman & Gallistel, 1978; Inhelder & Piaget, 1958) and to serve as a good predictor for their later use of counting strategies (Geary et al., 2018) and mathematical learning (i.e., cardinality; Geary et al., 2018).

Basic arithmetic principles are operationalised by additive composition, commutativity, associativity, and inversion. These skills are needed in different calculation tasks, mainly in understanding the part–whole relations (what entities to count), together with understanding mathematical operational symbols and place-value and base-10 systems. The understanding of these concepts and principles helps the child to judge the correctness of arithmetical tasks (Geary et al., 2007). However, children can use these basic arithmetical principles to determine simple arithmetical tasks before they invent computational strategies, for example, by using commutativity when mentally adding two parts in different orders and getting the same sum (Baroody et al., 2003), or even though they have difficulties in basic arithmetic tasks (Geary et al., 2007). The relation between children's conceptual understanding about numbers and their goal-based numerical activities is a dynamic process where procedures and principles develop side-by-side and support each other (Rittle-Johnson & Siegler, 1998).

The third group, counting skills, includes knowledge of number symbols, number word sequences, and enumeration with concrete objects. Knowledge of number words and symbols includes learning the numbers and number words, and the naming and recognition of numbers. Making the connection between quantities and numerals is a skill that appears to be important for developing increasingly sophisticated strategies for basic arithmetic combinations (facts) (Vanbist et al., 2012). Number word sequence skills involve knowing the correct order of number words and being able to recite the number words forward and backward in chronological order by counting each number or skipping numbers, as well as starting from any given number. When the string of number words is used to enumerate a collection of items, a numerical operation is performed (Sophian, 1998). Enumeration is the ability to count the numerosity of a concrete set and is an essential skill for developing further arithmetical skills. With increasing age and experience with numbers, children start to use more effective and varied counting strategies (Chu et al., 2018). At first, children count objects one by one, and later develop and rely on more effective strategies, such as counting on or up from addends (Baroody, 1999). Counting is a powerful way of learning about numerical relations (Sophian, 1998), as the counting skills (rote sequence) and counting principles (the stable order principle, the one-to-one principle, and the cardinality principle; Gelman & Gallistel, 1978) support each other. Counting skills (knowledge of number symbols, number word sequences, and enumeration with concrete objects) have been emphasised also in other frameworks of early numeracy skills development (Fritz et al., 2013; Krajewski & Schneider, 2009; Sarama & Clements, 2009; Steffe, 1992; Wright et al., 2006).

The fourth group, basic skills in arithmetic, focuses on mastering addition and subtraction tasks with number symbols, and later also multiplication and division tasks. Young children start to solve arithmetic tasks by counting concrete elements (e.g., fingers and manipulatives) and verbal strategies (i.e., reciting number words) and increasingly by using more effective memory-based strategies (e.g., decomposition and fact-retrieval) (Clements & Sarama, 2014; Chu et al., 2018). With development and schooling, children shift from using counting-based strategies to memory-based strategies, and their solving of arithmetical tasks gets more fluent (Geary et al., 2007). Especially solving single-digit arithmetic tasks requires various strategies and memory-based processes (Clements & Sarama, 2014; Siegler, 1996). Sophisticated arithmetical skills are characterised by an ability to flexibly use different strategies, depending on the problem (Clements & Sarama, 2014). Currently only Sarama and Clements (2009) and Wright and colleagues (2006) directly mention arithmetical skills in their frameworks. The other frameworks (Fritz et al., 2013; Krajewski & Schneider, 2009; Steffe, 1992) focus more on counting skills (number word sequence and object counting) and arithmetical principles (e.g., ordinality, cardinality, and decomposition) than on directly mastering arithmetical tasks using addition and subtraction.

Based on longitudinal studies and items in existing test batteries Aunio and Räsänen (2016) tackled the complexity of early numeracy and constructed a model of four core early numeracy skills that are considered important in developing a good foundation for later mathematical learning. The strength of the core factor model is that it offers educators a clear evidence-based framework to be used in structuring their educational practice in identifying and supporting children at risk for mathematical learning difficulties, in contrast to the more extensive frameworks that existed. This kind of evidence-based model was also highly needed in the Finnish national context where prevention and early support in learning was emphasised in the three-level support model, but evidence-based practice and support programs were not available (Björn et al., 2016).

1.1.3 Early Numeracy Skills Development

In order to identify and support children at risk for mathematical learning difficulties, evidence of the early numeracy skills that form the foundation of later learning in mathematics and how these skills are interrelated and how they develop is needed (Desoete et al., 2009; Penner et al., 2019). The early numeracy skills have been described in the existing frameworks of early numeracy skills development, but it is still not clear how these early numeracy skills are related over time or how individual learning trajectories develop. Longitudinal studies (e.g., Aunio & Niemivirta, 2010; LeFevre et al., 2006; Locuniak & Jordan, 2008; Vanbinst et al., 2019) have contributed with findings of the predictors and developmental trajectories, but many of these studies have either focused on the relations between specific early numeracy skills and later overall mathematics performance or on the longitudinal relations within a specific early numeracy subskill (e.g., counting or symbolic magnitude comparison). However, longitudinal research on the relations between early numeracy subskills is still lacking. Cross-sectional studies (e.g., Gray & Reeve, 2016; Purpura & Lonigan, 2013; Jordan et al., 2010; Li et al., 2018) have mainly focused on how the skills are related concurrently and examining differences between different performance groups. Both variable-centred (relations among early numeracy skills) and person-centred approaches (differences among individuals) have been used in these longitudinal and cross-sectional studies.

Findings from cross-sectional studies have supported the relations between the core early numeracy skills and mathematical learning: symbolic and non-symbolic number sense, such as approximate counting and non-symbolic number comparison (Cai et al., 2018; Libertus et al., 2016) and symbolic number comparison and magnitude processing (Li et al., 2018); mathematical relational skills, such as numbering, relations and arithmetic operations (Purpura & Lonigan, 2013); counting skills, such as reciting number word sequences (Desoete et al., 2009); and basic skills in arithmetic, such as addition and subtraction, counting strategies and fluency (Jordan et al., 2010). Cross-sectional person-centred studies have particularly highlighted symbolic number sense as an indicator for children at risk in their mathematical learning (Gray & Reeve, 2016).

The predictive value of the core early numeracy skills on later mathematical learning has been supported by findings from longitudinal variable-centred studies. First, symbolic and non-symbolic number sense, such as approximate counting and non-symbolic number comparison (Hannula-Sormunen et al., 2015; Libertus et al., 2013) and symbolic number comparison and magnitude processing (Vanbist et al., 2018), is seen as a predictor for later mathematical learning. Second, the predictive value of mathematical relational skills, such as comparison and seriation (Desoete et al., 2009) and counting principles (Stock et al., 2007), including cardinality (Geary et al., 2018), have been supported. Third, counting skills, such as reciting number word sequences (Desoete et al., 2009; Krajewski & Schneider, 2009) and enumeration (Geary et al., 2018), are found to be a predictor for later mathematical learning. Fourth, the predictive value of basic skills in arithmetic, such as addition and subtraction, counting strategies and fluency (Aunio & Niemivirta, 2010; LeFevre et al., 2006; Locuniak & Jordan, 2008) have been supported. Furthermore, longitudinal person-centred studies have highlighted the predictive value of symbolic and non-symbolic number sense (Chew et al., 2019; Reeve et al., 2012; Salminen et al., 2018) on later mathematical learning, especially the importance of symbolic number sense (Vanbist et al., 2015). However, most of the existing longitudinal person-centred studies have only investigated the predictive value over a relatively short time period (e.g., one year; transition period from kindergarten to first grade).

1.2 Identifying Children at risk for Mathematical Learning Difficulties

Children start formal education with differing skills. Already in kindergarten there are differences in children's early numeracy skills and performance. Some children experience difficulties acquiring basic numerical skills, with problems ranging from mild to severe (Geary, 2011). Children with problems in their mathematical learning in kindergarten and the early school years are often regarded as children at risk for mathematical learning difficulties. Early identification of children at risk for learning difficulties is the first step in supporting these children (Fuchs et al., 2007; Penner et al., 2019). However, the assessment tool should not only identify the lowest performing children, but also give more information on children's strengths and needs. This puts high demands on the assessment tools that are used in the educational practice for identifying children at risk for mathematical learning difficulties.

1.2.1 Children at risk for Mathematical Learning Difficulties

Even before formal schooling individual differences in children's early numeracy skills can be identified (Bisanz et al., 2005; Dyson et al., 2011). For some children the learning, mastering and development of early numeracy seem unproblematic, whereas for some children it is more challenging to acquire the basic early numeracy skills. The terminology concerning those children who perform lower than their average or typically performing peers varies in the literature. Mathematical learning difficulties, low performance, or low achievement are used to describe the group of children who perform lower than their peers. The prevalence of the low-performing children varies in the literature between 15 and 35 percent, depending on what assessments and criteria are used (e.g., Butterworth et al., 2011; Geary 2013; Mazzocco & Räsänen, 2013). A common criterion used for mathematical learning difficulties is a performance at or below the twenty-fifth percentile in at least two consecutive validated mathematics assessments (Geary, 2011; Geary, 2013). Additionally, it is notable that assessments at several time points within or across years are required for decisions on at-risk status (Mazzocco & Räsänen, 2013). In kindergarten and early school years, children performing lower than their peers are often described as children at risk for mathematical learning difficulties.

Of those children performing at the lowest continuum, some show severe and persistent problems in their mathematical learning. Mathematical disability (MD), mathematical learning disability (MLD) and developmental dyscalculia (DD) are used interchangeably to describe this group of children. In diagnostics also the terms 'specific learning disorder with impairment in mathematics' (DSM-5, 315.1; American Psychiatric Association, 2013; previously in DSM-4, 315.1: mathematics disorder) and 'specific disorder of arithmetical skills' (ICD-10, F81.2; World Health Organization, 2016) are used. The prevalence of this group of children is about four to seven percent and refers to a specific learning disorder and severe difficulties in acquiring basic mathematical skills (Butterworth et al., 2011; Geary, 2013; Zhang et al., 2020). The markers for developmental dyscalculia can be found at the neural level with impairments in the brain activity, primarily in frontal lobes, the intraparietal sulci and left angular gyrus, are considered to have origins in biological causes (Butterworth et al., 2011). Children with developmental dyscalculia show problems in understanding counting concepts, rely on counting-based strategies, have deficits in arithmetic fact retrieval, and commit more procedural errors than their peers (Geary & Hoard, 2003; Zhang et al., 2020).

In recent research the core deficit in developmental dyscalculia has been found to lie in the symbolic numerical magnitude processing, more precisely in understanding sets and their numerosities and operating with the number symbols with an understanding of the logical relation between them (Geary, 2013; Vanbinst et al., 2016; Schneider et al., 2016). Two explanations of this deficit have been suggested: the defective number module hypothesis, meaning that the problem's origin is in processing numerosities (e.g., Butterworth, 2005; Reigosa-Crespo et al., 2012) and the access deficit hypothesis, meaning that the problems are in difficulties accessing magnitude information from numerical symbols rather than in processing numerosities (e.g., De Smedt & Gilmore, 2011; Desoete et al., 2012; Rousselle & Noël, 2007; Wilson & Dehaene, 2007). Also, low-performing children have deficits in processing numbers, but the problems may originate in cognitive and motivational causes, as well as sociocultural and educational factors (Price & Ansari, 2013). Additionally, low-performing children tend to benefit from standard interventions, contradictory to children with developmental dyscalculia, who are in need of more individualised support (Desoete et al., 2012; Mazzocco & Räsänen, 2013).

Even though there is no consensus concerning the terminology, diagnostic criteria or cut-offs, there is evidence that children at risk for mathematical learning difficulties can be placed in two categories: (1) low-performing children and (2) children with mathematical learning disability or developmental dyscalculia (e.g., Geary, 2011; Price & Ansari, 2013; Zhang et al., 2020). These groups of children are often contrasted in research against the group of average performing children. However, even if these two categories of mathematical learning difficulties show differing performance patterns and learning trajectories, it is notable that within the groups there are individual differences (Mazzocco & Räsänen, 2013). Furthermore, the distinction between these categories is not yet defined (Zhang et al., 2020). It is possible that the importance of specific early numeracy subskills differs depending on where children are in their learning process (Desoete et al., 2012; Geary, 2013). Consequently, it is still unclear how in early age the differentiation between mathematical learning difficulties and developmental dyscalculia can be identified, what the learning trajectories are for these two groups of children, and how numeracy assessments can differentiate these two groups. Also, the effects of early interventions for children with mathematical learning difficulties, particularly developmental dyscalculia, are still unclear and more research is needed in this area (Butterworth et al., 2011; Dowker & Sigley, 2010; Mazzocco & Räsänen, 2013). Altogether, this points out the importance of educators having insight into the core early numeracy skills as well as risk factors in order to identify and support in the early stages those children at risk for mathematical learning difficulties.

1.2.2 Assessment for Identifying Children at Risk for Mathematical Learning Difficulties

In order to provide children with sufficient support, it is important to identify children at risk for mathematical learning difficulties at an early stage (Geary, 2011; Penner et al., 2019). Identifying children at risk for mathematical learning difficulties is often based on assessments, a process of gathering information regarding the children's skills, needs, strengths and performance in order to plan and conduct adequate instruction based on the information (Pegg, 2003). Furthermore, valid and reliable assessment tools should not only differentiate children at risk for mathematical learning difficulties but also give more detailed information regarding children's performance and the development needed (Aunio, 2019; Purpura & Lonigan, 2015). Repeated assessments to continuously follow children's mathematical learning are also important (Aunio, 2019), as decisions of at-risk status should be based on assessments at several time points within or across years (Mazzocco & Räsänen, 2013). This raises the need for appropriate assessment tools that are evidence-based and designed for a particular purpose.

Currently, there is a wide range of tools for educators to use for assessment of early numeracy skills in kindergarten and early school years (e.g., Child Math Assessment, CMA; Starkey et al., 2004; MARKO-D test:

Ricken et al., 2013; Number Sense Screener, NSS™: Jordan et al., 2012; Number Sets test: Geary et al., 2009; Research-based Early Mathematics Assessment, REMA: Clements et al., 2008; Test of Early Mathematics Ability, Third Edition, TEMA-3: Ginsburg & Baroody, 2003; The Early Numeracy Test, WENT: Wright et al., 2006; The Number Knowledge Test, NKT: Okamoto & Case, 1996; The Utrecht Test of Early Numeracy, ENT-test: Van Luit et al., 1994). Common to these tests is that they are theoretically underpinned and constructed for identifying children at risk for mathematical learning difficulties. However, to the best of our knowledge, levels for internal consistency are published only for some of these tests. Of these tests, only the Number sets test (Weiland et al., 2012) is constructed to be conducted in groups; the rest are conducted individually.

The strength of individual assessments is that the administrator can guide and monitor the performance during the assessment, whereas the group-administered assessment is generally less time-consuming (Gregory, 2015). Therefore group-administered assessment can be considered efficient for identifying children at risk for mathematical learning difficulties in a heterogenic classroom. However, group-administered assessments (i.e., screening) often have the purpose of easily identifying those children who need support but do not provide deeper and more specific information in what particular skills children need support (Gregory, 2015). Furthermore, these assessments mainly aim to identify children at risk for mathematical learning difficulties, not to differentiate the lowest performing children into the two categories (i.e., low-performing children and children with developmental dyscalculia). For diagnostic assessment of developmental dyscalculia, a more detailed diagnostic evaluation is needed. On other hand, it is unclear if behavioural measures are able to differentiate the two groups in the low end of the normal distribution, as so many factors (e.g., cognitive, motivational, sociocultural and educational) affects the performance in the test situation. However, in the educational practice, particularly in early school years, the focus is on identifying the lowest performing children and their strengths and needs, in order to provide these children with sufficient support.

Many assessment tools that are designed to be used in the educational practice either broadly measure mathematical skills and concepts (e.g., CMA; Starkey et al., 2004; WENT: Wright et al., 2006) or focus more deeply on specific mathematical skills (e.g., NKT: Okamoto & Case, 1996; Number sets test: Weiland et al., 2012). Additionally, most of the tests have only a unidimensional construct. Describing and interpreting sub-skills as separate factors gives researchers and educators possibilities to examine early numeracy performance, skills, strengths, and needs more specifically (Purpura & Lonigan, 2013). Curriculum-based measurement (CBM) has been suggested as a reliable and valid measure of children's performance, as it combines the advantages of norm-referenced achievement tests and curriculum-based assessments developed by teachers, and it strives to minimize the gap between the measurement and instruction (Fuchs, 2016).

In order to determine if a test is evidence-based and fit for its purpose, psychometric properties have been set. A theoretical underpinning provides evidence that the skills in focus are relevant and gives an opportunity to understand the underlying structure of the early numeracy skills (Aunio et al., 2019). The validity and reliability of a test can be examined by different criteria for construct validity (structural, cross-cultural, known-group and convergent validity), criterion validity (concurrent and predictive validity) and reliability (item consistency, test-retest and inter-rater reliability) (Consensus-based Standards for the selection of health Measurement Instruments (COSMIN) methodology: Terwee et al., 2017; Gregory, 2015; Standards for educational and psychological testing: AERA, APA, NCME, 2014). The predictive validity is especially important when determining how effective the test is at identifying at risk children (Gersten et al., 2011; Penner et al., 2019).

1.3 Supporting Children at Risk for Mathematical Learning Difficulties

Early support can prevent or decrease later mathematical learning difficulties. In school low-performing children seem to benefit less from ordinary instruction than their average-performing peers (Zhang et al., 2020). Therefore, it is important to provide the children that have been identified as at risk for mathematical learning difficulties with adequate support. Screening all students and providing those children identified as at risk for mathematical learning difficulties with adequate support are the first steps in the recommendations of effective ways to support children (Gersten et al., 2009). Additionally, the recommendations include focus on core numerical skills, explicit and systematic instruction, working with underlying structures and visual representations, enhancing fluency in basic arithmetical facts, monitoring progress and including motivational strategies (Gersten et al., 2009). Explicit instruction along with peer-assisted instruction, concrete-representational-abstract, CAI and games have also been pointed out in review studies of effective instructions for improving early numeracy skills for children at risk for mathematical learning difficulties (Mononen et al., 2014).

Children at risk for mathematical learning difficulties are likely to benefit from additional support, but interventions that focus on particular early numeracy subskills and individual differences seem to be the most effective (Dowker & Sigley, 2010). Interventions are systematic educational programs designed to give long-term improvements on specific skills (Clements & Sarama, 2011). An effective intervention includes several sessions over a specific time period (e.g., six weeks to six months) (Gersten et al., 2008). Promising results have been reported from computer-assisted interventions to improve the early numeracy skills of children at risk for mathematical learning difficulties (e.g., Benavides-Varela et al., 2020; Mononen et al., 2014). Additionally, computer technology and digital tools are a part of our everyday life and classrooms, therefore the questions of how to use these in the best way are important to consider both for researchers and educators (Cheung & Slavin, 2013). Consequently, it is important to carefully and critically choose which software to use in a computer assisted intervention (Gersten et al., 2008). One commonly used piece of software is the Number Race game, an adaptive computer game that is evidence-based and freely available in many languages, including Swedish.

1.3.1 Computer Assisted Intervention for Supporting Children at Risk for Mathematical Learning Difficulties

Computer technology and digital tools have been suggested to complement average classroom instruction by providing intensive, individualised training for children in need of extra support (Cheung & Slavin, 2013; Praet & Desoete, 2014; Räsänen et al., 2009). One way to utilise computer technology is through computer games, as they generally motivate children and can provide attractive possibilities for training in an entertainment context (Kroesbergen & Van Luit, 2003; Wilson, Dehaene et al., 2006). Adaptive computer programs have an additional advantage, as they maintain the difficulty of an educational task, providing the child with exactly the required difficulty level (Räsänen, 2015; Wilson et al., 2006a). However, even though many computer games, programs and applications are available, only few of them are evidence-based (Young, et al., 2012; Knops, 2019).

Computer-assisted intervention (CAI) uses a computer to present instructional material and monitor learning. The advantages of CAI include one-to-one interaction, high motivation, instantaneous response, the possibility to proceed at the child's own pace and level, individual attention, and multimodal presentation of concepts (Fengfeng, 2008). Findings from recent meta-analysis of CAI and digital-based interventions are contradictory. Some studies have concluded that CAI can be used successfully to improve children's numeracy skills (Benavides-Varela et al., 2020; Chodura et al., 2015; Li & Ma, 2010). Other studies have reported no additional effects compared to other interventions (Seo & Bryant, 2009) or even that it is less effective than teacher instruction (Dennis et al., 2016; Kroesbergen & Van Luit, 2003). When comparing specifically digital games Benavides-Varela and colleagues' (2020) findings did not suggest games to be more effective than other digital programs with drill-practices. However, the CAIs have been very different concerning target groups (age and performance level), software (digital applications or games) and research design (setting, control groups, sample size, intensity, duration, and measurements). This highlights the importance of careful selection of the software to be used in computer-assisted interventions for children at risk for mathematical learning difficulties (Gersten et al., 2008).

CAI in mathematics in the early grades has mainly focused on enhancing low-performing children's basic numerical skills. To the best of our knowledge, the existing mathematics programs and games that have been used in research-based CAIs in kindergarten and the early grades include Calcularis (Käser et al., 2013), GraphoGame Math (Salminen et al., 2015), Lola's World (Aunio & Mononen, 2018), and The Number Race (Wilson, Revkin et al., 2006; Wilson et al., 2009; Räsänen et al., 2009; Salminen et al., 2015). The Number Race is research based and widely used in both research and educational practice. However, only Räsänen and colleagues (2009) and Salminen and colleagues (2015) conducted their studies in authentic preschool settings.

1.3.2 The Number Race Game

The Number Race (NR) computer game was originally developed by a French research group for remediation of dyscalculia in children ages 5–8 (Wilson, Dehaene et al., 2006). The NR is adaptive and can vary in three ways: the distance between the numerical representations, the speed and response deadline, and the conceptual complexity (Wilson, Dehaene et al., 2006). At the initial level, the game focuses on non-symbolic number skills, such as recognizing numerical quantities and comparing numbers by choosing the larger of two quantities (Wilson, Dehaene et al., 2006). At subsequent levels, symbolic number skills are more prominent, as the game improves the children's fluency in arithmetic and mapping numbers to quantities by adding or subtracting in order to make comparisons. The additions and subtractions are conceptually oriented, concrete operations instead of drills of arithmetical facts (Wilson, Dehaene et al., 2006).

There exist a few studies on the effectiveness of the NR (version 2.0) that have reported promising results from interventions with the NR game to improve mathematical skills in children with mathematical learning disabilities (Wilson, Revkin et al., 2006; Wilson et al., 2009; Räsänen et al., 2009). The first study of the effectiveness of NR was an open trial study with a small group of children (only nine children ages 7-9 years) and without a control group (Wilson, Dehaene et al., 2006). In most of the previous studies, the participating children have been kindergarteners (Räsänen et al., 2009; Salminen et al., 2015; Sella et al., 2016; Wilson et al., 2009), from a specific target group (low socio-economic status: Wilson et al., 2009; with MLD: Salminen et al., 2015; Wilson, Revkin et al., 2006), or the interventions were conducted in a highly controlled learning environment (Obersteiner et al., 2013). One previous study (Brankaer et al., 2014) did not find any effects of the NR intervention, compared to a passive control group. Also, Räsänen and colleagues (2009) found only small and relatively short-lasting effects from the NR intervention. The results from previously published NR studies have been critically discussed by Szűcs and Myers (2017), indicating that some previous studies have an inadequate design, either in that they lack a control group or that they contrast NR training with non-mathematical training (reading training or drawing activity).

1.4 The Present Study

The importance of early numeracy skills for later mathematical learning has been pointed out in previous research literature. As there are already large individual differences in children's early numeracy performance in kindergarten and early school years (Aunio & Niemivirta, 2010; Jordan et al., 2009) and the gap between children tends to persist or even grow wider across grades (Geary, 2011), it is highly important to identify and support children for mathematical learning difficulties at an early stage. Early support is proposed to prevent or at least reduce later mathematical learning difficulties (Fuchs et al., 2007; Zhang et al., 2020). In order to know who are in need of support in their early numeracy development, educators need to first identify the children at risk for mathematical learning difficulties.

The assessment process requires valid and reliable assessment tools that not only differentiate the children but also give information on children's performance, skills, strengths and needs (Aunio, 2019). This requires that the tests measure early numeracy skills broadly with a multidimensional structure. Many of the existing early numeracy tests measure early numeracy broadly, without providing specific information regarding in what subskills children need support, or focus only on a particular early numeracy subskill, leaving out information about other subskills. Neither of these approaches can provide educators with adequate information on children's strengths and needs that are required in the assessment process. Furthermore, the assessment for identifying children at risk for mathematical learning difficulties should be a continuous process, including several assessment points, which additionally is an important aspect when developing and validating evidence-based assessment tools to be used in the educational practice. Also, in supporting children, there are demands of targeted and research-based interventions.

When identifying and supporting children at risk for mathematical learning difficulties, it is important to focus on the early numeracy skills that form the foundation of later learning in mathematics and to know how these skills are interrelated and how they develop (Desoete et al., 2009). Both person-centred and variable-centred cross-sectional and longitudinal studies have provided knowledge of the developmental trajectories and interrelations of early numeracy skills, but longitudinal research on the relations between early numeracy subskills is still lacking and the person-centred studies have mainly focused on a specific early numeracy subskill (e.g., non-symbolic magnitude processing) instead of several subskills. Early numeracy skills have been emphasised and operationalised in the existing theoretical frameworks of early numeracy skills development. However, many of these frameworks have been extensive for educators to use in their educational practice and highlight the importance of frameworks that not only operationalise the early numeracy skills, but also can be used as a working model in the educational practice.

1.4.1 Aims and Research Questions

This thesis aims to contribute to the research on early numeracy development by focusing on identifying and supporting children at risk for mathematical learning difficulties in kindergarten and early school years with a focus on improving evidence-based educational practice. This thesis highlights core early numeracy skills, based on the core factor model (Aunio & Räsänen, 2016) constructed to be used as a working model for educators in the process of identifying and supporting children at risk for mathematical learning difficulties. Based on the model early numeracy was operationalised through four early numeracy subskills: symbolic and non-symbolic number knowledge, mathematical relational skills, counting skills and basic skills in arithmetic.

Longitudinal data from three age groups (kindergarten, first grade and second grade) were collected in order to answer the research questions. To reach a deeper understanding of the development and interrelation of the core early numeracy skills both a person-centred and a variable-centred approach were used. The reliability and validity evidence of the Early Numeracy test (EN-test; Koponen et al., 2011a, 2011b, and 2011c) was investigated in order to confirm that the test could be used as a research-based assessment tool both for educators and for researchers to identify children at risk for mathematical learning difficulties. The EN-test was used in all three substudies to measure children's early numeracy performance in the three age groups. Thereto, an intervention study was conducted to examine the effectiveness of a CAI intervention for supporting children that are identified as at risk for mathematical learning difficulties in first grade. In the CAI, a research-based computer game was used.

The following three research questions were addressed:

1. How do the early numeracy core skills develop during one school year in first and second grade? (Study I and II)
2. How reliable and valid is the Early Numeracy test for identifying children at risk for mathematical learning difficulties in kindergarten, first and second grade? (Study I and II)
3. How can children at risk for mathematical learning difficulties be supported in a computer-assisted intervention in first grade? (Study III)

2. Method

2.1 Context

In Finland compulsory formal education consists of nine years of comprehensive school, starting in August the year the child turns seven. To prepare children for formal schooling, they enter a one-year kindergarten education. The compulsory formal education and the one-year kindergarten education follow the national curriculum framework. The national curriculum frameworks for kindergarten education emphasise learning in formal and informal situations. In kindergarten education, the aims for learning are described more on an overall level than in first and second grades. In first and second grades children have specific subjects studied during separate lessons with aims for learning specified in the national curriculum guidelines.

In kindergarten mathematics, the curriculum guidelines from 2010 (Finnish National Board of Education, 2010) focus on giving children a base for their mathematical learning in varying everyday situations. The only specific mathematical content that is mentioned concerns concepts as classification and seriation of objects based on their shape, quantity, and other characteristics. The newer curriculum framework in mathematics from 2014 (Finnish National Board of Education, 2014a) also emphasises problem solving, combining numerals with quantity and number word, and aspects of early geometry.

In first and second grade mathematics, the curriculum framework from 2004 (Finnish National Board of Education, 2004b) focuses on learning, understanding, and performing operations with numbers, number symbols, and number words, mainly in the number range of 0–20. By the end of the first school year, numbers up to 100 are also introduced. The key areas are basic addition and subtraction, algebra, statistics, measurements, and geometry. The newer curriculum framework in mathematics from 2014 (Finnish National Board of Education, 2014b) has the same core content, with some parts described in more detail than in the previous curriculum framework. Additionally, programming and digital tools are emphasised in the newer curriculum.

In both kindergarten and comprehensive school, the educational support is organised based on the three-tiered support model: general support (Tier 1), intensified support (Tier 2), and special support (Tier 3) (Finnish National Board of Education, 2014b). The three-tiered educational support model in the Finnish education system is influenced by the Response to Intervention model (RtI) and has both similarities and dissimilarities with the RtI model. Assessment for identifying children at risk for mathematical learning difficulties is important at Tier 1 in the RtI, as the first step to support children is to identify all children at risk (Björn et al., 2018; Gersten et al., 2009). Early identification and intervention are emphasised in the Finnish three-tiered educational support model as well, but systematic assessment for identifying children at risk for mathematical learning difficulties is not included in the general support level as in the RtI model (Björn et al., 2018). This is true for the educational support in the Finnish three-tiered educational support model as well; there are no demands and guidelines for the intensity and content of the interventions (Björn et al., 2018). The Finnish three-tier support model focuses on early identification of learning difficulties, early intervention, educational differentiation, and collaboration between professionals in education and health care. Children are provided with extra support in successive tiers of increasing intensity.

The participants in this study come from Swedish-speaking kindergartens and schools in the Swedish-speaking parts of Finland. The Swedish-speaking population is a minority (5.2%, according to Official Statistics of Finland, 2019) in Finland, and they live mainly on the west coast and in southern parts of Finland. Finland has two official languages, Finnish and Swedish. There are equal opportunities concerning education for both Finnish and Swedish speaking populations. In national and international assessments there have not been any significant differences between Finnish and Swedish speaking children at the beginning of comprehensive school (national student assessment: Ukkola & Metsämuuronen, 2019), nor at the end of the compulsory school (international student assessment, i.e., PISA; Leino et al., 2019).

2.2 Participants and Procedure

In the three original studies included in this thesis, the participants were drawn from the same longitudinal study with a sample of 361 kindergarteners (Study I), 321 first graders (Studies I, II and III) and 457 second graders (Studies I and II). The data were collected during the academic year 2011-2012 from different-sized kindergartens and schools in urban and rural areas of Swedish-speaking Finland. All children attended neighbourhood schools, and no special education classes were included in the study.

In the initial stage of the longitudinal study, teachers and principals were contacted and informed about the purpose and procedure of the longitudinal study. A total of 21 kindergartens and 23 schools were interested in participating, and permission was obtained from the principals and teachers. Thereafter, parents received an information letter with the descriptions of our study purpose, procedure, and contact information. Permission for children to participate in this study was obtained in writing from their parents. Teachers and children participated voluntarily in the data collection, with the possibility to decline or discontinue participation at any point. No personal information about the children was collected other than children's first name, birth date and name of kindergarten or school. Teachers were able to give additional information (e.g., home language, level of support) in connection with the assessment and intervention. However, this information was provided only for some of the children. The reason for the limited background information was that this was not originally designed as a longitudinal research study; instead the focus in the initial stage was to develop an assessment tool for educational practice and background variables were only collected in order to organise the data. The children's names as well as kindergarten and school names were replaced with number codes in the data coding stage to secure children's anonymity. A separate permission for the intervention study was collected from the participating children's parents before the intervention period started.

The children's early numeracy skills were assessed three times in first and second grade during one academic school year. The first assessment took place at the beginning of the school year (September), the second in the middle of the school year (January-February) and the third at the end of the school year (May). In kindergarten children's early numeracy skills were only assessed at the beginning of the kindergarten year (September). The intervention in first grade took place in April and May, right before the third assessment.

2.3 Constructing the Early Numeracy test

The validation of the Swedish language version of the Early Numeracy test (EN-test) was a part of this thesis. The EN-test was constructed within the LukiMat project, a project funded by the Finnish Ministry of Education and Culture and a collaborative project with researchers at Åbo Akademi University, University of Jyväskylä and Niilo Mäki Institute. The overall aim of the LukiMat project was to provide educators, other school personnel and parents a public web-based information service (lukimat.fi) with research-based information on children's reading and mathematical learning and difficulties in mastering those skills in early school years. One part of the project was to develop an assessment tool that teachers could use for identifying children at risk for mathematical learning difficulties – the EN-test. A multi-professional and multilingual team constructed the EN-test both in Finnish and in Swedish during the academic year 2010-2011 and piloted the test this same year. My role in the LukiMat-project was to construct the Swedish information service concerning mathematical learning and the Swedish language version of the EN-test together with the two other Swedish speaking team members (Johan Korhonen and Karin

Linnanmäki). The Swedish language version of the EN-test was constructed based on and partly parallel with the Finnish version. I was responsible for the planning and implementation of the data collection with the Swedish EN-test. The validation of the Swedish EN-test was conducted outside the LukiMat-project. The main data for this thesis were collected during the academic year 2011-2012.

The aim with the EN-test is to identify children at risk for mathematical learning difficulties in kindergarten and first and second grades (Koponen et al., 2011a, 2011b, 2011c). The test is based on the theoretical model of core numerical skills for learning mathematics in children aged 5 to 8 years and focuses on four skill groups: symbolic and non-symbolic number knowledge, understanding mathematical relations, counting skills, and basic skills in arithmetic (Aunio & Räsänen, 2016). In the original model the third subskill was named symbolic and non-symbolic number sense, whereas in the EN-test it was named symbolic and non-symbolic number knowledge.

The EN-test is designed to be an easily administered paper-pen based group-test that teachers can use as a part of their regular schoolwork. The teachers follow a manual with detailed written instructions and give verbal instructions based on the instructions. The children answer the task on their individual papers. In kindergarten and in the beginning of first grade, eight items measuring verbal counting skills are administered individually. Within each age level, three versions are available: one extended version at the beginning (T1), one follow-up in the middle (T2) and one follow-up at the end (T3) of the school year. In the extended version at the beginning of the school year all four early numeracy subskills are included, but in the follow-up versions some subskills are measured more narrowly than others and some subskills are left outside. This construct of the EN-test was based on the idea that the test should be easily administered and not too time-consuming or extensive to be used in classroom practice. The total number of items, the number of items measuring each subskill, and the number range within tasks differ between the age groups. Example tasks are presented in the Appendix (Table A1). The test battery and test handbooks are freely available in Swedish and Finnish from the LukiMat web service.

Kindergarten. In kindergarten, symbolic and non-symbolic number knowledge are measured at the beginning and in the middle of the school year and understanding mathematical relations at the beginning and the end of the school year. Only counting skills are measured at all three time points. Symbolic and non-symbolic number knowledge is assessed with tasks where children are asked to choose the largest or smallest number out of three alternatives (T1 and T2). In understanding mathematical relations tasks, children are asked to compare the number of objects and to choose the picture with more, most, fewer or same number of objects (T1), to put a cross on a specific object in a seriation based on the ordinal number

(T1 and T3) and to place an object on the right place in a seriation based on the size of the object (T1). In the counting skills tasks, children are asked to combine number words and number symbols, number words and magnitudes, and number symbols and magnitudes (T1). Counting skills are also assessed with individually administered tasks where children are asked to count as far as they can, starting from 1 or a given number and counting backwards (T1, T2 and T3). For basic skills in arithmetic, children are assessed with non-symbolic story problems where they are asked to determine how many objects (e.g., coins or apples) they have when objects are added or taken away. The internal consistency for the EN-test in kindergarten is presented in Table 2. However, in this thesis only the EN-version for the beginning of the kindergarten year was used for measuring children's early numeracy skills at the beginning of the kindergarten year (Study II).

First grade. In first grade, symbolic and non-symbolic number knowledge, understanding mathematical relations and basic skills in arithmetic are measured at all three time points. Counting skills are measured only at the beginning of the school year, including eight tasks measuring verbal counting skills that are administered individually. In the symbolic and non-symbolic number knowledge tasks, children are asked to compare the number of coins in two baskets with two boxes with coins in each basket and to determine in which basket they have more coins (T1), to determine in which number set the numbers are arranged from smallest to biggest number (T1), to compare two numbers and determine how much bigger or smaller one number is than the other (T2) and to compare two numbers and choose the biggest or smallest number (T3). In understanding mathematical relations tasks, children are asked to compare the number of objects and to choose the picture with more and fewer objects (T1), to choose the picture with the correct number of objects based on one-to-one correspondence (T1), and to determine the next number in an increasing and decreasing number sequence (T1, T2 and T3). In counting skills tasks, children are asked to count numerosities of sets when they know the total number of objects, but some objects are hidden, or determine how many is one or two more or one or two less (T1). Counting skills are also assessed with individually administered tasks where children are asked to count as far as they can, starting from 1 or a given number and counting backwards (T1). For basic skills in arithmetic, children are assessed with non-symbolic story problems where they are asked to determine how many objects (e.g., coins or apples) they have when objects are added or taken away (T1) and symbolic addition and subtraction calculation tasks (T2 and T3). The internal consistency for the EN-test in first grade is presented in Table 2.

Second grade. In second grade, basic skills in arithmetic and symbolic and non-symbolic number knowledge are measured at all three time points, while understanding mathematical relations and counting skills are only measured at the beginning of the school year. In the symbolic and non-symbolic number knowledge tasks, children are asked to compare four numbers and choose the biggest or smallest number (T1, T2 and T3) and to determine in which number set the numbers are arranged from smallest to biggest (T1). In understanding mathematical relations tasks, children are asked to determine the next number in an increasing and decreasing number sequence (T1). Counting skills are assessed with tasks where children are asked to write with number symbols the number the teacher is saying (T1). For basic skills in arithmetic, children are assessed with speedy symbolic addition and subtraction calculation tasks (T1, T2 and T3). The internal consistency for the EN-test in second grade is presented in Table 2.

Table 2

Number of Items and Reliability Coefficients in the Early Numeracy Test.

	Kindergarten			First grade			Second grade		
	T1	T2	T3	T1	T2	T3	T1	T2	T3
Symbolic and non-symbolic number knowledge (NK)	8 (.86)	6 (.85)	-	8 (.70)	6 (.82)	6 (.80)	8 (.75)	4 (.66)	9 (.68)
Understanding mathematical relations (MR)	16 (.76)	-	4 (.53)	16 (.71)	6 (.81)	6 (.89)	8 (.93)	-	-
Counting skills (CS)	16 (.82)	19 (.81)	15 (.81)	16 (.83)	-	-	8 (.81)	-	-
Basic skills in arithmetic (BA)	8 (.63)	-	8 (.72)	16 (.78)	24 (.90)	20 (.94)	40 (.94)	56 (.95)	56 (.94)
Total	48 (.90)	25 (.84)	27 (.87)	56 (.90)	36 (.88)	32 (.95)	64 (.95)	60 (.95)	65 (.94)

Note. α = Cronbach's alpha.

2.4 The Intervention Program with the Number Race Game

In this thesis the updated Swedish version of the Number Race (NR) computer game (*Tal i farten*, version 3.0) was used as an intervention method to investigate the effects for the low-performing children in first grade (Study III). The aim with NR is to remediate dyscalculia by practicing various number presentations and the transformations between symbolic and non-symbolic number representations, with a special focus on the representation of quantities and approximate numerical comparison (Wilson, Dehaene et al., 2006). NR is primarily aimed at children aged five to eight years. The game is adaptive, and the difficulty of the tasks is modulated by varying the numerical distance between sets, the time limit for responding, and the format of the displayed quantities (from simple non-symbolic comparison to more complex symbolic calculation). The difficulty is constantly adjusted by an adaptive algorithm so that the child's performance stays at a 75% accuracy level.

The NR starts with a view where the child chooses between two visual contexts: jungle or underwater world. In both contexts the game idea is the same; only the visual layout differs. The child is competing against the software in numerical comparison tasks where the task is to compare quantities (i.e., dots), numbers (4 vs. 2) or number combinations (addition and subtraction calculations, e.g., 3 + 5 vs. 2 + 1). To win, the child has to choose the larger quantity, larger number or larger sum or difference. Immediate response is given by applause and after the comparison both the child's and the opponent's (i.e., the software) player characters move on the number line as many steps as points are given. After this, a new numerical comparison task is given. The numerical representations (dots, numbers, or number combinations) are adapted based on the child's individual level.

In the new version of the NR (version 3.0), basic arithmetic is denoted more visibly on the number line, and the number line is now continuous instead of divided into segments, as in the earlier version (version 2.0). Moving on the number line, the focus is on counting on instead of starting from the beginning. The total view is made clearer, and everything is on the same page: the number line, the two number boxes to choose from, and the comparison of magnitudes. The updated version is in better agreement with recent studies on the format of the number line (Siegler & Ramani, 2009) and focuses more on strengthening the connection between counting and basic addition than does the previous version of NR. The NR software is open source (GNU Public License) and can be freely downloaded from http://www.thenumberrace.com.

2.5 Data Analysis

In all three substudies a quantitative approach was used to analyse the data in order to answer the research questions. In the analysis we utilised the MPLUS programme (version 8; Muthén & Muthén, 1998–2015) to conduct the confirmatory factor analyses, latent profile analyses and multivariate regression analyses. Analysis of variance, multivariate analysis of variance, chi-square tests, and reliability analyses were conducted using SPSS statistical software.

2.5.1 Analysis of Variance and Multivariate Analyses of Variance

Analysis of variance (ANOVA) and multivariate analysis of variance (MANOVA) are used to examine the statistical differences among means. In ANOVA only one dependent variable is allowed, whereas MANOVA is an extension of ANOVA and allows comparing group means on several variables (Tabachnick & Fidell, 2007). In Study I, ANOVAs and MANOVAs with post hoc comparisons (Bonferroni correction and Games-Howell post hoc-test) were used to examine the differences in early numeracy performance between the three early numeracy profiles both in first grade and in second grade. With one-way ANOVA we investigated the group differences in overall early numeracy performance and age at three time points. To look more closely at group differences in the early numeracy subskills at the three time points we conducted MANOVAs for each time point. Unstandardised sum scores from the EN-test at three time points in two grades were used. In Study III, ANOVAs and MANOVAs with post hoc comparisons (Bonferroni correction and Tamhane's post hoc test) were used to examine the differences in early numeracy performance between the NR-intervention group, the low-comparison group and the average-performing group at the three time points in first grade. ANOVAs were used to examine the group differences in the overall early numeracy performance and age at the three time points. MANOVAs were used to examine more specifically the group differences in the early numeracy subskills. In Study III, mixed factorial ANOVA were used to examine the effectiveness of the NR-intervention. Partial eta square (η_p^2) was used as a measure of the effect size.

2.5.2 Confirmatory Factor Analysis and Multigroup Confirmatory Factor Analysis

Confirmatory factor analysis (CFA) is a type of structural equation modeling (SEM) that is used to investigate the causal relations among latent and observed variables (e.g., test items and test scores) in a theory-derived prespecified model (Byrne, 2012; Schumacker & Lomax, 2004). In a CFA the researcher has to prespecify the model, the number of factors and item loadings, in contrast to exploratory factor analysis, where the researcher seeks to discover the underlying construct of the items and factors. The strength of CFA is the strong base in a theoretical model and it is therefore commonly used to examine the latent structure of a test (Brown, 2006) and for examining the structural validity of a test (Terwee, et al., 2018). In Study II, CFAs were used to examine whether the factor structure of the EN-test supports the factor structure in the early numeracy core factor model of Aunio and Räsänen (2016) in order to seek evidence for the structural validity of the EN-test. Structural validity refers to whether the items represent the theoretical constructs that they are designed for (Gregory, 2015). Separate CFAs were conducted for kindergarten, first grade and second grade. The model fit of three different models, a four-factor model, a three-factor model, and one-factor model were compared. Due to categorical data, Weighted Least Squares Means and Variance Estimation

(WLSMV) were used (Brown, 2006). The goodness of the model fit is evaluated using the chi-square test, the Comparative Fit Index (CFI), the Tucker-Lewis Index (TLI), and the Root Mean Square Error of Approximation (RMSEA). A non-significant result on the chi-square test and values greater than 0.95 for CFI and TLI, and less than 0.05 for RMSEA indicate an excellent model fit (Marsh, Hau, & Wen, 2004). When comparing nested models, an improvement of more than .01 in CFI and .015 in RMSEA indicates that the more complex model (i.e., the model with more factors) fits the data better than the more parsimonious model (Chen, 2007).

In Multigroup CFA several groups can be tested simultaneously, testing if the same underlying structure is supported across different groups of individuals (Brown, 2006). In Study II, multigroup CFAs were used to examine the known-group validity and the cross-cultural validity. Known-group validity was tested by investigating measurement invariance across gender (girls vs. boys) and age (median split: younger vs. older), whereas cross-cultural validity was tested by investigating measurement invariance across language groups (Finnish vs. Swedish speaking children).

2.5.3 Cronbach’s Alpha

Cronbach’s alpha is a common measure for calculating the internal consistency reliability of a test (Gregory, 2015; Terwee et al., 2017). Internal consistency tells how items are related to each other and how well they measure the same construct. In Study II, Cronbach’s alpha was used to measure the reliability of the EN-test in kindergarten, grade 1 and grade 2. Additionally, in Study I and Study III, Cronbach’s alpha was used to calculate the internal consistency of the EN-test for the specific samples. The recommended acceptable value of Cronbach's alpha coefficient is 0.70 (Peterson, 1994; Nunnally, 1978).

2.5.4 Independent Samples Chi-square Tests

Independent sample chi-square tests (χ^2 tests) are used to examine whether there is an association between two categorical variables (Tabachnick & Fidell, 2007). In Study I, chi-square tests were used to examine the association between the early numeracy profiles and gender, as well as the association between the early numeracy profiles and language in first grade and in second grade. In Study III, chi-square tests were used to examine the association between the intervention group or the two comparison groups and gender, as well as the association between the groups and language. These associations were examined in order to determine if either gender (boys or girls) or language (Swedish or other home language) category was overrepresented in one of the early numeracy profiles (Study I) or intervention or comparison groups (Study III).

2.5.5 Latent Profile Analysis

Latent Profile Analysis (LPA) is a person-centred method for identifying homogenous subgroups within the sample based on certain sets of variables. LPA aims to identify the smallest number of latent classes that best describes the association between the variables. In Study I, LPAs were used to identify clusters of individuals with similar patterns of early numeracy performance. In the analyses one class at time is added stepwise until the model fits the data. The best model was chosen based on the statistical criteria, Bayesian information criterion (BIC), Vuong-Lo-Mendell-Rubin (VLMR) likelihood ratio test and entropy value. A decrease in BIC when an additional class is added indicates a better model fit. A resulting p-value of less than 0.05 for VLMR suggests that the estimated model is preferable over the reduced model (Lo et al, 2001). Entropy with values approaching 1 indicates clear delineation of classes (Celeux & Soromenho, 1996). Furthermore, the class size, usefulness and interpretability of the latent classes in the models were also considered as criteria for the best fitting model.

2.5.6 Multivariate Regression Analysis

Multivariate regression analysis is a type of regression analysis. Regression analysis aims to examine whether there is a statistical relation between the independent and the dependent variable and whether the independent variable can predict the dependent variable. Multivariate regression analysis allows examination of the relations between multiple independent variables and dependent variables (Tabachnick & Fidell, 2007). In Study I, a series of multivariate regression analysis models was fitted to the data to explore the relations between the early numeracy factors (NK, MR, CS, and BA) in both first grade and second grade over three time points. Maximum Likelihood with robust standard error estimation was used in the analysis, as it is more robust to non-normality and non-independence of the observations. The goodness of the model fit was evaluated using the chi-square test ($\chi 2$), the Comparative Fit Index (CFI), the Tucker-Lewis Index (TLI), the Standardised Root Mean Square Residual (SRMR) and the Root Mean Square Error of Approximation (RMSEA). For CFI, values of .90 and above were considered to represent a good model fit, while values over .95 represented an excellent fit. TLI > .90 indicates an acceptable fit (Bentler & Bonett, 1980). For SRMR, a value less than .08 is generally considered a good fit (Hu & Bentler, 1999). For RMSEA, values below .08 are considered to represent acceptable fit and values below .05 an excellent fit (Marsh et al., 2004).

3. Overview of the Original Studies

In this chapter an overview of the three original articles is presented. A summary of the aims, research questions, participants, measures, analyses, and results in the original studies are presented in Table 3.

Table 3

Summary of Aims, Research Questions, Participants, Measures, Statistical Methods, and Results of the Original Studies.

Study and title	Keywords	Aim	Research questions	Participants	Measures	Main statistical methods	Main results
Study I: Development of early numeracy skills during first and second grade	At risk for mathematical learning difficulties, early numeracy, numerical development, person-centred analysis, variable-centred analysis	A variable-centred and a person-centred approach to investigate the development of early numeracy skills within one year in first and second grade	1. How are early numeracy skills interrelated over time? 2. What kind of early numeracy profiles can be found among children in the beginning of the school year? 3. How are these early numeracy profiles related to early numeracy skills in the middle and in the end of the school year?	321 first graders and 457 second graders	EN-test	Multivariate regression analysis, Latent profile analysis, one-way ANOVA, MANOVA, Independent sample χ^2-tests	CS in the beginning of first grade was an important predictor of later early numeracy skills while BA emerged as a key predictor of later early numeracy skills in both grades. Three substantially different early numeracy profiles were identified: well-performing, average-performing and at risk for mathematical learning difficulties.
Study II: Reliability and validity evidence of early numeracy test for identifying children at risk for mathematical learning difficulties	Assessment, early numeracy, mathematical learning difficulties, screener, validation	Investigate the reliability (internal consistency) and validity (structural validity, known group validity, and cross-cultural validity) evidence of the EN-test	1. Validity: structural validity, known group validity, and cross-cultural validity 2. Reliability: internal consistency	361 kinder-garteners, 321 first graders, and 457 second graders	EN-test	Confirmatory factor analysis (CFA), multigroup CFA, reliability analysis (Cronbach's alpha coefficient)	The EN-test was found to have adequate structural validity, known-group validity, cross-cultural validity, and internal consistency. The empirical data support the aim of measuring four early numeracy skills described in the core factor model (Aunio & Räsänen, 2015).
Study III: The Number Race – computer-assisted intervention for mathematically low-performing first graders	Computer-assisted interventions (CAI), low performance, mathematical learning, mathematical learning difficulties, mathematics game	Investigate the effectiveness of the Number Race (NR), computer game in enhancing low-performing first grade children's mathematical performance	1. How does the NR intervention group develop in its overall mathematical performance, compared to the low- and average-performing comparison groups? 2. How does the intervention group develop in the different mathematical subskills (NK, MR, CS, and BA), compared to the low- and average-performing comparison groups?	334 first graders: NR-intervention group *n* = 29, low-comparison group *n* = 27, average-comparison group *n* = 278	EN-test	ANOVA (one way; factorial), MANOVA	No statistically significant intervention effect on children's mathematical performance. At none of the time points did the NR-intervention group differ significantly from the low-comparison group, which only received ordinary classroom instruction in mathematics during the four-week intervention period. The average-performing children outperformed the low-performing children at all time points.

Note. NK = symbolic and non-symbolic number knowledge; MR = understanding mathematical relations; CS = counting skills; BA = basic skills in arithmetic. EN-test = Early Numeracy test.

3.1 Study I

3.1.1 Aims

The aim of this study was to examine the development of children's early numeracy skills during one school year in first grade and in second grade. The study used both a variable- and person-centred approach to more closely investigate the interrelation between four early numeracy subskills (symbolic and non-symbolic number knowledge, understanding mathematical relations, counting skills and basic skills in arithmetic) and what kind of early numeracy profiles can be found among children in the beginning of the school year and how the early numeracy skills develop within these profiles over a school year.

3.1.2 Participants, procedure, and measures

The participants in this study were 321 first graders (162 girls) and 457 second graders (240 girls) from 23 schools in Swedish-speaking areas of Finland. The children ranged in age in first grade from 80 months to 111 months (M_{age} = 86.87; SD = 3.94) and in second grade from 91 months to 115 months (M_{age} = 98.38; SD = 3.65). Permission for children to participate in this study was obtained in writing from their parents. Children's early numeracy skills were assessed with the Early Numeracy test at three time points within the school year: in the beginning, in the middle and in the end. The assessment was administered by the classroom or special education teacher and was completed in an ordinary classroom setting during one or two lessons.

3.1.3 Analysis

The variable-centred approach started with exploring the relations between the early numeracy factors (NK, MR, CS and BA) over time in both first and second grades. A series of multivariate regression analysis models were fitted to the data. First, a baseline model consisting of autoregressive paths by the theoretical framework of the four early numeracy factors were set. Second, we tested a model consisting of both autoregressive paths and cross-lagged paths between the early numeracy factors and compared it to the baseline model. Third, we tested a model consisting of only the significant paths from the second model and compared it to the baseline model. Maximum Likelihood with robust standard error estimation was used in the analysis. The goodness of the model fit was evaluated using the chi-square test ($\chi 2$), the Comparative Fit Index (CFI), the Tucker-Lewis Index (TLI), the Standardised Root Mean Square Residual (SRMR) and the Root Mean Square Error of Approximation (RMSEA).

In the person-centred approach early numeracy profiles were identified through latent profile analyses (LPA; Muthén & Muthén, 1998–2015). Standardised factor scores for each early numeracy factor (NK, MR, CS and BA) at the first time point (T1) were created based on confirmatory factor analysis and used in the LPAs. The differences in early numeracy performance between the early numeracy profiles at the three time points were examined through analysis of variance (ANOVA) and multivariate analyses of variances (MANOVA). The partial eta squared (η_p^2) was used to measure the effect size. Chi-square tests and ANOVA were performed to examine the background variables: age, gender, and native language.

3.1.4 Results

A series of multivariate regression analysis was conducted: (1) a model consisting of autoregressive paths, where each early numeracy factor (NK, MR, CS and BA) was predicted by the same early numeracy factor at the previous time point (e.g., NK3 on NK2 and NK2 on NK1), (2) a model with cross-lagged paths, where each early numeracy factor was predicted by all other early numeracy subskills at the previous time point (e.g., NK3 on BA2 and MR2), (3) model that consisted of only the significant paths from the second model. The third model was chosen as the model fit indices supported the model fit altogether. In first grade, counting skills in the beginning of the school year was an important predictor of later early numeracy skills while basic arithmetic skills emerged as a key predictor of later early numeracy skills in both grades.

Latent profile analysis identified three early numeracy profiles that differed from the early numeracy skills in the beginning of the school year: well-performing (first grade: 33.3%; second grade: 39.6%), average-performing (first grade: 43.6%; second grade: 45.5%) and at risk for mathematical learning difficulties (first grade: 23.1%; second grade: 14.9%). In both grades, all three profiles substantially differed from each other at all three time points, except in the end of the school year in the first grade, wherein the differences between the well-performing children and the average-performing children were non-significant. Age, gender, and language were not related to group-belonging in the first grade. In the second grade, older children and boys were overrepresented in the well-performing profile, while girls were overrepresented in the average-performing profile.

3.1.5 Discussion

This study aimed to examine the development of children's early numeracy skills during one school year in first grade and in second grade with both a variable- and person-centred approach. Children's counting skills in the beginning of first grade was an important predictor of later early numeracy skills while basic arithmetic skills emerged as a key predictor of later early numeracy skills in both grades.

In first grade, the significant correlations within time points between the early numeracy measures are in line with previous cross-sectional studies (Purpura & Lonigan, 2013; Jordan et al., 2010; Li et al., 2018). Concerning the longitudinal relations, our study shows that children's counting skills in the beginning of first grade predict subsequent early numeracy skills above the effects of the autoregressive paths. These results highlight the importance of counting skills for later mathematical learning (Desoete et al., 2009; Geary et al., 2018; Krajewski & Schneider, 2009). However, already in the middle of first grade, basic arithmetic skills were the single most important subskill for later mathematical learning. This trend was evident among second graders as well, in the beginning of the school year. This is consistent with previous longitudinal studies on the effect of basic arithmetic skills on symbolic number knowledge (Vanbinst et al., 2019) and mathematical learning in general (Aunio & Niemivirta, 2010; LeFevre et al., 2006; Locuniak & Jordan, 2008).

In both first and second grade, latent profile analysis identified three early numeracy profiles that differed from the early numeracy skills in the beginning of the school year: well-performing, average-performing and at risk for mathematical learning difficulties. The three groups differed from each other at all three time points, except from the end of school year in first grade, wherein the differences between the well-performing children and the average-performing children were non-significant. These results indicate that the low-performing children have deficits in all early numeracy subskills and were not able to close the gap to their average- or well-performing peers. Our finding is supported by previous studies suggesting that low-performing children seem to benefit less from ordinary education than the average performing children (Aunola et al., 2004; Fuchs & Fuchs, 2006) and that the gap between the low-performing children and the average-performing children tends to persist and may even grow wider throughout the school years.

3.2 Study II

3.2.1 Aims

The aim of this study was to investigate the reliability and validity evidence of the Early Numeracy test for identifying children at risk for mathematical learning difficulties in Swedish-speaking kindergartens and schools in Finland. The validity of the test was examined by collecting evidence related to structural validity, known group validity, and cross-cultural validity. The reliability evidence of the test was analysed based on internal consistency.

3.2.2 Participants, procedure, and measures

The participants in this study were 1139 children (587 girls) from 23 kindergartens and 26 primary schools in Swedish-speaking areas of Finland. The sample consisted of 361 children in kindergarten (185 girls), 321 children in first grade (162 girls), and 457 children in second grade (240 girls). The children ranged in age in kindergarten from 61 months to 85 months (M_{age} = 73.97; SD = 3.65), in first grade from 80 months to 111 months (M_{age} = 86.87; SD = 3.94), and in second grade from 91 months to 115 months (M_{age} = 98.38; SD = 3.65). Permission for children to participate in this study was obtained in writing from their parents. Children's early numeracy skills were assessed in the beginning of the school year with the Early Numeracy test. The assessment was administered by the classroom or special education teacher and was completed in an ordinary classroom setting during one or two lessons.

3.2.3 Analysis

To test the structural validity of the EN-test we conducted confirmatory factor analysis (CFA) with the weighted least squares means and variance estimation (WLSMV). The goodness of the model fit was evaluated using the chi-square test, the Comparative Fit Index (CFI), the Tucker-Lewis Index (TLI), and the root mean square error of approximation (RMSEA). Multigroup CFAs were conducted to test both known-group validity and cross-cultural validity. Cronbach's alpha was used to calculate the internal consistency.

3.2.4 Results

Structural validity was established through CFA, were the overall goodness-of -fit indices of the test suggested that the four-factor model fit the data significantly better than a one-factor or two-factor model in all three age groups. In kindergarten the strongest correlation was found between CS and BA, whereas in first grade the strongest correlations were found between CS and NK and between CS and BA. In second grade the strongest correlation was found between CS and NK. Known-group validity and cross-cultural validity were invariant across gender, age, and language groups, indicating that the four-factor model fit the gender, age and language groups equally well. Internal consistency for the tests and sub-skills varied from good to excellent in all three age groups.

3.2.5 Discussion

This study aimed to investigate the reliability and validity evidence of the early numeracy test for identifying children at risk for mathematical learning difficulties in Swedish-speaking kindergartens and schools in Finland. The EN-test was found to have adequate structural validity, known-group validity, cross-cultural validity, and internal consistency. The empirical data support the aim of measuring four early numeracy skills described in the model (Aunio & Räsänen, 2016): symbolic and non-symbolic number knowledge, understanding mathematical relations, counting skills, and basic skills in arithmetic in the three age groups in kindergarten, first grade and second grade. Based on our results, the EN-test can be considered as an appropriate assessment to identify children at risk for mathematical learning difficulties.

The strength of the EN-test is that different early numeracy subskills are described and interpreted as different factors, in order to more specifically describe children´s performance. Many of existing measurements have a unidimensional approach (e.g., MARKO-D test, Ricken et al., 2013; CMA, Starkey et al., 2004) and therefore do not enable a more comprehensive information of children's performance. A multidimensional construct has been found in some tests that have originally been seen as unidimensional, the Finnish ENT-test (Aunio et al., 2006) and TEMA-3 (Ryoo et al., 2015). A multifactorial approach to early numeracy is presented in frameworks that describe the conceptual structure of early numeracy and mathematical development in early school.

3.3 Study III

3.3.1 Aims

The aim of this study was to examine the effect of the new version of the adaptive computer game, the Number Race (NR), to enhance basic mathematical skills for children in first grade who were deemed to be low-performing in mathematics. A quasi-experimental research design was used with a low-performing intervention group, a low-performing comparison group, and an average-performing comparison group.

3.3.2 Participants, procedure, and measures

The participants in this study were 334 first graders (171 girls) from 23 different schools in Swedish-speaking areas of Finland. Permission for children to participate in this study was obtained in writing from their parents. Children's early numeracy skills were assessed three times during the school year with the Early Numeracy test: in the beginning, in the middle and in the end. The assessment was administered by the classroom or special education teacher and was completed in an ordinary classroom setting during one or two lessons.

Based on the first assessment low performing children (cut-off point at the lowest 20th percentile) were identified and the children were assigned to three groups: a low-performing intervention group (n = 29; M_{age}= 86.24; *SD* = 4.79), a low-performing comparison group (n = 27; M_{age} = 86.04; *SD* = 3.91) and an average-performing comparison group (n = 278; M_{age}= 86.53; *SD* = 3.79). The trained research assistants visited the schools to install the NR and to guide teachers towards accomplishing the intervention. The intervention was supervised by the classroom or special education teacher or class assistant. In addition to ordinary teacher instruction in mathematics, the intervention group received NR training for 15-minute sessions, 3–4 days per week, during a four-week period. During the intervention, the teachers were asked to keep a logbook of the process (child's name, date of play, minutes of play, and some voluntary note), for fidelity purposes. The NR intervention took place between the second and third assessments.

3.3.3 Analysis

Separate ANOVAs with post hoc comparisons were used to compare the overall early numeracy performance within and between groups at the three time points (T1, T2, and T3). The effectiveness of the NR intervention between T2 and T3, when the intervention took place, was analysed by conducting a 2 (T2 and T3) x 3 (NR group, low-comparison, and average-comparison) mixed factorial ANOVA. The partial eta squared (η^2_p) was used to measure the effect size. MANOVAs were conducted for each time point to compare the NR group to the two comparison groups in the subskills.

3.3.4 Results

The ANOVAs revealed significant differences and large effect sizes between the groups in overall mathematical performance at T1, T2, and T3. However, the difference between the NR group and the low-comparison group in overall mathematical performance was not significant at any of the three time points. Mixed factorial ANOVA did also reveal a non-significant time * group interaction, indicating that the intervention did not have a significant effect. MANOVAs revealed a large statistically significant main effect for the groups when looking at the subskills separately. The difference between the NR group and the low-performing comparison group at the three time points were not statistically significant in any of the subskills. No statistically significant correlations were found concerning duration (minutes of play) and performance at T3, nor between intensity (number of occasions of play) and performance at T3.

3.3.5 Discussion

In this study a quasi-experimental research design to investigate the effectiveness of the NR computer game in enhancing low performing first grade children's mathematical performance. The group comparisons between the low-performing intervention group, the low-performing comparison group, and the average-performing comparison group revealed no evidence for the effect of the intervention on children's early numeracy performance. At none of the three time point assessments did the low performing groups differ significantly from the low-performing comparison group, which only received ordinary classroom instruction in mathematics during the intervention period. The average-performing comparison group did outperform children in both low-performing groups at all time points. These results suggest that the low-performing children in our study did not gain from playing the NR-game that focus on enhancing the children's non-symbolic and symbolic number knowledge, practicing the links between number representations, conceptualizing and automating arithmetic, and emphasizing approximate non-symbolic and symbolic comparison.

Our results are contradictory to previous CAI-studies using the NR-game that has given promising results (Obersteiner et al., 2013; Räsänen et al., 2009; Sella et al., 2016; Wilson, Dehaene et al., 2006; Wilson et al., 2009). However, in previous studies the children receiving intervention has either been kindergarteners (Räsänen et al., 2009; Salminen et al., 2015; Sella et al., 2016; Wilson et al., 2009), children from a specific target group (low socio-economic status: Wilson et al., 2009; with MLD: Salminen et al., 2015; Wilson, Revkin et al. 2006) or the intervention has been conducted in a more controlled setting (Obersteiner et al., 2013). It is possible that the children in the intervention group had already mastered these skills on a higher level than the NR could provide practice or that the EN-test was not sensitive enough to capture or match the skills trained in the NR-game. Our study was conducted in an authentic school setting, highlighting the importance of targeted interventions that are easily to conduct in the general classroom to enhance children's early numeracy skills.

4. General Discussion

The aim of this thesis was to contribute to the research on early numeracy development by focusing on identifying and supporting children at risk for mathematical learning difficulties in kindergarten and early school years, with a focus on improving evidence-based educational practice. Overall, the results of this thesis demonstrated support for the importance of using evidence-based tools for assessment and identification of children's early numeracy skills. Our findings support a multidimensional construct of early numeracy skills, where counting skills and basic arithmetical skills were found to be important predictors of later early numeracy skills in the early school years. The validity and reliability evidence of the EN-test were supported, indicating that the EN-test fit its purpose. In the computer-assisted intervention with the NR-game the low-performing children did not improve their early numeracy skills compared to the control groups, indicating the need for targeted interventions.

4.1 Development of Early Numeracy Core Skills during One School Year

The first research question for this thesis focused on how the early numeracy core skills are related and how they develop during one school year in first and second grades. Counting skills at the beginning of first grade were found to be an important predictor of later early numeracy skills while basic arithmetic skills emerged as a key predictor of later early numeracy skills in both grades. Counting skills has been emphasised as a foundational skill in existing frameworks of early numeracy skills development (Fritz et al., 2013; Krajewski & Schneider, 2009; Sarama & Clements, 2009; Steffe, 1992; Wright et al., 2006). Furthermore, counting skills have been found to be related to overall mathematical learning (Desoete et al., 2009) and a predictor for later mathematical learning (e.g., Krajewski & Schneider, 2009; Geary et al., 2018). The importance and the predictive value of counting skills are understandable as counting skills include, for example, enumeration, counting sequence and mapping numbers and numerals, skills that are considered essential for developing further arithmetical skills. Furthermore, counting skills are a key deficit in mathematical learning disabilities. Deficits in understanding counting concepts and relying on counting-based strategies have been identified in children with developmental dyscalculia, along with deficits in arithmetic fact retrieval and committing more procedural errors than their peers (Geary & Hoard, 2003; Zhang et al., 2020).

Recent research literature has highlighted the importance of symbolic and non-symbolic number knowledge for later mathematical learning.

Understanding and operating with number symbols and quantities are also a key aspect in many existing early numeracy frameworks (e.g., Krajewski & Schneider, 2009; Sarama & Clements, 2009). However, the core early numeracy subskills symbolic and non-symbolic number knowledge and mathematical relational skills did not play a major role in predicting subsequent early numeracy skills in first and second grade. Although especially symbolic number knowledge is suggested to have predictive value, it might be that symbolic and non-symbolic number knowledge is more critical before formal education (Chew et al., 2019; Reeve et al., 2012). Understanding mathematical relations, especially arithmetical principles (e.g., ordinality, cardinality, and decomposition) has been emphasised in existing frameworks of early numeracy development (e.g., Fritz et al.,2013; Krajewski & Schneider, 2009) and in recent research literature (Geary et al., 2018; Stock et al., 2007). It is possible that these skills were more critical in the arithmetical tasks, as understanding these concepts and principles helps children in counting and judging the correctness of arithmetical tasks (Geary et al., 2007). Being able to examine the relation of symbolic and non-symbolic number knowledge and mathematical relational skills to other early numeracy skills also in kindergarten could have provided different and additional value.

In both first and second grades three distinct early numeracy profiles were found: well-performing, average-performing and at risk for mathematical learning difficulties. In both grades, all three profiles were substantially different from each other at all three time points, except at the end of first grade, wherein the differences between the well-performing children and the average-performing children were non-significant in terms of symbolic and non-symbolic number knowledge and basic skills in arithmetic. This may indicate that the average-performing children improved their non-symbolic and symbolic number knowledge and basic skills in arithmetic, or that the well-performing children did not improve their skills as much as their peers. On the other hand, this could be an indicator that the EN-test was not able to differentiate between the two profiles any longer at the end of first grade. However, the EN-test is constructed to differentiate low-performing children from their average performing peers, and so focusing on children who are at risk for mathematical learning difficulties, and its sensitivity and specificity are not designed to differentiate children's performance on the other end of the spectrum. The developmental trajectories of the profiles are in line with previous findings. Most of the previous longitudinal person-centred studies have focused on the development of a specific early numeracy subskill over one school year and have been conducted with younger children (e.g., Gray & Reeve, 2016; Salminen et al., 2018). The variable-centred longitudinal studies have focused on the predictive value of a specific early numeracy subskill instead of investigating different components of early numeracy, or on the longitudinal relations within a specific early numeracy subskill (e.g., counting or symbolic magnitude comparison). As early numeracy is not a unidimensional factor this approach gives only a limited picture how the early numeracy skills develop and are interrelated.

The complexity of early numeracy skills and the fact that subskills are developmentally intertwined with each other presents challenges for research on early numeracy. However, in order to not only differentiate low-performing children from their peers, but also get more information on children's strengths and needs, it is important to describe and interpret subskills as separate factors (Purpura & Lonigan, 2013; Ryoo et al., 2015).

4.2 Identifying Children at Risk for Mathematical Learning Difficulties with Evidence-based Assessment

The second research question focused on the reliability and validity of the EN-test for identifying children at risk for mathematical learning difficulties in kindergarten, first grade and second grade. In Study II, the psychometric properties of the EN-test for the three age groups were investigated at the beginning of the school year. Identifying children at risk for mathematical learning requires valid and reliable assessment tools that not only differentiate the low-performing children but also give more specific information regarding what particular subskills should be supported (Aunio, 2019; Purpura & Lonigan, 2015). The EN-test was found to have adequate structural validity, supporting the multidimensional structure of the EN-test with four distinct subskills, indicating that the EN-test can provide both information on children's early numeracy skills on an overall level and more detailed information on children's performance concerning the four early numeracy subskills. A multidimensional structure of early numeracy skills with different subskills enables researchers and educators to examine children's performance, strengths, and needs more specifically and plan more targeted interventions based on this information (Purpura & Lonigan, 2013; Ryoo et al., 2015). The EN-test can provide more extensive and detailed information about children's performance than can be offered in narrower scales with a unidimensional structure.

The cross-cultural validity and known-group validity provided evidence that the EN-test is suitable for use in the average heterogeneous classroom, as the cross-cultural validity supported that the EN-test works equally well and measures the same constructs across girls and boys and younger and older children within grade, and the known-group validity supported the comparability in the two language groups. Furthermore, the reliability of the EN-test was supported by evidence for the internal consistency, indicating that the items measure the same construct. The values of internal consistency were similar to other existing early numeracy tests that have reported evidence for internal consistency (e.g., Geary et al., 2009; Jordan et al., 2012).

Findings from Study I additionally supported the predictive validity of the EN-test in early school years, as profiles of low-performing children who significantly differed from the average- and well-performing children were identified in both first and second grades. In both grades, the profiles of low-performing children showed lower performance level than their peers at all subskills at all three time points. Also, in Study III a group of low-performing children in first grade were identified based on their performance in the EN-test in the beginning of school year. In this study a cut-off at the lowest 20th percentile was used to differentiate the lowest performing group of children from their average-performing peers. The low-performing children performed lower than the average-performing peers across the school year in all early numeracy subskills. Even though the low-performing children received an intervention at the end of the school year, they were not able to improve their early numeracy skills to the same level as their average peers. At none of the three time point assessments did the intervention groups differ significantly from the low-performing comparison group, which received ordinary classroom instruction in mathematics during the intervention period. However, when comparing the children in the intervention group to their low-performing peers, neither did the low-performing peers close the gap with their average-performing peers.

Our finding is supported by previous studies suggesting that low-performing children seem to benefit less from ordinary education than typically performing children (Aunola et al., 2004; Fuchs & Fuchs, 2006) and that the gap between the low-performing children and the average-performing children tends to persist and may even grow wider throughout the school years (Aunola et al., 2006; Jordan et al., 2007; Zhang et al., 2020). This highlights the importance of continuously assessing children's early numeracy skills at several assessment points within and across grades, which the EN-test for three age groups and three subtests within a grade enables. Assessments at several time points are not only important in order to follow up and identify low-performing children, but they are also required for decisions on at-risk status (Geary, 2013; Mazzocco & Räsänen, 2013). Study I provided support for the possibility of using the different versions of the EN-test for identifying children at risk for mathematical learning difficulties in early school years.

The EN-test was constructed to assess children's early numeracy skills and identify children at risk for mathematical learning difficulties in kindergarten, first grade, and second grade simply and efficiently. The psychometric criteria evidence that EN-test fits its purpose was found, as a profile of low-performing children that differed from their average- and well-performing peers was identified. However, the LPAs were not able to differentiate the low-performing children in two groups (i.e., low-performing and MLD/developmental dyscalculia), which may either indicate that the EN-test is not sensitive enough to differentiate the lowest performing children that might have more severe mathematical learning disabilities or that it may not be possible to differentiate these two groups in early school years.

The first argument is based on the findings that symbolic magnitude processing skills have been suggested to be a strong predictor for later mathematical learning (De Smedt et al., 2013; Schneider et al., 2016) and the core of mathematical learning disabilities (Price & Ansari, 2013). However, the symbolic and non-symbolic number knowledge skills were measured and operationalised narrowly in the EN-test both concerning number of tasks and the content in the tasks. The other argument is based on findings that the importance of specific early numeracy subskills differs depending on where children are in their learning process (Desoete et al., 2012; Geary, 2013). Additionally, a developmental shift from focus on non-symbolic number knowledge to symbolic number knowledge in early school years is suggested (Desoete et al., 2012). The development of early numeracy skills and the relation between the early numeracy skills were not investigated concerning kindergarten. This could have contributed to this question of whether it is possible to identify children with mathematical learning disabilities before formal education and if there is a developmental shift in these skills. However, answering these questions may need a more sensitive assessment than the group- and paper-and-pen-based EN-test that aims to identify those children that are in need of support.

4.3 Supporting Children at Risk for Mathematical Learning Difficulties with Computer Assisted Interventions

The third research question focused on how children at risk for mathematical learning difficulties can be supported with a computer-assisted intervention using the research-based computer game NR. In Study III, the group comparisons between the low-performing intervention group, the low-performing comparison group, and the average-performing comparison group in first grade revealed no evidence for the effect of the intervention on children's early numeracy performance. We suggest that the differences between our results and findings from previous studies originate in the characteristics of our target group (age, low performance on EN-test) as well as the early numeracy skills that were practiced in the NR (symbolic and non-symbolic number knowledge, especially comparing magnitudes and numbers) compared to the early numeracy skills measured by the EN-test (symbolic and non-symbolic number knowledge, understanding mathematical relations, counting skills and basic skills in arithmetic).

Contradictory to previous studies with promising results of the NR game, our study was conducted with first graders in an authentic school setting. Previous studies have either been conducted with younger children (kindergarteners: Räsänen et al., 2009; Salminen et al., 2015; Sella et al., 2016; Wilson et al., 2009), with children from a specific target group (low socio-economic status: Wilson et al., 2009) with children with severe mathematical learning disabilities (Salminen et al., 2015; Wilson, Revkin et al., 2006) or in a more controlled setting (Obersteiner et al., 2013). The intensity and duration of the NR intervention did not differ from previous NR studies. However, it is suggested that an effective intervention should include several sessions over a specific time period (e.g., six weeks to six months) (Gersten et al., 2008). A consideration for further studies is to consider the length of the intervention more clearly. However, if the content in the intervention is not individualised based on children's strengths and need, increasing the length would not be sufficient per se.

In our study the NR-intervention group were compared to their peers receiving ordinary instruction (i.e., business as usual) during the intervention period. Unfortunately, it is not clear if the intervention was conducted as a complement to or instead of ordinary classroom instruction in mathematics, or how the instruction in class was conducted during the time of the intervention. However, during the school year no significant differences were found between the intervention group and their low-performing peers at any of the time points. A delayed post-test could have shown some significant differences, or at least strengthen the evidence for, or lack of, sustainable learning effects as a result of the NR-intervention.

In previous studies where CAIs have been contrasted to ordinary classroom instructions there have been contradictory findings, ranging from it is effective to it is less effective than teacher instruction (Dennis et al., 2016). Even though CAI can offer one-to-one interaction, high motivation, instantaneous response, the possibility to proceed at one's own pace and level, individual attention, and multimodal presentation of concepts (Fengfeng, 2008), there are limitations even with research-based and adaptive CAI programs. It might be that the educator can more flexibly modify the instruction based on children's strengths, needs and progress, compared to a software program. Even though NR is an adaptive game that is constructed to maintain the difficulty of the tasks and provide the child with exactly the required difficulty level the focus is still on a specific early numeracy skill. It might be that the children in the NR group did not benefit from practicing the non-symbolic and symbolic number knowledge skills; instead, they could have benefitted from systematic and explicit instructions on counting skills or mathematical relational skills. The findings from Study I indicated that the majority of children by the end of first grade have acquired solid non-symbolic and symbolic number knowledge, which indicates that the NR game could be adequate for those children with most severe mathematical learning difficulties in first grade and not for low-performing children per se.

Also, previous studies have highlighted the importance of targeted interventions. Even though an intervention tool is evidence-based and includes features that are supported in research (e.g., systematic and explicit instruction, concrete-representational-abstract, CAI or games) it does not necessarily mean it will be effective for all children. It is important to carefully and critically choose which software to use in a computer-assisted intervention (Gersten et al., 2008). Overall, children benefit from extra support, but individualised interventions that pin specific skills seem to be more effective than interventions that disregard individual differences (Dowker & Sigley, 2010).

4.4 Strengths and Limitations

This thesis has strengths that offer a contribution to the research on early numeracy skills development and how to identify and support children at risk for mathematical learning difficulties in kindergarten and early school years with evidence-based tools. However, there are also some limitations concerning both methodological and practical issues that should be addressed. The strength of this study is in the longitudinal data from three time points from first and second grades and additionally one time point in kindergarten, offering the possibilities to examine both the relations between early numeracy skills (variable-centred approach) and differences among early numeracy profiles (person-centred approach). Early numeracy skills were operationalised with a multifactorial approach, highlighting the multidimensional construct of early numeracy skills that enable a deeper understanding of how the early numeracy skills are related over time and how individual learning trajectories develop compared to studies with a more narrow approach to early numeracy.

The use of the EN-test for measuring children's early numeracy performance is both a strength and a limitation in this thesis. However, it is notable that the aim for this thesis would not have been possible to achieve without an appropriate early numeracy test. Examining the psychometric criteria for EN-tests provides valuable information about the assessment tool and evidence that the test can fulfil its purpose to identify children at risk for mathematical learning difficulties in kindergarten and first and second grades. The reliability and validity of the EN-test was examined and compared to many other tests and several psychometric criteria reported. This thesis provides evidence for the construct validity in terms of structural validity, cross-cultural validity and known-group validity, criterion validity in terms of predictive validity, and reliability in terms of internal consistency. Investigating other psychometric criteria (e.g., test-retest and inter-rater reliability, concurrent and convergent validity) was left outside this thesis. However, for example, concurrent validity was not possible to examine as no other norm-referenced test was available for the target groups (age and language). Additionally, the predictive validity was investigated only across one school year in first and second grades. A longitudinal design over several years would have given a deeper understanding of the children's learning trajectories and additional evidence for the predictive validity.

The limitations concerning the use of the EN-test for measuring children's early numeracy performance mainly tap the construct of the EN-test. Due to the construct of the EN-test all four subskills could not be measured at all the time points and the content and number of tasks per subskill differed across grades. Therefore, it was not possible to investigate the relations between all the four subskills at the three time points. In kindergarten only counting skills were measured at the three time points and therefore this age group was not included in Study I, as it was not possible to analyse the relations between subskills. Another reason for excluding kindergarten from Study I is the differences between the content and aims in the kindergarten and the compulsory formal education. However, examining the predictive validity of the EN-test in kindergarten would be of interest, as predictive validity is especially important for a test that aims to identify children at risk for mathematical learning difficulties.

An additional limitation concerning the EN-test is how symbolic and non-symbolic number knowledge was operationalised and measured. In the EN-test symbolic and non-symbolic number knowledge is measured with tasks requiring knowledge of symbols and number words, for example, when comparing numbers, listening to number words and using number symbols to answer the paper-and-pen tasks. The focus is mainly on the symbolic numerical processes and the accuracy in answering the tasks. In other related studies, reaction times are often used as a more sensitive measure of numerical magnitude processing, as reaction time can tap the distance effect (i.e., the numerical distance between the magnitudes compared effect the speed and accuracy in the performance) and size effect (i.e., the numerical value and the number range effect the speed and accuracy in the performance). Reaction time is most often measured with computerized tasks (De Smedt et al., 2013). In a study by Brankaer and colleagues (2016) paper-and-pen tests were suggested to be an adequate measure of symbolic magnitude processing. However, in their test (Symbolic Magnitude Processing Test, SYMP test) the number of tasks was more extensive, and the test focused only on this particular early numeracy subskill. As a consequence of the operationalisation and measurement of symbolic number comparison and magnitude processing and based on recent research, the concept of symbolic and non-symbolic number knowledge was used instead of symbolic and non-symbolic number sense that Aunio and Räsänen (2016) used in their model of core early numeracy skills.

It is possible that these issues concerning the construct of the EN-test and the operationalisation of symbolic and non-symbolic number knowledge may have reflected the results of studying the effect of the NR intervention to enhance children's early numeracy skills. The effectiveness of the NR intervention was measured based on the performance in the EN-test before and after the intervention. The NR game focuses on practicing various number presentations and the transformations between symbolic and non-symbolic number representations and the children solve the tasks individually in the software environment, where the tasks are presented multimodally, whereas the EN-test focuses on four core early numeracy skills and the children solve the tasks individually on paper based on verbal instructions from the educator. Closely matching the focus in the intervention and the assessment could have led to different effects for the intervention. The results from Study I also indicate that the intervention for the low-performing children in this age group should focus on counting skills and basic arithmetic skills rather than on numeric comparisons. As the intervention was planned and carried out before analysing the development and interrelations of the early numeracy skills, these findings were not utilised for the intervention. Consequently, this points out the importance of using the information from assessments for planning interventions.

Another limitation in the NR intervention is that the children in the NR intervention group were identified as low-performing based on the assessment at the beginning of the school year (i.e., 7–8 months before the intervention started). However, in educational practice this kind of solution can be authentic, which only highlights the importance of continuous and reliable assessments that provide extensive information regarding the children's needs that is useful in planning and conducting targeted interventions. Information on how the intervention was conducted (e.g., as a part of or complement to ordinary classroom instruction, did the children get additional support during the intervention period) would have provided more information needed to interpret the efficacy of the intervention.

Concerning identifying children at risk for mathematical learning difficulties, this study only focused on measuring early numeracy performance and limited background information on the participants was collected. It is known that various factors, for example cognitive, emotional, motivational, sociocultural and emotional factors affect the early numeracy performance (Geary, 2013; Price & Ansari, 2013). Especially language skills, executive functions (e.g., working memory and inhibition) and kindergarten attendance have shown to predict early numeracy skills development (Aunio et al., 2019). The test situation requires attention, working memory and language skills, especially quantity language (i.e., more, less, fewer and double) which can affect the child's early numeracy performance. In future studies, including these factors in studies of early numeracy development would be of interest both in understanding the underlying causes of low performance and construct of early numeracy, but also how it affects individual learning trajectories and the intervention effects. However, these questions are not easy to address in studies conducted in authentic school settings but are important for diagnostics and highlight the current research interest of early numeracy predictors and risk factors, and whether it is possible to identify children with mathematical learning disabilities before formal education. However, the purpose of the EN-test is expressly to be an assessment for identifying children at risk, which means differentiating those children that are in need of additional support from their average-performing peers. In educational practice identifying children in need of support and providing them with adequate support are more important than differentiating the children with mathematical learning disabilities from their low-performing peers (Mazzocco & Räsänen, 2013).

4.5 Implications for Educational Practice

This thesis has an aspiration to improve educational practice by emphasising early numeracy skills, early identification and support and providing educators with examples of evidence-based tools for identifying and supporting children at risk for mathematical learning difficulties. Identifying and supporting children at an early stage has the potential to reduce the risk for mathematical learning difficulties (Gersten et al., 2011), but it requires valid and reliable tools for identification and support (Aunio, 2019; Purpura & Lonigan, 2015). Based on the findings the EN-test can be suggested as a simple and adequate group-based assessment tool for identifying children at risk for mathematical learning difficulties within and across grades. The three subtests across grades enable continuous assessment at several time points. Findings from Study I support that the assessment should be a continuous process and children's early numeracy skills should not be assessed only at the beginning of the school year.

Additionally, the EN-test is based on a theoretical framework that can provide educators with an evidence-based working model and improve their understanding of early numeracy development and relations between early numeracy skills in order to identify and support core skills in children's early numeracy development. It is argued that educators need evidence-based guidance to improve their knowledge regarding what to focus on in supporting children's early numeracy skills (Clements & Sarama, 2014; Desoete et al., 2009). This thesis provides empirical validation for the model of core early numeracy skills (Aunio & Räsänen, 2016), suggesting it to be an adequate working model for educators. A framework suitable as a working model in educational practice should be both specific in the operationalisation of the core skills but flexible enough to offer educators the possibility to choose the best way to support these skills (Pegg, 2003). The model of core early numeracy skills by Aunio and Räsänen provides specific information on the core early numeracy skills and their relations but gives the educators possibilities to choose how to work with these skills. The model of core early numeracy skills can provide an evidence-based working model for educators that brings the assessment and instruction closer to each other.

In the Finnish three-tier system assessment for identification is emphasised. A problem in Finland has been the lack of evidence-based assessment tools for identifying and interventions for supporting children at risk for mathematical learning difficulties (Björn et al., 2018). The EN-test is the first and currently still the only research-based assessment tool for identifying children at risk for mathematical learning difficulties for Swedish speaking children in Finland. An additional advantage with both the EN-test and the NR game are that both are freely available for all educators. The EN-test is available in two languages, whereas the NR game is available in several more. However, the findings from the NR intervention highlight the importance of targeted interventions, as a particular intervention may not be effective for all children. The Number Race game can provide good support for some children but for providing low-performing children with adequate support, there should be a selection of freely available evidence-based tools.

4.6 Conclusions

Identifying and supporting children at risk for mathematical learning difficulties are important in order to provide all children with foundational early numeracy skills in kindergarten and early school years. For this, evidence-based tools for identification and support are needed. This thesis highlights the importance of which early numeracy skills are the most fundamental, which children we need to identify and support, and how these children can be identified and supported with an evidence-based assessment and an intervention.

In this thesis early numeracy was studied based on a multidimensional structure and operationalised through four early numeracy subskills: symbolic and non-symbolic number knowledge, understanding mathematical relations, counting skills and basic skills in arithmetic (Aunio & Räsänen, 2016). This thesis indicates that the EN-test is an efficient and reliable assessment that both identifies the children in need of support and provides information on what particular subskills should be in supported in the targeted interventions. This thesis supports earlier findings on the importance of identifying and supporting children at risk for mathematical learning difficulties at an early stage and highlights the importance of a multidimensional approach to early numeracy in identifying and supporting children at risk for mathematical learning difficulties.

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Appendix

Table A1

Examples of Test Items in the EN-test in Kindergarten, First Grade and Second Grade

Level	Skill group	Content	Example item	Number of items		
				T1	T2	T3
Kindergarten	Symbolic and non-symbolic number knowledge (NK)	Understanding magnitudes embedded in the number word sequence (number range: 0-10)	"Here you see three numbers. Which is the largest number?" (e.g., 3, 5, and 2)	4	3	-
			"Here you see three numbers. Which is the smallest number?" (e.g., 3, 5, and 2)	4	3	-
	Understanding mathematical relations (MR)	Comparing magnitudes and seriation (number range: 0-10)	"Look at these four squares. In which square do you have most stars?"	2	-	-
			"Look at these four squares. In which square do you have more stars than trees?"	2	-	-
			"Look at these four squares. In which square do you have the same number of stars and trees?"	2	-	-
			"Look at these four squares. In which square do you have fewer stars than trees?"	2	-	-
			"Look at this row with stars. Put a cross on the fourth star."	4	-	4
			"These pyramids are arranged from largest to smallest. Where does this pyramid fit?"	4	-	-
	Counting skills (CS)	Number identification, recognition and writing	"Look at these four squares with dots. I will say a number. Mark the square with the same number of dots as the number I say."	2	2	-
		Counting up and back, count up from a given number (number range: 0-30)	"Look at these four numbers. I will say a number. Mark the same number symbol as the number I say."	2	2	-
			"Look at these dots. How many dots do you see? Mark the number symbol that is the same as the number of dots."	2	2	-
			"Look at this number symbol. Mark the square with the same number of dots as the number you see."	2	2	-
			"In this box, we have coins. There are three coins in the box. In which box is it two more?" (The numbers are presented with number symbols.)	-	-	2
			"In this box, we have coins. There are three coins in the box. In which box is it two less?"	-	-	2
			"I would like to know how high you can count. Start counting from number 1. I will tell when to stop."	3	3	3
			"I would like to hear you count more. Count as high as you can, but now start counting from number 4. I will tell when to stop."	3	3	3
			"I would like to know if you can count backwards. Start from number 6, and I will tell when to stop."	2	2	2

			"In this task, we have stars. Some stars are visible, some are hidden. How many stars are hidden when there are five stars altogether?"	-	-	3
	Basic skills in arithmetic (BA)	Addition with verbal stories Subtraction with verbal stories (number range: 0-10)	"You have two coins (the child sees a picture with two coins) and you get two more. How many coins do you have now? Choose your answer from the given numbers."	4	-	4
			"You have three coins (the child sees a picture with three coins) and give away one. How many coins do you have left? Choose your answer from the given numbers."	4	-	4
First grade	Symbolic and non-symbolic number knowledge (NK)	Understanding magnitudes embedded in the number word sequence (number range: 0-25)	"In this basket, you have two boxes with money. In the first box, you have three coins and in the other box two coins. Altogether there are five coins in this basket. In this basket, you have also two boxes. In the first box, you have four coins and in the other box no (zero) coins. In which basket, do you have more coins?"	4	-	-
			"Look at these three squares. In which square are the numbers arranged from smallest to largest?"	4	-	-
			"Look at these two numbers (e.g., 7 and 4). 7 is bigger than 4, but how much bigger is it? Choose your answer from the given alternatives."	-	3	-
			"Look at these two numbers (e.g., 4 and 7). 7 is smaller than 4, but how much smaller is it? Choose your answer from the given alternatives."	-	3	-
			"Look at these numbers (e.g., 31 52 36 47). Which is the biggest number?"	-	-	3
			"Look at these numbers (e.g., 31 52 36 47). "Which is the smallest number?"	-	-	3
	Understanding mathematical relations (MR)	Comparing magnitudes, seriation, one-to-one-correspondence (number range: 0-70)	"Look at these four squares with stars and trees. In which square do you have more stars than trees?"	2	-	-
			"Look at these four squares with stars and trees. In which square do you have fewer stars than trees?"	2	-	-
			"Look at these four squares with gloves. Three boys are going out. How many gloves do they need?"	4	-	-
			"Look at these three numbers. Can you find out which is the next number?" (e.g., 3, 4, 5, _)	4	3	3
			"Look at these three numbers. Can you find out which is the next number?" (e.g., 6, 5, 4, _)	4	3	3
	Counting skills (CS)	Counting numerosity of a set, counting part of a whole	"In this task, we have stars. Some stars are visible, some are hidden. How many stars are hidden when there are five stars altogether?"	4	-	-
		Counting up and back, count up from a given number (number range: 0-30)	"In this box, we have coins. There are three coins in the box. In which box is it two more?" (The numbers are presented with number symbols.)	2	-	-
			"In this box, we have coins. There	2	-	-

			are three coins in the box. In which box is it two less?"			
			"I would like to know how high you can count. Start from number 1, and I will tell when to stop."	3	-	-
			"I would like to know if you can count backwards. Start from number 6, and I will tell when to stop."	3	-	-
			"I would like to know if you can count up with skipping one each time, like this: two, four, six... Now it is your turn, start from number 2, and I will tell when you can stop."	2	-	-
	Basic skills in arithmetic (BA)	Addition stories and with symbols	You have two coins, and you get two more. How many do you have now?"	4	-	-
		Subtraction stories and with symbols (number range: 0-15)	"Look at this row. Can you solve this task? Choose your answer from the given numbers." (e.g., 4+3 alternatives 5, 6, and 7)	4	-	-
			"You have three coins and give away one, how many do you have left?"	4	-	-
			"Look at this row. Can you solve this task? Choose your answer from the given numbers." (e.g., 7–3, alternatives 3, 4, and 5)	4	-	-
			"Look at this row. Can you solve this task? Write your answer on the line." (e.g., 4+3)	-	8	6
			"Look at this row. Can you solve this task? Write your answer on the line." (e.g., 7–3)	-	8	6
			"Look at this row. Which number is missing? Write your answer on the line." (e.g., 7–_=4)	-	4	4
			"Look at this row. Which number is missing? Write your answer on the line." (e.g., 7–_=4)	-	4	4
Second grade	Symbolic and non-symbolic number knowledge (NK)	Understanding magnitudes embedded in the number word sequence (number range: 0-9000)	"Look at these numbers (e.g., 31 52 36 47). Which is the biggest number?"	4	4	6
			"Look at these numbers (e.g., 31 52 36 47). "Which is the smallest number?"	4	-	-
			"Look at these three numbers. Arrange the numbers from smallest to largest?"	-	-	3
	Understanding mathematical relations (MR)	Seriation (number range: 0-50)	"Look at these numbers (e.g., 3, 4, 5, _). Can you find out which is the next number?"	4	-	-
			"Look at these numbers (e.g., 6, 5, 4, __). Can you find out which is the next number?"	4	-	-
	Counting skills (CS)	Number identification, recognition, and writing (number range: 0-300)	"I will tell you a number. Write the number with number symbols."	8	-	-

Basic skills in arithmetic (BA)	Addition with symbols Subtraction with symbols (time limit) (number range: 0-20)	“You will now get some addition tasks. Solve a task and write your answer in the square, and then go to the next task. Solve as many tasks as you can.” (e.g., 8+1 and 7+5; time limit)	20	20	20
		“You will now get some subtraction tasks. Solve a task and write your answer in the square, and then go to the next task. Solve as many tasks as you can.” (e.g., 8–1 and 15–6; time limit)	20	20	20
		“Here are some addition tasks. Solve a task and write your answer in the square. (e.g., 18+21 and 27+35)	-	8	8
		“Here are some subtraction tasks. Solve a task and write your answer in the square. (e.g., 18–12 and 51–36)	-	8	8

Heidi Hellstrand

Early Numeracy Development:

Identifying and Supporting Children at Risk for Mathematical Learning Difficulties

Identifying and supporting children at risk for mathematical learning difficulties are important in order to provide all children with foundational early numeracy skills in kindergarten and early school years. Consequently, evidence-based tools for identification and support and an understanding of distinct early numeracy skills are needed. The present thesis investigates how early numeracy skills develop and are interrelated over time, examines the psychometric criteria of the Early Numeracy test and investigates the effects of a computer-assisted intervention program on children's early numeracy skills. Overall, the results highlight the importance of using evidence-based tools for identifying and supporting children's early numeracy skills development in kindergarten, first and second grades. Furthermore, the results support a multidimensional construct of early numeracy skills, where counting skills and basic arithmetical skills seems to be important predictors of later early numeracy skills in the early school years. Further, the validity and reliability evidence of the Early Numeracy test was supported, indicating that the test fulfil its purpose to identify children at risk for mathematical learning difficulties. Lastly, in the computer-assisted intervention including the Number Race-game, low-performing children did not improve their early numeracy skills compared to the control groups, indicating a need for interventions targeting specific early numeracy skills. This thesis contributes to the research on early numeracy development by focusing on identifying and supporting children at risk for mathematical learning difficulties in kindergarten and early school years, with a focus on improving evidence-based educational practice.

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