

# Fat Tails due to Variable Renewables and Insufficient Flexibility

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# Fat Tails due to Variable Renewables and Insufficient Flexibility

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## Abstract

The large-scale integration of renewable energy sources requires flexibility from power markets in the sense that the latter should quickly counterbalance the renewable supply variation driven by weather conditions. Most power markets cannot (yet) provide this flexibility effectively as they suffer from inelastic demand and insufficient flexible storage capacity.

Research accordingly shows that the volume of renewable energy in the supply system affects the mean and volatility of power prices. We extend this view and show that the level of wind and solar energy supply affects the tails of the electricity price distributions as well, and that it does so asymmetrically. The higher the supply from wind and solar energy sources, the fatter the left tail of the price distribution and the thinner the right tail.

This implies that one cannot rely on symmetric price distributions for risk management and for valuation of (flexible) power assets. The evidence in this paper suggests that we have to rethink the methods of subsidizing variable renewable supply such that they take also into consideration the flexibility needs of power markets.

*Keywords:* Intermittent renewable supply, flexibility, power prices, fat tails, asymmetric probability distribution.

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## 1. Introduction

Electricity markets have experienced radical structural changes over the past few decades. During this period of time, many countries liberalised their electricity sector and set the path to the creation of competitive power markets. Besides that, most of these markets experienced drastic reforms during the energy transition, with the most prominent being the increasing penetration of energy supply from renewable energy sources (hereafter we refer to those with RES). The rapid and large-scale integration of intermittent RES, however, induces significant impact on power prices and substantially increases the demand for power system flexibility, as intermittent energy supply comprises non-controllable variability and partial predictability (Perez-Arriaga and Batlle (2012) and Kyritsis et al. (2017)).

Partial predictability is predominantly driven by the fact that weather-dependent RES do not perfectly adapt their output as a reaction to economic incentives, and therefore to the flexibility demand from the energy system. A better understanding of the impact that intermittent RES have on electricity prices are of great concern to managers, who must take better long- and short-term decisions in the operating on electricity markets, but also to policy makers who endeavour to adjust the electricity market design in order to increase power system flexibility, and thereby accelerate the reduction of emissions in the power sector.

Focusing on the German electricity market, a prominent example of a market integrating variable energy supply from RES, Kyritsis et al. (2017) show that both solar and wind power generation have an impact on the probability distribution function of electricity prices by decreasing the average price, which is - in other words - the a merit-order effect. According to Kyritsis et al. (2017), electricity prices decline when the share of RES in the power system increases. Würzburg et al. (2013) discuss several studies, of which nine focus on the German electricity market, and all provide evidence for the merit-order effect. More recent studies that yield the same conclusion are, among others, Tveten et al. (2013), Ketterer (2014), Paraschiv et al. (2014), and Dillig et al. (2016).

Tveten et al. (2013), Ketterer (2014), and Kyritsis et al. (2017) go one step further and examine how changes in intermittent renewable energy supply affect the volatility of electricity prices. In fact, Kyritsis et al. (2017) study both solar and wind power generation technologies and, considering the recent period of high renewable penetration, they show that solar and wind have a different impact on the volatility of electricity prices; while solar power generation reduces the volatility of electricity prices and the probability of electricity price spikes, wind power volatility increases electricity price volatility and introduces electricity price spikes. The same relation between solar and the volatility of electricity prices manifests in Tveten et al. (2013) and between wind and

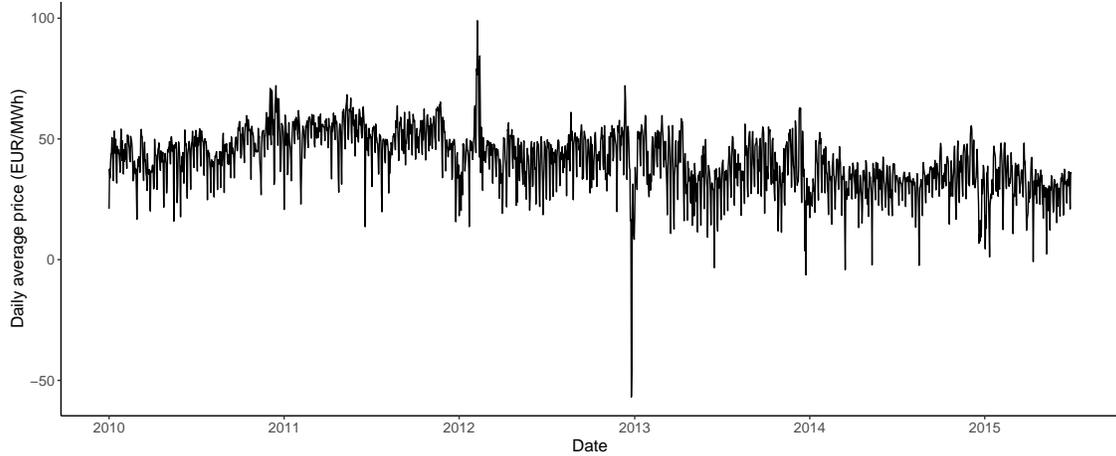


Figure 1: Average daily price on the German day-ahead market 2010-2015.

electricity prices volatility in Ketterer (2014), during the first period of RES integration. A more recent study by Johnson and Oliver (2019) analyzes wind and solar supply together and also find that RES is increasing power price variance. Increased price variance induced by RES calls for more knowledge about managing energy price risk and valuing real options, such as the option to store power in batteries and alternative power storage systems and the option to flexibly adjust consumption to respond to changes in supply from RES.

The view from the literature is that power prices decline (*ceteris paribus*) as a result of an increase in RES and that the volatility of power prices changes (*ceteris paribus*) as a result of solar and wind energy supply variations. This view motivates us to further examine the impact of intermittent energy supply on the probability distribution of power prices. We question whether the increasing share of variable solar and wind power generation also affects the tails of power price distribution. The motivation for our research question becomes clear from Figure 1 which shows the development of the day-ahead average daily prices in €/MWh in Germany from January 2010 to June 2015. In this figure, the typical characteristics of day-ahead power prices, such as mean reversion and extremely high and low prices, become apparent. Kyritsis et al. (2017) focus on how power price variation (volatility), being the second moment of a probability distribution function, is affected by changes in RES supply. As extreme prices, which are kurtosis events, influence the fourth moment of a probability distribution function, we think that these observations cannot be captured only by variation (or the second moment as it were). Therefore, the present study examines extreme power prices.

The expansion of variable or intermittent RES requires an increasing effort from the non-

intermittent suppliers to counterbalance abrupt changes in production volumes. This may result in increased supply frictions, which become more prominent during periods of limited power system flexibility, in terms of adjusting the production volumes by the non-intermittent suppliers. Thus, the lower the flexibility of the power system, the higher the probability of extreme prices to occur. Hence, beyond the mean and variance of the electricity price distribution, the shape of the probability distribution in the tails is also driven by the penetration of RES into the power system.

This reasoning relates the tail structure of electricity price distribution closely to power system flexibility, which is the key challenge towards the large-scale integration of RES. However, there is not a consensus view in the literature on the relation between intermittent wind and solar energy supply and the tails of the power price probability distribution. Limited evidence comes from studies that marginally touch on the link between extreme electricity prices and intermittent supply. For instance, Paraschiv et al. (2014) do not find conclusive evidence for the case of solar supply, but their results show that upward price spikes occur mostly when wind energy supply is low. By comparing the tail fatness of the empirical power price distributions between emerging and developed economies, LeBaron and Samanta (2005) show that one of the factors influencing the distribution of electricity prices is the different penetration level of intermittent renewable generators. From a similar point of view, Lindstrom and Regland (2012) study six European electricity markets through the employment of a regime switching model, and find a positive relation between the frequency of extreme price events and the penetration of renewable energy sources in the power system; hence, they provide evidence of renewable energy supply increasing the tail fatness of the electricity price distribution. In contrast, Keles et al. (2016) apply an AR-GARCH model on EPEX day-ahead market data and indicate that the tail fatness of the power price distribution is reduced over the period from 2008 to 2014. Although the authors do not make a strong claim, they suggest that their results are possibly driven by the increasing share of RES, and particularly wind, in the power generation mix.

Kyritsis et al. (2017) demonstrate the different impact of wind and solar energy supply on power price variation and provide some main distributional properties of electricity prices related to price spikes, for different solar and wind power penetration levels. Those price spikes (being both extreme high and low prices) are not studied in particular. Extreme Value Theory (EVT) is a field within statistics that focuses on the probability structure of extreme observations only. As extreme high and low prices occur due to abrupt changes in supply from RES and the inflexibility of the power system to cope with these changes, prices behave different than when such changes do not occur. This motivates us to apply EVT as we believe that the probability distribution of extreme prices could not necessarily be caused by higher variance or volatility only. In this study,

we therefore proceed a step further and investigate whether the results Kyritsis et al. (2017) found for volatility also hold with regard to the tail fatness of the power price distribution. Not only we look at the effect of solar and wind on the tails, but the main advantage is that we disentangle the effects of each of them on the left and right tail of the power price distribution. This paper contributes to the literature by extending Kyritsis et al. (2017). Using their data and methodology, we examine the impact that the penetration of intermittent RES in the German power supply mix has on both tails of the power price probability distribution, but also separately on the left and right tail.

The distribution of electricity prices can significantly deviate from the normal distribution, and one needs to incorporate information about the tails to correctly model the shape of the distribution. The tail fatness of the electricity price distribution has direct implications for risk management, energy policy making in the sense that supply from RES in combination with insufficient flexible storage capacity and inelastic demand lead to extreme electricity prices, and for the real options valuation of flexible power suppliers for which price variation is a key-input variable.

The remainder of the paper is structured as follows. Section 2 introduces our methodology. Section 3 discusses the data, and section 4 presents the empirical findings. Section 5 concludes.

## **2. Methodology**

Motivated by the aforementioned discussion, we investigate the impact of energy supply from RES on the tail fatness of the empirical power price distribution. Due to price inelastic short-term demand and insufficient storage capacity, power prices exhibit mean reversion, high volatility, and frequent upward and downward price spikes. As a consequence, the probability distribution of power prices is non-normal and exhibits fat tails. This has been recognized by, among others, Huisman and Hurman (2003), Byström (2005), Walls and Zhang (2005), Chan and Gray (2006), and Herrera and González (2014), who apply extreme value theory (EVT) to examine extremely high and low power prices.

None of the aforementioned studies focus on the relation between the probability and magnitude of extreme prices and the fundamentals of the electricity markets, such as power generation mix, flexible storage capacity, expected demand, and available supply. We agree with Paraschiv et al. (2014) stating that stochastic models are often built on simplistic assumptions and that one should focus more on the role of fundamentals in the analysis of power prices. This motivates us to examine the relationship between the probability and magnitude of extreme power prices and wind and solar energy supply. In addition, we examine whether changes in intermittent supply from

renewable energy sources have a different effect on the right side of the (empirical) power price distribution than on the left side, as previous studies found mixed evidence for this tail asymmetry. Frestad et al. (2010), for instance, do not find sufficient evidence for tail fatness asymmetry in the Nordic Electricity Swap Market. González-Pedraz et al. (2014), however, suggest that positive price spikes are more frequent in electricity prices than drops, thereby indicating tail asymmetry. We aim to contribute to this literature by examining the relation between the volume supplied by renewable energy sources and extreme power prices.

To formulate our expectations about this relationship, we think of a power market with intermittent RES, non-intermittent suppliers who can adjust production volumes, price inelastic consumers, and insufficient flexible storage capacity. With intermittent RES we mean, for instance, wind and solar power producers who have limited capacity to adjust volumes; intermittent can also be called variable in that sense, and we shall use both terms interchangeably. In such a power market, non-intermittent suppliers have to increase or decrease production when energy supply from variable RES decreases or increases so as to keep the system in balance.<sup>1</sup>

Now, consider a period in time when the energy market is in balance: the non-intermittent producers and RES supply the customers' demand. We question what would happen with extreme power prices when supply from RES increases or decreases, for instance due to a change in weather conditions. We distinguish between periods of high or low demand from customers and supply from RES.

Increased demand for flexibility arises when supply from RES changes. The non-intermittent producers are the only ones who can supply flexibility as they can adjust production. The prices that the non-intermittent suppliers charge for this flexibility depend on the competition that they face. When reserve margin, being ready to produce spare capacity, is low, demand for increasing production can be supplied only by a few non-intermittent producers, and this might result in very high prices. Demand for reducing production can be supplied by many producers and extremely low prices are not likely. Therefore, extremely high prices are more likely than extremely low prices when reserve margin is low. When reserve margin is high, only a few non-intermittent power plants produce. When a decrease in production is demanded, only a few producers can fulfil that demand and this lack of competition might lead to extremely low prices. When an increase

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<sup>1</sup>In most markets, non-intermittent output is predominantly generated using coal, nuclear, gas or hydro technologies and each of these technologies have different flexibility levels regarding ramping up and down production.

in production is demanded, many producers that are standing idle can increase production and extremely high prices are therefore less likely.

Summarising, extreme high (low) prices are more likely to occur than extreme low (high) prices when consumer demand is high (low). When examining power prices we expect that the probability distribution function of power prices has a fatter right (low) than low (high) tail, when demand is high (low). This is summarised as:

1. during periods of high demand, the right tail is fatter than the left tail;
2. during periods of low demand, left tail is fatter than the right tail.

So far, we have only looked at the relation between tail fatness and demand (or reserve margin). But the supply from RES plays a crucial role as well. When demand is low and the share of RES supply is high, less non-intermittent power plants produce than when the share of RES is low. As a consequence, there are even less non-intermittent producers that can provide flexibility through decreasing production. Therefore, very low prices even become more likely when the share of RES is high. When demand is high and the share of RES supply is low, a decrease of supply from RES can be met by only a few producers that have spare capacity left to increase production. Consequently, they might even charge higher prices than when the share of RES supply is high. The combined effect can be summarised through our hypotheses below:

1. during periods of low RES supply, the right tail is fatter than the left tail and the difference in fatness will be more pronounced when the demand is higher;
2. during periods of high RES supply, the left tail is fatter than the right tail and the difference in fatness will be more pronounced when the demand is lower.

These statements summarise our expectations about the tails of the empirical power price probability distribution function. The statements directly relate tail fatness on one side and demand and RES supply fundamentals on the other.

### *2.1. Measuring the fatness of the tails*

To observe the fatness of the tails, we apply extreme value theory (EVT) and measure what is called the tail-index. The tail-index is a measure for tail fatness. The following discussion is based on Huisman et al. (2001) unless otherwise stated.

EVT investigates the distribution of tail observations. Fat-tailed distributions are probability distributions whose tails do not exhibit exponential decay such as the normal distribution. Instead they have fatter tails. In the limit, the tail shape follows a Pareto distribution or power law for a general class of fat-tailed distributions. This power law is  $x^{-1/\gamma}$  when  $x$  becomes large. The

parameter  $\gamma$  is the tail-index. The higher  $\gamma$  is, the fatter the tail becomes, i.e. the slower the probability density function decays to zero. This definition is good for the purpose of this paper, but for a more general discussion we refer, for instance, to Huisman et al. (2001) and to Keles et al. (2016). The latter is more recent and applied this to power prices.

Hill (1975) proposed a maximum likelihood estimator for the tail-index of a conditional Pareto distribution. Consider a sample of  $n$  positive and independent observations drawn from some fat-tailed distribution. Let  $x(i)$  be the  $i$ th-order statistic such that  $x(i) > x(i - 1)$  for  $i = 2 \dots n$ . Hill (1975) proposed the following estimator for  $\gamma$ :

$$\gamma(\kappa) = \frac{1}{\kappa} \sum_{j=1}^{\kappa} \ln(x(n - j + 1)) - \ln(x(n - \kappa)). \quad (1)$$

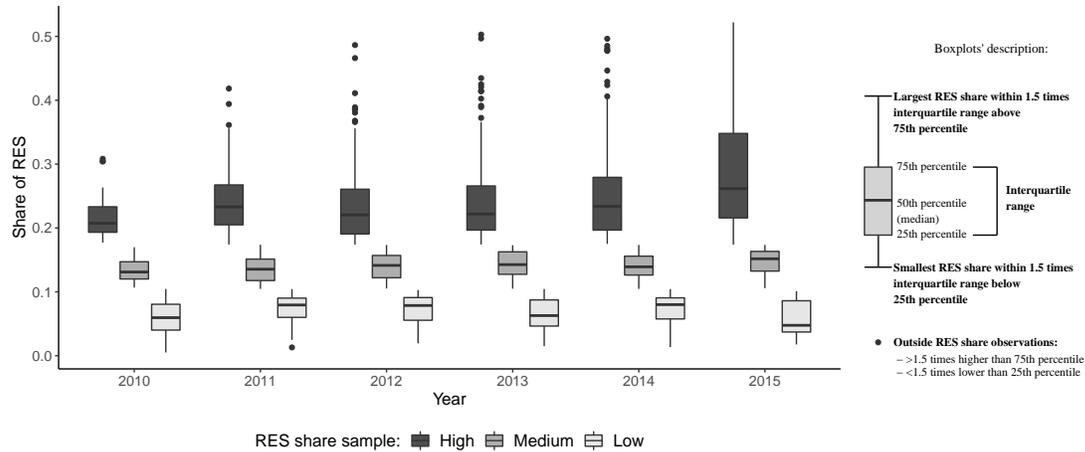
The idea behind the Hill (1975) estimator is that one selects to include the  $\kappa$  largest observations and that one starts at the threshold observation  $x(n - \kappa)$  and that one measures the distance between the other tail observations and that threshold. This estimator is simple and frequently applied, but suffers from the problem that the researcher has to select  $\kappa$ , being the number of tail observations to include in the estimate.

Huisman et al. (2001) suggest a variation of the Hill estimator that reduces the impact of the choice for  $\kappa$ . They observe that the  $\gamma$  estimates from the Hill (1975) estimator increase almost linearly in  $\kappa$  for several fat-tailed distribution functions. They propose the following regression equation:

$$\gamma(k) = \beta_0 + \beta_1 \times k + \epsilon(k), \quad (2)$$

for  $k = 1 \dots \kappa$ . The first reason for using this method is that the evaluation of equation (2) for  $k$  approaching zero yields that  $\beta_0$  becomes an unbiased estimate of  $\gamma$ . We refer to Huisman et al. (2001) for the derivation, a weighted least squares variant, and the calculation of standard errors.

Huisman et al. (2001) argue that their approach, which is less dependent on a subjective choice for  $\kappa$ , provides more robust estimates even in smaller samples. This is the second reason that we choose to adopt this methodology, as we want to observe tail-index estimates from several smaller sub-samples of our data (left and right tails and periods with high/low demand). We do not consider others as we are not interested in the exact levels of the tail-index estimates. Following our hypotheses, we want to observe whether tail-index estimates increase or decrease as a result of the market share of RES and demand, and we are therefore more interested in differences, in fact only higher or lower, and not so much in exact levels. This makes our results being less dependent on the particular tail-index estimation method employed.



Individual boxplots comprise the daily share of RES observations in either the Low, Medium or High share of RES samples, categorized by year. Low, Medium and High RES share samples each contain 669 observations, spread across the five and a half years of data analyzed. All investigated years contain data points from each of the three RES categories.

Figure 2: Share of RES per selected subsample.

Keles et al. (2016) offer an in-depth investigation on the thresholds  $\kappa$  to be used for German electricity prices. They show that the tail-index remains relatively stable between selecting the biggest/smallest 10% and 15% of the observations. We follow their result and select  $\kappa$  to be set at 10% and 15% thresholds, but also include the 20% threshold for robustness reasons.

A last note on estimating the tail-index is that Hill (1975) assumes that the tail observations  $x(i)$  are positive. Assuming that the mean of the distribution function under consideration is zero, this assumption implies that we can only measure the tail-index of the right tail. However, by using the absolute values of all  $x(i)$ 's, we can measure the tail-index for both tails simultaneously, and by taking  $-x(i)$ , we can measure the tail-index for the left tail.

### 3. Data

We use the data from Kyritsis et al. (2017). Their sample consists of German day-ahead (Phe-lix) power prices, solar and wind power generation, and total electricity load from January 2010 to June 2015. We have three reasons why we chose to use their data. First, the time span is characterised by a rapid and large-scale integration of RES. Second, this is a period for which we know that intermittent wind and solar supply explains variation in electricity price volatility. Third, the data enables us to test the relationship between the tails of the power price probability distribution and the share of renewable energy supply.

It is important however to note that our results obtained from the German power market do not necessarily hold in other markets, in particular for those with different market structure and different power generation mixes. Besides the power mix particularities, each power market has also different regulations in place. For the German market during the period investigated, as Patrick et al. (2019) explain, through Renewable Energy Sources Act (Erneuerbare-Energien-Gesetz – EEG) passed in 2000 and further strengthened by Energiewende in 2010, RES output is prioritised for dispatch. These legislative measures led to a feed-in tariff system for the wind and solar supply, a system that incentivises RES producers to maximize their production, regardless of the impact on power prices. For German RES producers, since they are subsidised at a fixed positive EUR/MWh rate, curtailment of supply is not beneficial even at negative day-ahead prices. This aspect, combined with the fact that there is no curtailment obligation imposed at day-ahead market level, leads to RES supply being always placed first in the merit order curve on the German day-ahead market <sup>2</sup>. Another peculiarity of the German power markets is that negative prices are allowed to be bid in the market, and in this way, extreme negative low prices can appear in certain moments.

As we want to examine the differences in tail fatness for high and low demand periods and observe how these differences alter when the share of RES changes, we create different samples. To distinguish between high and low demand periods, we separate between average daily prices (average over 24 hours), average prices during off-peak hours (average over the prices for delivery during hours 21-8) - being low demand periods - and average prices during peak hours (average over the prices for delivery during hours 9-20) - being high demand periods.

To observe the tail fatness for different market shares of RES, we follow the methodology of Kyritsis et al. (2017) and Nicolosi (2010) using actual power generation data for the total, wind and solar output. However, their sampling method creates too many categories and too small samples to draw conclusions. Therefore, we adjust their sampling method to create equally sized sub-samples, which are large enough to measure the tail-index estimates.<sup>3</sup> To differentiate between left and right tail of the distribution of electricity prices, we place the (absolute) values below the median into the left tail observations and the values above the median into the right tail observa-

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<sup>2</sup>As both Michael and Iain (2018) and Patrick et al. (2019) show, it is only by 2016 that curtailment regulation was implemented in the German power market and it affects only the real time markets. Through an amendment to the Renewable Energy Act, up to 3% of RES installed capacity can be curtailed yearly in each Distribution System Operator area and only if such a measure is needed for maintaining grid stability.

<sup>3</sup>Using initially the sampling method of Kyritsis et al. (2017), we obtained similar qualitative results as these that we discuss later.

tions. We are able to work with such a sampling strategy since our methodology treats data as a cross-section where the exact location in time of each data point is not relevant.

Figure 2 illustrates a summary of the share of RES in Germany for the period investigated in each of the three subsamples considered. The sub-sample “Low” contains prices from those days where the market share of supply from RES is between 0.5% and 10.5%. The second sub-sample “Medium” contains those days with supply from RES between 10.5% and 17.4%. The third sub-sample “High” contains the days with supply from RES between 17.4% and 52.2% (the maximum observed market share from RES). By construction, all sub-samples have equal size, and we use the same number of observations to estimate the left and right tail, as well as both tails simultaneously. Each of the three sub-samples contains 669 observations, of which 334 are used to estimate the left-tail and 334 are used to estimate the right tail<sup>4</sup>. Each of the years comprised in the dataset includes days with “Low”, “Medium” and “High” share of RES.

#### 4. Empirical results

Table 1 presents tail-index estimates for various sub-samples assuming that 15% of the observations are tail observations (i.e.  $\kappa$  is 15%). The estimates for  $\kappa$  equal to 10% and 20% are also shown in Table A.1. Since in our hypotheses we develop expectations only for the “Low” and “High” subsamples at the right and left tail of the power price distribution, we focus in this section on analysing these results. The tail-index estimates for the “Medium” samples are also available in Table A.1. With these estimates, we want to test our two statements.

Table 1: Tail-index estimates for samples equally sized by RES supply.  $\kappa$  is 15%.

Share of RES supply	All hours		Peak hours		Off-peak hours	
	Left	Right	Left	Right	Left	Right
Low (0.5-10.5%)	0.16 (0.148)	0.36 (0.001)	0.19 (0.006)	0.31 (0.001)	0.19 (0.006)	0.19 (0.005)
	$t=-1.37$		$t=-19.21^{***}$		$t=0.03$	
High (17.4-52.2%)	0.39 (0.001)	0.06 (0.009)	0.31 (0.001)	0.08 (0.007)	0.53 (0.001)	0.11 (0.022)
	$t=37.19^{***}$		$t=33.03^{***}$		$t=19.04^{***}$	

*Significant at  $p^{***}<0.01$ ,  $p^{**}<0.05$ ,  $p^*<0.1$ ; standard errors in parentheses.  
 $t$  is the  $t$ -value of the difference between the estimates of the left and the right tail.*

<sup>4</sup>Since in each of the three subsamples we have an odd number of observations, the median data point is excluded from the tail index calculations.

Our first statement is that *during periods of low RES supply, the right tail is fatter than the left tail and the difference in fatness will be more pronounced when the demand is higher*. As demand during peak hours is higher than during off-peak hours, the peak hours represent the high demand observations. Let's focus on the first row with numbers. This row contains the tail-index estimates from the sample of power prices from days when the share of intermittent RES supply is lowest. The column headed *peak hours* shows that the tail-index  $\gamma$  for the right tail is 0.31 and for the left tail is 0.19. The difference between the left and right tail-index is highly significant with a t-value equal to -19.21. The higher the tail-index, the fatter the tail is, and we can therefore safely conclude that the right tail is fatter than the left tail. This provides evidence of tail asymmetry, which is in line with our aforementioned expectations.

We also expected that the difference in fatness will be more pronounced when the demand is higher. This is what Table 1 shows as well. We see - as expected - that in the "Low" RES rows for the *off-peak hours*, when demand is typically lower than during peak hours, the right tail-index estimate becomes smaller. In fact, the tail asymmetry appears to not be present in the *off-peak hours* for the "Low" RES samples, and the difference between the tail-indexes of the left and right tail is not statistically significant. From the results regarding our first statement, we observe that, on the German day-ahead market, when RES supply is low, high price spikes are more likely to occur than low price spikes, and are more pronounced when the demand is higher.

Our second statement is that *during periods of high RES supply, the left tail is fatter than the right tail and the difference in fatness will be more pronounced when the demand is lower*. When we move to the "High" tail index row, we see that the relation becomes the opposite: the left tail is now significantly fatter than the right tail. Apparently, in periods with high demand and high share of RES supply, an increase in RES supply, which yields a demand to ramp down supply from non-intermittent producers, leads more frequently to extremely low prices than when a decrease in RES supply occurs. The energy market here is less flexible to deal with RES supply increases than decreases, as only a few non-intermittent power plants produce and therefore can ramp down their supply. Those power plants are more likely to be inflexible producers for whom at certain moments in time ramping down production can be technically infeasible or economically not beneficial. Such a situation might lead to extremely low prices.

During off-peak hours demand is typically lower than during peak hours, and we'll use the samples with prices from off-peak hours as low demand observation. For the periods with an average lower demand, in the "High" sample, where the share of RES supply is high, we clearly see that the left tail is fatter than the right tail, and that the difference between the two tail-indexes is highly

significant with a t-value equal to 19.04. This finding confers additional robustness to our previous result on tail fatness asymmetry, which is in accordance with our expectations. Table A.1 in the appendix shows the same estimates, but for different settings for  $\kappa$ , supporting that the previous conclusions are robust with respect to whether we set  $\kappa$  equal to 10%, 15%, or 20%.

The results that we found are all in line with our expectations. During periods of low share of RES supply, the right tail is fatter than the left tail and the difference in fatness will be more pronounced when demand is higher. During periods of high share of RES supply, left tail is fatter than the right tail and the difference in fatness will be more pronounced when demand is lower.

As mentioned earlier, Kyritsis et al. (2017) show that supply from RES drives power price volatility and that wind and solar energy supply have a different impact. As energy supply from solar sources only occurs during day-time, it increases the reserve margin during peak hours. So far we have examined the impact of aggregate supply from RES on the tails. We also examined the impact of energy supply from wind and solar separately, as in Kyritsis et al. (2017), to observe whether the two impact tails differently. To do so, we sample the data in groups with low and high supply from wind or solar sources. Tables 2 and 3 show how we constructed those samples for wind and solar, respectively.<sup>5</sup> Estimating the tail-index for each sample, and following these two tables, enables us to observe whether wind or solar affects our results differently, as what was observed for volatility by Kyritsis et al. (2017).

Table 2 shows the results for supply from wind sources. We observe the same pattern as what we found before: i) a fatter right than left tail during peak hours and low share of wind supply, and the opposite tail structure when the share of wind supply is at the highest level, and ii) a fatter left than right tail during off-peak hours with a high share of wind supply, which difference disappears when the share of wind supply is lower. The results from only wind supply are no different than the results observed from samples with aggregated energy supply, from both wind and solar sources.

When we measure the tail-index for both tails of the electricity price distribution and thus do not disentangle the effect on the left and right tail, we clearly notice that the higher the share of wind supply, the fatter the tails of the power price probability distribution are. For instance, during all hours the tail-index estimate for low share of wind supply is 0.20 and for high share of

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<sup>5</sup>Tables A.2 and A.3, similar as with A.1 presented in the appendix of this manuscript, include the complete tail-index estimates set for the wind and solar subsamples.

Table 2: Tail-index estimates for samples equally sized by wind supply.  $\kappa$  is 15%.

Share of wind supply	All hours		Peak hours		Off-peak hours	
	Left	Right	Left	Right	Left	Right
Low (0.4-4.9%)	0.09 (0.042)	0.27 (0.002)	0.08 (0.013)	0.31 (0.001)	0.11 (0.012)	0.13 (0.024)
	$t=-4.25^{***}$		$t=-17.76^{***}$		$t=-0.95$	
High (11.2-50.6%)	0.37 0.001	0.02 0.009	0.26 0.002	0.09 0.007	0.53 0.001	0.05 0.010
	$t=36.82^{***}$		$t=26.12^{***}$		$t=46.06^{***}$	

*Significant at  $p^{***}<0.01$ ,  $p^{**}<0.05$ ,  $p^*<0.1$ ; standard errors in parentheses.  
 $t$  is the  $t$ -value of the difference between the estimates of the left and the right tail.*

wind supply is 0.29, which difference is statistically significant <sup>6</sup>. This finding confers additional robustness to our conclusions as it aligns with the findings of previous studies in the literature, for instance Ketterer (2014) and Kyritsis et al. (2017), who employ conditional heteroskedasticity models and find evidence of wind power generation increasing the volatility of electricity price in Germany, and thus the fatness of the tails.

Table 3 shows the tail-index estimates from groups sampled on the share of solar supply. For peak hours, we again observe the same pattern: a fatter right than left tail for low share of solar supply, which difference becomes not significant for higher shares of solar supply. For the “High” solar sample, we observe higher but not statistically significantly different left than right tail-index estimates. This result was drawn from a period when the penetration of solar supply in the German power market was relatively low. Comparing with the results from the wind supply samples, we would expect that for a higher penetration of solar supply to observe a significantly fatter left tail compared to the right one for the German day-ahead power prices during peak hours. This consequence of high solar supply is closely linked with the “duck curve” phenomenon, where, in power markets with a high solar penetration rate, conventional producers are forced during the peak hours to ramp down significantly their production and ramp up again during the off-peak hours. High levels of solar supply generated during peak hours, put pressure on the flexibility of the power markets and in the same time they lower the prices, decreasing the probability of high spike occurrences. As opposed to tables 1 or 2, in table 3 we do not present the results for the off-peak hours and we choose to do this because of the date limitations that we face. The data used includes the level of solar supply as a total daily output without distinguishing between the hours when it was produced. As solar supply is generated in mostly during peak hours, and in

<sup>6</sup>These results are displayed in Table A.2 within the appendix of this manuscript.

many winter days exclusively during peak hours, we cannot draw expectations and conclusions based on the off-peak solar samples.

Both Tables 2 and 3 show the estimates in which we include 15% of the observations ( $\kappa$ ) to estimate the tail-index. Tables A.2 and A.3 in the appendix also show the tail-index estimates from samples with different settings for  $\kappa$ .

Table 3: Tail-index estimates for samples equally sized by solar supply.  $\kappa$  is 15%.

Share of solar supply	All hours		Peak hours	
	Left	Right	Left	Right
Low (0.0-2.2%)	0.21 (0.004)	0.26 (0.002)	0.09 (0.012)	0.24 (0.002)
	$t=-11.63^{***}$		$t=-12.97^{***}$	
High (6.7-20.9%)	0.23 (0.002)	0.18 (0.013)	0.28 (0.001)	0.15 (0.185)
	$t=3.63^{***}$		$t=0.68$	

*Significant at  $p^{***}<0.01$ ,  $p^{**}<0.05$ ,  $p^*<0.1$ ; standard errors in parentheses.  
 $t$  is the  $t$ -value of the difference between the estimates of the left and the right tail.*

Our results depend on the assumption that the energy market is not flexible enough to respond easily to changes in the supply of RES, and therefore shed light on the value of electricity storage solutions. Fat tails make the value of an option to store power, or an option to curtail production, higher than when tails are thin. For power storage facilities our results imply that one wants to have them charged when demand is high, and especially when the share of RES is low. One wants them to be discharged, being ready to charge, when demand is low and the share of RES is high. This charging strategy that follows from our results makes perfect sense, as it follows that the power storage facility is charged during periods when RES supply is abundant (low demand and a high share of RES) and discharged when RES supply is relatively scarce (high demand and a low share of RES).

## 5. Concluding remarks

RES supply, being a variable source of power production, poses challenges to power markets as they are often not flexible enough to counterbalance RESs variation in production volumes, since power storage is insufficient and power demand is inelastic. Non-intermittent producers are the only ones that can provide this flexibility, and we argue that they either do exercise market power during times when the supply of flexibility is low or that they are technically constrained in such moments being not capable of supplying the needed flexibility. As a consequence, in such mo-

ments, extremely high or low prices are more likely to occur.

Using extreme value theory, we demonstrate that the tails of the power price probability distribution are fatter when the supply of flexibility is low. Such moments of low power flexibility occur when both the reserve margins of non-intermittent suppliers and RES supply are either at low or at high levels. More specifically we find support for our claims that i) during periods of high share of RES, the left tail is fatter than the right tail and the difference in fatness will be more pronounced when demand is lower, and ii) during periods of low share of RES, right tail is fatter than the left tail and the difference in fatness will be more pronounced when the demand is higher. When we focus separately on the share of wind and solar supply, instead of aggregate supply from RES, we find the same results for wind and for solar.<sup>7</sup>

Although it was already known that power prices are not normally distributed, this paper shows that the amount of non-normality in the tails, i.e. the tail fatness, can be forecasted by demand and volume of RES. For risk managers, this implies that risk models should be made conditional on those variables and one should use models in which the tail structure can be flexibly adjusted to the supply and demand conditions. This will also impact hedging decisions as one would like to hedge more for those periods when extreme losses may be expected. Another implication of this is that those who assess the value of storage facilities or determine storage strategies may want to include these conditional tail estimates in their models. By doing so, they will bring their charge/discharge decisions more in line with the demand for flexibility.

In order to achieve large-scale integration of RES in the power system, policy makers and market participants should have a clear understanding of the requirements for power system flexibility. This study provides insights into when, and to what extent, extreme prices occur depending on the electricity demand and RES supply, and thereby the demand for flexibility to adjust electricity supply through non-intermittent producers. This will directly affect the value of power storage facilities or, options to curtail production from RES, during periods of time when the power system cannot provide sufficient flexibility to adjust production while RES supply increases. Considering that with the increasing share of RES we will have more variability in the power system and more frequent extreme low prices, we call for rethinking the way new RES installed capacities are subsidised in Germany. Besides encouraging the increase in share of RES, it is also important to

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<sup>7</sup>Although, to be more precise, not for solar during off-peak hours. This is, however, a result that we do not consider relevant, as the sun does not shine during night time and the observation most likely cannot be attributed to solar supply.

promote and protect power market flexibility and the stability of the grid. One type of policies could put us closer to this path would be encouraging only instalments of RES that have attached storage capacity so that in the critical moments, RES can ramp down or ramp up production. Moreover, further rethinking the options of economic curtailment of RES as a measure of adding flexibility to the market could also be part of the solution for adding more flexibility on the lower part of the distribution function of German day-ahead power prices.

We call for additional research based on higher frequency data and on a much wider variety of countries with varying level of flexible power generation and intermittent renewable energy sources. The latter would provide a more accurate picture of the impact of intermittent RES on the power system. Another path that is worth investigating is looking at how the integrated assessment models can be improved based on the information that RES supply is changing the probability of low and high price spike occurrence.

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## 6. Appendix

Table A.1: Tail-index estimates for samples equally sized by RES supply

Share of RES supply	Selected threshold	All hours			Peak hours			Off-peak hours		
		Left	Both	Right	Left	Both	Right	Left	Both	Right
Low (0.5-10.5%)	tail at 10%	0,17	0,27	0,35	0,19	0,32	0,39	0,14	0,17	0,20
	se at 10%	(0,000)	(0,000)	(0,000)	(0,000)	(0,000)	(0,000)	(0,001)	(0,000)	(0,000)
	<i>(t-statistic)</i>	<i>t=-337***</i>			<i>t=-460,17***</i>			<i>t=-54,62***</i>		
	tail at 15%	0,16	0,26	0,36	0,19	0,28	0,31	0,19	0,19	0,19
	se at 15%	(0,148)	(0,001)	(0,001)	(0,006)	(0,001)	(0,001)	(0,006)	(0,004)	(0,005)
	<i>(t-statistic)</i>	<i>t=-1,37</i>			<i>t=-19,21***</i>			<i>t=0,03</i>		
tail at 20%	0,17	0,25	0,34	0,16	0,25	0,34	0,18	0,19	0,18	
se at 20%	(0,026)	(0,008)	(0,003)	(0,016)	(0,008)	(0,003)	(0,040)	(0,007)	(0,027)	
<i>(t-statistic)</i>	<i>t=-6,4***</i>			<i>t=-10,73***</i>			<i>t=0,11</i>			
Medium (10.5-17.4%)	tail at 10%	0,09	0,1	0,09	0,05	0,05	0,06	0,11	0,05	0,03
	se at 10%	(0,030)	(0,017)	(0,012)	(0,008)	(0,008)	(0,005)	(0,037)	(0,008)	(0,011)
	<i>(t-statistic)</i>	<i>t=-0,13</i>			<i>t=-1,59</i>			<i>t=1,95*</i>		
	tail at 15%	0,11	0,10	0,11	0,11	0,11	0,11	0,13	0,08	0,03
	se at 15%	(0,017)	(0,039)	(0,007)	(0,744)	(0,011)	(0,016)	(0,018)	(0,006)	(0,010)
	<i>(t-statistic)</i>	<i>t=0,29</i>			<i>t=0,01</i>			<i>t=4,82***</i>		
tail at 20%	0,13	0,1	0,11	0,12	0,12	0,12	0,13	0,09	0,07	
se at 20%	(0,007)	(0,103)	(0,216)	(0,055)	(0,018)	(0,030)	(0,022)	(0,015)	(0,010)	
<i>(t-statistic)</i>	<i>t=0,08</i>			<i>t=0,05</i>			<i>t=2,56**</i>			
High (17.4-52.2%)	tail at 10%	0,43	0,34	0,03	0,32	0,25	0,06	0,65	0,45	0,05
	se at 10%	(0,000)	(0,000)	(0,008)	(0,000)	(0,000)	(0,014)	(0,000)	(0,000)	(0,062)
	<i>(t-statistic)</i>	<i>t=48,01***</i>			<i>t=18,58***</i>			<i>t=9,63***</i>		
	tail at 15%	0,39	0,29	0,06	0,31	0,21	0,08	0,53	0,38	0,11
	se at 15%	(0,001)	(0,001)	(0,009)	(0,001)	(0,002)	(0,007)	(0,001)	(0,001)	(0,022)
	<i>(t-statistic)</i>	<i>t=37,19***</i>			<i>t=33,03***</i>			<i>t=19,04***</i>		
tail at 20%	0,37	0,25	0,07	0,25	0,18	0,09	0,48	0,34	0,11	
se at 20%	(0,003)	(0,008)	(0,010)	(0,014)	(0,019)	(0,008)	(0,002)	(0,002)	(0,007)	
<i>(t-statistic)</i>	<i>t=29,49***</i>			<i>t=9,85***</i>			<i>t=49,49***</i>			

Significant at  $p^{***}<0,01$ ,  $p^{**}<0,05$ ,  $p^*<0,1$ ; tail = tail-index estimate; se = standard errors.

$t$  is the  $t$ -value of the difference between the estimates of the left and the right tail.

Table A.2: Tail-index estimates for samples equally sized by supply from wind sources

Share of wind	Selected threshold	All hours			Peak hours			Off-peak hours		
		Left	Both	Right	Left	Both	Right	Left	Both	Right
Low (0.4-4.9%)	tail at 10%	0,08	0,13	0,29	0,04	0,15	0,29	0,06	0,11	0,12
	se at 10%	(0,010)	(0,001)	(0,000)	(0,011)	(0,000)	(0,000)	(0,006)	(0,016)	(0,003)
	<i>(t-statistic)</i>	$t=-20,67^{***}$			$t=-21,33^{***}$			$t=-10,1^{***}$		
	tail at 15%	0,09	0,2	0,27	0,08	0,17	0,31	0,11	0,14	0,13
	se at 15%	(0,042)	(0,003)	(0,002)	(0,013)	(0,008)	(0,001)	(0,012)	(0,018)	(0,024)
	<i>(t-statistic)</i>	$t=-4,25^{***}$			$t=-17,76^{***}$			$t=-0,95$		
tail at 20%	0,15	0,22	0,27	0,12	0,18	0,29	0,14	0,14	0,11	
se at 20%	(0,008)	(0,059)	(0,008)	(0,036)	(0,023)	(0,006)	(0,016)	(0,010)	(0,014)	
<i>(t-statistic)</i>	$t=-10,29^{***}$			$t=-4,56^{***}$			$t=1,16$			
Medium (4.9-11.1%)	tail at 10%	0,13	0,23	0,32	0,13	0,22	0,35	0,11	0,15	0,21
	se at 10%	(0,002)	(0,000)	(0,000)	(0,002)	(0,000)	(0,000)	(0,008)	(0,001)	(0,000)
	<i>(t-statistic)</i>	$t=-114,35^{***}$			$t=-124,06^{***}$			$t=-12,29^{***}$		
	tail at 15%	0,13	0,18	0,26	0,08	0,19	0,35	0,11	0,14	0,16
	se at 15%	(0,021)	(0,006)	(0,002)	(0,007)	(0,004)	(0,001)	(0,046)	(0,038)	(0,041)
	<i>(t-statistic)</i>	$t=-5,99^{***}$			$t=-39,93^{***}$			$t=-0,73$		
tail at 20%	0,11	0,17	0,25	0,08	0,19	0,33	0,14	0,12	0,15	
se at 20%	(0,007)	(0,022)	(0,012)	(0,010)	(0,014)	(0,004)	(0,019)	(0,242)	(0,037)	
<i>(t-statistic)</i>	$t=-10,83^{***}$			$t=-24,21^{***}$			$t=-0,41$			
High (11.2-50.6%)	tail at 10%	0,42	0,37	0,04	0,31	0,26	0,08	0,64	0,47	0,03
	se at 10%	(0,000)	(0,000)	(0,007)	(0,000)	(0,000)	(0,014)	(0,000)	(0,000)	(0,008)
	<i>(t-statistic)</i>	$t=53,54^{***}$			$t=15,79^{***}$			$t=77,16^{***}$		
	tail at 15%	0,37	0,29	0,02	0,26	0,22	0,09	0,53	0,36	0,05
	se at 15%	(0,001)	(0,001)	(0,009)	(0,002)	(0,002)	(0,007)	(0,001)	(0,001)	(0,010)
	<i>(t-statistic)</i>	$t=36,82^{***}$			$t=26,12^{***}$			$t=46,06^{***}$		
tail at 20%	0,35	0,25	0,03	0,24	0,21	0,07	0,47	0,32	0,06	
se at 20%	(0,003)	(0,009)	(0,011)	(0,018)	(0,029)	(0,015)	(0,002)	(0,003)	(0,011)	
<i>(t-statistic)</i>	$t=28,34^{***}$			$t=7,1^{***}$			$t=35,49^{***}$			

Significant at  $p^{***}<0,01$ ,  $p^{**}<0,05$ ,  $p^*<0,1$ ; tail = tail-index estimate; se = standard errors.

$t$  is the  $t$ -value of the difference between the estimates of the left and the right tail.

Table A.3: Tail-index estimates for samples equally sized by supply from solar sources

Share of solar supply	Selected threshold	All hours			Peak hours			Off-peak hours		
		Left	Both	Right	Left	Both	Right	Left	Both	Right
Low (0-2.2%)	tail at 10%	0,23	0,20	0,24	n.a.	0,10	0,19	0,38	0,32	0,20
	se at 10%	(0,000)	(0,000)	(0,000)	n.a.	(0,000)	(0,000)	(0,000)	(0,000)	(0,000)
	( <i>t</i> -statistic)	<i>t</i> =-31,62***			<i>n.a.</i>			<i>t</i> =454,31***		
	tail at 15%	0,21	0,25	0,26	0,09	0,17	0,24	0,36	0,32	0,18
se at 15%	(0,004)	(0,001)	(0,002)	(0,012)	(0,025)	(0,002)	(0,001)	(0,001)	(0,012)	
( <i>t</i> -statistic)	<i>t</i> =-11,63***			<i>t</i> =-12,97***			<i>t</i> =14,56***			
tail at 20%	0,25	0,26	0,24	0,17	0,22	0,26	0,32	0,31	0,17	
se at 20%	(0,015)	(0,006)	(0,022)	(0,022)	(0,022)	(0,010)	(0,004)	(0,003)	(0,028)	
( <i>t</i> -statistic)	<i>t</i> =0,33			<i>t</i> =-3,49***			<i>t</i> =5,19***			
Medium (2.2-6.7%)	tail at 10%	0,27	0,30	0,31	0,08	0,22	0,33	0,44	0,34	0,17
	se at 10%	(0,000)	(0,000)	(0,000)	(0,050)	(0,000)	(0,000)	(0,000)	(0,000)	(0,000)
	( <i>t</i> -statistic)	<i>t</i> =-124,64***			<i>t</i> =-5,16***			<i>t</i> =482,25***		
	tail at 15%	0,25	0,32	0,26	0,11	0,25	0,32	0,35	0,36	0,17
se at 15%	(0,002)	(0,001)	(0,002)	(0,081)	(0,001)	(0,001)	(0,001)	(0,001)	(0,042)	
( <i>t</i> -statistic)	<i>t</i> =-5,18***			<i>t</i> =-2,56**			<i>t</i> =4,28***			
tail at 20%	0,28	0,32	0,22	0,15	0,24	0,29	0,34	0,36	0,14	
se at 20%	(0,006)	(0,003)	(0,080)	(0,019)	(0,011)	(0,006)	(0,003)	(0,002)	(0,028)	
( <i>t</i> -statistic)	<i>t</i> =0,74			<i>t</i> =-6,8***			<i>t</i> =7,45***			
High (6.7-20.9%)	tail at 10%	0,24	0,19	0,10	0,33	0,23	0,12	0,11	0,08	0,04
	se at 10%	(0,000)	(0,000)	(0,080)	(0,000)	(0,000)	(0,004)	(0,008)	(0,011)	(0,010)
	( <i>t</i> -statistic)	<i>t</i> =1,77*			<i>t</i> =47,78***			<i>t</i> =5,92***		
	tail at 15%	0,23	0,20	0,18	0,28	0,22	0,15	0,14	0,11	0,10
se at 15%	(0,002)	(0,002)	(0,013)	(0,001)	(0,002)	(0,185)	(0,040)	(0,006)	(0,018)	
( <i>t</i> -statistic)	<i>t</i> =3,63***			<i>t</i> =0,68			<i>t</i> =1,01			
tail at 20%	0,21	0,21	0,19	0,26	0,22	0,16	0,18	0,13	0,11	
se at 20%	(0,011)	(0,144)	(0,038)	(0,010)	(0,040)	(0,027)	(0,012)	(0,020)	(0,010)	
( <i>t</i> -statistic)	<i>t</i> =0,66			<i>t</i> =3,39***			<i>t</i> =4,87***			

Significant at  $p^{***}<0,01$ ,  $p^{**}<0,05$ ,  $p^*<0,1$ ; tail = tail-index estimate; se = standard errors.

*t* is the *t*-value of the difference between the estimates of the left and the right tail.