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DYNAMIC BEHAVIOUR OF THE FIRM UNDER DUAL INCOME TAXATION

Abstract: This study examines certain incentive aspects of the dual income tax system (DIT) operated in the four major Nordic countries since the beginning of the 1990s. In this tax system capital income is taxed at a flat rate, whereas earned income is subject to a conventional progressive schedule. The analysis focuses on the splitting of dividend income received from a closely held firm into capital and earned income parts. A deterministic, dynamic investment model as developed by Sinn (1991) is applied. This basic model is extended in several directions. The study in chapters 2 and 3 reveals that Nordic DIT may have strong effects on the firm's investment and financing behaviour. In extreme cases the firm's cost of capital can even be negative. These effects may also have efficiency and welfare consequences. The study also shows that the distortions are very sensitive to the content of the capital base concept. For instance, when financial assets are included in the capital base the firm's marginal return on capital is at the level of the nominal interest rate, thus eliminating the real distortions discussed above.

Key words: dual income taxation, corporate taxation, investment incentives
JEL classification: H25, H32

Tiivistelmä: Tutkimus tarkastelee Pohjoismaissa 1990-luvun alussa käyttöönotetun eriytetyn tuloverotuksen eräiden piirteiden ohjausvaikutuksia yrityksen investointi- ja rahoituspäätöksiin. Kyseinen järjestelmä verottaa pääomatuloa suhteellisella verokannalla, kun taas ansiotuloihin kohdistuu progressiivinen verotus. Tutkimus keskittyy tarkastelemaan suppeassa omistuksessa olevan osakeyhtiön osingon verotusta. Tutkimus soveltaa Sinnin (1991) determinististä, dynaamista investointimallia. Perusmallista esitetään useita variaatiota. Luvuissa 2 ja 3 osoitetaan, että eriytetyllä tuloverotuksella voi olla voimakas vaikutus yrityksen investointi- ja rahoituspäätöksiin. Yrityksen pääomakustannus voi olla verotuksen vaikutuksesta jopa negatiivinen. Tämä voi heikentää talouden tehokkuutta ja alentaa hyvinvointia. Tutkimus osoittaa kuitenkin myös, että ohjausvaikutukset ovat herkästi riippuvia osingon pääomatulo-osuuden laskennassa sovellettavan pääomakäsitteen yksityiskohtaisesta sisällöstä. Jos esimerkiksi sijoitukset finanssivarallisuuteen luetaan mukaan pääomapohjaan, pääoman rajatuotto asettuu markkinakoron tasolle ja reaalitaloudellista vääristymää ei synny.

Asiasanat: eriytetty tuloverotus, yritysverotus, investoinnit
JEL classification: H25, H32

Preface

The idea of this study first came in to me in the mid 1990s from a personal need to obtain a deeper and more analytical understanding of the functioning of certain features of the recent tax reforms. Taking on the task entailed moving to an academic environment after a period of more than a decade in more practical work at the Ministry of Finance and the National Board of Taxation. Without the help and support of several persons I would never have been able to complete this project.

My warmest thanks go to my supervisors, Professors Seppo Salo and Jouko Ylä-Liedenpohja, for their professional guidance and for their patience and encouragement in times of difficulty. They came to the rescue on several occasions, when I had reached the limits of my analytical abilities. I am also heavily indebted to Professor Pertti Haaparanta, who was ready to discuss research problems whenever I needed it. Moreover, without his encouragement I would not have pitched the study at this level.

I also owe much to my official pre-examiners, Professor Søren Bo Nielsen and Doctor Luis H.R. Alvarez, for their expert comments. Luis took on the task of reading the manuscript with extreme care. His detailed comments did not make the final weeks easier, but undoubtedly improved the end result. Any remaining errors and inelegancies in mathematical presentation are of course entirely due to me.

I am grateful to Doctor Martti Hetemäki, whose encouragement and help were decisive in getting the study off the ground. I also owe much to my superiors at the Tax Department of the Ministry of Finance for their understanding. My growing interest in theoretical analysis undoubtedly affected my work effort when at the department. I hope now that this investment will bring benefits for all of us. Thanks also to my colleague Terhi Järvikare at the Tax Department for her valuable comments.

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1. Introduction

1.1 Background to the Study

In the late 1980s and early 1990s the Nordic countries implemented ambitious reforms of their individual and corporate income taxation. They abandoned the principle of global income taxation that had long guided the evolution of their income tax systems and adopted a system of so-called dual income taxation (DIT).

Whereas in an ideal global income tax system all economic income is subject to a single progressive tax schedule, the Nordic innovation made a sharp distinction between capital income and earned income (labour income, pensions and social benefits). The former category of income is taxed at a proportional and fairly low rate while the latter is subject to a conventional progressive tax schedule.[1]

The Nordic model had the very ambitious objective of solving many of the traditional problems of capital income taxation and at the same time guiding the Nordic high-tax welfare economies to a world of increasing capital mobility. The major domestic goals of the reform were to improve savings incentives, alleviate the problems arising from taxing inflationary gains, limit opportunities for tax arbitrage and reduce the distortions caused by the non-uniform treatment of different kinds of capital income in the old system.

The idea of dualistic taxation was first developed and implemented in Denmark, but the benefits of a separate proportional tax on capital income have also been emphasised outside the Nordic countries, e.g. by King (1987a, 1987b). Theoretical support for this tax system is also given by the Johansson-Samuelson theorem, expanded upon by Sinn (1987), which says that a uniform tax on all capital income is neutral with respect to investment.[2,3]

Much of the criticism levelled against dual income taxation emphasises the violation of the principles of horizontal and vertical equity. As shown by Sörensen (1994), these arguments are seriously weakened if one assumes an inflationary economy and takes a lifetime perspective, as applied frequently by proponents of consumption tax, instead of the static and nominalistic approach typical of the popular discussion.

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[1] See Viherkenttä (1993), Sörensen (1994,1998) and NSFR (1993,1996).

[2] Note that DIT has also raised interest outside the Nordic countries, see Head (1997) and Cnossen (1996, 1998). Both authors compare DIT to the CEBIT reform proposal of the US Treasury.

[3] Note that the following conditions must be satisfied: fiscal depreciation equals true economic depreciation and there is no double taxation of corporate profits. In an uncertain environment symmetric treatment of taxable profits and losses is also required.

Another strand of criticism against DIT concerns its treatment of small and medium-sized firms (SMEs). One important part of the Nordic systems is that income received from different kinds of firms is apportioned to earned income and capital income. This is done to satisfy the requirement of equal treatment of wage earners and persons receiving their earned income from their own firms. Since the two income components are not readily distinguishable in the case of business income, this division is performed on an estimated basis by defining capital income as an imputed return on the capital invested in the firm and by taxing the residual as earned income.

Critics have pointed out that this division entails a system that increases the administrative costs of taxation and that despite considerable preventive legislation DIT opens new opportunities for tax planning in the area of SMEs, especially through the transfer of income earned on labour inputs to lightly taxed capital income.

This tax planning opportunity is of key importance for the present study. In the popular tax debate it is usually seen as a problem of horizontal equity and also as one of government revenues. Much less attention has been drawn to the question of how this tax system distorts the behaviour of firms in the SME sector.[4] This neglect is remarkable in the light of the strong emphasis on the neutrality principle in the discussion surrounding the tax reforms. Another, but related, problem in the debate has been its static nature. More interest has been devoted to the question of how to prevent one-off manipulations of the capital base used in calculating the imputed capital income, for example by bringing private low-yielding assets into the firm, than to the question of how the opportunity to affect the capital base distorts the decision-making of firms in the long run.

The aim of this study is to examine the behavioural effects of the division of business income under DIT using a microeconomic dynamic model framework. We try to identify the incentives that the basic structures of the system have for the firm's investment and financing behaviour. We confine the study to the taxation of a closely held corporation and have in mind the Finnish tax system but perform the study in a somewhat more general framework.

The next section of this chapter gives an overview of the corporate tax systems applied in the OECD countries and describes the Nordic dual income tax systems in some detail. Section 1.3 presents a short review of the theory of corporate

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[4] The Nordic tax system is examined from an economic point of view e.g. by Sannarnäs (1995), Södersten (1996), Fjaerli and Lund (1997), Nielsen and Sörensen (1997), Valkonen (1997) and Sörensen (ed.) (1998). Furthermore, Sörensen and Hagen (1996) discuss at some length the division of income received from small businesses. However, none of these gives an analytical treatment of the division system. Kari et al. (1998) presents some preliminary results on the effects of the division system using the approach of this thesis.

taxation that forms the methodological basis for our analysis in chapters 2 - 4. The last section gives the outline of this study.

1.2 Institutional Framework

In this section we explain the central features and concepts of the institutional framework of this study. We first give a review of the different forms of corporate tax systems and then introduce the central principles of the Nordic dual income taxation. We will concentrate on the main rules behind the tax systems. A more thorough treatment of the institutional details can be found in OECD (1991), Messere (1993) and NSFR (1993, 1996).

The capital income tax systems of the western countries are widely divergent, both in their tax rates and structures. One of the unifying features between them is the treatment of corporations as separate entities in taxation.[5] This practice, together with the principle of global income taxation, has led to double taxation of corporate profits, meaning that profits are subject to taxation first at the firm level and second as dividends or capital gains at the shareholder level. This practice is seen as a source of many kinds of distortions.

Most countries have introduced some instruments to mitigate this double taxation. Table 1.1 gives a classification of the different systems used in the OECD countries. There are basically three different categories: first, a system with no relief for double taxation, often called the classical system, second, a system with relief at the firm level, and finally, a system where relief is given at the shareholder level.

Several countries, among them Sweden and the USA, use the classical system. However, the great majority of the OECD countries mitigate double taxation by providing relief at the shareholder level. The imputation system, as applied by the largest EU countries, France, Italy, Germany and United Kingdom, is the most widely applied method in this category. Under the imputation system partial or full credit for the corporate tax on distributed profit is deducted from the shareholder's personal tax bill. There are two distinguishing features associated with the imputation system. First, the shareholder is taxed on the dividend grossed up by the credit for the corporate tax. Second, the system contains a compensatory tax which prevents credit being given if dividends are paid from untaxed profits. Application of this compensatory tax assumes very different and complicated forms in different countries. It may include a carry-back or a carry-forward system, or both, as in the UK, according to which the compensatory tax can be

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[5] See OECD (1991) or Messere (1993) for reasons why there is a separate corporation tax.

charged against previous or future tax surpluses.[6] Finland applies the carry-back method. We discuss these features later in chapter 3.

Several countries provide dividend relief at the shareholder level using somewhat simpler methods of taxing dividends at a low special rate or giving tax credit for dividends without any direct link to the amount of the corporate tax. This latter method does not include the grossing up of the dividend and/or the compensatory tax just mentioned.

Relief at the corporate level is today quite rare but was used previously by many countries, e.g. West Germany, Sweden and Finland.[7]

One source of double taxation of corporate profits is capital gains taxation. Norway seems to be one of the very few countries with a systematic method to eliminate double taxation of retained profits.[8] One potential reason for this rarity is that the effective tax rate on capital gains is fairly low in most countries.

It should be noted that the Nordic countries have very different systems. In 1996 Sweden operated a classical system with strong non-neutralities.[9] Denmark applied modest relief at the shareholder level and Norway and Finland had full imputation systems. The Norwegian and Finnish systems differed due to the elimination of double taxation of retained earnings in Norway. Another special feature of the Norwegian system is the use of the partnership method in the taxation of closely held corporations in certain cases.

Next we turn to a review of some central features of the Nordic dual income tax system. As stated above, this tax system departs from the principle of global income taxation and separates the taxation of capital income from other types of income, such as labour income, pensions and government transfers. In this study this latter category of income is called earned income. All capital income, such as dividends, rents, interest income and realised capital gains, is subject to the same low proportional tax rate, in contrast to the progressive tax scheme with high top rates applied to earned income.[10] Corporate profits are also taxed at the capital

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[6] A tax surplus means here a corporate tax that is not credited to the shareholders, i.e. taxes paid on profits not distributed as dividends.

[7] West Germany moved from a split rate system to full imputation in 1976. Finland changed its partial dividend deduction system to full imputation in 1990. Sweden abolished its dividend deduction system in 1994.

[8] Mexico also eliminates double taxation of retained earnings, see Gjems-Onstad (1996).

[9] Actually Sweden has changed its corporate tax system frequently. Up to 1993 it applied a dividend deduction system, in 1994 a system with a zero tax rate for dividends and from 1995 a classical system. From the beginning of 1997 Sweden introduced restricted dividend relief granted to dividends received from unlisted corporations.

[10] Note that we speak here of nominal rates. The effective tax rate on capital income is not necessarily 'low' in the Nordic countries. Ylä-Liedenpohja (1997) and Sörensen (1994) discuss this question.

income tax rate. Sweden departed from this principle in 1995 by introducing a rate structure where the tax rate on capital income is 30% and the tax rate on corporate profits 28%.

Table 1.1   Corporate Tax Systems in 1996

Classical system (No relief)	Relief at shareholder level		Relief at company level
	Imputation system	Other[1]	
Netherlands	Australia	Austria	Germany[2]
Sweden	Canada	Belgium	Iceland
Switzerland	Finland	Denmark	
Turkey	France	Greece	
United States	Germany[2]	Japan	
	Ireland	Spain	
	Italy	Portugal	
	New Zealand		
	Norway		
	United Kingdom		

Sources: Cnossen (1998), Messere (1993)
1) Portugal and Spain: simple credit without gross-up. Other countries: special tax rate on dividends.
2) Germany is shown twice since it has a combination of an imputation system and a split-rate system.

Nominal capital gains are subject to taxation at the full capital income tax rate under DIT. There is no indexation for inflation or other special provisions. The system also aims at symmetric treatment of income and expenses.

One stylised feature of the Nordic model, stressed e.g. by Sörensen (1994) and Nielsen and Sörensen (1997), is that the proportional tax rate on capital income is set at the level of the lowest marginal tax rate for earned income. In Norway and Sweden this feature has corresponded to the reality quite exactly. In Finland, however, an increase in the tax rate for capital income by three percentage points from 25 per cent to 28 per cent in 1996 raised the capital income tax rate slightly above the lowest rate for earned income.[11]

Table 1.2 illustrates the rate structures of the income systems in the Nordic countries. Note that the top rate on earned income is more than twice as high as the nominal tax rate on capital income in Finland, the ratio being a little lower in

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[11] In Finland the income tax liability at income levels lower than the threshold for the central government tax schedule consists of local tax, church tax and employees' social security contributions. The average combined rate for these taxes in 1993-1998 has been:

Year	1993	1994	1995	1996	1997	1998
Tax rate, %	24.7	26.1	25.9	25.3	25.4	25.2

Sweden and Norway. Denmark, which was the first to introduce a dual income tax system, today operates a schedular system, with several separate rate schemes for different types of capital income.

Table 1.2   *Marginal Income Tax Rates in the Nordic Countries in 1995, %*

| | Marginal tax rate on earned income | | Tax rate on capital income | Tax rate on corporate income |
| | Level of income, FIM [2] | Top rate | | |
	103,000	193,000			
Denmark	47.0	59.0	63.5	40/63.5[1]	34.0
Finland	45.0	56.3	64.3	28.0	28.0
Norway	36.0	48.0	49.5	28.0	28.0
Sweden	34.2	55.0	55.0	30.0	28.0

Sources: Cnossen (1998), Laakkonen and Lehtinen (1997), Lodin (1996), Viitamäki (1995)
1) Top marginal tax rate on dividends and long-term capital gains 40% and on interest income 63.5%.
2) Single-earner couple with two children, Laakkonen and Lehtinen (1997). Figures are for 1997.

An important aspect of the Nordic income tax systems is the division of business income received from small and medium-sized enterprises into capital and labour income components. This division is made on an estimated basis by calculating first an imputed return on the capital invested in the firm, regarding this as capital income, and treating the residual of business income as earned income. This division applies to income derived from all organisational forms: profits of sole proprietorships and partnerships as well as dividends from closely held corporations.

The aim of this estimated division is to improve the neutrality of the tax system with respect to the organisation of economic activity. This neutrality principle requires that the effective tax burden of capital should be independent of the form of ownership, and, similarly, that the effective burden on labour inputs should be the same whether they are employed in the supplier's own firm or are hired by other firms. This principle is automatically achieved in a global comprehensive income tax, with a uniform tax schedule. However it is hard to implement in a tax system where different rate schedules are applied to different types of income.[12]

In the Nordic systems the imputed return on capital is calculated as the product of a presumed rate of return and the capital invested in the firm. These two concepts have been defined differently in the Nordic countries. Table 1.3 illustrates the main differences concerning the definition of the capital base. The comparison concentrates on closely held corporations. Denmark is not included because it

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[12] See Sörensen and Hagen (1996) for a discussion of the theoretical and practical issues in this area.

applies the division rules only to sole proprietors and partnerships. Closely held corporations are taxed according to the general rules applying to all corporations.

Table 1.3   Characteristics of the Division Systems, Closely Held Corporations

	Capital base	Financial assets included in the base	Valuation[1]	Income division applied for corporations
Finland	net assets	yes	wtv	Unlisted corporations
Norway	gross assets	no	wtv	Closely held corporations with active shareholders
Sweden	net assets	yes	ap[2]	Closely held corporations with active shareholders

Source: Lodin (1996)
1) wtv = value used in wealth taxation, ap = historical acquisition price of the shares.
2) For sole proprietors, however, the capital base in Sweden is valued at wtv.

Norway employs a so-called gross method, where the capital base is defined as the firm's gross business assets. One special feature of the Norwegian system is that financial assets are excluded from the base. Assets are assessed at the value used in wealth taxation. This means that many asset types are included in the capital base assessed at book value. The presumed rate of return is defined in Norway as the long-term interest rate on government bonds plus a fixed risk premium of 6% (in 1995). In Norway an income division is applied to corporations if at least two-thirds of the firm's shares are owned by active shareholders, i.e. owners who also sell labour inputs to the firm. Due to the partnership method operated by Norway this division is not applied to dividends but to the portion of corporate profits that is taxed as the income of the active owner.

Finland has a pure net method, where the capital base is the firm's current net assets (gross assets minus debt) assessed at the taxable value of those assets. Financial assets are included in the capital base. The presumed rate of return is fixed and has been considerably higher than long-term interest rates (15% in 1993-1998, 13.5% from the beginning of 1999). A division is applied to the profits of partnerships and sole proprietors and dividends received from domestic corporations not listed on the Helsinki Stock Exchange.

One feature of the systems employed by Finland and Norway is that the capital base includes physical assets fairly comprehensively but does not include all categories of intangibles, such as capitalised expenditure of R&D activities.[13]

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[13] See Sörensen and Hagen (1996) and Järvikare (1997).

Thus the systems of these two countries seem to treat different types of investment in a non-neutral fashion.

Sweden uses a net method and the presumed rate of return is defined in a similar fashion to that in Norway. The division of income is applied to partnerships, sole proprietors and closely held corporations with active shareholders. In the case of corporations the asset base is the acquisition value of the shares.

As the above brief review shows, the dual income tax systems of the Nordic countries are based on some common leading principles but differ very much in detail. This complicates any analysis of the Nordic model to some extent. In chapters 2 - 4 we will analyse a stylised (or theoretical) version of the Nordic DIT. We will assume that the capital base is the firm's current net assets (gross assets minus debt) valued at book value. Concerning the rate structure of the system we will assume that the marginal tax rate on earned income is strictly higher than the flat rate on capital.

1.3 Model Framework - Review of the Theory of Corporate Taxation

Our study of the effects of dual income taxation uses an approach that we call here the neo-classical theory of corporate taxation. We will give a brief review of the framework and results of this theory, putting more emphasis on the areas relevant for our study.

The roots of this research tradition lie in the dynamic theory of the firm pioneered in the 1960s by Jorgenson (1963, 1967) and Hall and Jorgenson (1967).[14] The models used an intertemporal continuous-time framework and assumed perfect capital markets. This latter feature allowed researchers to resort to Fisher's separation theorem and assume that a firm maximises the wealth of its owners.

These early studies contained some features of the tax system, but they were not sufficiently thorough in this respect. They failed to include personal taxation and were imprecise concerning the legal form of the firm. They also ignored the variety of financing forms and the differences in the tax treatment of these. A broader approach in this respect came somewhat later in Stiglitz (1973) and was completed in the studies by King (1974a, 1974b, 1975), Auerbach (1979) and Boadway and Bruce (1979), and in a more exact form in Sinn (1987). These authors introduced several new features to the Jorgensonian framework: corporate and personal taxation modelled in a generalised form, institutional constraints for borrowing and dividend policies and, perhaps most notably, an equilibrium condition for financial markets reflecting consistently the effects of capital income taxation.

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[14] We emphasise here the dynamic nature of the theory. However, many of the results of the theory can also be derived in a static framework.

The original framework was later improved and extended in many directions. Kanniainen and Södersten (1995a) clarified the analysis, stressing the importance of the exact form of the institutional constraints. In contrast to the traditional approach that focused on equilibrium analysis, Sinn (1991), van Schijndel (1988) and van Hilten et al. (1993, ch.5) studied the impacts of taxation during the growth phase of the firm. Auerbach (1989), Kanniainen and Södersten (1995b) and Alvarez et al. (1998) studied the dynamic adjustment of a firm to tax reform, the latter in a stochastic framework. Mayer (1986), Keen and Schiantarelli (1991) and Virolainen (1998) used a stochastic version of the original model in studying the effects of certain asymmetries of corporate taxation.

To describe more closely the approach pioneered by King (1974a) and others, later called the 'new view' of corporate taxation, we will set up a simplified version of the model in that tradition. We assume perfect foresight and no market imperfections other than taxation (no informational asymmetries, no dividend preferences etc.). Prices and interest rates are time-independent and the fiscal depreciation equals the true economic depreciation. Moreover, we assume for simplicity a classical corporate tax system, where there is no relief from the double taxation of dividends. The model is as follows:[15]

(1.1a) $\quad\quad max_{\{I, R, Q\}}\ V(t_0)$

(1.1b) $\quad\quad$ s.t. $\dot{K}(t) = I(t) - \delta K(t)$

(1.1c) $\quad\quad \dot{B}(t) = R(t)$

(1.1d) $\quad\quad f(K(t)) + Q(t) + R(t) = D(t) + I(t) + rB(t) + T(t)$

(1.1e) $\quad\quad D(t) \leq f(K(t)) - \delta K(t) - rB(t) - T(t)$

(1.1f) $\quad\quad B(t) \geq 0,\ D(t) \geq 0,\ Q(t) \geq 0$

(1.1g) $\quad\quad T(t) = \tau_f[f(K(t)) - rB(t) - \delta K(t)]$.

(1.1h) $\quad\quad K(t_0) = K_0,\ B(t_0) = B_0$

where $V(t)$ is the value of the firm's equity at time t, D is the amount of dividends distributed (we omit here the dependency on time), R is the amount of new debt issues, Q is the amount of new share issues, K is the real value of the firm's capital stock, B is the real value of the firm's stock of debt and T is the amount of

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[15] Note that as in the original Jorgensonian approach there are no adjustment costs in the model. This is one of the differences between a typical new view model and the later neoclassical investment theory. However, there are several tax studies that include adjustment costs, eg. Ylä-Liedenpohja (1978), Robson (1989), Auerbach (1989) and Kanniainen & Södersten (1995b).

taxes paid by the firm. $\dot{K}$ and $\dot{B}$ depict the changes in the value of the firm's capital stock and the stock of debt respectively. The operating income (earnings before depreciation, interest and taxes) of the firm, depicted by $f(.)$, is a function of the capital stock only[16] and has the following properties: $f' > 0$, $f'' < 0$. The term δ denotes the constant, exponential rate of depreciation, r denotes the risk-free market rate of interest and, finally, τ_f denotes the rate of corporate tax.

Equation (1.1a) states that the firm is assumed to maximise the value of its equity at time t_0. In the literature this value is assumed to be determined in the capital markets by the following equilibrium condition (see King (1974a), Sinn (1991)):

$$(1.2) \qquad (1-\tau_p)rV(t) = (1-\tau_d)D(t) + (1-\tau_g)[\dot{V}(t)-Q(t)]$$

where τ_p is the marginal tax rate on interest income, τ_d is the marginal tax rate on dividends and τ_g is the marginal effective tax rate on capital gains (accrual basis). This condition requires that the firm's equity is priced in the markets so that the after-tax return on equity equals the after-tax return on bonds. By solving this differential equation we obtain the following formula for $V(t_0)$[17]

$$(1.3) \qquad V(t_0) = \int_{t_0}^{\infty} [\gamma D - Q]e^{-r'(t-t_0)}dt$$

where $r' = (1-\tau_p)r/(1-\tau_g)$ and $\gamma = (1-\tau_d)/(1-\tau_g)$. The term r' is called the tax-adjusted discount rate of the owner.

Equations (1.1b)-(1.1f) determine the constraints on the firm's policy. Equation (1.1b) is the capital accumulation constraint that determines the course of the value of the firm's capital stock over time. Equation (1.1c) in turn describes the evolution of the firm's debt stock. Equation (1.1d) is the firm's budget constraint. The sources of funds are operating income, new debt and new equity issues (on the left-hand side). The uses of funds are dividends, investment, costs of borrowing and firm-level taxes (on the right-hand side).

It is a well known fact that in a framework with perfect foresight and perfect capital markets taxation may induce a value-maximising firm to pursue some extreme behaviour in its financing policy. For example it can finance its investments entirely with borrowing or even issue an unlimited amount of debt and pay out the proceeds as an infinitely large dividend. There are several ways to deal with this problem. One of them incorporates some structures in the model that produce an endogenously determined internal optimum for debt.

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[16] This is a common assumption in corporate tax literature and can be made without any loss of generality, as shown in e.g. Ylä-Liedenpohja (1976) pp. 26-27.

[17] To assure the existence of $V(t_0)$ we assume that $\lim_{t\to\infty} e^{-r't}V(t) = 0$.

A second alternative, and perhaps the most commonly used in tax literature, is to set exogenous constraints on borrowing. Such constraints can be based on flow variables limiting new debt financing with respect to investments or on state variables limiting the stock of debt with respect to the firm's capital stock.[18]

In fact the legal systems of all western countries impose such constraints. Equation (1.1e) is one version of legal dividend constraints whereby dividends may not exceed the after-tax profit of the firm.[19] By combining (1.1d) and (1.1e) we notice that this constraint implies a constraint on external financing

$$(1.1e') \qquad\qquad R + Q \leq I - \delta K$$

and prevents perverse policies of excessive issues of external financing combined with similar dividend distributions. Our simple model allows firms to finance their net investment totally from borrowing. This case is empirically unrealistic but is allowed here, because our aim is not to study the determinants of the internal financial optimum of a firm.[20] But our framework lends itself rather well to a study of the direction of incentives in taxation on firms' financial policy.

The constraints in (1.1f) set lower bounds for variables B, Q and D. The first constraint ensures that the firm cannot act as a lender. The second constraint excludes the distribution of profits using share repurchases.[21] The latter constraint is necessary in order to prevent firms receiving funds as a negative dividend subsidised by the government at the tax rate for dividends. Finally, equation (1.1g) is a determination of corporation tax and equation (1.1h) is the value of the firm's capital stock at the beginning of the planning period inherited from the past history of the firm.

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[18] For a discussion see e.g. Sinn (1987), van Hilten et al. (1993) and Lesourne and Leban (1982).

[19] If the tax system allows accelerated depreciation, the exact form of the dividend constraint depends on the reporting convention applied. The majority of OECD countries operate so-called uniform reporting, where the tax accounts presented to the fiscal authorities must coincide with the public accounts drawn up for the shareholders and debtors. In this case dividends are constrained to accounting profits after taxes and accelerated depreciation. A separate reporting convention is applied primarily in Anglo-Saxon countries whereby tax accounts are not tied to public accounts. In this case dividends are constrained by profits after taxes, economic depreciation and the increase in 'tax debt' generated by the excess fiscal depreciation. See Kanniainen and Södersten (1995a) and Sörensen (1995) for more on this issue. The dividend constraint above is written in terms of flow variables. In some countries, e.g. in Finland, dividends are constrained with respect to the free reserves composed of current and accumulated past after-tax accounting profits. Virolainen (1998) models this system and introduces a new state variable to describe the free reserves.

[20] For a discussion concerning the internal financial optimum, see surveys in Virolainen (1998) and Harris and Raviv (1991).

[21] In most European countries firms are not allowed to repurchase their own shares. This was previously also the case in Finland. Quite recently, however, the law has been changed and corporations today have permission, subject to some restrictions, to repurchase their shares.

Thus the approach considers a value-maximising incorporated firm that chooses its financing and investment policies over an infinite horizon. The value of the firm's shares is determined in the financial markets and the firm operates in an environment of non-neutral taxation with some institutional constraints on its policy.

Next we will state some central results of this research tradition obtainable from the model (1.1a)-(1.1h). We will study the firm's policy in the long-run equilibrium. Using the optimality conditions it can be shown that the firm finances its investment in this phase entirely by:[22]

$$(1.4) \quad \begin{bmatrix} borrowing \\ new\ share\ issues \\ retained\ earnings \end{bmatrix} \quad if \quad \begin{cases} \tau_p < \min\left[\tau_f + (1-\tau_f)\tau_d,\ \tau_f + (1-\tau_f)\tau_g\right] \\ \tau_f + (1-\tau_f)\tau_d < \min\left[\tau_p,\ \tau_f + (1-\tau_f)\tau_g\right] \\ \tau_f + (1-\tau_f)\tau_g < \min\left[\tau_p,\ \tau_f + (1-\tau_f)\tau_d\right] \end{cases}$$

Here τ_p, $\tau_f + (1-\tau_f)\tau_d$ and $\tau_f + (1-\tau_f)\tau_g$ are the total effective tax rates on interest income, dividends and capital gains respectively, reflecting the combined firm- and investor-level tax treatment. The rate of corporation tax is not included in the effective tax rate on interest income due to the deductibility of debt costs in taxation. Condition (1.4) sates that borrowing is the optimal policy in the steady state if the total tax rate on debt interest is lower than the total tax rate on dividends or capital gains. Similarly, new share issues are the optimal form of financing if the total tax rate on dividends is lower than that on interest income and capital gains and finally retained earnings are the cheapest form of financing if the total tax rate on capital gains is lower than that on interest and dividends.

The cost of capital after depreciation for the three financing regimes is[23]

(1.5)   borrowing: $f'(K) - \delta = p_B \equiv r$

new share issues: $f'(K) - \delta = p_{NS} \equiv \dfrac{1 - \tau_p}{(1 - \tau_f)(1 - \tau_d)} r$

retained earnings: $f'(K) - \delta = p_{RE} \equiv \dfrac{1 - \tau_p}{(1 - \tau_f)(1 - \tau_g)} r$

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[22] King (1974a) and Atkinson and Stiglitz (1980) derive the same results in a more static framework using a so-called perturbation argument. They examine the effect of a feasible change in the firm's financial policy on the consumption possibilities (wealth) of its shareholders. Our approach here is a little different. The conditions are feasibility conditions for the three different financial regimes obtained directly from the first-order conditions for the model.

[23] These are derived from the marginal conditions that determine the firm's steady-state capital stock in cases where the financing form in question is the optimal marginal financing form.

From inspecting (1.4) and (1.5) we can draw the following conclusions.

- the before-tax cost of the firm's borrowing equals the market interest rate; this means that taxation does not distort investment decisions when the investment is financed by borrowing[24]

- the cost of equity-financed investment is different for new share issues and retained earnings

- the cost of retained earnings is independent of the marginal tax rate on dividends, but is affected by the effective tax rate on capital gains

- with $\tau_g < \tau_d$, as is usually assumed to be the case, retained earnings are a cheaper form of financing than external equity; this means that external equity is not used in financing investments.[25]

The last result, according to which the cost of retained earnings is different and lower than that of new share issues, can be clarified using the following example. Consider a firm that decides to finance additional investment by increasing retained profits and lowering dividends. For a shareholder the value of one markka of after-tax profits is $(1-\tau_d)$ when this profit is distributed and $(1-\tau_g)g$ when it is retained, the term g denoting the amount by which the market value of the firm's shares appreciates when the profit is retained. In market equilibrium the two values must be equal. This equilibrium is produced by adjustment of g to the value:

$$(1.6) \qquad g^* = (1-\tau_d)/(1-\tau_g)$$

Suppose now that the one markka investment financed with retained earnings has a before-tax return p (net of depreciation) and that this return is paid out as taxable dividends. The after-tax return to the shareholder from this investment is $p(1-\tau_f)(1-\tau_d)$. The shareholder compares this to an investment in bonds of amount g^*. The after-tax rate of return on this investment is $r(1-\tau_d)$. The firm is indifferent between the two investments if:

$$(1.7) \qquad p(1-\tau_f)(1-\tau_d) = [(1-\tau_d)/(1-\tau_g)]r(1-\tau_p)$$

which reduces to $p = (1-\tau_p)r/(1-\tau_f)(1-\tau_g)$. Note that p, the minimum required return on investment in the firm, is independent of dividend taxation and corre-

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[24] In the 1970s there was a school of thought stressing the neutrality of the corporate tax system mostly due to the deductibility of interest. This 'neutrality view' was expounded most prominently by Stiglitz (1973, 1976). See also Sörensen (1995).

[25] The tax rate τ_g is an effective rate and takes into account any special treatment of capital gains in taxation, any advantage from the deferral of taxation to the moment of realisation of the gain, as well as any disadvantage from taxing nominal rather than inflation-indexed gains.

sponds to p_{RE} in (1.5). The intuition behind this independence result is that dividend taxation affects this investment in two ways. It raises the tax rate on the return of the investment and it lowers the opportunity cost of using the profit for investment in the firm. These two effects are of opposite signs and cancel each other. (In equation (1.7) the two $(1-\tau_d)$ terms cancel out.)

As pointed out by Zodrow (1991), the mechanism here is analogous to that in the taxation of savings under a cash flow-type expenditure tax. The cash flow tax combines a deduction of all savings with full taxation of the returns and the amount invested upon withdrawal. The consequence of this is that savings are effectively exempt from taxation. The benefits from the original allowance exactly offset the subsequent taxation of interest and principal. In our dividend tax case the counterpart to the original allowance is the freedom from dividend taxation resulting from use of the one-markka profit for investment in the firm instead of dividend distributions. Later, when paid out as dividends, the original investment and return on it are subject to full taxation at the rate τ_d. As a result the return on the investment is effectively exempt from dividend taxation.

These results are in striking contrast to the 'old view' associated most notably with Harberger (1962, 1966). That view maintains that double taxation of distributed profits distorts the decisions of corporations and allocates the capital to the non-corporate sector, causing severe welfare losses. Proponents of the new view challenged that view by arguing that the marginal source of equity is retained earnings, and, accordingly, dividend taxation has no or minor distortionary effects. The potential source of distortions, if any, is capital gains taxation. The observation that retained earnings is a cheaper form of equity financing was well in line with the empirical fact that on average investments are financed mostly with debt and retained earnings and only to a small extent by external equity.

The new view was challenged in the 1980s by Poterba and Summers (1985) with a rehabilitation of the old view. They imputed an imperfection of the capital markets in the form of preference for dividends to a model similar to the new view models. This was justified inter alia by empirical findings according to which dividend pay-out ratios do not behave as the new view implies i.e. as a residual item in the application of profits, and are quite independent of variations in profits. This extra gain from dividends establishes an internal optimum for the two forms of equity. As a result the cost of equity is a weighted average of the costs of internal and external equity and reflects the dependence of the cost of capital on the dividend tax rate.

One strand of corporate tax research studies the effects of tax incentives on investment. Hall and Jorgenson (1967, 1971) contributed strongly to this literature in its early stage. Especially interesting from the European and Nordic perspectives is the literature that examines the effects of accelerated depreciation. The

studies by Boadway (1978) and Boadway and Bruce (1979) gave a solid basis for this analysis. Later Södersten (1982), Ylä-Liedenpohja (1984) and Kanniainen and Södersten (1995a) applied and improved this framework. In this approach fiscal depreciation is modelled by adding a condition to the standard model describing the development of the book value of capital and by letting the fiscal depreciation be deductible from the taxable income of the firm. To review the results and concepts of this analysis it is useful to look at the firm's cost of capital (after depreciation) when internal equity is the marginal form of financing. The expression for the cost of capital in this case is:

$$(1.8) \qquad p = \frac{1}{1-\tau_f} r' \left[1 - \frac{\tau_f(\varepsilon - \delta)}{r'+\varepsilon} \right]$$

where ε is the rate of accelerated depreciation and $r' = (1-\tau_p)r/(1-\tau_g)$ is the owner's discount rate. Note that $r'/(1-\tau_f)$ is the cost of capital of internal equity, p_{RE}, in (1.5). With $\varepsilon > \delta$ the expression in the brackets in (1.8) is less than 1 and p in this case is less than p_{RE}. This means that accelerated depreciation lowers the firm's cost of capital.

Södersten (1982) gave an economic interpretation to this expression of the cost of capital. The term $\tau_f(\varepsilon - \delta)/(r'+\varepsilon)$ is the present value of the tax saving from the excess depreciation and it can be interpreted as giving the average part of the investment that is financed by deferred taxes (interest-free tax debt). The investment of one markka is then financed from two sources, retained after-tax earnings and deferred taxes. Since the cost of tax debt is zero, the total cost of capital in this case is the unit cost of internal equity weighted by the share of this financing form, given by the expression in the brackets in (1.8) and being less than 1.

If the marginal source of financing is not equity but debt, which is the case when $\tau_p < \tau_f + (1-\tau_f)\tau_g$, the outcome is a little different. In this case there is a 'rivalry' between debt costs and accelerated depreciation with respect to deductions from taxable income. Kanniainen and Södersten (1995a) show that the cost of capital in this case is dependent on the exact form of the dividend constraint. As mentioned above, this constraint is different under systems of uniform reporting and separate reporting. In the former case the upper limit on dividends is after-tax profit after fiscal depreciation and in the latter case it is the after-tax profit after economic depreciation and an increase in tax debt.

Kanniainen and Södersten (1995a) show that under separate reporting the cost of capital is the weighted cost of debt, the expression for the weight of debt being the same as the weight of equity in (1.8). In the uniform reporting case the weight of debt, α, becomes different:

$$(1.9) \qquad \alpha = 1 - \frac{(\tau_p - \tau_g)(\varepsilon - \delta)}{r' + \varepsilon}$$

α is now independent of the corporate tax rate, τ_f, but depends instead on the tax rates on interest income and capital gains.[26]

Most of the literature cited here studies investment and financing decisions under long-run equilibrium. One important exception to this rule is Sinn (1991), who studies the investment and financing decisions in the birth, growth and steady-state phases of the corporate firm. Sinn applies a simplified version of the new view model, in which the firm is purely equity-financed and owner-level dividend tax is the only form of taxation. Sinn adds to the framework a cost function for the initial equity invested at the birth-phase of the firm.

The solution to this model shows that with positive dividend taxation the initial capital stock of a wealth-maximising firm financed with an initial equity issue is strictly lower than the long-run equilibrium capital stock. The size of this initial capital stock is adversely affected by the dividend tax rate τ_d. After the birth phase the firm grows to the equilibrium level by financing its growth with internal financing only. Slow adjustment to the equilibrium level is produced by the non-neutrality of the tax system, not by any explicit adjustment costs.

The study by Sinn confirms the validity of the new view results in the steady-state phase but suggests that in the birth and growth phases the cost of capital is dependent on dividend taxation and may be higher than the cost of capital of external equity given by (1.5). This means that dividend taxation is not neutral but distorts investment in a growing firm. This result has been considered as a potential explanation for the gap between new view and empirical findings as regards the effects of dividend taxation on corporate investment, Zodrow (1991).

Another class of studies of the dynamics of the firm is that of van Schijndel (1988) and van Hilten et al. (1993) using the model framework by Lesourne and Leban (1978) and van Loon (1983). Despite the fact that these studies do not make use of any equilibrium condition similar to (1.2), they are useful references in our analysis of the firm's debt policy performed in chapter 4.

1.4 Outline of the Study

This study analyses the effects of dual income taxation on the investment and financing behaviour of a closely held corporation. Of particular interest is the division of dividends into earned income and capital income, a central feature of

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[26] Sörensen (1995) gives a thorough discussion of the cost of capital under different reporting conventions.

dual income tax (graduated dividend tax). We confine the analysis to a variant of this system in which the asset base of the division is the firm's current net assets.

We use a dynamic model of the firm similar to that applied by Sinn (1991). Apart from the mainstream tax analysis we do not restrict ourselves to studying the effects under long-run equilibrium, rather our aim is to examine the effects over the whole growth cycle of the firm. We feel that this is especially important here because the tax system under investigation is one applied to small and medium-sized firms that, which, logically perhaps, have typically not reached their maturity phase. This emphasis on dynamics has, however, some consequences for the approach applied. To be able to derive the firm's optimal dynamic behaviour we have to keep the model extremely simple. This leads us to a procedure where we make several small variations on the basic model, each being an attempt to shed light on some features of the whole.

In chapter 2 we construct the basic model of the study and present a way to model the graduated dividend tax system of the DIT. The basic model is extremely simple. Its tax system includes only one tax form, owner-level dividend taxation. The firm is fully equity-financed and there is homogenous capital. However, the model proves to be very useful. It succeeds in clarifying many of the behavioural effects of the graduated dividend tax system, and, in being closely related to the model by Sinn (1991), it makes it possible to compare the firm's behaviour under a graduated dividend tax system to that under a linear system. We define linear dividend taxation as a tax system where dividends are subject to the same proportional rate at all levels of dividend income. Using this comparison we can illustrate the ceteris paribus effects of establishing a graduation in the tax scheme.

In Chapter 3 we extend the framework by introducing a more comprehensive tax system including firm-level taxation, taxation of capital gains and a system of alleviating the double taxation of dividends. We show that these extensions do not appreciably affect the basic results derived in chapter 2.

Chapter 4 introduces three extensions to the basic model. The first examines the firm's policy in a model with two assets, financial capital and real capital. The second concerns the firm's debt policy under dual income taxation. The final extension introduces depreciable capital with accelerated fiscal depreciation. The unifying feature of these extensions is that they all affect the firm's capital base used in dividing dividends into capital income and earned income. The aim is to give a more versatile view of the effects of the tax system.

Chapter 5 concludes the study with a summary of the main findings and some general conclusions drawn from them.

2. Dual Income Tax in a Dynamic Model of the Firm

In this chapter we start with an analysis of the dual income tax system. The first section presents a dynamic model of the investment and financing decisions of the firm in the presence of taxation. This model serves as a basis for all subsequent models in this study. After introducing the framework we describe our approach to the modelling of graduated dividend tax. A specific method for handling the problems caused by the non-linearity of the tax system is presented. In sections 2.3 - 2.6 we give a full analysis of the basic model. The final section summarises the main findings.

2.1 The Model

The model constructed below closely follows the approach of Sinn (1991) and is a simplified version of the typical new view model presented in section 1.3. It is a deterministic partial equilibrium model in continuous time.

The firm is a limited company operating in competitive markets. It finances its activities with internal and external equity and its objective is to maximise the wealth of its shareholders. The firm has no access to the capital markets but its shareholders are able to borrow and lend at the market rate of interest.

As in the new view literature we assume that the value of the firm's equity is determined according to the equilibrium condition (1.2). As explained in section 1.3, this condition requires that the firm's equity is priced at each point of time so that the expected after-tax return on the shares equals the expected after-tax return on interest-bearing assets. We thus assume that even though our firm is not a listed company, its shares may be traded and the valuation of these shares satisfies equilibrium pricing.

By solving equation (1.2) with respect to V, we can write the value of the firm's equity at time t_0 as:

$$(2.1) \qquad V(t_0) = \int_{t_0}^{\infty} \left\{ \gamma' D_n(t) - Q(t) \right\} e^{-r'(t-t_0)} dt,$$

where D_n is the shareholders' net after-tax dividend income and $\gamma' = 1/(1-\tau_g)$ and $r' = (1-\tau_p)r/(1-\tau_g)$. Due to the deterministic nature of the model r is the risk-free market rate of interest. The terms τ_p and τ_g denote the tax rate on interest income and the effective tax rate on capital gains respectively. In this chapter we study a simplified tax system where $\tau_g = \tau_p = 0$ and consequently $\gamma' = 1$, $r' = r$.

The firm produces using a single input, capital, denoted by K. The firm's net operating income is described by the function $f(K)$ and is defined as $f(K) = F(K)-\delta K$, where $F(K)$ is gross operating income and δ is the constant rate of economic depreciation. $F(K)$ has the usual properties $F(0) = 0$, $F' > 0$, $F'' < 0$ and also satisfies the Inada conditions $\lim_{K \to 0} F'(K) = \infty$ and $\lim_{K \to \infty} F'(K) = 0$.

Note that $f'(K)$ is allowed to be negative. The reason for this is that the marginal condition defining the firm's steady-state capital stock can be satisfied even in an extreme case where the firm's cost of capital is negative. In section 2.5.1 we will argue that in the tax system studied the cost of capital can be negative in certain cases.

The firm's budget constraint is:

(2.2) $f(K) + Q = D + I,$

where the left-hand side denotes the sources of funds, net operating income $f(K)$ and new equity Q, and the right-hand side the uses of funds, dividends D and investments I. There are no tax terms in (2.2) because the basic model does not include firm-level taxation. This feature is introduced in chapter 3.

As in Sinn (1991), we assume that capital is homogenous. Furthermore, since we describe the firm's operating profit as $f(K)$, capital appears in the model as if it were non-depreciating. These two features are relaxed in chapter 4, where we study first a model with two assets and later the case where depreciation of capital is modelled explicitly.

Note that the variable K has two roles in this study. First, through the function $f(K)$ we can regard it as a capacity measure. Second, by normalising the price of capital to 1, we can also use K as a measure of the value of the assets of the firm.

The firm's capital stock develops as follows: $\dot{K} = I$. By substituting I here from the budget constraint (2.2) we obtain the state equation for the capital stock:

(2.3) $\dot{K} = f(K) + Q - D$

To rule out subsidised equity injections, dividends are required to be non-negative. We also assume that distributions of profits via share repurchases are not allowed. The rationale of and justification for these assumptions was discussed in section 1.3.

(2.4) $\qquad D \geq 0$

(2.5) $\qquad Q \geq 0$

Furthermore, we assume that K is always non-negative. In order to simplify the solution process, we do not include this constraint explicitly in the model. Note also the content of the first Inada condition. It states that the productivity of capital is extremely high at very low levels of capital. In our framework this has the obvious implication that after the establishment phase the firm's capital stock is always kept strictly positive, $K > 0$. We benefit from these features in several places, for example when we trace unfeasible policies and also when we investigate the sufficient conditions for the model.

Based on Sinn (1991), the model includes optimisation of the initial value of the firm's capital stock. This feature allows us to assess the impact of dividend taxes in the birth phase of a firm. Technically this is introduced into the model by adding to the objective function an initial cost function, denoted by $\varphi(K)$.[27]

The model framework consists of equations (2.1) and (2.3) - (2.5), the objective function in (2.1) being supplemented by the initial cost function.

2.2 The Tax System

This section describes the modelling of the graduated dividend taxation of DIT. As in Sinn (1991) we assume that the dividend tax is the only form of taxation. A more comprehensive tax system is introduced later in chapter 3. The shareholders' dividend tax liability, T, under DIT is as follows:

$$(2.6) \qquad T = \begin{cases} \tau_c D & \text{if } D \leq bN \\ \tau_c bN + \tau_e(D - bN) & \text{if } D > bN, \end{cases}$$

where τ_c and τ_e denote the marginal tax rates on capital income and earned income, b is the imputed rate of return used in calculating the imputed yield on capital invested in the firm, N is the capital base used in calculating this yield and D is the cash dividend received by the shareholders.

We will assume that the tax rates are constant,[28] strictly positive and non-confiscatory. We further assume that the marginal tax rate on earned income is

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[27] For deriving the optimal initial value of a state variable, see Seierstad and Sydsaeter (1987) p.185 or Leonard and Long (1993) p. 248.

[28] The constancy of the marginal tax rate on earned income may seem a strong simplification. It can, however, be justified by a further assumption that the shareholders have a fixed amount of wage income that is independent of the firm's dividend distribution decisions, and by the observation that the mar-

strictly higher than the tax rate on capital income. This assumption deviates only slightly from the stylised feature of DIT, emphasised by Sörensen (1998, 1994), whereby the lowest rate for earned income is set at the level of the proportional tax rate for capital income.

(A2.1) $\qquad\qquad 1 > \tau_e > \tau_c > 0$

The shareholders' after-tax net dividend income is:

$$\text{(2.7)} \qquad D_n = D - T = \begin{cases} (1-\tau_c)D & \text{if } D \le bN \\ (1-\tau_e)D + \tau_x bN & \text{if } D > bN \end{cases}$$

where $\tau_x \equiv \tau_e - \tau_c$. Note that assumption (A2.1) implies $\tau_x > 0$.

As in Huber (1994), the net dividend D_n is a 'kinked' function of the cash dividend, D. This function has two linear sections and a kink at the point $D = bN$. The inclusion of a variable of this kind in the continuous-time optimal control framework presented in section 2.1 raises certain problems. The ordinary tools for solving these models used by economists usually assume that the objective function is continuously differentiable. In our case the partial derivative of D_n with respect to D is discontinuous. We can, however, get round this problem using the procedure described below.

Let us first define a new variable:

(2.8) $\qquad\qquad D_x \equiv D - bN$

D_x gives the portion of the dividends taxable as earned income. From the economic content of D_x it follows that this variable must be restricted so as to be non-negative:

(2.9) $\qquad\qquad D_x \ge 0$

Using the new notation, the net dividend, D_n, can be written as follows:

$$\text{(2.10)} \qquad D_n = \begin{cases} (1-\tau_c)D & \text{if } D \le bN \\ (1-\tau_c)D - \tau_x D_x & \text{if } D > bN \end{cases}$$

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ginal tax rate actually stays constant over a fairly wide range in the tax schedules of the Nordic countries. In Finland the marginal tax rate in 1996 was, for example, 51% between FIM 104 000 - 163 000 and 57% between FIM 163 000 - 290 000 (proportional local tax of 25% is included). Thus the constancy assumption does not conflict with practice, especially if dividend income is small compared to wage income.

The next step is to set up the following extended model to describe the tax system consisting of equation (2.11) and inequality (2.12):

(2.11) $D_n = (1-\tau_c)D - \tau_x D_x$,

(2.12) $D - D_x \leq bN$

The first equation defines D_n and the second sets an upper limit for the dividend taxable as capital income. One role of constraint (2.12) is that it introduces into the model a dependence between the tax liability and the capital base. Note that we do not use here the definition for D_x in (2.8). Thus, we do not define the relationship between D and D_x closer than in (2.12). Note also that D_n is now continuously differentiable with respect to D (and D_x).

When we compare this extended model to the original model (2.10) we observe that the former allows the combination $D < bN$ and $D_x > 0$, which is not possible under (2.10). This combination is nevertheless not economically meaningful for the shareholders, since the marginal tax rate on dividends is τ_e and the total tax liability is higher than if $D < bN$ and $D_x = 0$, for example.

When solving the model we observe that the policy depicted by the combination $D < bN$ and $D_x > 0$ is unfeasible. From this it follows that in the case of the optimal solution our extended model (2.11) - (2.12) fulfils the true model (2.10).

It is worth summarising the above-described procedure for modelling graduated dividend tax:

1. Depict dividends with two variables, D and D_x.

2. Describe graduated dividend taxation with a slightly extended model, consisting of equations (2.11) and (2.12).

3. Show using the first-order conditions for the problem, that in the case of the optimal solution the extended model satisfies the correct description of the tax system.

Thus we bring graduated dividend taxation into the model framework by adding the constraints in (2.9) and (2.12) and writing D_n as in (2.11).

We have not discussed the concept of the capital base yet. Generally we define the capital base in this study as net capital measured in book value terms, i.e. the difference between the book value of the firm's gross assets and the book value of its debt. In this chapter we make two important assumptions: first, the book value and real value of the capital are equal, and second, the firm has no debt.

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From these and the assumption that the price of one unit of capital is one markka it follows that we can express the capital base as simply:

(2.13) $$N = K$$

K can now be substituted for N in constraint (2.12).

2.3 Solution to the Basic Model

2.3.1 Optimality Conditions

The basic model is as follows:

(2.14a) $$\max_{\{K_0,D,D_x,Q\}} \int_{t_0}^{\infty} \{(1-\tau_c)D(t) - \tau_x D_x(t) - Q(t)\}e^{-r(t-t_0)}dt + \varphi(K(t_0))$$

(2.14b) $$\dot{K}(t) = f(K(t)) + Q(t) - D(t), \qquad K(t_0)=K_0$$

(2.14c) $$h_1 = bK(t)+D_x(t)-D(t) \geq 0$$

(2.14d) $$h_2 = D(t) \geq 0$$

(2.14e) $$h_3 = D_x(t) \geq 0$$

(2.14f) $$h_4 = Q(t) \geq 0$$

(2.14g) $$\varphi(K) = -K$$

The firm's problem is to decide its initial capital, K_0, and the time paths for the dividend, D, dividend taxable as earned income, D_x, and new equity financing, Q, so that the firm's objective function (2.14a) and constraint conditions (2.14b) - (2.14f) are satisfied. The firm's planning period is $[t_0, \infty]$. Technically the model is a continuous-time optimal control problem with three control variables, D, D_x, and Q, and one state variable, K. In equations (2.14c) - (2.14f) we denote the constraint formulas by $h_1,...,h_4$.

The model can be solved using the maximum principle.[29] It can further be shown that the model fulfils the so-called sufficient conditions, which implies that a so-

[29] The model fulfils the regularity conditions of the maximum principle (see Chiang (1992, 278)). The integrand in (2.14a), the right side of the equation of motion for capital (2.14b) and the inital cost function, $\varphi(.)$, are continuous and continuously differentiable. In addition the constraint functions in (2.14c) - (2.14f) are linear.

lution satisfying the first-order conditions is an optimal solution to the problem. The sufficient conditions are studied in appendix 1.

The current-value Hamilton function for the model is

$$(2.15) \qquad H = (1-\tau_c)D - \tau_x D_x - Q + \lambda[f(K) + Q - D]$$

where λ is the so-called co-state variable, i.e. the shadow price of the state, K. This depicts the impact of a marginal change in capital on the value of the firm's equity. The current-value Lagrange function for the model is

$$(2.16) \qquad L = H + q_1[bK + D_x - D] + q_2 D + q_3 D_x + q_4 Q$$

where $q_1,\ldots,q_4$ are the shadow prices of constraints (2.14c) - (2.14f). The interpretation of the shadow prices is discussed in more detail in section 2.4. It is significant that unlike in static models, the shadow prices are not necessarily constants, but may change over time. In order to simplify the notation we depict the time-dependency of these and other variables only when necessary. The optimality conditions of the model are

$(2.17a) \qquad \partial L/\partial D = 1 - \tau_c - \lambda - q_1 + q_2 = 0$

$(2.17b) \qquad \partial L/\partial D_x = -\tau_x + q_1 + q_3 = 0$

$(2.17c) \qquad \partial L/\partial Q = -1 + \lambda + q_4 = 0$

$(2.17d) \qquad q_1 \geq 0, \; bK + D_x - D \geq 0, \; q_1(bK + D_x - D) = 0$

$(2.17e) \qquad q_2 \geq 0, \; D \geq 0, \; q_2 D = 0$

$(2.17f) \qquad q_3 \geq 0, \; D_x \geq 0, \; q_3 D_x = 0$

$(2.17g) \qquad q_4 \geq 0, \; Q \geq 0, \; q_4 Q = 0$

$(2.17h) \qquad \dot{\lambda} = r\lambda - \partial L/\partial K = r\lambda - f'(K)\lambda - bq_1$

$(2.17i) \qquad \dot{K} = f(K) + Q - D$

$(2.17j) \qquad \lambda(t_0) = - d\varphi(K)/dK = 1$

Conditions (2.17a) - (2.17g) evaluate the maximisation of the Hamilton function with respect to the control variables D, Q and D_x. Condition (2.17h) is the equation of motion for the co-state variable and (2.17i) is the equation of motion for

the state variable. Equation (2.17j) is the initial time transversality condition, which is used when deriving the optimal stock of initial capital.

2.3.2 Alternative Policies of the Firm

The complementarity conditions (2.17d) - (2.17g) give a total of 16 different combinations of control variables, each of which forms one specific control regime (policy) of the firm. The optimal solution to the problem consists of one or several consecutive periods where one regime prevails in each period. The regimes included in the optimal solution have to fulfil the first-order conditions of the model. In addition, the state and co-state variables have to be continuous in the solution throughout the firm's planning period $[t_0, \infty]$.[30]

Our analysis proceeds as follows. First we enumerate the different regimes. In the second stage we discern the feasible regimes from the potential regimes and in the final stage we construct the chain or string of regimes which describes the optimal solution.[31] The different regimes are listed in table 2.1.

Table 2.1   *Policy Alternatives of the Basic Model*
      (0 = binding constraint, + = non-binding constraint)

Regime no.	h_1	h_2	h_3	h_4
1	0	0	0	0
2	0	0	0	+
3	0	0	+	0
4	0	0	+	+
5 (A3)	0	+	0	0
6	0	+	0	+
7 (A4)	0	+	+	0
8	0	+	+	+
9 (A1)	+	0	0	0
10	+	0	0	+
11	+	0	+	0
12	+	0	+	+
13 (A2)	+	+	0	0
14	+	+	0	+
15	+	+	+	0
16	+	+	+	+

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[30] In problems where there are pure state constraints, the co-state variable may in certain cases be discontinuous. This does not occur here, since all the constraints include control variables.

[31] For a more systematic solution method in constructing an optimal solution to a model that contains several constraints, see van Loon (1983) Appendix 3 or van Hilten et al. (1993) Appendix 2.

Many of the potential regimes are unfeasible. Below we list the regimes that are omitted and give the reasons for excluding them.

Regimes 1 - 4. In these regimes the first two constraints are binding. This means that the dividend is simultaneously at its lower and upper bounds, i.e. $D = 0$ and $D = bK+D_x$, implying $K \leq 0$. This conflicts with the consequence of the Inada condition according to which K is always kept strictly positive. Regimes 1-4 are omitted on this basis.

Regimes 11, 12, 15 and 16. In section 2.2 we saw that the way of modelling dividend taxation we use, allows the following combination of dividend variables: $D_x > 0$ and $D < bK$. That is not feasible according to the 'true' definition of net dividends in (2.10). This combination prevails in regimes 11, 12, 15 and 16. The shadow prices for the first and third constraint in these regimes are $q_1=q_3=0$. By substituting these into the first-order condition (2.17b), we obtain the result that these regimes are valid only under the condition $\tau_x = 0$, i.e. when the tax system does not include any graduation of the marginal tax rate. This is of course in conflict with the tax rate assumption (A2.1) and means that the regimes can be rejected as unfeasible. Thus, the imprecision inherent in our way of modelling graduated dividend tax is eliminated on optimality grounds.

Regimes 6, 8 and 14. In these regimes the firm obtains new financing by issuing new shares, i.e. $Q > 0$. From this it follows that $q_4 = 0$ and further from equation (2.17c) that $\lambda = 1$. In regimes 6 and 8, it holds that $q_2 = 0$ and $q_1 > 0$. By substituting these into equation (2.17a), we obtain $\lambda \leq 1-\tau_c$. Combining these results, we find that regimes 6 and 8 are feasible only if $\tau_c \leq 0$. In a corresponding manner we can find that regime 14 is feasible only if $\tau_c = 0$. We can reject the regimes because of assumption (A2.1), which requires that $\tau_c > 0$.

Regime 10. Also in this regime the firm issues new equity. By substituting the values of the shadow prices into the first-order conditions (2.17c) and (2.17h), we obtain $\lambda = 1$ and $\dot{\lambda} = 0 => f' = r = $ constant. This marginal condition implies that the capital stock stays constant in this regime, $\dot{K} = 0$. Using (2.17i) and taking into consideration that $D = 0$, we obtain $\dot{K} = f(K) + Q = 0$. There is, however, a contradiction, since $Q > 0$ by assumption and $f(K) > 0$ by the properties of $f(K)$. Regime 10 can be rejected on this basis.

We have shown that of the 16 potential regimes, 12 are unfeasible. Of the four remaining regimes, one common denominator is that the company does not raise equity financing. This shows that one of the central findings of Sinn (1991) prevails in an environment of graduated dividend tax. According to this result, in a pure dividend tax system a firm will use share issues only to finance a small amount of initial capital; however, it will not use them during the growth phase or in the steady-state phase.

2.3.3 Analysis of Feasible Policies

In this sub-section we investigate the first-order conditions for the four feasible regimes of the model. We rename these policies as follows: A1=9, A2=13, A3=5 and A4=7. We start from regime A1, where the dividend is at its lower limit.

Regime A1: $(D = D_x = Q = 0; q_1 = 0; q_2, q_3, q_4 > 0)$[32]

Combining the definition of the regime with the first-order conditions (2.17a) - (2.17c) and (2.17i), we obtain:

(2.18) $$1 - \tau_c \leq \lambda \leq 1 \quad \text{and}$$

(2.19) $$\dot{K} = I = f(K)$$

From the properties of $f(K)$ it follows that $f(K) > 0 \Rightarrow \dot{K} > 0$. The regime is thus a growth regime where the firm uses all of its internal financing for investments. No dividends are paid.

Next we investigate the following regime where the dividend is between its upper and lower limits.

Regime A2: $(0 \leq D \leq bK, D_x = Q = 0; q_1 = q_2 = 0; q_3, q_4 > 0)$

When we substitute $q_1 = q_2 = 0$ into equation (2.17a) we obtain:

(2.20) $$\lambda = 1 - \tau_c$$

This indicates that the co-state is a constant. By taking the derivative of equation (2.20) with respect to time we obtain $\dot{\lambda} = 0$ and by substituting this into (2.17h) and solving the equation with respect to f' we obtain:

(2.21) $$f'(K) = r \qquad \Rightarrow \quad K = K^*$$

This is a marginal condition that defines a constant value of the firm's capital stock. The constancy implies that $\dot{K} = 0$. By substituting this and $Q = 0$ into (2.17i) we obtain $D = f(K)$. The firm does not invest but uses all of its profits for dividend distributions. In terms of Sinn (1991) this regime describes a firm that has reached its maturity phase.

Note that equation (2.21) is exactly the same as the marginal condition defining the firm's steady-state capital stock under linear dividend taxation obtained by

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[32] The values of the shadow prices and control variables give a definition for the regime.

Sinn (1991). A remarkable feature of this condition is that the firm's cost of capital depicted by the right-hand side is independent of the dividend tax rate.[33]

Equation (2.17i) and the values of the control variables imply that this regime is feasible on the condition $f(K) \leq bK$. By defining $\sigma(K) \equiv f(K)/K$ we may express this as follows:

$$(2.22) \qquad\qquad \sigma(K^*) \leq b \qquad\qquad (regime\ A2)$$

Regime A2 is feasible if the firm's average rate of return on capital is smaller or equal to the imputed rate of return, studied at the level of capital $K=K^*$.

Below we will examine a regime where the dividend is equal to the imputed return on capital invested in the firm. Hence the upper constraint on the dividend is binding, as is the non-negativity constraint on D_x.

Regime A3: $(D = bK, D_x = Q = 0;\ q_1, q_3, q_4 > 0,\ q_2 = 0)$

The values of the shadow prices combined with the first-order conditions imply:

$$(2.23) \qquad\qquad 1 - \tau_e \leq \lambda \leq 1 - \tau_c$$

$$(2.24) \qquad\qquad \dot{K} = f(K) - bK$$

From (2.24) we observe that the firm expands its capital stock if $f(K) > bK$. If, on the other hand, $f(K) = bK$ the firm uses all of its profits for dividends and does not invest.

Finally, in what follows, we investigate the last of the four feasible regimes. Here the dividend is once again at the upper limit, but now, unlike regime A3, $D_x > 0$.

Regime A4. $(D = bK + D_x,\ D_x \geq 0,\ Q = 0;\ q_1, q_4 > 0,\ q_2 = q_3 = 0)$

Using the definition of the regime and the first-order conditions we obtain:

$$(2.25) \qquad \lambda = 1 - \tau_c - \tau_x = 1 - \tau_e \ \ \& \ \ q_1 = \tau_x$$

The co-state is once again a constant. By substituting $\dot{\lambda} = 0$ and the values of λ and q_1 from (2.25) into the first-order condition (2.17h) and solving this equation with respect to f', we obtain the following marginal condition

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[33] In chapter 3 we will argue that in a broader tax system the firm's cost of capital is no longer totally independent of taxation. Nevertheless, the independence of dividend taxation will prevail.

$$(2.26) \qquad f'(K) = r - \frac{\tau_x}{1-\tau_e} b \qquad \Rightarrow \quad K = K_A*$$

By taking the derivative of the equation with respect to time and using equation (2.17i), we obtain $\dot{K} = f(K) - D = 0 \Rightarrow D = f(K)$. As in regime A2, the firm's capital stock is a constant, and the firm does not invest but rather distributes all of its profits as dividends. The steady-state capital stock defined by (2.26) is denoted K_A*.

Comparing (2.26) with (2.21) we can conclude that the cost of capital is lower in regime A4 than in regime A2. This implies $K_A*>K*$, i.e. the capital stock of regime A4 is larger than that of regime A2 and the steady-state capital stock under linear dividend taxation.

It is worth noting that the cost of capital defined in (2.26) can even be negative. Here too, however, the condition defines a unique steady-state capital stock provided that the rate of depreciation is high enough.[34]

The results obtained above together with equation (2.17i) imply a feasibility condition of $f(K) \geq bK$. Using the notation $\sigma(K) \equiv f(K)/K$, we can write

$$(2.27) \qquad \sigma(K_A*) \geq b \qquad (regime \ A4)$$

Regime A4 exists if the firm's average rate of return on capital exceeds (or is equal to) the imputed rate of return when $K = K_A*$.

The central properties of the four feasible regimes are summarised in Table 2.2.

Table 2.2   Characteristics of the Feasible Regimes

Regime	D	$\dot{K}$	λ	Feasibility	Regime type
A1	$= 0$	$= f(K)>0$	$1-\tau_c \leq \lambda \leq 1$	Always	growth
A2	$= f(K)$	$= 0$	$\lambda=1-\tau_c$	$\sigma(K*) < b$	steady-state
A3	$= bK$	$= f(K)-bK \geq$	$1-\tau_e \leq \lambda \leq 1-\tau_c$	$\sigma(K*) > b$	growth/steady-st.
A4	$= f(K)$	$\overset{\wedge}{= 0}$	$\lambda=1-\tau_e$	$\sigma(K_A*) \geq b$	steady-state

[34] Attention is drawn to the discussion in section 2.1 concerning the definitions and properties of $F(K)$ and $f(K)$. The former concept was defined to be $F(K) = f(K) - \delta K$ and one of its properties was $F' >0$. From these features it follows that equation (2.26) defines a unique steady-state capital stock provided that $\delta > r - \tau_x b/(1-\tau_e)$.

2.3.4 Optimal Solution

Final Regimes:

The solution to our basic model must be some combination of the four feasible regimes studied above. We will start by first reviewing the final stage of the solution and continue by constructing a regime chain to describe the firm's optimal policy during the growth path. We will apply the principle that the co-state and the state variable must be continuous throughout the firm's planning period.

A steady state can prevail in three of the four feasible regimes. In regimes A2 and A4 the firm is permanently in a steady state. In regime A3, in turn, a steady state occurs when $f(K) = bK$, i.e. when the firm's gross operating income is exactly equal to the imputed return on capital (see equation (2.24) and the discussion that follows). This implies that the model has three mutually exclusive optimal solutions, each of which ends with one of the three regimes.

Solution Ending in Regime A4 (case $\sigma(K_A{}^*) \geq b$):

The solution ends in regime A4 if the average rate of return on invested capital satisfies the condition $\sigma(K_A{}^*) \geq b$. Using the first-order conditions and the continuity requirement for the state and co-state variables we can construct the following regime chain to describe the firm's optimal policy

$$\text{Initial investment} \Rightarrow \text{Regime A1} \Rightarrow \text{Regime A3} \Rightarrow \text{Regime A4}$$

The optimality of this solution can be justified as follows. In regime A4 the co-state obtains the value $\lambda = 1 - \tau_e$. The tax rate assumption (A2.1) and equations (2.17j), (2.18), (2.20) and (2.23) imply that in the optimal solution regime A4 must be preceded by regime A3. The continuity requirement for the co-state variable excludes cases where A4 would be the only regime and also cases where it would be preceded by regime A1 or A2.

On the same grounds we conclude that regime A3 cannot be the first regime. Regime A2 is excluded owing to the following argument. According to (2.22) regime A2 is feasible only if $\sigma(K^*) < b$. However, from the concavity of $f(K)$ it follows that $\sigma(K_A{}^*) \geq b \Rightarrow \sigma(K^*) > b$. The feasibility condition for regime A2 is in contradiction with this latter property, which implies that regime A2 cannot be part of the solution.

A regime chain where regime A1 is the predecessor of regime A3 satisfies the first-order and continuity conditions. Comparing (2.18) with (2.17j) we observe that the initial time transversality condition is also satisfied.

In appendix 2 we show that a switch from regime A1 to regime A3 occurs when the firm's capital stock is lower than K^*. We will denote this level of capital as $K^\#$.

Thus, at the birth of the firm, at time t_0, the owners invest an amount K_0 of equity capital in the firm. After this phase the firm grows using internal financing. This growth phase consists of two different periods. In the first the firm uses all of its internal financing for investment (regime A1). No dividends are paid. After the firm's capital stock has reached the level $K^\#$ the firm switches to the second growth phase in which the firm pays dividends according to the rule $D = bK$ and invests the rest of its profits (regime A3). This policy is obeyed until the firm's capital stock reaches the level K_A^*. At this point the firm stops investing and begins to pay out all of its profits as dividends (regime A4). As was noted above, $K_A^* > K^*$, i.e. the firm's steady-state capital stock is greater than that under a neutral tax system.

Of the dividends in the steady-state phase, the amount bK_A^* is taxed as capital income and the amount $f(K_A^*) - bK_A^*$ is taxed as earned income. Thus the marginal tax rate on dividends is τ_e.

Figure 2.1 illustrates this solution.

Solution Ending in Regime A2 (case $\sigma(K_A^*) < b$):

Regime A2 is the final regime of the optimal solution if the average rate of return on invested capital satisfies the condition $\sigma(K^*) \leq b$. The firm's steady-state capital stock in this case is K^*, i.e. the same as that under linear dividend taxation.

The continuity requirements discussed above rule out regime chains where regime A3 or A4 is the predecessor of regime A2. Using the same argument, A2 cannot be the only regime. The only regime that can fill the gap between the initial state and regime A2 is regime A1. Hence, we obtain the following solution:

$$\text{Initial investment} \quad \Rightarrow \quad \text{Regime A1} \quad \Rightarrow \quad \text{Regime A2}$$
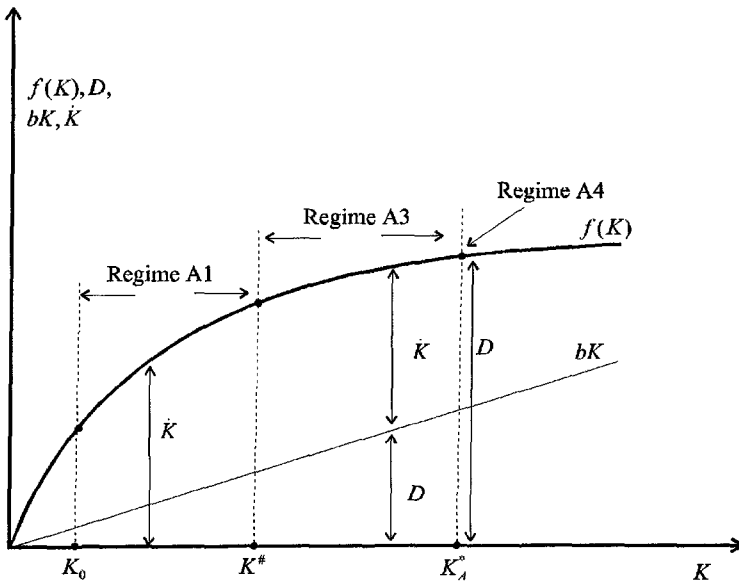
Figure 2.1   *Solution, Case $\sigma(K_A^*) \geq b$*

The solution now includes only one phase of internally financed growth. In this phase the firm uses all of its profits for investments and does not pay dividends. In fact, the firm's policy here is exactly the same as in the case of linear dividend taxation as reported by Sinn (1991). The similarity of the two solutions can be explained as follows. The dividends paid out by our low-profitability firm will be taxed in their entirety at the proportional capital income tax rate, τ_c. In the deterministic environment of our model the firm knows this with certainty, ignores the graduation in the tax scheme and behaves as if the dividend tax system were completely linear.

Figure 2.2 below illustrates the solution.

Solution Ending in Regime A3 (case: $\sigma(K^*) < b < \sigma(K_A^*)$):

Above it was shown that the model has a third solution ending in regime A3 and prevailing when $\sigma(K^*) < b < \sigma(K_A^*)$. This condition implies that the firm cannot switch from regime A1 to regime A2 but can very well continue to regime A3. In this regime the firm pays a dividend, $D = bK$, and uses the rest of its profits for investments, $I = f(K) - bK$. The firm continues this policy until at some level of capital, K', satisfying $K' < K_A^*$, it is faced with a situation where $f(K') = bK'$. The firm distributes all of its profits as dividends and its financing is not sufficient for investments. Thus K' is the firm's steady-state capital stock.

The firm's optimal policy is described by the following regime chain:

$$\text{Initial investment} \quad \Rightarrow \quad \text{Regime A1} \quad \Rightarrow \quad \text{Regime A3}$$

Note that $K' > K^*$, i.e. the steady-state capital stock, is greater than under linear dividend taxation. Note also that dividends will be taxed in their entirety at the proportional capital income tax rate, τ_c.

Figure 2.3 below illustrates the solution. Observe that the firm's capital stock remains at the level where the function describing the firm's operating profit, $f(K)$, intersects the line depicting the imputed return on capital, bK.

Summary of the Solutions

We found three different solutions to the basic model. Each of them entails a positive initial investment financed with equity, one or two growth phases where investment is financed with retained earnings and finally a steady-state phase with a constant capital stock. The solutions differ in the size of the steady-state capital stock and also in the qualitative features of the growth phase. Which of these solutions is effective was shown to be dependent on the magnitude of the firm's average rate of return on capital. Table 2.3 summarises the main characteristics of the solutions.

Table 2.3   Solutions to the Basic Model

Solution	Steady-state regime	Growth regimes	Steady-state capital stock	Feasibility condition
A I	A4	A1, A3	K_A^* $(>K^*)$	$\sigma(K_A^*) \geq b$
A II	A2	A1	K^*	$\sigma(K^*) \leq b$
A III	A3	A1, A3	K' $(>K^*, <K_A^*)$	$\sigma(K^*) < b < \sigma(K_A^*)$
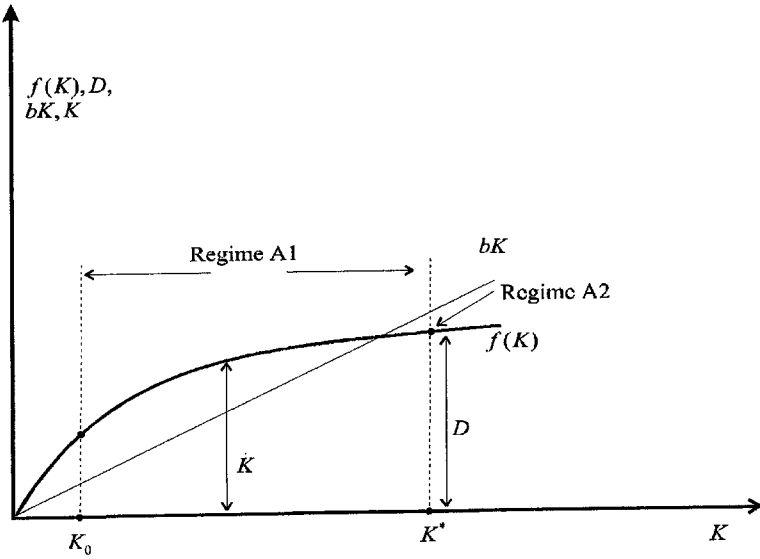
Figure 2.2   *Solution, Case* $\sigma(K^*) \leq b$
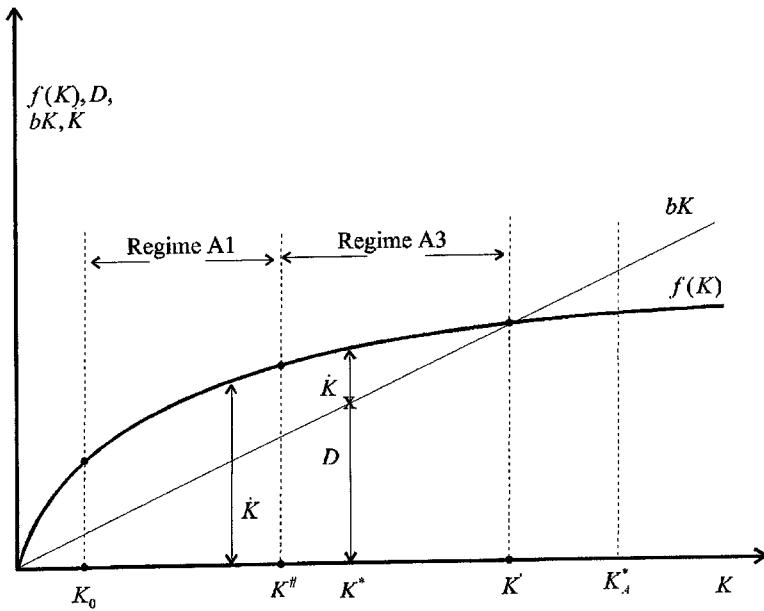
Figure 2.3   *Solution, Case* $\sigma(K^*) < b < \sigma(K_A^*)$

2.4 Economic Interpretation

This section aims to give an economic interpretation of the results obtained in the preceding section. The analysis concentrates on solution A I, which is perhaps the most interesting of the different cases. In this DIT clearly affects the firm's policy and the steady-state capital stock is determined by a marginal condition comparing the costs and benefits of investment, not as a boundary solution, as in solution A III for example.

Interpretation of the shadow prices

Let us return to the following optimality conditions

(2.17a) $\partial L/\partial D = 1 - \tau_c - \lambda - q_1 + q_2 = 0$

(2.17b) $\partial L/\partial D_x = -\tau_x + q_1 + q_3 = 0$

(2.17h) $\dot{\lambda} = r\lambda - f'(K)\lambda - bq_1$

In these equations q_1 is the shadow price of the first constraint, $D - D_x \leq bK$. Mathematically it represents the extra value of the Hamiltonian gained if the upper bound of the dividend taxed as capital income is increased by one unit. Thus it is the net benefit gained from a one-markka increase in imputed capital income.

The shadow price q_3, in turn, gives the change in the value of the Hamiltonian induced from changing the lower bound of dividends taxed as earned income, D_x. An increase in the potential amount of D_x imposes a decrease in D and this prompts an increase in the flow of financing that may be used for investing. Thus, we can interpret q_3 as describing the value of the benefits from investing one markka. The shadow price q_2 has the same general interpretation.

It is well known from optimal control literature that the co-state variable, λ, is the shadow price of the state variable. In this framework it tells us the impact of a one-unit increase in the capital stock on the value of the firm's equity.

Following van Hilten et al. (1993), we will give a further and economically interesting interpretation of the shadow prices q_2 and q_3. Let us study the conditions (2.17a) and (2.17b) in regimes A3 and A4, where $q_2 = 0$. Substituting q_1 from (2.17b) into (2.17a) we obtain

(2.28) $q_3 = \lambda - (1 - \tau_e)$

On the right-hand side of this equation λ is the gross benefit and $(1-\tau_e)$ a forgone after-tax dividend caused by using one markka of profits for investment instead of paying it as dividends to the shareholders. Thus, $(1-\tau_e)$ is the opportunity cost

of investing. Both benefits and costs are measured here in present-value terms. We can now interpret q_3 as giving the net present value (NPV) of a marginal investment. The same interpretation is also applicable to the shadow price q_2.

Using (2.17a), (2.17b) and (2.17h) we can obtain the following equation for λ in regimes A3 and A4[35]

$$(2.29) \quad \lambda(t) = \int_{t}^{\infty} \left\{ (1 - \tau_e) f'(K(s)) + q_3(s) f'(K(s)) + \left[\tau_x - q_3(s) \right] b \right\} e^{-r(s-t)} ds$$

λ is expressed here as the present value of the income stream generated by a one-unit increase in capital discounted at the discount rate of the shareholder. The benefits from an investment consist of three components. The first is the additional after-tax operating profit generated by investments. The second term, using the interpretation given by van Hilten et al. (1993), represents the indirect effect of additional investments at time s. It gives the benefit caused when the extra cash flow accrued at time s is invested back into the firm. The value of the benefits from this reinvestment is the invested cash flow, $f'(K)$, multiplied by the shadow price of investment (reduced dividend), q_3. The third term gives the net benefit from the increase in the capital base resulting from the original investment made at time t. This term has two parts. The expression $\tau_x b$ gives the instant tax saving brought about by the increase in the capital base. In this term b is the amount of dividend taxed as capital income instead of earned income due to the increase in the capital base and term τ_x is the change in the marginal tax rate for this income. The second part, the term $q_3 b$, again gives an indirect effect. It represents the value of decreased investment at time s caused by the increase in the capital base and the ensuing increased dividend payments. Note that the third term in (2.29) is non-negative in regimes A3 and A4.

Interpretation of firm's policy

In the following we will interpret the solution we derived in the previous section. We start with the steady-state phase of the solution, i.e. with regime A4. In this regime $\lambda = 1 - \tau_e$, $q_1 = \tau_x$, $q_3 = 0$. The last feature tells us that in that regime the NPV of investing is zero. The standard theory of financial economics tells us that it is not profitable for the firm to invest in this phase. This conclusion is in line with the optimal policy that we derived in the previous section. We saw there that in regime A4 the firm is in a steady state and pays out all of its cash flow as dividends.

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[35] First substitute λ in equation (2.17a) into the term that includes f' in equation (2.17h) and solve the resulting differential equation with respect to λ (see van Hilten et al. (1993) pp. 245-246).

We obtain a more detailed picture of this when we substitute the values of the shadow prices into equation (2.29), solve the integral and multiply both sides by r. We obtain the following equation describing the equilibrium of regime A4.[36] Note that (2.30) is written in terms of rates of return, not present values, as for example equation (2.28).

$$(2.30) \qquad (1 - \tau_e)f'(K) + \tau_x b = (1 - \tau_e)r$$

Equation (2.30) is a static non-arbitrage condition similar to the condition (1.7) in section 1.3 from which, by solving it with respect to f', we can obtain the marginal condition (2.26) determining the firm's steady-state capital stock.[37]

The left-hand side of (2.30) gives the gains from investing one markka. They include the marginal after-tax operating profit (first term) and the tax saving caused by the increase in the capital base (second term). These two terms are familiar to us from equation (2.29).[38] The right-hand side gives the opportunity cost of investing in terms of rates of return. It is the effective return on one markka of pre-tax dividends invested by the shareholder from after tax-dividends. The amount invested is $(1-\tau_e)$ and the nominal rate of return on it is r.

Thus, there are two sources of benefits from investments in DIT: first, the marginal increase in operating profit and second, the tax saving from increasing the capital base. Because of this tax saving it may be profitable to invest even if the pre-tax marginal return on capital is lower than its opportunity cost, the discount rate of the owner.

We gave above an interpretation of the firm's policy in the steady-state phase. Next we turn to an analysis of the growth phase. We start with regime A3, where the firm pays a dividend equal to $D = bK$ and invests the remainder of its profits, i.e. $I = f(K) - bK$. In regime A3 $q_3 > 0$. Using equation (2.28) and the NPV interpretation of q_3, this tells us that it is profitable to invest in regime A3, especially when the opportunity cost of investing is $1-\tau_e$. In the following equation we can see why the firm nevertheless pays out its cash flow as dividends up to the amount bK. We can rewrite (2.17a) as follows:

$$(2.31) \qquad -q_1 = \lambda - (1-\tau_c) < 0$$

where $-q_1$ gives the NPV of increasing investment (lowering dividends) when the opportunity cost of investing is $1-\tau_c$, i.e. when the marginal dividend is taxed as capital income. This value is negative, which implies that it is not profitable to

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[36] The integral in equation (2.29) can be solved easily since f' is a constant in regime A4 and τ_e, τ_x and b are parameters that are always constants.

[37] Note that our aim here is not to derive (2.26), but to clarify its interpretation.

[38] There is no indirect effect in this case because the firm does not invest.

invest the profits. Thus, the firm pays out the profits as dividends up to the amount bK.

Imputing q_1 from (2.17b) to (2.17a) we obtain

(2.32) $\qquad 1 - \tau_c - \lambda + q_2 - \tau_x + q_3 = 0$

In regime A1 $q_2 > 0$ and $q_3 = \tau_x > 0$. From these and from (2.32) we obtain the following two formulas for the NPV in regime A1.

(2.33) $\qquad$ a) $\qquad q_2 = \lambda - 1-\tau_c$

$\qquad\qquad$ b) $\qquad q_2 + q_3 = \lambda - 1-\tau_e$

The first equation gives the NPV of marginal investment when the opportunity cost of capital is $1-\tau_c$, i.e. when the marginal dividend, if paid out, is taxed as capital income. The second gives the NPV when the dividend, if paid out, would be taxed as earned income. In both cases the NPV is positive, which implies that it is profitable to use all funds for investments and pay no dividends. This is in line with the result obtained in section 2.4.

2.5 Further Analysis

2.5.1 Size of the Incentive

We have shown that under graduated dividend taxation a firm with a high average rate of return grows its capital stock larger than in an ordinary linear tax system. In this subsection we make numerical calculations that show how large this tax incentive is. In equation (2.26) this incentive is represented by the second term on the right-hand side. The term expresses how much of a lower return before taxes the owner will accept under DIT on investments made in the firm compared to the discount rate, r. The size of the incentive seems to be dependent on the tax rate on earned income, τ_e, the capital income tax rate, τ_c, and the imputed rate of return, b. Table 2.4 presents calculations for the incentive term, w, and for the firm's cost of capital, p, for different marginal tax rates on earned income and for different interest rates. Finnish parameter values for b and τ_c are used ($b = 15\%$, $\tau_c = 28\%$).

Table 2.4   *How Large is the Incentive?*

		Marginal tax rate on earned income				
		40 %	45 %	50 %	55 %	60 %
w, % [*]		3.0	4.6	6.6	9.0	12.0
p, % [*]	$r = 10$ %	7.0	5.4	3.4	1.0	-2.0
	$r = 7,5$ %	4.5	2.9	0.9	-1.5	-4.5
	$r = 5$ %	2.0	0.4	-1.6	-4.0	-7.0

[*] $w = \tau_s b/(1-\tau_s)$, $p = r - w$

The table shows that the incentive of DIT described by w is strongly dependent upon the marginal tax rate for earned income and can be quite large. The firm's cost of capital is lowered by 6.6 percentage points at the 50 per cent marginal tax rate and by 12 percentage points at the 60 per cent marginal tax rate. In this latter case, for example, the firm can accept an investment yielding 12 percentage points lower than the market rate of interest rather than distribute the profit as a dividend. From the table it can be seen that the incentive makes the cost of capital very low, at high marginal tax rates even strongly negative.

The calculations are made using Finnish parameter values. It is worth noting, however, that the model contains some strong assumptions, and therefore the figures in table 2.4 do not describe the Finnish case quite correctly. The tax system, for example, is unrealistically simple in that it only includes dividend taxation. In chapter 3 we will observe that the firm's cost of capital is affected inter alia by capital gains tax. Moreover, the capital base contains only homogenous capital with decreasing returns. The analysis in section 4.2 will show us that including financial capital in the model changes the picture substantially.

2.5.2 Allocative Distortion Generated by DIT

This study focuses primarily on the effect of taxation on firm behaviour, i.e. it uses a partial equilibrium framework. In this section we diverge from this principle and investigate in a simple general equilibrium framework how the graduated tax system applied to unlisted companies may distort the allocation of capital in the economy. We will apply a two-sector model developed by Harberger and often used in the context of tax incidence analysis.[39] Here we follow Sinn (1987) and use the model purely to examine the effects of taxation on the intersectoral allocation of capital.

We assume that there are two production sectors in the economy, listed firms (Y) and unlisted firms (X). Both sectors use capital, K, and labour, L, as inputs and they are

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[39] For more complete descriptions of the framework, see McLure (1974) or Myles (1995) pp. 235-242.

provided with the same production function, $f(K,L)$. Capital is assumed to be perfectly mobile and labour is perfectly immobile between the sectors. The two sectors compete for the economy's given capital stock: $K^+ = K_X + K_Y$, where $K^+ = $ constant. We also assume that investment financing is offered only on equity terms. While dividends distributed by listed firms are taxed linearly, dividends received from unlisted firms are subject to a graduated dividend tax scheme. With a market rate of interest, r, the two sectors invest in capital until the following marginal conditions are satisfied:

(2.34)
$$f_{K_Y}(K_Y, L) = r \qquad\qquad \textit{listed sector}$$

$$f_{K_X}(K_X, L) + \frac{\tau_x}{1 - \tau_e} b = r \qquad\qquad \textit{unlisted sector}$$

In equilibrium the efficient allocation of capital that maximises total production would require the marginal returns on capital in the various sectors to be equal. Owing to the graduation in taxation the unlisted firm's capital will nevertheless be allocated so that in equilibrium:

(2.35)
$$f_{K_X}(K_X, L) + \frac{\tau_x}{1 - \tau_e} b = f_{K_Y}(K_Y, L)$$

In equilibrium the marginal return will displace the tax wedge, $w = \tau_x b/(1-\tau_e)$, preventing the efficient allocation of capital. Owing to the tax wedge too much of the given capital stock is allocated to unlisted firms and correspondingly a less than optimal amount to listed firms. As a consequence of this, part of the total capital is used inefficiently, which means that the economy's total production is lower than in an economy where different sectors are treated neutrally in the tax system.

Figure 2.4. illustrates the allocation. The horizontal axis depicts the constant capital stock in the economy for which the two sectors compete. The unlisted sector's capital, K_X, is measured from left to right and the listed sector's capital, K_Y, from right to left. The curves intersecting at point D describe the MRR on capital for the two sectors. Point B on the horizontal axis describes the allocation of capital in a tax-less economy and in an economy with linear dividend taxation. The amounts of capital in the two sectors are in that case $K_X = $ AB and $K_Y = $ CB.

The taxation of dividends paid by unlisted firms under a graduated system will nevertheless lead to an allocation of capital that is described by point B' in the figure where $K_X = $ AB' and $K_Y = $ CB'. Due to the tax system, the unlisted sector's production is now greater than the optimal allocation by the amount BDFB', while the production of the listed sector is smaller than the optimal allocation by

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the amount BDEB'. In net terms these mean a production loss of DFE for the overall economy. Note also that due to increased investment demand the equilibrium interest rate is now higher than in the case of a neutral tax system.
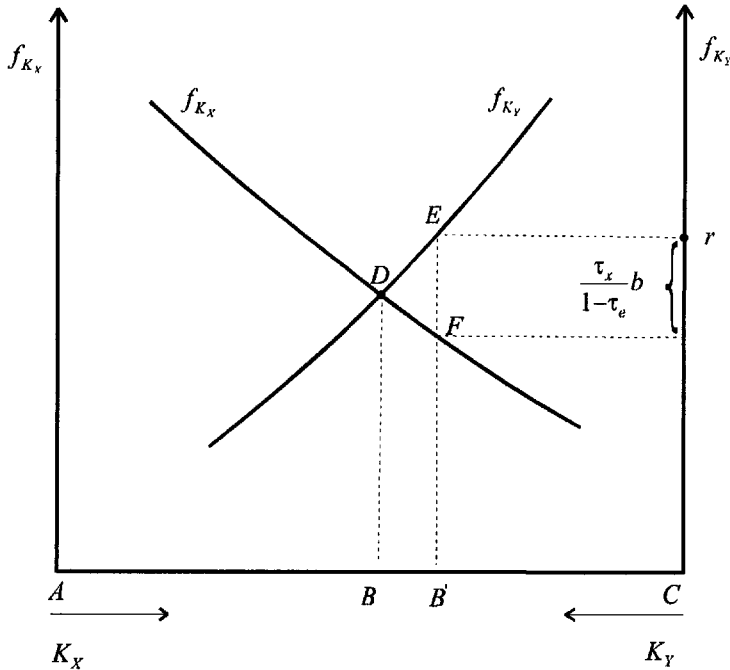
Figure 2.4   Allocative Distortion and Resulting Production Loss

The original two-sector analysis by Harberger considered the allocational implications of double taxation of corporate firms. This analysis showed that double taxation will reallocate capital from the corporate sector to unincorporated firms subject to more lenient taxation. Observe the difference between the results of the two models. While in the original two-sector analysis capital is reallocated towards the more lightly taxed sector, in our case capital is induced to increase in the unlisted sector where the average and marginal tax rates are higher than in the other sector.

The analysis was marked by the argument that graduated taxation may lead to a distortion of capital allocation and this may consequently lead to a loss of production and welfare. The analysis contained several strong assumptions. In section 4.2 we will argue, for example, that relaxing the homogeneity of capital assumption of our basic model changes the efficiency conclusions remarkably.

2.6 Sensitivity Analysis

2.6.1 Introduction

This section studies the impact of changes in the environmental factors on the firm's optimal policy. The analysis concerns changes in the parameters b, τ_c, τ_e and r as well as marginal operating income, $f'(K)$,[40] ceteris paribus. The focus of interest is the impact of these changes on critical levels of capital: the firm's initial capital, K_0, steady-state capital, K_A^*, and the level of capital at the switching point between regimes A1 and A3, $K^\#$. In addition we will evaluate the impact of the changes on the share of external equity of the total financing, denoted here by e and defined as $e = K_0/K_A^*$.

Note that in the reference case of linear dividend taxation the steady-state capital stock is independent of the dividend tax rate and that the higher this tax rate is, the smaller the initial stock of equity, i.e. $\partial K^*/\partial \tau_d = 0$ and $\partial K_0/\partial \tau_d < 0$ (see Sinn (1991)).

We will concentrate our analysis again on the case $\sigma(K_A^*) > b$, where the firm's steady-state capital stock is K_A^* and where dividends are taxed on the margin as earned income. This restriction excludes not only the analysis of the other two solutions, but also a situation where the change in environment would lead to a shift from one solution to the next. For example, a rise in parameter b may lead to a situation where the condition $\sigma(K_A^*) > b$ is not valid after the change, leading the firm to switch its policy to solution A II or A III. Similarly, a rise in productivity of capital might induce the firm to switch from solution A II to solution A III or AI. We ignore these possibilities in the analysis but comment on one implication of such changes at the end of this section.

The impact of environmental changes on the steady-state capital can be studied using ordinary means of comparative statics. Using the notation $\tau_x = \tau_e - \tau_c$ we can rewrite the marginal condition (2.26) as

$$(2.26') \qquad f'(K) = r - \frac{\tau_e - \tau_c}{1 - \tau_e} b$$

The impact of changes in environmental factors on the levels of the capital stock K_0 and $K^\#$ can be examined using the methods of comparative dynamics. We will first investigate how these changes shift the adjustment path in the (K, λ) space and, finally, at what level of capital the new path satisfies the given values of the co-state variable, λ. In this analysis we make use of the marginal condition

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[40] To be more exact the marginal productivity of capital will be assumed to move up at all levels of K.

(2.26') as well as the following differential equations describing the slope of the co-state variable:

$$\text{(2.36)} \qquad \frac{d\lambda(K)}{dK} = \frac{\dot{\lambda}(t)}{\dot{K}(t)} = \frac{(r - f'(K))\lambda}{f(K)} \qquad \text{(regime A1)}$$

$$\text{(2.37)} \qquad \frac{d\lambda(K)}{dK} = \frac{\dot{\lambda}(t)}{\dot{K}(t)} = \frac{(r + b - f'(K))\lambda - b(1 - \tau_c)}{f(K) - bK} \qquad \text{(regime A3)}$$

Figure 2.5 describes the solution in the (K, λ) space (cf. Sinn 1991, p. 280). The adjustment path approaches the equilibrium point $(K_A^*, 1-\tau_c)$ in three phases. First the capital grows from zero to the level K_0 via initial investment. The adjustment path is plotted to be horizontal in this phase.

In the next phase the adjustment occurs in regime A1. The co-state in this stage is a declining function of K, which can be demonstrated as follows. Analysis in appendix 2 implies that $f' > r$ in regime A1. From this it follows that the numerator of the right-hand side of (2.36) is negative. Since the denominator is always positive, we can conclude that $d\lambda/dK < 0$. By taking the derivative of the slope expression of the co-state with respect to K, we observe that $d^2\lambda/dK^2 > 0$, i.e. the rate of decline in the adjustment path decreases as K grows.

In regime A3 the co-state is declining and its slope approaches zero towards the end of the regime. That $\lambda(K)$ is declining in the interval $[K^\#, K^*]$ is easy to show. From the properties $f' \geq r$, $\lambda(t) > 0$ and $q_1 = (1-\tau_c)-\lambda > 0$ it follows that $\dot{\lambda}(t)$, and since the denominator of (2.37) is positive, $d\lambda/dK$ is also negative in that interval (including point $K=K^*$). Furthermore, by substituting the value of $\lambda(t)$ at the end of regime A3 (where $K= K_A^*$) from (2.26) to (2.37) it can be easily observed that $d\lambda/dK = 0$ at that point. This means that λ approaches its steady-state value smoothly.

The negativity of $d\lambda/dK$ in the interval $[K^*, K_A^*]$ cannot be studied in quite the same way. However, by differentiating $d\lambda/dK$ with respect to K we can observe that this second derivative is $d^2\lambda/dK^2 > 0$ at the point where the first derivative is $d\lambda/dK = 0$. This result reveals two important features. First, $\lambda(K)$ must be declining when it approaches the end-point of this regime (at $K=K_A^*$). Second, the sign of $d\lambda/dK$ cannot fluctuate from positive to negative when K grows. These results, combined with the fact that $\lambda(K)$ is declining at $K = K^*$, exclude the

cases where $d\lambda/dK \geq 0$ in the interval $[K^*, K_A^*]$. Thus, the slope of $\lambda(K)$ must be negative throughout regime A3 except at the final point $K = K_A^*$.[41]
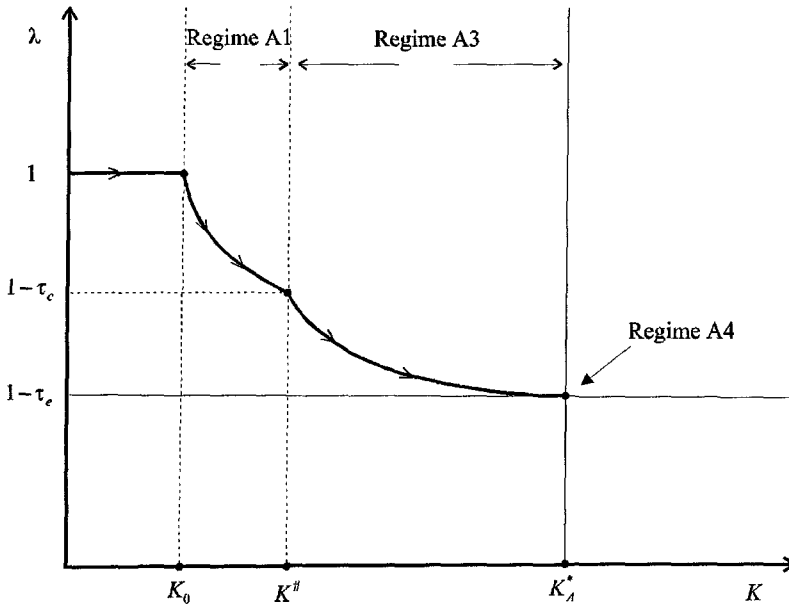
Figure 2.5   Solution in the (K, λ) space

2.6.2 Effects on Steady-State Capital, K_A^*

The dependence of K_A^* on exogenous variables and $f'(K)$ is obtained by differentiating (2.26') implicitly. We obtain the following results

$$(2.38) \qquad \partial K_A^*/\partial \tau_e > 0, \ \partial K_A^*/\partial \tau_c < 0, \ \partial K_A^*/\partial b > 0, \ \partial K_A^*/\partial r < 0, \ \partial K_A^*/\partial f' > 0$$

A rise in the tax rate on earned income, parameter b and the productivity of capital increase the steady-state capital stock, while a rise in the tax rate on capital income and the interest rate lowers it. From the effects of changes in the tax rates we can infer that a rise in the tax rate differential increases the steady-state capital stock.

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[41] The adjustment path is similar also in the case in which $f'(K)$ is negative when K approaches its steady state value.

These results are quite intuitive. An increase in τ_e, b and a decrease in τ_c increase the tax saving from a rise in the capital base and make investment more profitable. A rise in r increases the costs of investment and has an adverse effect on the steady-state capital stock. A rise in $f'(K)$ in turn raises the marginal productivity of investment and increases the level of the capital stock that is profitable to use in production.

2.6.3 Effects on $K^\#$

Tax Rate on Earned Income τ_e

As the earned income tax rate, τ_e, rises, the new equilibrium is attained at a higher level of capital and smaller value for the co-state. Let us denote the new equilibrium values K_A^*'and $\lambda'(K_A^*')$. In the (K, λ) space the equilibrium shifts to the right and downwards. From equation (2.37) we see that $\partial\lambda/\partial K$, which depicts the slope of the adjustment path, is not directly dependent upon τ_e. Thus, the new and the old path follow the same differential equation. Only the boundary condition differs.

We will first analyse the new path at the level of the old steady-state capital stock, K_A^*. Let us assume that the new path is above the old path at this point, i.e. $\lambda'(K_A^*) > 1-\tau_e$. From this assumption it follows that the first term of the expression $\dot{\lambda}(t)$, denoted by $(b + r - f')\lambda$, is higher in the new path than in the old path. Because the term $(1-\tau_c)b$ is the same in both paths, we can conclude that $\dot{\lambda}(t)$ must be positive in the new path. From this it follows that $d\lambda/dK$ is positive.

If the new path were exactly on the old path when $K = K_A^*$, the slope of the new path at this point should be $d\lambda/dK = 0$. However, it was shown above that the co-state is negative in regime A3 at all points except at the equilibrium point. From this we can conclude that the new adjustment path cannot be on or above the old path at the level of the capital stock K_A^*. Instead it must be below the old path, i.e. $\lambda'(K_A^*) < \lambda(K_A^*)$.

The new adjustment path must actually be below the old adjustment path for all values of K in regime A3. We can make this conclusion as follows. If the new path were above the old path at some level of capital, there would have to be a point where the new path intersected the old path. Above it was demonstrated that the old and new adjustment paths are characterised by the same differential equation (2.37). This differential equation nevertheless would have to have a unique solution, and interpreting this hypothesised intersection point as an initial value for the equations would mean that the old and new paths had to be on the same path. We have nevertheless shown that at least at the capital level $K=K_A^*$

the new path is below the old path. This means that no common point can exist and further that the new adjustment path must be below the old path for all values of K in regime A3.

A change in parameter τ_e affects the capital level $K^{\#}$ through the shift in the adjustment path. The initial value of λ in regime A3 will remain unchanged as τ_e rises, i.e. $\lambda(K^{\#})= \lambda'(K^{\#'}) = 1-\tau_c$. From the downward shift in the adjustment path it follows that the new path crosses this horizontal line at a lower level of capital. Thus, a rise in the tax rate τ_e lowers the level of capital at the switch between regimes A1 and A3, i.e. $\partial K^{\#}/\partial \tau_e < 0$. Figure 2.6 illustrates this effect.

Capital Income Tax Rate τ_c

A rise in parameter τ_c affects both the equilibrium point and the slope of the adjustment path. The new equilibrium will occur at a lower level of capital, but at the old value of the co-state variable. In the (K, λ) space the equilibrium point will shift horizontally to the left. From equation (2.37) we can observe that a rise in τ_c reduces the negativeness of the last term in the numerator of the expression for $d\lambda/dK$. This means that the change reduces the negativeness of the slope of the adjustment path. Together these changes mean that the new adjustment path in regime A3 is below the old path for all values of K. The impact of a rise in τ_c on the level of capital at the switching point, $K^{\#}$, is nevertheless not unique since the change in this parameter also affects the value of λ at the switching point reducing it. The new adjustment path can intersect the changed initial value at the original point as well as to the left or to the right of it.

Parameter b

A rise in parameter b will steepen the adjustment path and shift the equilibrium horizontally to the right. On the basis of these changes we can infer that the new adjustment path is above the old path for all values of K in regime A3. Because a change in b does not affect the initial value of λ in regime A3, we can deduce that the new path will satisfy this unchanged initial value at a higher level of capital than the old path. Thus a rise in b prompts an increase in the level of capital at the switching point between regimes A1 and A3, i.e. $\partial K^{\#}/\partial b > 0$.

Interest Rate r

A rise in interest rates will affect both the equilibrium point and the slope of the adjustment path. It will reduce the steady-state capital, shifting it horizontally to the left in the (K, λ) space and will reduce the negativeness of the slope of the adjustment path. These changes imply that the new path is below the old path for all values of K. This implies that the new adjustment path satisfies unchanged initial value of λ at a lower level of capital than the old path. Therefore $\partial K^{\#}/\partial r < 0$.

Productivity of Capital

A rise in productivity for all levels of capital will raise the steady-state capital stock, keeping the value of the co-state unchanged. The effect on the slope of the adjustment path remains unclear, however. This is because both the numerator and the denominator of the slope expression in (2.37) change. The numerator declines (becomes more negative) and the denominator increases. The total effect is undetermined. It turns out that the same problems also apply in regime A1. Thus the effect of a change in f' on $K^{\#}$ as well as on K_0 remains unclear.

2.6.4 Effects on Optimal Initial Capital, K_0^*

The impact of the change in the parameters and the productivity of capital on the optimal initial capital can be obtained by studying the shift in the adjustment path in regime A1. From equation (2.36) we can see that now the slope of the adjustment path is not dependent on the tax parameters, τ_e, τ_c and b, but is still dependent upon the interest rate and productivity of capital.

Earned Income Tax Rate τ_e

Above it was demonstrated that an increase in the parameter τ_e will reduce the capital at the switching point between regimes A1 and A3. The switching point, which can also be seen as the end-point of the adjustment path in regime A1, shifts horizontally to the left in the (K, λ) space. Because the adjustment path is negatively sloped in regime A1, the new end-point must be below the old adjustment path. From equation (2.36) we see that the differential equation of the adjustment path is the same for the old and new paths, from which it follows that the new path must be below the old path at all levels of K. The new path satisfies the unchanged initial value of the co-state, $\lambda = 1$, at a lower level of capital. A rise in the earned income tax rate, τ_e, will thus reduce the optimal initial capital, i.e. $\partial K_0 / \partial \tau_e < 0$. Figure 2.6 illustrates this effect.

Capital Income Tax Rate τ_c

Above it was shown that a rise in the capital income tax rate, τ_c, will shift the point of switching between regimes A1 and A3 downward and that the horizontal shift remains uncertain. From equation (2.37) we see that the change in parameter τ_c will not affect the slope of the adjustment path. If the switching point does not shift horizontally at all or if it shifts to the left, it is clear that the new switching point will be located below the old adjustment path. In this situation the new adjustment path is below the old adjustment path for all K in regime A1.

The case where the point shifts to the right is more difficult. We know, however, that the new point is located below the old adjustment path in regime A3.[42] From equations (2.36) and (2.37) we see that at the switching point where $\lambda = 1-\tau_c$ it holds that $d\lambda_{A1}/dK > d\lambda_{A3}/dK$, which implies that the continuation of the old path in regime A1 must be above the old path of regime A3. From these features we can conclude that the new switching point must be below the continuation of the old adjustment path of regime A1. Because the new and old adjustment paths have the same differential equation, the new path is below the old path for all values of K and intersects the line $\lambda = 1$ at a lower value of K than the old path. Thus, a rise in τ_c reduces the optimal initial capital, i.e. $\partial K_0/\partial\tau_c < 0$.

Parameter b

An increase in parameter b will shift the end point of the adjustment path in regime A1 horizontally to the right but leaves the slope unchanged. This implies that the new path will be above the old path and will intersect the line $\lambda = 1$ at a higher value of K than the old path, i.e. $\partial K_0/\partial b > 0$.

Interest Rate r

A rise in the rate of interest affects the end-point as well as the slope of the adjustment path in regime A1. The end-point shifts horizontally to the left and the slope becomes less negative. From this we can conclude that the new adjustment path is below the old path and fulfills the initial condition of the co-state at a lower level of capital. Thus a rise in r reduces the initial capital, i.e. $\partial K_0/\partial r < 0$.

2.6.5 Summary of Sensitivity Analysis

Table 2.5 summarises the findings of the sensitivity analysis. A rise in the earned income tax rate, parameter b and productivity of capital will increase the long-run equilibrium capital stock, whereas a rise in the capital income tax rate and interest rate will lower it. Thus unlike under linear dividend taxation the size of the steady-state capital stock now depends on dividend tax parameters.

Correspondingly a rise in the earned income tax rate, capital income tax rate and interest rate will reduce the optimal initial capital, K_0, whereas a rise in parameter b will increase it. Note that the direction of the effect of dividend tax rates on the stock of initial capital is the same as under linear dividend taxation.

A rise in the earned income tax rate and interest rate will reduce the level of capital at the regime switch between regimes A1 and A3, i.e. it will spur the earlier payment of dividends, whereas a rise in parameter b will increase this level of

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[42] A rise in τ_c shifts the adjustment path downward in regime A3, see above.

capital. The table also presents the impact of a change in parameters on the share of financing via external equity of the steady-state capital stock, variable e. The sign of this effect is unique only with respect to the earned income tax rate. A rise in τ_e will reduce e.

Table 2.5   Results of the Sensitivity Analysis

Changing parameter	K_A*	$K^{\#}$	K_0	e
τ_e	+	−	−	−
τ_c	−	?	−	?
b	+	+	+	?
r	-	-	-	?
$f'(K)$	+	?	?	?

Above we excluded environmental changes that would induce a move from one solution to another. One interesting example of such changes is a shock that alters the productivity of capital. It is worth examining this case in some detail. Consider a situation where a (permanent) productivity-improving shock changes the firm's type from a "solution A II" firm to a "solution A I" firm. This means that due to this shock the firm's equilibrium capital stock changes from $K*$ to K_A*. This increase in f' actually affects the firm's behaviour in two ways. First, it prompts the firm to exploit the new opportunities for profitable production caused directly by the improved productivity. Second, it also triggers the special investment incentives of the graduated tax system. The combined effect on investment may be much stronger than the effect of a similar shock in the environment of an ordinary tax system. Note that the opposite may also occur. A profitability-decreasing shock may change the type of the firm, lower the desired capital stock and hamper investment. This discussion suggests that the dual income tax system may strengthen the effects of business cycles on investment.

As a final topic we will present a result that provides some further information on the solution to the basic model. It is not a finding of sensitivity analysis, but it can be presented here because it is based on the same approach applied above. Our aim is to compare the size of the optimal initial capital under linear and graduated dividend taxation.[43] We assume that the dividend tax rate of the linear system, τ_d, equals the tax rate on capital income under DIT, i.e. $\tau_d = \tau_c$. In appendix 2 it is shown that the level of the capital stock, at which the first growth

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[43] We can also interpret this comparison as being between cases $\sigma(K*) < b$ and $\sigma(K_A*) > b$ in our basic model.

phase of our model, regime A1, ends, satisfies $K^\# < K^*$, where $K^\#$ is the end-value of K in regime A1. The value of λ at the end-point of this growth regime is the same in both tax systems. In the (λ, K) space this means that the end-point of regime A1 in the graduated system is horizontally to the left of the equilibrium point of the linear system. In both systems the slope of λ is characterised by the same differential equation given by equation (2.36). Relying on the same arguments as above we can conclude that the adjustment path of the graduated system must be below the path of the linear system for all levels of capital and that it satisfies the initial condition for λ, which is the same for both systems, at a lower level of capital than that of the linear system. It therefore holds that $K_0^A < K_0^L$.[44] The graduated system thus crowds out external equity as a financing form at a higher rate than linear dividend taxation and leads to lower investment of initial equity and slower growth in the early phases of a firm.
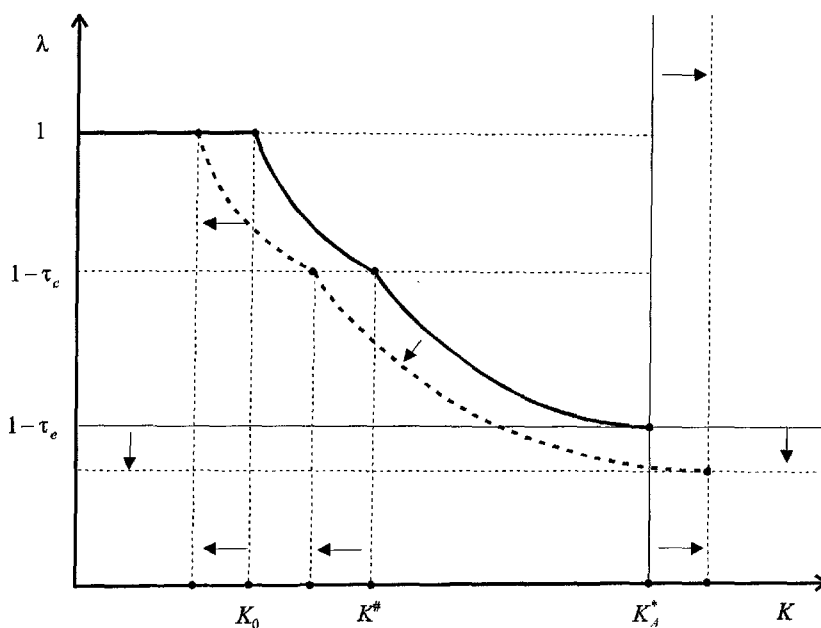
Figure 2.6   Effects of a Rise in Tax Rate on Earned Income τ_e

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[44] The superscript 'A' indicates graduated taxation and the superscript 'L' linear dividend taxation.

2.7 Summary

This chapter started our analysis of graduated dividend taxation using a simple, dynamic investment model developed by Sinn (1991). Our aim was two fold. We sought to set up a model framework that could serve as a basis for elaboration in later chapters. We also aimed to obtain some basic results that can be extended and qualified later, and, perhaps most importantly, be compared to the well known results by Sinn (1991) concerning firms' policy under linear dividend taxation.

We first introduced the dynamic model framework and after that developed a way to model the graduated dividend tax system. This modelling method removes the non-linearities caused by the tax system and allows us to use standard mathematical tools in the analysis.

The major finding of this chapter was that the neutrality of dividend taxation with respect to investment decisions shown by Sinn (1991) breaks down when a graduated tax scheme is introduced. There are, however, three different solutions to the problem depending on the size of the firm's average rate of return on capital. In each case the firm's steady-state capital stock is defined according to a different rule. We observed that after introducing the graduated dividend tax scheme a firm with a low average rate of return ($\sigma(K^*)<b$) behaves entirely as in linear taxation. But the behaviour of a high-rate-of-return firm ($\sigma(K_A^*)>b$) is affected instead. Graduated taxation decreases the firm's cost of capital, which implies an increase in the steady-state capital stock. An intermediate case was also found ($\sigma(K^*)< b< \sigma(K_A^*)$) where the incentive exists but is smaller than in the high-profitability case.

The introduction of the graduated tax scheme was also observed to change the firm's adjustment to the steady state. The growth phase of a high-rate-of-return firm consists of two different regimes. In the first the firm uses all its funds to finance growth similarly to the idle growth phase in Sinn (1991). In the second regime the firm pays out part of its internal financing as dividends and uses the residual to finance investments.

The non-neutrality with respect to investment follows from the fact that in the graduated tax system the firm can by investing reduce the share of dividends subject to earned income taxation and increase the part that will be taxable as capital income. Thus, there are two sources of benefits that investment brings for the shareholders in this tax system: (a) an increase in operating income (and future dividends) and (b) a tax saving via increasing the share of dividends taxable as capital income. The latter benefit lowers the firm's cost of capital and increases the optimal steady-state capital stock compared to linear dividend taxation.

Using numerical calculations and plausible parameter values for the Finnish tax system we showed that the effect on the cost of capital can be large. At high levels of marginal tax rates on earned income the firm's cost of capital can well be driven below zero.

Using comparative statics we showed that in the high-rate-of-return case the firm's steady-state capital stock is dependent on three tax parameters: (a) positively on the marginal tax rate on earned income, (b) positively on the imputed return parameter b and (c) negatively on the tax rate on capital income. The size of the firm's initial equity was shown to be negatively dependent on tax rates. It was also shown that the discrimination against external equity is more severe in a graduated dividend tax system than in linear taxation.

Using a simple textbook two-sector model we also demonstrated that the tax incentive implies a distortion of the use of resources. In the framework applied the graduated dividend tax system induces an over-accumulation of capital in unlisted companies at the cost of other firms and thus leads to inefficient investment and welfare losses.

The observation according to which the firm's optimal investment policy depends on the firm's relative profitability led us to conclude that graduated dividend taxation may cause pro-cyclical changes in investment.

3. A More Comprehensive Tax System: Firm-Level Taxation and Capital Gains Taxation

3.1 Introduction

Chapter 2 examined graduated dividend taxation in a very rudimentary environment where both the firm's policy alternatives and tax system were extremely simplified. Here we will take a step toward a more realistic framework. We will include a broader tax system containing both firm-level taxation and taxation of capital gains and interest income. Double taxation of dividends is assumed to be mitigated by an imputation system.

Our aim is to study how the changing of the tax system affects the firm's optimal policy as observed in the preceding chapter. We will ask if the special incentives of dual income taxation are still present, and if so, how do the changes in the tax system affect the scale of these. We are also interested in seeing how the new tax factors affect the adjustment phase of the firm, especially the relative importance of internal financing and initial equity in financing the firm's long-run equilibrium capital stock.

We will focus our analysis on the case in which dividends are subject to taxation on the margin as earned income.[45] It turns out that the condition for this case is:

(A3.1) $$\sigma(K_G^*) > b,$$

where $\sigma \equiv (1-\tau_f)f(K)/(1-s)K$. K_G^* depicts the optimal capital stock in the long-run equilibrium for the case studied and τ_f and s are the corporate tax rate and the rate of imputation credit.

We assume that the new tax parameters are constant and satisfy:

(A3.2) $$1 > \tau_g, \tau_f > 0, \quad \tau_f \geq s \geq 0$$

where τ_g is the marginal effective tax rate on capital gains (accrual basis) and τ_f and s are as defined above. In a full imputation system $s = \tau_f$ and in a partial imputation system $\tau_f > s > 0$. We assume moreover that interest income is taxed as capital income at the rate τ_c.

We will not restrict the analysis to the Finnish case of unified capital income taxation with full imputation credit, where $\tau_c = \tau_f = s$, but study the more general

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[45] The case of a high-rate-of-return firm. See above section 2.3.4.

case where these parameters can differ. However, the Finnish system is an important reference case in the analysis.

3.2 Changes in the Tax System

3.2.1 Imputation System

As discussed in section 1.2, the mitigation of double taxation of dividends is a central component of the tax systems of most OECD countries. We incorporate this feature in our tax system in the form of an imputation system. We model it using the approach by King (1977) and Sinn (1987). In doing so we deliberately omit many of the complex details of the systems such as rules on the consequences if the dividend exceeds the firm's taxable income (or imputation credit exceeds corporate tax calculated on the firm's taxable profit).[46] Some authors, especially Keen and Schiantarelli (1991), Huber (1994) and Weichenrieder (1998), have highlighted the potential influences of these regulations.

To model the imputation system we first introduce some new concepts. The owner's taxable dividend income is now the cash dividend grossed up with the imputation credit (gross dividend) and is defined as $G = D/(1-s)$. Let G_x denote that part of gross dividend income taxable as earned income. This income concept is obtained by comparing the gross dividend to the imputed return on capital, i.e. $G_x = G - bN$. The shareholder's dividend tax after imputation credit, T_{ac}, is now obtained by first calculating the tax based on the gross dividend and deducting from this the imputation credit, sG. We obtain:

$$(3.1) \qquad T_{ac} \; = \; \tau_c bN + \tau_e G_x - sG \; = \; \tau_c bN + \tau_e \left(\frac{D}{1-s} - bN \right) - \frac{sD}{1-s}$$

Let us define $D_x = (1-s)G_x$. Using this and the definition for G_x we obtain:

$$(3.2) \qquad\qquad D_x = D - (1-s)bN.$$

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[46] In Finland imputation credit is compared to the corporate tax calculated on the firm's taxable profit ('mainstream corporate tax'), whereas in the UK the comparison is between the gross dividend and the taxable profit of the firm. In the Finnish system the firm has to pay a compensatory tax if the credit exceeds the 'mainstream corporate tax'. This compensatory tax can nevertheless be charged against tax surpluses accumulated during the previous ten years. The tax surplus is the positive difference between the 'mainstream corporate tax' and the credit. In the UK the firm pays a type of withholding tax based on the dividend, 'advanced corporation tax' (ACT), which can be deducted from corporate tax. The portion of the ACT that cannot be deducted immediately can be carried forwards or backwards. The time constraint for the offsetting of ACT and the charging-off of tax surpluses in Finland, as well as the fact that they do not earn interest, make the effective marginal tax rate of the dividend non-linear and affect the firm's dividend and financing policies. As far as we know, however, these latter features have not yet been studied thoroughly.

Thus, the part of the cash dividend subject to taxation as earned income is now obtained by comparing the cash dividend to the after-credit imputed return. Utilising equations (3.1) and (3.2) we obtain the following expression for the net dividend:

$$(3.3) \qquad D_n = \frac{1-\tau_c}{1-s}D - \frac{\tau_x}{1-s}D_x$$

3.2.2 Other Changes

The changes in the tax system also affect the firm's objective function, $V(t_0)$. The owner's discount rate, r', and the coefficient γ' are now defined as:

$$(3.4) \qquad r' = \frac{1-\tau_c}{1-\tau_g}r, \qquad \gamma' = \frac{1}{1-\tau_g}$$

Internal financing is accrued as after-tax operating income, $(1-\tau_f)f(K)$. The firm's budget constraint is thus:

$$(3.5) \qquad (1-\tau_f)f(K) + Q = D + I$$

The first control constraint is also affected. Using the above definition for D_x in (3.2) it is obtained as:

$$(3.6) \qquad D - D_x \le (1-s)bN$$

3.3 Model, Optimality Conditions and Analysis of Feasible Policies

By substituting equations (3.3) - (3.6) into the basic model, we obtain the following problem:

$$(3.7a) \qquad \max_{\{K_0,D,D_x,Q\}} \int_{t_0}^{\infty} \left\{ \frac{1-\tau_c}{(1-s)(1-\tau_g)}D - \frac{\tau_x}{(1-s)(1-\tau_g)}D_x - Q \right\} e^{-\frac{1-\tau_c}{1-\tau_g}r(t-t_0)} dt + \varphi(K(t_0))$$

$$(3.7b) \qquad \dot{K} = (1-\tau_f)(f(K)+Q-D), \qquad K(t_0)=K_0$$

$$(3.7c) \qquad h_1 = (1-s)bK + D_x - D \ge 0$$

$$(3.7d) \qquad h_2 = D \ge 0$$

$$(3.7e) \qquad h_3 = D_x \ge 0$$

$$(3.7f) \qquad h_4 = Q \ge 0$$

(3.7g) $\varphi(K) = -K$

The model has the same regularity, convexity and convergence properties as the basic model. The same solution process as in chapter 2 can be applied and the solution that satisfies the first-order conditions is the optimal solution to the model.

The current-value Lagrangian of the model is:

(3.8) $L = \dfrac{1-\tau_c}{(1-s)(1-\tau_g)} D - \dfrac{\tau_x}{(1-s)(1-\tau_g)} D_x - Q + \lambda[(1-\tau_f)f(K) + Q - D] +$

$$q_1[(1-s)bK + D_x - D] + q_2D + q_3D_x + q_4Q$$

The first-order conditions are:

(3.9a) $\partial L/\partial D = \dfrac{1-\tau_c}{(1-s)(1-\tau_g)} - \lambda - q_1 + q_2 = 0$

(3.9b) $\partial L/\partial D_x = -\dfrac{\tau_x}{(1-s)(1-\tau_g)} + q_1 + q_3 = 0$

(3.9c) $\partial L/\partial Q = -1 + \lambda + q_4 = 0$

(3.9d) $q_1 \geq 0, \ (1-s)bK + D_x - D \geq 0, \ q_1[(1-s)bK + D_x - D] = 0$

(3.9e) $q_2 \geq 0, \ D \geq 0, \ q_2D = 0$

(3.9f) $q_3 \geq 0, \ D_x \geq 0, \ q_3D_x = 0$

(3.9g) $q_4 \geq 0, \ Q \geq 0, \ q_4Q = 0$

(3.9h) $\dot{\lambda} = \dfrac{1-\tau_c}{1-\tau_g} r\lambda - (1-\tau_f)f'(K)\lambda - (1-s)bq_1$

(3.9i) $\dot{K} = (1-\tau_f)f(K) + Q - D$

(3.9j) $\lambda(t_0) = -d\varphi(K)/dK = 1$

The four control constraints in (3.7c)-(3.7f) define again sixteen different control regimes. These are listed in table 2.1 in chapter 2. We proceed as in chapter 2 and begin the analysis by first going through the infeasible regimes.

Regimes 1-4, 11-12 and 15-16 can be shown to be infeasible on the same grounds as in section 2.3.2. Regimes 6, 10 and 14 are contradictory with assumption (A3.1). All of the aforementioned regimes are omitted from further analysis.

Regime 8 is not contradictory with assumptions (3A.1) - (3A.2), nor does it prove to be infeasible on the grounds applied in section 2.3.2. We will assess this regime a little more closely. By substituting the values of the shadow prices $q_2=q_3=q_4=0$ into the first-order conditions (3.9a) - (3.9c), we find that in the regime:

$$(3.10) \qquad \lambda = \frac{1-\tau_e}{(1-s)(1-\tau_g)} = 1$$

This leads us to two observations. First, the co-state is constant, from which it follows that $\dot{\lambda} = 0$, and further that $\dot{K} = 0$. By substituting the latter result into equation (3.9i), we obtain $D = (1-\tau_c)f(K) + Q$. The firm does not invest, but rather pays a dividend, financing this with after-tax profits and share issues. Second, equation (3.10) gives a feasibility condition, $(1-\tau_e)/(1-s) = (1-\tau_g)$. The regime can be effective only if the owner's after-tax income from one markka of profits is independent of the form in which this income is received, as a dividend (taxed as earned income) or a capital gain. Expressed in terms of tax rates, the condition requires that the effective marginal tax rate on dividends is as great as the effective tax rate on capital gains. This shows that the regime is a special case and valid only under a razor's edge condition.[47]

Looking at the policy in regime 8 more carefully, we observe that the amount of the dividend payment and equity financing are not uniquely defined. The dividend has a lower limit $(1-s)bK$. Within the bounds of this lower limit the firm is indifferent with respect to the size of the dividend. The amount of equity financing, Q, can be interpreted as being determined by the difference between dividends and internal financing (when the dividend is given). The firm thus finances the portion of the dividend that exceeds internal financing with a share issue.

As discussed in section 1.3., most OECD countries set legal constraints on dividend distributions. Usually dividends are constrained with respect to annual after-tax profits and in some countries accumulated earnings. The firm's policy in regime 8 is not in conformity with this regulation. Adding a dividend constraint to our model, for example in the form of equation (1.1f), would rule out this regime. We omit regime 8 on these grounds. The razors' edge feature of the tax rate condition also reduces interest in this regime.

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[47] This razor's edge property is emphasized by the fact that τ_g is the effective tax rate for capital gains, which is dependent on the lag in realising capital gains and the discount rate r'. The magnitude of τ_g thus varies on a case-by-case basis.

Next we will analyse the four remaining regimes. At issue are the same policy alternatives that constituted the solutions to the basic model. We will now denote these regimes by G1...G4 and start the analysis with regime G1.

Regime G1. ($D = D_x = Q = 0$; $q_1 = 0$, q_2, q_3, $q_4 \geq 0$)

Regime G1 is like regime A1, a growth regime where the company uses all of its financing for investments. Using the definition of the regime and the optimality conditions, we obtain the following feasibility condition $(1-\tau_c)/(1-s) \leq (1-\tau_g)$, where the left-hand side is after-credit net income from a one-markka dividend when this dividend is taxed as capital income. The right-side gives the after-tax income from one markka of capital gain. The regime is feasible if the net dividend is smaller than the net capital gain. In terms of effective tax rates the same condition can be written as:

$$(3.11) \qquad\qquad \tau_g \leq \tau_c^{ef}$$

where $\tau_c^{ef} = (\tau_c - s)/(1-s)$.[48] According to this representation the regime is feasible if the effective tax rate (after imputation credit) on dividends taxable as capital income is greater than the effective tax rate for capital gains, i.e. if the tax system discriminates against dividends as a form of paying out profits (or against share issues as a financing form). In the Finnish type tax system $\tau_c^{ef} = 0$, which implies that (3.11) is not satisfied.

Next we turn to analyse regime G2.

Regime G2. ($0 \leq D \leq (1-s)bK$, $D_x = Q = 0$; $q_1 = q_2 = 0$, q_3, $q_4 \geq 0$)

When we substitute $q_1 = q_2 = 0$ into equation (3.9a) we obtain:

$$(3.12) \qquad\qquad \lambda = \frac{1 - \tau_c}{(1 - s)(1 - \tau_g)}$$

Equations (3.12) and (3.9c) imply that regime G2 is feasible only on condition (3.11). Thus, regime G2 is not feasible under the parameters for the Finnish tax system.

Equation (3.12) implies that $\dot{\lambda} = 0$. When substituting this into the first-order condition (3.9h) and solving the equation with respect to f, we obtain the following marginal condition:

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[48] $\dfrac{1-\tau_c}{1-s} = \dfrac{1-s-(\tau_c-s)}{1-s} \leq 1-\tau_g \quad \Leftrightarrow \quad \tau_g \leq \dfrac{\tau_c - s}{1-s} \equiv \tau_c^{ef}$

$$(3.13) \qquad f'(K) = \frac{1-\tau_c}{(1-\tau_f)(1-\tau_g)} r$$

The right-hand side of (3.13) is a constant, which implies that regime G2 is a steady-state regime.

Using the firm's budget constraint we obtain the following feasibility condition for the regime $(1-\tau_f)f(K) < (1-s)bK \Rightarrow \sigma < b$. This contradicts assumption (A3.1), which implies that regime G2 is infeasible in the present setting.

Ignoring the infeasibility for a while and using the link to regime A2 in chapter 2, we can infer that the right-hand side of (3.13) gives us the firm's cost of capital in a linear dividend tax system in the case where retained earnings are the marginal source of financing. The cost of capital seems to be dependent upon the marginal tax rates τ_c, τ_g and τ_f, but not upon the tax treatment of dividend income. The significance of capital gains taxation will show up very clearly when we study the Nordic-type of system where $\tau_f = \tau_c$. In this case:

$$(3.13') \qquad f'(K) = \frac{1}{1-\tau_g} r \qquad (>r)$$

Capital gains tax is the only tax factor affecting the cost of capital. It raises it higher than market rates of interest, lowering the firm's optimal long-term capital stock compared to a neutral system.

It is worth bearing in mind, however, that (3.13) and (3.13') are relevant conditions for the steady state capital stock only in the case where condition (3.11) is satisfied. As was stated, this does not occur in Finland.

Regime G3. $(D = (1-s)bK, D_x = Q = 0; q_1, q_3, q_4 \geq 0, q_2 = 0,)$

From first-order conditions (3.9a) - (3.9b) and the values of the shadow prices we obtain the following range for the co-state variable:

$$\frac{1-\tau_e}{(1-\tau_g)(1-s)} \leq \lambda \leq \min[\frac{1-\tau_c}{(1-\tau_g)(1-s)}, 1].$$

This implies the following feasibility condition:

$$(3.14) \qquad \tau_g \leq \tau_e^{ef}$$

where $\tau_e^{ef} = (\tau_e - s)/(1-s)$. Regime G3 is feasible only if the effective marginal tax rate on dividends taxable as earned income is greater than the effective rate on

capital gains. This condition is much easier satisfied than condition (3.11). It may well prevail in the Finnish tax system.[49]

The optimality conditions, together with assumption (A3.1) and the definition of the regime, imply that $\dot{K} > 0$. The regime is thus a growth regime where the firm pays a dividend equivalent to $(1-s)bK$ and invests the rest. Assumption (A3.1) now precludes the possibility that regime G3 might spur a steady state.

Regime G4. $(D = (1-s)bK + D_x, D_x > 0, Q = 0; q_1, q_4 \geq 0, q_2 = q_3 = 0)$

From the first-order conditions (3.9a) - (3.9c) we obtain:

$$\lambda = \frac{1 - \tau_e}{(1 - \tau_g)(1 - s)} \quad \& \quad \lambda \leq 1 \quad \& \quad q_1 = \frac{\tau_x}{(1 - \tau_g)(1 - s)}$$

The first two results together imply that the regime is feasible under condition (3.14). By substituting λ, the expressions for q_1 and $\dot{\lambda} = 0$ into the condition (3.9h) and by solving the equation with respect to f' we obtain the following marginal condition:

$$(3.15) \qquad f'(K) = \frac{1 - \tau_c}{(1 - \tau_f)(1 - \tau_g)} r - \frac{(1-s)}{(1 - \tau_f)} \frac{\tau_x}{(1 - \tau_e)} b$$

The right-hand side of this is constant, which implies that the firm is in a steady-state. The stock of capital defined by (3.15) is denoted by K_G^*.

Note that (3.15) defines a unique value of K as long as the level of the rate of depreciation is high enough. We assume that depreciation satisfies this requirement.[50]

On the basis of the budget constraint in (3.9i) we obtain the following feasibility condition for regime G4:

$$(3.16) \qquad \frac{1 - \tau_f}{1 - s} \frac{f(K)}{K} \equiv \sigma(K_G^*) \geq b$$

Assumption (A3.1) is satisfied and regime G4 can be stated to be feasible.

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[49] For parameter values $\tau_g = 0.15$ and $s = 0.28$, condition (3.14) holds if $\tau_e > 0.39$. This requirement is not very stringent since, for example, the Finnish marginal tax rate on earned income is on average slightly higher than 0.5 (see Viitamäki (1998)).

[50] See the discussion after equation (2.26) in chapter 2.

3.4 Optimal Solution

We found that there are three regimes that satisfy the requirements (3A.1)-(3A.2). Regime G1 is feasible if the effective tax rate on capital gains is lower than the marginal effective tax rate on dividends, taxable as capital income. It was shown that this condition is not satisfied in uniform capital income taxation with full imputation, as enforced e.g. in Finland. Regimes G3 and G4 are feasible under a less strict condition requiring that the effective rate on capital gains is lower than the effective rate on dividends subject to taxation on the margin as earned income. Regime G2 was shown not to satisfy assumption (A3.1). The properties of the regimes are summarised in table 3.1. Note that there is only one regime that produces a steady state, regime G4. The other two regimes are the potential growth regimes of the solution.

Table 3.1   Characteristics of the Feasible Regimes

Regime	Feasibility condition for the regime	Co-state variable	Regime type
G1	$\dfrac{\tau_c - s}{1-s} = \tau_c^{ef} \geq \tau_g$	$\dfrac{(1-\tau_c)}{(1-s)(1-\tau_g)} \leq \lambda \leq 1$	Growth
G3	$\dfrac{\tau_e - s}{1-s} = \tau_e^{ef} \geq \tau_g$	$\dfrac{(1-\tau_e)}{(1-s)(1-\tau_g)} \leq \lambda \leq \min[\dfrac{(1-\tau_c)}{(1-s)(1-\tau_g)},1]$	Growth
G4	$\dfrac{\tau_e - s}{1-s} = \tau_e^{ef} \geq \tau_g$	$\lambda = \dfrac{(1-\tau_e)}{(1-s)(1-\tau_g)} \leq 1$	Steady state

Depending on the size of the tax parameter τ_g relative to parameters s, τ_c and τ_e, three solutions can be found. These are listed in table 3.2. The difference between the solutions lies in the length of the adjustment path. Solution G I is the solution with the longest adjustment path and it occurs on condition $\tau_c^{ef} > \tau_g$, i.e. when the effective tax rate on capital gains is smaller than the effective rate on dividends when the dividend is taxed as capital income. After the initial investment the firm begins its internally financed growth in regime G1, continues growing later in regime G3 and switches finally to the steady-state regime G4. This solution is very similar to the solution A I of the basic model (see Table 2.3).

Solution G II is obtained when $\tau_e^{ef} > \tau_g > \tau_c^{ef}$, i.e. when the effective tax rate on capital gains is between the two effective marginal tax rates on dividends. The growth path consists of just one regime. After the initial equity investment the firm starts internal growth directly in regime G3 and subsequently shifts to the steady-state regime G4.

The third feasible solution is the one where $K_0 = K_G*$. The firm's capital stock is invested to the steady state level with the initial investment. This alternative is effective on the razor's edge condition $\tau_e^{ef} = \tau_g$, i.e. when the effective tax rate on capital gains is exactly as high as the effective rate for earned income. In this case taxation does not discriminate against either of the two income channels, dividends or capital gains.

Table 3.2   Solutions to the Model

Solution	Regime chain	Condition
G I	$K_0 \rightarrow G1 \rightarrow G3 \rightarrow G4$	$\tau_c^{ef} > \tau_g$
G II	$K_0 \rightarrow G3 \rightarrow G4$	$\tau_e^{ef} > \tau_g \geq \tau_c^{ef}$
G III	$K_0 \rightarrow G4$	$\tau_e^{ef} = \tau_g$

Note that the lower the effective tax rate on capital gains relative to the marginal tax rate on dividends, i.e. the greater the tax discrimination against dividends and share issues, the longer the growth path. In the extreme case, where taxation is effectively neutral with respect to the financing decision, the adjustment phase disappears completely (solution G III). Note also that the tax rate condition for solution G I is not satisfied in a uniform capital income tax system. The conditions for solutions G II and G III are nevertheless satisfied. Among other things this means that in the Finnish-type system our wealth maximising firm starts distributing dividends immediately after its birth.

When we look at the feasibility conditions of the three solutions, we observe that they do not allow the case where $\tau_g > \tau_e^{ef}$, i.e. where taxation discriminates against internal financing (dividends subject to taxation on the margin as earned income). This parameter scheme is nevertheless fully possible in the tax system that we assumed and also in the system operated in Finland, even if it requires a fairly low marginal tax rate on earned income. Without any further restrictions on the firm's policy the tax system in this case spurs the firm to raise an unlimited amount of equity by issuing new shares and to pay out an unlimited amount of dividends. As we pointed out in section 1.3 the legal systems of all OECD countries rule out such policies and therefore we omit this case.[51]

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[51] A better way to handle this situation would have been to introduce an explicit constraint for dividends similar to (1.1e) in section 1.3. However, to keep the model simple this approach was not implemented.

3.5 Further Analysis

This section aims to draw attention to some interesting aspects of the solution presented in the preceding section. We have obtained the following expressions for the firm's cost of capital in the case where dividends are subject to taxation on the margin as earned income[52]

$$(2.26')\qquad p_A = r - \frac{\tau_x}{1-\tau_e}b \qquad\qquad \text{(basic model)}$$

$$(3.15')\qquad p_G = \frac{1-\tau_c}{(1-\tau_f)(1-\tau_g)}r - \frac{(1-s)\tau_x}{(1-\tau_f)(1-\tau_e)}b \qquad \text{(model in this chapter)}$$

Note that the first term in both equations equals the cost of capital in the corresponding linear tax system (cf. (2.21) and (3.13)). The second term brings the incentive of graduated taxation into the expression. This term is negative in both cases. These observations imply that graduated taxation reduces the cost of capital and increases the steady-state capital stock in both cases relative to the corresponding linear system.

The second term of the p_G expression is independent of the effective tax rate for capital gains, τ_g, but depends instead on the corporate tax rate, τ_f, and the rate of imputation credit, s. In the full imputation case ($s = \tau_f$) the second term of p_G is exactly the same as the corresponding term of p_A. In the partial imputation case ($s < \tau_f$) the incentive terms differ, the term of p_G being the greater.

From (2.26') and (3.15') we observe that the tax rate on capital gains raises the first term and thus the whole expression for the cost of capital. This implies that if for example, the tax system of this chapter contains uniform capital income taxation with full imputation, we obtain $p_G > p_A$. Consequently the steady-state capital stock is lower in the tax system of this chapter than in the tax system of chapter 2, i.e. $K_G^* < K_A^*$.

When comparing p_G, ceteris paribus, with the cost of capital, p, of an economy where taxation does not distort investment decisions,[53] it can be observed that, depending on the values of the tax parameters, p_G may be smaller or larger than p. On the basis of the first-order conditions we can nevertheless conclude that in a system with full imputation it always holds that $r < b \Rightarrow p_G < r$, i.e. the cost of capital is smaller than the interest rate if the value of parameter b exceeds the

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[52] Case $\sigma > b$, where $\sigma = f(K_A^*)/K_A^*$ for the basic model and $\sigma = (1-\tau_f)f(K_G^*)/(1-s)K_G^*$ for the present model.

[53] In this case $p=r$.

interest rate. Thus in this case we have $K_G^* > K^*$. To prove this result let us use equation (3.9h) and rewrite the inequality $p_G < r$ as:

(3.17)
$$\left\{\frac{1 - \tau_c}{(1 - \tau_f)(1 - \tau_g)} - 1\right\}r - \frac{q_1}{\lambda}b < 0$$

The first-order conditions (3.9a) and (3.9c) imply that the shadow prices λ and q_1 obtain the following values in regime G4: $\lambda \leq 1$ and $q_1 \geq (1-\tau_c)/[(1-s)(1-\tau_g)]-1$. These further imply that the second term on the left side of (3.17) is always greater than the expression in brackets. Thus, when $r < b$, inequality (3.17) always holds regardless of the values of the tax parameters.[54] In addition, it can be shown that given suitable tax parameters the situation $p_G < r$ may be obtained even if the interest rate is higher than b. Furthermore, full imputation is not a necessary requirement for this result.

Thus, the graduated dividend tax scheme reduces the distortion against investment caused by capital gains taxation, but does not take much for it to overcompensate for this distortion.

It can also be shown that inclusion of capital gains taxation and the corporate tax system in the model raises the amount of optimal initial equity, i.e. $K_0^G > K_0^A$.[55] This result, together with the previous one according to which the steady-state capital stock in the tax system of this chapter is smaller than that of the basic model, enables us to conclude that the share of external equity of the total amount of financing is greater here than in the tax system of the basic model. In other words, we have $e^G > e^A$, where $e^i = K_0^i/K_i^*$, $i=A,G$.

These results are clearly similar to the previous observations, according to which the expansion of the tax system shortens the adjustment path. In the extreme case, G III, both regimes of internally financed growth dropped out. The current result

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[54] Note, however, that the feasibility condition for regime G4, $\tau_e^{ef} \geq \tau_g$, is assumed. Therefore, combinations of high values of τ_g and low values of τ_e producing $\tau_e^{ef} < \tau_g$ are not allowed. As noted at the end of the preceding section such cases can exist e.g. in the Finnish system.

[55] The result prevails if r is not too high relative to parameter b. The result can be demonstrated by comparing the adjustment paths of the solutions to the basic model and to the model in this chapter in the (K, λ) space. First, under fairly undemanding conditions the equilibrium point of the solution G I can be observed to be above the adjustment path of the basic model. Second, the adjustment path of the solution G I can be observed to be more steeply declining for each K in both regime G3 and regime G1 than the path of the solution A I. Furthermore, at the switching points of growth regimes, it holds that $\lambda_G^\# > \lambda_A^\#$, i.e. the regime switch occurs at a higher co-state value in solution G I than in A I. From these we can conclude that the adjustment path of the model with the broader tax system must be above the adjustment path of the basic model for all values of K. Thus the former path intersects the initial value of the co-state at a higher level of capital than the latter path, i.e. $K_0^G > K_0^A$. This result does not require the assumption of full imputation. Demonstrating it in the case of partial imputation, however, is not an easy task.

$K_0^G > K_0^A$ indicates that the shortening of the adjustment path does not occur merely via the decline in steady-state capital, but rather via the increase in optimal initial capital. These results derive from the fact that incorporating capital gains taxation and the imputation system into the model tends to reduce the tax discrimination against equity financing and in an extreme case may even eliminate it completely.

3.6 Summary

This chapter examined graduated dividend taxation as a part of a more comprehensive tax system. The main findings of this analysis were as follows.

The special incentive of graduated taxation observed in chapter 2 exists in this expanded tax system as well. The incentive is independent of the taxation of capital gains, but dependent upon the rate of the dividend relief. In the case of full imputation the incentive is as large as in chapter 2. Under a partial imputation it is greater than in chapter 2.

The inclusion of the new tax parameters in the model also affects the first term of the cost of capital expression. Capital gains taxation, for example, raises this term and the cost of capital. However, even under rather undemanding conditions the cost of capital will remain lower in the tax system of this chapter than in a neutral tax system.

We observed that the duration of an internally financed growth period depends upon the relationship between the effective tax rate for capital gains and dividends. The harsher the taxation of capital gains in relative terms, the shorter the phase of internally financed growth and the larger the portion of the steady-state capital stock financed with the initial equity investment.

The analysis showed that in a Finnish-type tax system, where uniform capital income taxation is combined with full imputation, the firm starts paying dividends immediately after its birth. So, unlike in Sinn (1991) and chapter 2, there is no growth phase in which the firm uses all of its internal financing for investments.

4. Towards a More Realistic Concept of the Capital Base: Inclusion of Debt, Financial Assets and Accelerated Fiscal Depreciation

4.1 Introduction

The two previous chapters analyzed the effects of dual income taxation on the growth of a closely held firm. Chapter 2 introduced a rather simple basic model and used it to derive a number of key results. Chapter 3 expanded the model's tax system by bringing into the study a corporate tax system and capital gains taxation. The analysis in both chapters demonstrated that graduated dividend taxation may be strongly non-neutral.

These chapters also showed that the capital base used in calculating the share of capital income of the dividend is the key link by which the effects of graduated dividend tax are transmitted to the firm's behaviour. The framework made some heroic assumptions concerning the content of the capital base. It was defined as gross capital assessed at replacement value. As was pointed out in chapter 1, the capital base is nevertheless a much more complex concept in practice, thus raising a number of issues concerning for example the valuation and scope of it. Theoretically the capital base may be defined as containing all the business assets of the firm, but in reality narrower concepts are used. Intangible capital and financial assets may be left out. One further issue is whether debt should be deducted from the base or not. Moreover, due to the costs of information the replacement value is available only exceptionally in assessing the value of assets. Consequently the book value is a common starting point in valuations.

This chapter takes a step towards a more realistic analysis. The prime focus is not on comparing various definitions of the capital base, but rather on expanding the policy alternatives of the firm. The common denominator of these extensions, however, is that they all affect the content of the capital base and thus the firm's behaviour.

In section 4.2 we assess the impact of the homogeneity of the capital assumption by offering the firm access to the financial markets as an investor. In this model variant the firm has two types of assets: real capital generating declining marginal returns and financial capital generating constant returns. The firm's policy is studied under two alternative definitions of the capital base. The results of this model show that the inclusion of financial assets in the capital base is an important policy decision which alters the effects of graduated taxation significantly.

In section 4.3 we will allow the firm to have access to the debt market. The aim is to explain how net capital based graduated dividend tax affects the firm's borrowing policy. Section 4.4 investigates the effects of graduated dividend taxation in an environment of accelerated fiscal depreciation. The firm's policy is studied under two different systems of fiscal depreciation. In the first the depreciation is exogenous for the firm. In the second case the firm has some freedom in choosing the size of the depreciation allowances.

All three extensions in this chapter will bring significant new features regarding the effects of graduated dividend taxation. The analysis does not, nevertheless, seek to be exhaustive on issues related to the capital base.

4.2 Two Assets: Real Capital and Financial Capital

4.2.1 Incorporation of Financial Capital in the Model

In this section the firm has two forms of assets: real capital, K, which has a declining marginal return, $f(K)$, shown above in the basic model, and financial capital, F, with a constant marginal of return, ρ. The firm's behaviour is analysed under two different definitions of the capital base. In the first, the capital base includes the firm's total capital stock, $N = F+K$, and in the second only the stock of real capital, $N = K$.

The analysis has two goals. First, we ask in which way the results of the basic model are dependent on the assumptions concerning homogeneity of capital and concavity of the production function. Second, we aim to establish how the policy decision concerning the inclusion of financial capital in the capital base affects the firm's behavior. The latter question is of practical significance since, as we pointed out in chapter 2, there are different solutions to this question in existing tax systems. Financial wealth is not considered a part of the capital base in Norway. Similarly in Finland partnerships and private firms cannot expand their capital base via financial investments. On the other hand, in Finland and in Sweden the capital base for dividend income includes financial capital.

We retain most of the assumptions of the basic model. The firm is still financed purely with equity, depreciation of capital is modeled as if capital were nondepreciating and the tax system consists of dividend tax only.

We assume that there are well functioning financial markets in the economy. The firm's rate of return on financial investments is depicted by ρ. The shareholder's rate of return on alternative investments is r, as above. From the assumption of perfect financial markets it follows that the firm and the shareholder obtain the same interest rate on their investments in financial markets. Owing to the deterministic nature of the model, we further assume that r and ρ do not contain any

risk premia. This implies that the owners' discount rate and the firm's return on bonds are equal.

(A4.1) $$r = \rho$$

The firm's budget constraint is:

(4.1) $$f(K) + \rho F + Q = D + I + S$$

where the sources of funds are net operating income, $f(K)$, return on financial capital, ρF, and new share issues, Q, and the uses of funds are dividends, D, investments in real capital, I, and net investments in financial capital, S. By solving (4.1) with respect to I and substituting this into $\dot{K} = I$ we obtain the state equation for real capital as:

(4.2) $$\dot{K} = f(K) + \rho F + Q - D - S$$

The equation depicting the development of financial capital is:

(4.3) $$\dot{F} = S$$

From the exclusion of debt it follows that the firm's position in the financial markets is always non-negative. We take this into consideration by including the following constraint:

(4.4) $$F \geq 0$$

We retain the assumption of the basic model that an optimal amount of initial capital is invested in the firm at time t_0. Now, however, we further assume that this initial investment can be directed towards real capital as well as financial capital. We take this feature into consideration by including in the model an initial cost function for both types of capital (φ_1 and φ_2).

We make the same assumption as in chapter 3:

(A4.2) $$\sigma(K_A{}^*) > b ,$$

i.e. we focus our attention again on firms with a high average rate of return on capital, and whose dividend payments are subject to taxation on the margin as earned income in the long-run equilibrium. As we observed in chapters 2 and 3, in this case the graduated tax system has an especially interesting impact on the firm's behaviour.

We can now write the model as follows:

(4.5a) $\max_{\{K_0,F_0,D,D_x,Q,S\}} \int_{t_0}^{\infty} \{(1-\tau_c)D - \tau_x D_x - Q\}e^{-r(t-t_0)}dt + \varphi_1(K(t_0)) + \varphi_2(F(t_0))$

(4.5b) $\dot{K} = f(K) + \rho F + Q - D - S, \quad K(t_0) = K_0$

(4.5c) $\dot{F} = S, \quad F(t_0) = F_0 \geq 0$

(4.5d) $h_1 = bN + D_x - D \geq 0$

(4.5e) $h_2 = D \geq 0$

(4.5f) $h_3 = D_x \geq 0$

(4.5g) $h_4 = Q \geq 0$

(4.5h) $h_5 = F \geq 0$

(4.5i) $\varphi_1(K) = -K, \quad \varphi_2(F) = -F$

The problem now has two state variables, K and F, and four control variables, the new one being net investment in financial capital, S.[56] The model satisfies the regularity properties required by the maximum principle discussed in chapter 2. Furthermore, as demonstrated in appendix 1, the model has the concavity properties required by the sufficiency conditions. Thus, the solution that satisfies the first-order conditions and the transversality condition presented in section 4.2.3.2 is the optimal solution to the model.

4.2.2 Optimality Conditions and Feasible Regimes

The current-value Lagrangian for the problem is:

(4.6) $L = (1-\tau_c)D - \tau_x D_x - Q + \lambda_1[f(K) + \rho F + Q - D - S] + \lambda_2 S$

$q_1[bN + D_x - D] + q_2 D + q_3 D_x + q_4 Q + q_5 F$

where λ_1 and λ_2 are shadow prices for real and financial capital and $q_1, ..., q_5$ are the shadow prices for constraints (4.5d) - (4.5h). The first-order conditions without the complementarity conditions are

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[56] We restrict F_0 to be non-negative in the model. Following the practice of chapter 2 this is not done for K_0. However, the implicit non-negativity assumption for K discussed in section 2.1 also applies to the initial value of K.

(4.7a) $\qquad \partial L/\partial D = 1 - \tau_c - \lambda_1 - q_1 + q_2 = 0$

(4.7b) $\qquad \partial L/\partial D_x = -\tau_x + q_1 + q_3 = 0$

(4.7c) $\qquad \partial L/\partial Q = -1 + \lambda_1 + q_4 = 0$

(4.7d) $\qquad \partial L/\partial S = -\lambda_1 + \lambda_2 = 0$

(4.7e) $\qquad \dot{\lambda}_1 = r\lambda_1 - \partial L/\partial K = r\lambda_1 - f'(K)\lambda_1 - bq_1(\partial N/\partial K)$

(4.7f) $\qquad \dot{\lambda}_2 = r\lambda_2 - \partial L/\partial F = r\lambda_2 - \rho\lambda_1 - bq_1(\partial N/\partial F) - q_5$

(4.7g) $\qquad \dot{K} = f(K) + \rho F + Q - D - R$

(4.7h) $\qquad \dot{F} = S$

(4.7i) $\qquad \lambda_1(t_0) = - d\varphi_1(K)/dK = 1$

(4.7j) $\qquad \lambda_2(t_0) \le - d\varphi_2(F)/dF = 1, \quad F_0 \ge 0, \quad F_0[\lambda_2(t_0)-1] = 0$

Note that (4.7i) and (4.7d) imply the following condition for the initial values of K and F:

(4.8) $\qquad \lambda_1(t_0) = \lambda_2(t_0) = 1$

Our analysis proceeds as in chapters 2 and 3. First we will discern the feasible regimes from the potential regimes. After this we will analyse these regimes and construct the optimal solution. There are now five constraints in the model, one more than in chapters 2 and 3. This means that there are twice as many potential regimes, i.e. a total of 32. The regimes are listed in table 4.1. The first 16 of them are the same as the potential regimes of our basic model in chapter 2. The new constraint is binding in these regimes ($h_5 = 0$). The regimes where the new constraint is non-binding ($h_5 > 0$) are numbered 17, ..., 32. We also make use of table 4.1 in sections 4.3 and 4.4, since there are also five constraints in the models in those sections.

In the following we will evaluate the infeasible policies. Regimes 1 - 4 and 17 - 20 are rejected on the same grounds as the first four regimes in chapter 2. The policies are feasible only on the condition $K \le 0$, which is against the assumptions set for K in chapter 2.

Regimes 11-12, 15-16 27-28 and 31-32 are feasible under the first-order conditions only when $\tau_x = 0$ and regimes 6, 8, 14, 22, 24 and 30 only when $\tau_c \le 0$. Our tax rate assumptions in (A2.1) rule out these regimes.

Regime 10 is rejected on the same grounds as in chapter 2. The firm's budget equation is $f(K) + Q = 0$. Since $f(K) > 0$ and $Q > 0$ in the regime, this equation is contradictory and the regime can be omitted.

Table 4.1   *Policy Alternatives in the Case of Five Constraints*
           (0 = binding constraint, + = non-binding constraint)

Regime	h_1	h_2	h_3	h_4	h_5	Regime	h_1	h_2	h_3	h_4	h_5
1	0	0	0	0	0	17	0	0	0	0	+
2	0	0	0	+	0	18	0	0	0	+	+
3	0	0	+	0	0	19	0	0	+	0	+
4	0	0	+	+	0	20	0	0	+	+	+
5	0	+	0	0	0	21	0	+	0	0	+
6	0	+	0	+	0	22	0	+	0	+	+
7	0	+	+	0	0	23	0	+	+	0	+
8	0	+	+	+	0	24	0	+	+	+	+
9	+	0	0	0	0	25	+	0	0	0	+
10	+	0	0	+	0	26	+	0	0	+	+
11	+	0	+	0	0	27	+	0	+	0	+
12	+	0	+	+	0	28	+	0	+	+	+
13	+	+	0	0	0	29	+	+	0	0	+
14	+	+	0	+	0	30	+	+	0	+	+
15	+	+	+	0	0	31	+	+	+	0	+
16	+	+	+	+	0	32	+	+	+	+	+

Regime 26 differs from regime 10 only with respect to the fifth constraint. Now $F > 0$, which means that the firm can use retained earnings to invest in the financial markets. The budget equation is not contradictory. The policy satisfies the tax rate assumptions as well as assumptions (A4.1) and (A4.2). We can nevertheless show that the regime does not fulfil the transversality condition for F, nor does it fit with the feasible regimes evaluated later. On these grounds regime 26 cannot be part of the optimal solution.

Regime 13, in which $f(K) < bN$, does not satisfy assumption (A4.2) and can be rejected.

There are seven regimes remaining: 5, 7, 9, 21, 23, 25, 29. In the first three regimes the non-negativity constraint for F is binding, $F = 0$. These regimes are renamed A3', A4' and A1', retaining the reference to the corresponding regimes of the basic model. The next four regimes are called F3, F4, F1 and F2. In these

regimes the constraint for F is non-binding, implying that the firm has a positive amount of financial capital. The numeric part of the abbreviations indicates the firm's dividend policy in the same manner as in chapters 2 and 3 above.

In the next section 4.2.3 we will evaluate these seven regimes in a case where the capital base is defined as $N = K + F$. Section 4.2.4, in turn, concerns a case where $N = K$.

4.2.3 Financial Assets Included in the Capital Base

4.2.3.1 Analysis of Feasible Regimes

When we define the capital base as $N = K + F$ we obtain $\partial N/\partial K = \partial N/\partial F = 1$. By substituting these into the first-order conditions we obtain the equations of motion for the co-state variables in the following form:

(4.7e')
$$\dot{\lambda}_1 = r\lambda_1 - f'(K)\lambda_1 - bq_1$$

(4.7f')
$$\dot{\lambda}_2 = r\lambda_2 - \rho\lambda_1 - bq_1 - q_5$$

Let us first assess regimes A4' and F4. By substituting the values of the shadow prices $q_2 = q_3 = 0$ into conditions (4.7a) and (4.7b) we obtain $q_1 = \tau_x$ and $\lambda_1 = 1 - \tau_e$. Using these results and equations (4.7d) and (4.7f') we obtain:

(4.9)
$$\rho = r - \frac{\tau_x}{1-\tau_e}b - q_5 \qquad \text{(regimes A4' and F4)}$$

Due to assumptions (A2.1) and (A4.1) equation (4.9) is satisfied only on strictly negative values of q_5 ($q_5 = -\tau_x b/(1-\tau_e) < 0$). This shadow price, however, is always non-negative.[57] From this it follows that regimes A4' and F4 are infeasible in our present case.

Regimes A1' and A3' are similar in most respects to the corresponding regimes in chapter 2 (regimes A1 and A3). On the basis of the first-order conditions (4.7d) - (4.7f') and assumption (A4.1) we nevertheless obtain a new requirement $f_K = \rho + q_5/\lambda_1$. Since we have $q_5 \geq 0$ in these regimes, this equation implies that regimes A3' and A4' are feasible only when $K \leq K^*$.

Next we will assess the financial investment regimes F1, F2 and F3.

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[57] Note that in regime A4' $F = 0$ from which it follows that $q_5 \geq 0$. In regime F4 $F>0$ which requires that $q_5 = 0$.

Regime F1: $(D = D_x = Q = 0, F > 0;\ q_1 = q_5 = 0,\ q_2, q_3, q_4 \geq 0)$

By substituting the values of the shadow prices into the first-order conditions (4.7a) - (4.7d) we obtain:

(4.10) $\qquad\qquad 1 \geq \lambda_1 = \lambda_2 \geq 1 - \tau_c$

By subtracting equation (4.7f') from equation (4.7e') and using (4.7d) and assumption (A4.1) we obtain:

(4.11) $\qquad\qquad f'(K) = \rho = r \qquad\qquad \Rightarrow\ K = K^*$

Thus the firm's stock of real capital is a constant ($\dot{K} = 0$) taking the value $K = K^*$. By substituting $f'(K) = \rho = r$ into equations (4.7e')-(4.7f') we obtain:

(4.12) $\qquad\qquad \dot{\lambda}_1 = \dot{\lambda}_2 = 0$

By substituting $\dot{K} = 0$ and $D = 0$ into equation (4.7g) we obtain:

(4.13) $\qquad\qquad S = f(K^*) + \rho F$

The firm uses all its internal financing for financial investments. This implies that $S = \dot{F} > 0$, i.e. financial capital is strictly increasing in regime F1.

Regime F2: $(0 < D < bN, D_x = Q = 0, F > 0;\ q_1 = q_2 = q_5 = 0, q_3, q_4 \geq 0)$

When we substitute $q_1 = q_2 = 0$ into equation (4.7a) we obtain:

(4.14) $\qquad\qquad \lambda_1 = \lambda_2 = 1 - \tau_c$

which implies $\dot{\lambda}_1 = \dot{\lambda}_2 = 0$, and, moreover, using equations (4.7e) - (4.7f):

(4.15) $\qquad\qquad f'(K) = \rho = r \qquad\qquad \Rightarrow\ K = K^*$

which implies again that $\dot{K} = 0$. The firm's budget equation becomes:

(4.16) $\qquad\qquad f(K^*) + \rho F = D + S$

The firm uses internal financing, $f(K) + \rho F$, for dividends and financial investments. Its policy is not uniquely determined. The firm is indifferent between dividends and financial investments within the limits set by the dividend constraints. It can pay a dividend of an amount $0 < D < bN$ and invest in the financial markets an amount $f(K^*) + \rho F > S > f(K^*) + \rho F - bN$.

Regime F3: $(D = bN, D_x = Q = 0, F > 0; \; q_1, q_3, q_4 \geq 0, \; q_2 = q_5 = 0,)$

From the first-order conditions we obtain a range for the co-states:

(4.17) $$1 - \tau_e \leq \lambda_1 = \lambda_2 \leq 1 - \tau_c$$

Using the interest rate assumption $\rho = r$ and equations (4.7f') and (4.7d) we obtain:

(4.18) $$\dot{\lambda}_1 = \dot{\lambda}_2 = -q_1 b$$

From (4.7a) we see that $q_1 = 1 - \tau_c - \lambda_1$ in regime F3. This, together with (4.17) and (4.18), implies that at their upper limits, where $q_1 = 0$, the co-states are constants, $\dot{\lambda}_1 = \dot{\lambda}_2 = 0$. If, on the other hand, $\lambda_1 = \lambda_2 < 1 - \tau_c$, then $q_1 > 0$ and the co-states are declining, $\dot{\lambda}_1 = \dot{\lambda}_2 < 0$.

By subtracting (4.7f') from (4.7e') and making use of the interest rate assumption we obtain once again:

(4.19) $$f'(K) = \rho = r \qquad \Rightarrow \quad K = K^*$$

Real capital is thus a constant in this regime taking the value $K = K^*$. By substituting $\dot{K} = 0$ and $D = bN$ into the equation of motion for K we obtain:

(4.20) $$S = f(K^*) + \rho F - bN$$

The firm thus pays a dividend of $D = bN$ and invests the rest of its profits in the financial markets.

Table 4.2 presents the main properties of the feasible regimes.

Table 4.2   Characteristics of the Feasible Regimes, Case $N = K + F$

Regime	λ	K	$\dot{K}$	$\dot{F}$	D	Budget constraint
A1'	$1 \geq \lambda_{1,2} \geq 1 - \tau_c$	$\leq K^*$	>0	$=0$	$=0$	$I = f(K)$
A3'	$1 - \tau_c \geq \lambda_{1,2} \geq 1 - \tau_e$	$\leq K^*$	>0	$=0$	$=bK$	$I = f(K) - bK$
F1	$1 \geq \lambda_{1,2} \geq 1 - \tau_c$	$= K^*$	$=0$	>0	$=0$	$S = f(K^*) + \rho F$
F2	$\lambda_{1,2} = 1 - \tau_c$	$= K^*$	$=0$	?	$0 < D < bN$	$S = f(K^*) + \rho F - D$
F3	$1 - \tau_c \geq \lambda_{1,2} \geq 1 - \tau_e$	$= K^*$	$=0$	?	$=bN$	$S = f(K^*) + \rho F - bN$

4.2.3.2 Closer Analysis of the Financial Investment Regimes

Construction of the optimal solution is now somewhat more problematic than in the previous chapters, since the set of feasible regimes does not seem to contain any obvious candidates for the final regime. For example the firm is not in a steady state in any regime. However, it is clear that regime A1' or A3' cannot serve as the final regime. In these regimes real capital is increasing and bounded and would at some moment pass the upper bound, K^*, if the firm remained in either of them.

Thus it seems that the final phase occurs in some of the financial investment regimes. Note that real capital is constant and takes the value K^* in all these regimes. Moreover, these regimes are all simultaneously feasible when $\lambda_1 = \lambda_2 = 1 - \tau_c$, the difference between them being the dividend policy. While in regime F1 all financing is used for investment in financial assets, in regimes F2 and F3 at least part of the profits are distributed. Note that all three regimes satisfy the condition $D \leq bN$.

Let us study the growth path of F in regime F3. By substituting S in (4.20) into equation (4.5c) and using the definition for N and solving the resulting differential equation we can derive an equation depicting this path:[58],[59]

$$(4.21) \qquad F(t) = \frac{f(K^*) - bK^*}{\rho - b} e^{(\rho - b)t} - \frac{f(K^*) - bK^*}{\rho - b} \qquad \text{(regime F3)}$$

Note that the expression $(f(K^*)-bK^*)/(\rho-b)$ is constant due to the constancy of K^*. Equation (4.21) implies that F grows at the rate of $\rho - b$, and if $\rho > b$ growth will continue without any limit. If $\rho < b$ instead, financial capital will converge to the equilibrium value $f(K^*)-bK^*)/(\rho-b)$. Note that this value depends on the difference $f(K^*)-bK^*$, which gives us the amount used for financial investment when $F = 0$ and by assumption (A4.2) is positive.[60]

The transversality condition for the present case is as follows:[61]

$$(4.22) \qquad \underline{\lim}_{t \to \infty} e^{-rt} \left\{ \hat{\lambda}_1(t) \left[K(t) - \hat{K}(t) \right] + \hat{\lambda}_2(t) \left[F(t) - \hat{F}(t) \right] \right\} \geq 0$$

---

[58] We will first assume $\lambda_1 = \lambda_2 = 1 - \tau_c$ and comment later on the case $\lambda_1 = \lambda_2 < 1 - \tau_c$.

[59] We assume that $t'=0$ and $F(t')=0$, where t' is the moment at which the firm enters into regime F3.

[60] Note that $F(t) = [f(K^*)-bK^*]t$ when $\rho = b$. $F(t)$ is divergent in this case.

[61] See, for example, Leonard and Long (1993) theorem 9.3.1. Observe that $\underline{\lim}(x)$ denotes lim inf (x).

where $\hat{K}(t)$ and $\hat{F}(t)$ are the paths of the state variables and $\hat{\lambda}_1(t)$ and $\hat{\lambda}_2(t)$ are the paths of the co-states that satisfy the first-order conditions of the model. $K(t)$ and $F(t)$ are any feasible values of the state variables.

Note that in regime F3 real capital is always $K = K^*$ and the first co-state is by assumption $\lambda_1 = 1-\tau_c$, i.e. both variables are constants. Remember also our implicit assumption made in chapter 2, according to which $K \geq 0$. From these features it follows that the real capital part of the expression on the left-hand side of condition (4.21) is always non-negative.

When we substitute the expression $F(t)$ in (4.21) for $\hat{F}(t)$ in (4.22) we obtain the part of the transversality condition which considers financial capital as:

$$(4.23) \quad \underline{\lim}_{t \to \infty} \left[e^{-rt} \hat{\lambda}_2 F(t) - e^{-bt} \hat{\lambda}_2 \frac{f(K^{**}) - bK^*}{\rho - b} - e^{-rt} \hat{\lambda}_2 \frac{f(K^*) - bK^*}{b - \rho} \right] \geq 0$$

The second and third terms of the expression in the brackets both approach zero when $t \to \infty$. The first term and the whole expression remain non-negative. The condition is thus satisfied regardless of the sign of the growth rate, $\rho - b$, because this is smaller than the discount rate, r. Thus, regime F3 satisfies the transversality condition and is a serious candidate for the final regime of the solution.[62]

The role of regimes F1 and F2 in the potential solution where regime F3 is the final regime can be examined as follows. Consider the value of the firm in the case of regimes F1 - F3, where dividends satisfy $0 \leq D \leq bN$ and $D_x = Q = 0$. The value of the firm can be written as:

$$(4.24) \qquad V = \int_0^\infty (1 - \tau_c) D(t) e^{-rt} dt$$

Using the budget identity (4.3) the equation of motion for financial capital (4.5c) can be rewritten as $D = f(K^*) + \rho F - \dot{F}$. Substituting this into (4.24) and integrating we obtain

$$(4.25) \qquad V = (1 - \tau_c) \left[\frac{f(K^*)}{\rho} + F(0) - \lim_{t'' \to \infty} e^{-rt''} F(t'') \right]$$

---

[62] We assumed above $\lambda_1 = \lambda_2 = 1-\tau_c$. It can be shown that in the case $\lambda_1 = \lambda_2 < 1-\tau_c$ regime F3 cannot serve as the final regime of the solution. The co-states in this case are declining and at some moment they would cross their lower limit $\lambda_1 = \lambda_2 = 1-\tau_e$. Thus the firm cannot stay in regime F3 forever. Furthermore there is no other regime to which the firm could switch. So, regime F3 is excluded from the solution in this case.

Note that the first two terms inside the brackets are constants. The third term might be time-dependent. The transversality condition, however, rules out growth of F at a rate greater than or equal to r when $t \rightarrow \infty$. This implies that the limit term must be zero. Consequently, the firm's value, V, is independent of the timing of dividend distributions, which means that any time path of dividends satisfying the condition $0 \leq D \leq bN$ is optimal.

This indicates that because of regimes F1 and F2 the firm is in the financial investment phase indifferent with respect to dividend policy (and growth rate of F). This indifference is constrained by the dividend constraint and also by the transversality condition. It can be shown that not all feasible dividend polices satisfy the transversality condition. One such example is the policy $D = 0$ of regime F1. Another example is the policy of regime F2 where the dividend is kept constant, $D(t) = D' < bN(t)$. In both cases financial capital grows at the rate ρ and the transversality condition is not satisfied.

4.2.3.3 Optimal Solution

Based on the analysis above as well as the continuity requirements for the state and co-state variables we can derive the following regime chain describing the firm's optimal policy:

$$\text{Initial investment} \Rightarrow \text{Regime A1'} \Rightarrow \text{Regime F3}^i$$

The final phase is depicted here as regime F3[i] and it contains the policy of regime F3 and those policies of regimes F1 and F2 that satisfy the growth condition for F discussed above.

The first-order conditions and continuity requirements imply that the financial investment phase can only be preceded by regime A1'. Regime A3' is not suitable for this role. Before the potential regime switch between regimes A3' and F3[i] it should hold that $f' > r$ and $q_1 > 0$. As regards equation (4.7e') this would mean that the co-state variables are declining as they approach the regime switch. Since at the switching point we have $\lambda_1 = \lambda_2 = 1-\tau_c$, this would imply that $\lambda_1 = \lambda_2 > 1-\tau_c$ before this point. This nevertheless contradicts the range allowed for the co-state variables in regime A3'(see table 4.2).

Regime A1' satisfies the initial time transversality conditions (4.8), so it is the starting regime for the solution.

According to the solution the firm begins to invest after its birth in real capital, financing its investments with internally generated funds (regime A1'). The initial equity injected in the firm is invested wholly in real capital. The starting value for financial capital is zero. After the firm's stock of real capital has

reached the level K^*, the firm starts its career as a financial investor. Its policy is not uniquely determined in this phase. It might grow its financial capital at a lower or higher rate. At the early stage of this financial investment phase it also might not distribute dividends. The firm's stock of real capital remains permanently at the level K^*, that is equal to the steady-state level of capital for economies with linear dividend taxation.

This solution indicates that including financial capital in the model framework and defining the capital base as $N = K+F$ eliminates the incentive to expand real capital greater than in linear dividend taxation. Graduated dividend taxation nevertheless affects the behaviour of the firm, encouraging it to postpone distribution of dividends and to expand its stock of financial capital.

4.2.3.4 Further Analysis

Next we will clarify some features of the solution outlined in the previous subsection. We will first try to offer an explanation as to why it is optimal for the firm to become a financial investor at the point where the firm's real capital reaches the level K^*. Later we will shed more light on the behaviour of the firm in the financial investment phase.

In chapter 2 we proposed that under graduated dividend taxation the firm will strive to expand its capital base to increase the portion of future dividends that will be taxed as capital income. In the present environment the firm can expand its capital base equally well by investing in financial and real capital. Thus a marginal increase in either type of capital produces an equivalent tax saving for the shareholders. In this situation the firm should invest in the type of capital that offers the greater direct marginal increase in the operating income of the firm. The firm gains a return ρ on an investment of one unit of money in financial capital and a return f' on a similar investment in real capital. When the firm's stock of real capital is K^* it holds that the marginal return on financial capital is as high as the return on the last marginal investment in real capital. But due to the concavity of the firm's profit function, an additional one unit increase in real capital would give a marginal return lower than the return on financial capital, i.e. $f' < \rho$ for an additional investment. Accordingly it pays the firm to switch at this stage to investing in financial capital.

As we pointed out above, the firm's policy is not uniquely determined in the financial investment stage. The firm is indifferent between paying a dividend and making financial investments. Additional insight into the situation can be gained from the following example. Let us compare returns on two alternative uses of profits from the standpoint of the shareholder, assuming $D < bN$. In the first alternative the shareholder takes the profit as a dividend and invests the after-tax amount in the financial market for one period at a rate of return r. The size of the

owner's investment is 1- τ_c and the value of the entire project after one period is $(1-\tau_c)(1+r)$. In the second alternative the one markka of profit is invested in financial capital by the firm for one period at a rate of return of ρ and taken by the shareholder after this as a dividend. The size of the investment is now 1, the dividend to be distributed after one period is $1+\rho$ and the value of the project after taxes from the standpoint of the shareholder is $(1-\tau_c)(1+\rho)$. From assumption (A4.1), according to which $\rho = r$, it follows that in both alternatives the value to the shareholder is the same.

Thus, it is equally beneficial for the shareholders to make investments in the financial markets via the firm as it is to do it in their own names. This simple analysis indicates that the indifference stems from the equivalence of the shareholder's discount rate to the firm's after-tax return on financial investments. This, in turn, results from assumption (A4.1) and the equal tax treatment of interest income in the hands of the firm and its shareholders (no firm-level or shareholder-level taxation of interest income).

Emphasizing the firm's indifference position in the financial investment phase may cloud the fact that graduated taxation of dividends clearly affects the firm's behaviour in the present framework. This effect can be highlighted by assessing the specific policy where the firm pays the maximum dividend and correspondingly invests the minimum amount allowed by the constraints. This minimum growth path of financial capital, F, occurs in regime F3 and is depicted by equation (4.21) above. Let us rewrite it as:

$$(4.21') \qquad F_{\min}(t) = \begin{cases} \dfrac{f(K^*)-bK^*}{\rho-b} e^{(\rho-b)t} - \dfrac{f(K^*)-bK^*}{\rho-b}, & \text{if} \quad \rho \neq b \\ \left(f(K^*)-bK^*\right)t, & \text{if} \quad \rho = b \end{cases}$$

As was noted in the previous section, the nature of the time path of $F_{\min}$ is strongly dependent upon the size of the term $\rho - b$. We have actually three different cases, $\rho < b$, $\rho = b$ and $\rho > b$, which we will study in the following a little more closely.

Case $\rho < b$

In this case the stock of financial capital, $F_{\min}$, is convergent and approaches the following equilibrium value

$$(4.26) \qquad F_{\min}^* = \frac{f(K^*)-bK^*}{b-\rho}.$$

The variable $F_{min}*$ can be interpreted as the minimum target level of financial capital to which the firm expands its stock of financial capital. From assumption (A4.2) it follows that $F_{min}* > 0$.

Note that under this minimum growth policy dividends rise along with the capital base. The equilibrium amount of dividends can be obtained by multiplying both sides of (4.26) by $\rho - b$ and making use of the equation $D = bN = b(K^*+F_{min}*)$. Using notation $N_{min}* = K^*+F_{min}*$ we obtain:

(4.27) $$f(K^*) + \rho F_{min}* = bF_{min}* + bK^* = bN_{min}*$$

$$\Rightarrow \quad f(K^*) + \rho F_{min}* = D_{max}$$

This implies that in equilibrium the firm pays out all of its internally generated cash flow as dividends, and furthermore, in the hands of the shareholders, the entire profit will be taxed as capital income.

We can characterize the indifference of the firm by using the concept of the minimum target level of financial capital as follows (case $\rho < b$). This indifference means that

1. the firm can approach the minimum target level of financial capital, $F_{min}*$, faster than implied by (4.21'), and

2. the firm can expand its stock of financial capital to a level that is greater than the minimum target level, $F_{min}*$.

Figure 4.1 illustrates the uses for the internal financing in case $\rho < b$. The solid black curve depicts the firm's internal financing. On the left-hand side of the point $N=K^*$ this consists solely of operating income earned on real capital, $f(K)$, and on the right-hand side of operating income plus the return on financial investment, $f(K^*) + \rho F$. The dashed line depicts the maximum dividend at each level of the capital base. The dividend can also be lower than this maximum amount. The line depicted by D' indicates one such path.

The firm invests at least the difference between its internal financing and its maximum dividend. By following the maximum dividend policy the firm expands its financial capital to the level $F_{min}*$ so that the firm's capital base reaches the value $N_{min}* = F_{min}* + K^*$. This occurs at the intersection of the line bN and the curve depicting the firm's internal financing. Note the similarity to solution A III in chapter 2 (Figure 2.3). In both cases the firm stops its growth at the point where its internal financing equals the return on its capital base.

Figure 4.2 illustrates the growth in the firm's capital base, N. Over the interval $t \in [t_0, t_1]$ the firm invests in real capital and from t_1 onwards in financial capital. The lower solid line, $N_{\min}^{b>\rho}$, depicts the growth when the firm obeys the minimum financial investment rule (pays a maximum dividend). This line converges to the equilibrium value, $N_{\min}^* = K^* + F_{\min}^*$, depicted by a horizontal dashed line. The upper solid line in turn depicts the growth in capital base in the case where the firm invests all of its internal financing in financial capital, i.e. by expanding financial capital at the maximum rate. The growth in capital base induced by the feasible policies is located between these two limiting growth paths $N_{\min}^{b>\rho}(t)$ and $N_{\max}(t)$. The dashed line, $N_{\min}^{b<\rho}(t)$, depicts the growth in financial capital with a minimum growth policy when $\rho > b$.

Case $\rho > b$

According to equation (4.21') the minimum growth path for F diverges when $\rho > b$. This means that the firm continues expanding financial capital forever. At no point does the firm pay out all of its internal financing as a dividend, but rather only part of it, the amount $D = bN < f(K^*) + \rho F_{\min}$. As in the case $\rho < b$, the shareholders do not pay earned income tax on dividends now either.

Figure 4.2 also illustrates the growth of the firm's capital base in the case $\rho > b$. The minimum growth path of this case is depicted by the dashed line. The capital base is now growing continually. The growth paths generated by the feasible policies are located between the curves $N_{\max}(t)$ and $N_{\min}^{b<\rho}(t)$.

Case $\rho = b$

In this special case the minimum growth path of financial capital is divergent and grows linearly. The firm never pays out the whole profits as dividends. Dividend payments are taxed entirely as capital income. Due to the similarity to the case $\rho > b$ this case is not illustrated in figure 4.2.
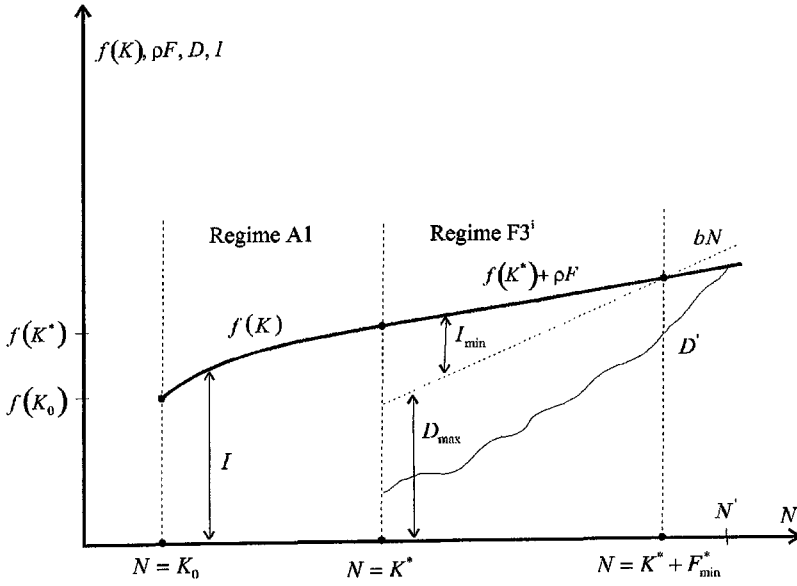
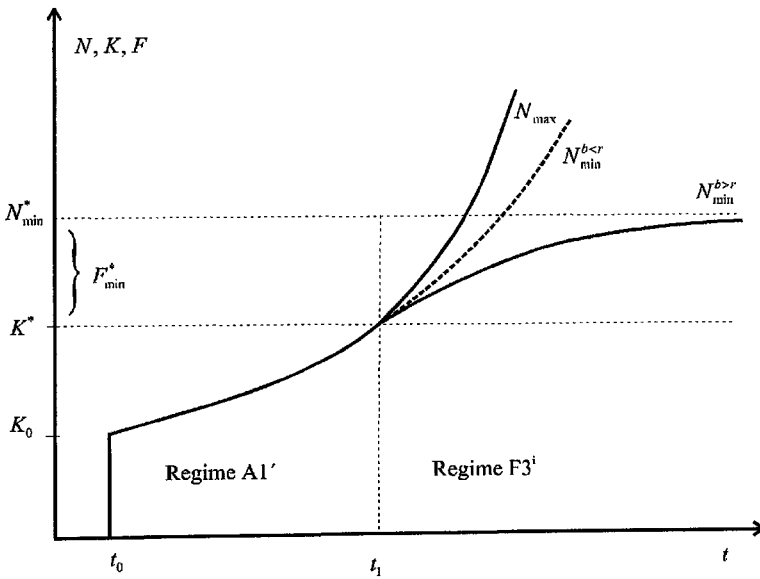*Figure 4.1*   *Firm's Policy, Case $b > \rho$*

Figure 4.2   *Growth of the Firm's Capital Base*

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4.2.4 Financial Assets Excluded from the Capital Base

4.2.4.1 Analysis of Feasible Regimes

The preceding subsection examined a case where financial capital is included in the capital base and showed that the real distortion found in chapter 2 vanishes in this setting. The present subsection aims to assess a case where the model contains financial capital but the capital base is defined as in chapter 2 and does not include financial assets, i.e. $N=K$. The question addressed is whether opening the firm access to the financial markets is sufficient to change the qualitative features of the incentives or whether the inclusion of financial capital in the capital base is also needed.

In the present case $\partial N/\partial K = 1$ and $\partial N/\partial F = 0$. Using these results equations (4.7e) and (4.7f) can be rewritten as

(4.7e'') $\dot{\lambda}_1 = r\lambda_1 - f'\lambda_1 - bq_1$

(4.7f') $\dot{\lambda}_2 = r\lambda_2 - \rho\lambda_1 - q_5$

When comparing these with the corresponding equations in the case $N = K+F$, we observe that the equations for λ_1 are exactly the same but the equations for λ_2 (equations (4.7f) and (4.7f'')) differ as equation (4.7f'') lacks the term bq_1. This difference has no consequences in regimes A1' and F1-F2, where $q_1 = 0$, but in regimes A3'-A4' and F3-F4, where $q_1 \geq 0$, the optimality conditions change.

The first control constraint also changes. It is now $h_1 = bK + D_x - D \geq 0$. The maximum amount of the dividend taxed as capital income is no longer dependent upon financial capital.

These changes affect regime F2 only with respect to the constraint. The dividend is now limited from above by the constant amount bK^*. The dividend distribution rule of regime A3' is now $D = bK^*$ and is independent of financial capital. Moreover, in this regime the upper limit for real capital changes from K^* to K_A^*.

Regime F3 has now the following properties. The co-state variables are constants and they can take values between $1-\tau_e \leq \lambda_1 = \lambda_2 \leq 1-\tau_c$. The firm's real capital is also a constant, $K = K'$, and can be in the range $K^* \leq K' \leq K_A^*$.[63] The firm pays a

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[63] By substituting ρ from (A4.1)into (4.7f'') and using (4.7d) we obtain $\dot{\lambda}_1 = \dot{\lambda}_2 = 0$. According to (4.7e'') it holds that $f' = r - q_1 b/\lambda_1$. The constancy of λ_1 implies that q_1 is a constant. From these features it also follows that f' is a constant. Equation (4.7a) now implies that in regime F3 $r \geq f' \geq r - \tau_x b/(1-\tau_e)$.

constant amount, $D = bK'$, as dividends and invests the rest of its profits in the financial markets, $S = f(K') + \rho F - bK'$.

As shown in the preceding subsection, in the case $N = K + F$ regimes A4' and F4 are infeasible. Now this result no longer holds. In the following these regimes are analysed in some detail. We will start with regime A4'.

Regime A4': $(D = bN + D_x, Q = F = 0, D_x > 0; \; q_1, q_3, q_4, q_5 \geq 0, \; q_2 = 0)$

The first-order conditions give us the following properties[64]

$$(4.28) \qquad f'(K) = r - \frac{\tau_x}{1 - \tau_e} b \quad \Rightarrow \quad K = K_A{}^*$$

$$(4.29) \qquad \rho = r - \frac{q_5}{1 - \tau_e}$$

$$(4.30) \qquad f(K) = D \qquad \qquad \text{(note: } D > bK, F = S = 0\text{)}$$

Equation (4.28) is a marginal condition that defines a constant stock of real capital. This condition is exactly the same as condition (2.26) in chapter 2. This means that the firm's capital stock takes the value $K = K_A{}^*$. Note that both state variables, K and F, are constants. This means that the firm is in a steady state in regime A4'. Equation (4.29) is a feasibility condition and is satisfied when $q_5 = 0$.[65] Equation (4.30) states that the firm uses all of its internal financing in dividend distributions and does not invest.

Next we will consider regime F4.

Regime F4. $(D = bN + D_x; Q = 0; D_x, F > 0; \; q_1, q_3, q_4 \geq 0; q_2 = q_5 = 0)$

From the first-order conditions we obtain:

$$(4.31) \qquad f' = r - \frac{\tau_x}{1 - \tau_e} b \quad \Rightarrow \quad K = K_A{}^*$$

$$(4.32) \qquad \rho = r$$

$$(4.33) \qquad f(K) + \rho F = D + S$$

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[64] Equation (4.28) defines a unique steady-state capital stock given a sufficiently high level of depreciation. Depreciation is assumed to satisfy this requirement. See the discussion after equation (2.26).

[65] Note that normally the property $q_5 = 0$ would indicate that $F \geq 0$. In the present case, however, the definition for the regime A4' requires that $F = 0$. These features indicate that regime A4' is here merely a boundary case.

Regime F4 produces a constant stock of real capital of $K = K_A{}^*$, i.e. the same as in regime A4'. Condition (4.32) tells us that regime F4 is feasible exactly on the condition $\rho = r$, which satisfies assumption (A4.1). Equation (4.33) implies that the firm's policy alternatives are now greater than in regime A4'. The firm may distribute only part of its profits and invest the rest in the financial markets. The firm is actually indifferent between paying a dividend and financial investment. This indifference, however, is constrained by the dividend constraint $D > bN$.

Regimes A4' and F4 are both feasible in the present case. In them, the firm is in a steady state, the stock of real capital being $K_A{}^*$. The regimes differ from one another only by the fact that in regime A4' $F = 0$ and in regime F4 $F > 0$.

Table 4.3 summarises the main properties of the feasible regimes.

Table 4.3   *Characteristics of Feasible Regimes, Case $N = K$*

Regime	λ	$\dot{\lambda}$	K	$\dot{K}$	F	Budget
A1'	$1 \geq \lambda_{1,2} \geq 1-\tau_c$	≤ 0	$K \leq K^*$	>0	$=0$	$I = f(K)$
F1	$1 \geq \lambda_{1,2} \geq 1-\tau_c$	$=0$	$K = K^*$	$=0$	>0	$S = f(K)+\rho F$
F2	$\lambda_{1,2}=1-\tau_c$	$=0$	$K = K^*$	$=0$	>0	$D+S = f(K)+\rho F$
A3'	$1-\tau_c \geq \lambda_{1,2} \geq 1-\tau_e$	≤ 0	$K^{\#} \leq K \leq K_A{}^*$	>0	$=0$	$I = f(K)-bK$
F3	$1-\tau_c \geq \lambda_{1,2} \geq 1-\tau_e$	$=0$	$K^* \leq K' \leq K_A{}^*$	$=0$	>0	$S=f(K')+\rho F-bK'$
A4'	$\lambda_{1,2}=1-\tau_e$	$=0$	$K=K_A{}^*$	$=0$	$=0$	$D = f(K_A{}^*)$
F4	$\lambda_{1,2}=1-\tau_e$	$=0$	$K=K_A{}^*$	$=0$	>0	$D+S=f(K_A{}^*)+\rho F$

4.2.4.2 Optimal Solution

Regimes A4' and F4 are obvious candidates to describe the firm's policy in the final phase. Regimes A1' and A3' and F1 are not suitable for this role for the same reasons as in the preceding section. Regimes F2-F3 cannot be final regimes, since they do not satisfy the transversality conditions of the model. This can be shown for regime F3 as follows. Here the firm's real capital and dividends are constants, with $K = K'$ and $D = bK'$. The equation of motion for financial capital is obtained as $\dot{F}(t) = f(K')+\rho F(t)-bK'$. This implies that financial capital grows at the rate ρ and violates the transversality condition, (4.22), which requires a growth rate smaller than r.

Thus, the firm's optimal policy in the final phase is described by regimes F4 and A4'. We depict this policy as 'regime A4^i ' the superscript referring to the indifference. This regime is assumed to satisfy the transversality condition (4.22).[66]

The regime chain that satisfies the first-order conditions as well as the continuity of the state and co-state variables is:

Initial investment $\Rightarrow$ Regime A1' $\Rightarrow$ Regime A3' $\Rightarrow$ Regime A4^i

On the basis of the continuity requirement the indifference regime A4^i can be preceded only by regime A3'. On the same grounds the predecessor of regime A3' can only be regime A1'. As above, regime A1' satisfies the initial time transversality condition (4.8) and can very well serve as the first regime in the optimal solution.

Thus, after the firm's initial capital is created the firm begins its real growth in regime A1', shifts at some point to regime A3', where it continues investing in real capital until this reaches the level $K = K_A{}^*$. At this stage the firm may start distributing all of its profits as a dividend.

The firm's behaviour is now almost the same as in the solution to the basic model (case $\sigma > b$). The long-run equilibrium level for real capital is $K_A{}^*$ and the firm grows in two stages (regimes A1' and A3'). The solutions to the two model variants are distinguished only by the possibility for financial investment in the final phase of the present case.

The analysis shows us that the inclusion of financial capital does not suffice to eliminate the incentive to expand real capital. Referring to the results obtained in the previous subsection, it seems to be necessary for financial capital to be included in the capital base for this incentive to be eliminated.

The factor behind the differences between the solutions to the two variants of the present model is quite obvious. In the case $N=K$ studied in this subsection a marginal increase in real capital generates a direct return, $f'(K)$, in addition to the tax saving $q_1 b$. In contrast, an increase in financial capital does not bring corresponding tax savings. Accordingly, the firm's optimal policy at the level of real capital K^*, is to continue investing its profits in real capital.

In the case of $N = K+F$ a marginal increase in both forms of capital gave the same tax saving. Due to the concavity of the profit function the firm stopped real investments at $K = K^*$ and switched to investing in financial capital.

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[66] Note that regime F4 includes policies where $F(t)$ grows at the rate ρ violating the transversality condition. Regime A4^i rules out such policies.

4.2.5 Summary

In this section the model was expanded to include the possibility of financial investments. The firm's behaviour was examined under two different definitions of the capital base.

It was observed that the real distortion caused by the graduated taxation of dividends found in chapter 2 is eliminated when financial investment is included in the capital base. The incentive to expand the capital base nevertheless remains, and leads to investment in the financial markets. In the case $\rho < b$ the firm expands its financial capital to at least the level where the firm can distribute all of its profits as a dividend subject to taxation as capital income. In the case $\rho \geq b$ the firm expands its financial capital infinitely, while still paying out part of its profits as a dividend.

When financial capital is excluded from the capital base the firm's optimal policy and the real distortion caused by the graduated tax system remains broadly the same as that we found in chapter 2.

4.3 Debt and Dual Income Tax

4.3.1 Introduction

In this section we analyse the impact of graduated dividend tax on the firm's borrowing policy. The previous chapters evaluated a firm financed purely with equity. Of the firm's financing decisions, only the choice between external and internal equity can be assessed in that framework. The analysis in chapter 3 showed that graduated dividend tax does not have a direct impact on the position of these forms of financing. The analysis nevertheless demonstrated that the central property of DIT, i.e. that dividends may be subject to a high tax rate on earned income, may lead to discrimination against external equity. This distortion, however, was shown to be highly dependent on the relative sizes of the effective tax rates on capital gains and earned income (τ_g and τ_e^{ef}).

This study focuses on a variant of DIT where the firm's capital base is defined as current net assets (cf. section 4.1). It is fairly evident that in this setting graduated dividend tax has an impact on the relative cost of debt. We can see this by considering a purely debt-financed investment. For this case $dN/dK = 0$, i.e. the firm's capital base is not affected by the investment.[67] This implies that such an investment does not generate the tax savings which constitute the incentive effects of graduated taxation.

However, this discrimination against debt does not mean that the firm will reject debt totally in financing investments. Our analysis in this section shows that the firm's debt policy takes different forms at different stages of the firm's growth cycle and that the discrimination is not seen until its later phases.

Before presenting the changes we make to the basic model we will conduct a short review of the research of van Schijndel (1988) and van Hilten et al. (1993) who have studied the dynamics of the firm's debt policy. Later on we will compare our results with those of these authors and make use of some of their interpretations.

These authors study the investment and financing behaviour of a firm financed with debt and internal equity in a framework of perfect foresight and perfect capital markets. Their framework is based on the models by Lesourne and Leban (1978) and van Loon (1983). They allow the costs of debt and equity to differ due to non-neutral taxation. They show that in a case where equity is cheaper than debt, $r'<\rho'$,[68] the firm uses debt until the condition $f'(K)=\rho'$ is satisfied.

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[67] The firm's capital base is defined now as $N=K-B$, where B is the stock of debt.

[68] The terms r' and ρ' depict here the after-tax discount rate of the owner and the firm's after tax-cost of debt.

They call this first part of the firm's growth cycle the phase of maximum growth. After this phase the firm starts a consolidation phase where it repays its debt and keeps the capital stock constant. This policy continues until the firm is fully debt-free. After the consolidation phase the firm starts expanding its capital stock again, now financing investment with retained earnings. This phase continues until the marginal condition determining the firm's steady-state capital stock, $f'(K) = r'/(1-\tau_f)$, is satisfied.[69]

The most interesting feature of this solution is that the firm maximises its use of debt in the early phase of the growth cycle despite the fact that debt is the more expensive form of financing. The authors explain this result by noting that in the phase where the marginal return on debt-financed investment exceeds the cost of debt this policy will maximise the growth of the firm's equity. This is due to the leverage effect of debt, which increases net profits and consequently allows for an increase in the growth of internally financed equity. When the firm enters the position where $f'(K) = \rho'$, this effect disappears and the firm stops issuing new debt.

We will diverge from van Schijndel (1988) and van Hilten et al. (1993) and allow also the case $r' \geq \rho'$, but, as we will observe, our results come out to be very similar to those of these authors. Furthermore their interpretation of the results is applicable in our case as well.

4.3.2 Inclusion of Debt in the Model

Debt can be included in our model framework generally in the same manner as financial investment in the previous section, i.e. by including a new equation of motion to depict the change in the stock of debt and by supplementing the firm's budget identity with cash flow associated with debt (funds from issuing new debt and cash flow used for servicing costs of debt). Debt must also be taken into account in the first constraint, where the dividend taxed as capital income is compared to the imputed return on the firm's net capital.

Nevertheless the inclusion of debt in models like the one applied in this study raises certain problems that were not faced in section 4.2. As discussed more thoroughly in section 1.3, in a framework of perfect foresight and perfect capital markets taxation may induce the firm to pursue some extreme behaviour in its borrowing policy. There are several ways to handle this problem in models like the one used in this study, one of them being exogenous constraints on borrowing. Another is to assume increasing marginal costs of borrowing, which may produce an endogenously determined internal optimum for debt.

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[69] A similar pattern of the firm's borrowing behaviour (maximum borrowing - repayment of debt - equity financed growth) was found already by Bensoussan et al. (1974) and Ylä-Liedenpohja (1976).

In order to keep the model simple and avoid divergence from the basic model of this study, as well as from the models of Sinn (1991) and van Hilten et al. (1993), we will choose the first approach and include in our model the following exogenous constraint, using the state variables:

$$(4.34) \qquad B \leq aK, \qquad 0 < a < 1,$$

where a is the firm's exogenously set maximum debt-to-capital ratio. To simplify the analysis we will assume that the firm cannot invest in the financial markets. This implies the following constraint:

$$(4.35) \qquad B \geq 0$$

One important feature of existing tax systems is the deductibility of costs of debt when calculating taxable profit. In order to incorporate this factor into our analysis we assume a tax system which includes firm-level taxation. To retain some consistency in the tax system we also assume an imputation system and taxation of interest income.

As in our previous models, capital is homogenous and its value in taxation corresponds to its replacement value. To simplify the model we exclude new share issues. This means that debt is now the only form of external financing.

The firm's budget constraint is:

$$(4.36) \qquad f(K) + R = I + D + \rho B + T,$$

where the sources of funds are operating income before taxes and costs of debt (and after depreciation), $f(K)$, and new debt financing, R. The uses of funds are investments, I, dividends, D, interest expenses on debt, ρB, and firm-level taxes, T. Taxes are calculated as follows: $T = \tau_f[f(K)-\rho B]$. Using the budget identity we now obtain the equation of motion for capital as:

$$(4.37) \qquad \dot{K} = (1-\tau_f)[f(K)-\rho B] + R - D$$

The equation of motion for debt is:

$$(4.38) \qquad \dot{B} = R$$

The firm's capital base is defined in the present case as:

$$(4.39) \qquad N = K - B$$

From the inclusion of imputation credit in the model it follows that the net dividend is written here as in chapter 3:

$$(4.40) \qquad D_n = \frac{1-\tau_c}{1-s}D - \frac{\tau_x}{1-s}D_x$$

We will retain assumption (A4.1), which now means that the shareholder's pre-tax discount rate and the firm's interest rate on debt are equal. Furthermore, we will retain assumption (A4.2), meaning that we will focus our attention on a high-rate-of-return firm whose dividend payments are subject to taxation on the margin as earned income. The variable σ is defined in the present model as

$$(4.41) \qquad \sigma \equiv \frac{(1-\tau_f)[f(K)-\rho B]}{(1-s)N}$$

i.e. the tax-adjusted average rate of return on equity (on net capital).

One key element of the basic model is that the amount of the firm's initial capital is determined as part of the firm's optimisation problem. We assume in this section that the initial value of debt is also defined on optimality grounds. Thus, as in the preceding section, we have two initial cost functions, φ_1 and φ_2, now for capital and debt.

We can write the model in its entirety as follows:

$$(4.42a) \quad \max_{\{K_0,B_0,D,D_x,R\}} \int_{t_0}^{\infty}\left(\frac{1-\tau_c}{1-s}D - \frac{\tau_x}{1-s}D_x\right)e^{-(1-\tau_c)r(t-t_0)}dt + \varphi_1(K(t_0)) + \varphi_2(B(t_0))$$

$$(4.42b) \qquad \dot{K} = \left(1-\tau_f\right)\left[f(K)-\rho B\right] + R - D, \quad K(t_0)=K_0$$

$$(4.42c) \qquad \dot{B} = R, \quad B(t_0)=B_0 \geq 0$$

$$(4.42d) \qquad h_1 = (1-s)b(K-B)+D_x-D \geq 0$$

$$(4.42e) \qquad h_2 = D \geq 0$$

$$(4.42f) \qquad h_3 = D_x \geq 0$$

$$(4.42g) \qquad h_4 = B \geq 0$$

$$(4.42h) \qquad h_5 = aK - B \geq 0$$

$$(4.42i) \qquad \varphi_1(K) = -K, \quad \varphi_2(B) = B$$

This model has the regularity properties required for applying the maximum principle discussed in chapter 2. As shown in appendix 1, the model also fulfils the

concavity and convergence properties required by the sufficiency condition. Thus the solution that fulfils the model's first-order conditions is the optimal one.

4.3.3 Analysis of Feasible Policies

The current-value Lagrangian for the problem is:

$$(4.43) \qquad L = \frac{1-\tau_c}{1-s}D - \frac{\tau_x}{1-s}D_x + \lambda_1\{(1-\tau_f)[f(K) - \rho B] + R - D\} + \lambda_2 R$$

$$q_1[(1-s)b(K-B) + D_x - D] + q_2 D + q_3 D_x + q_4 B + q_5(aK - B)$$

where λ_1 and λ_2 are the shadow prices for capital and debt and $q_1,...,q_5$ are the shadow prices for constraints (4.42d) - (4.42h). The first-order conditions without complementarity conditions are:

$$(4.44a) \qquad \partial L/\partial D = \frac{1-\tau_c}{1-s} - \lambda_1 - q_1 + q_2 = 0$$

$$(4.44b) \qquad \partial L/\partial D_x = - \frac{\tau_x}{1-s} + q_1 + q_3 = 0$$

$$(4.44c) \qquad \partial L/\partial R = \lambda_1 + \lambda_2 = 0$$

$$(4.44d) \qquad \dot\lambda_1 = (1-\tau_c)r\lambda_1 - \partial L/\partial K = (1-\tau_c)r\lambda_1 - (1-\tau_f)f'(K)\lambda_1 - (1-s)bq_1 - aq_5$$

$$(4.44e) \qquad \dot\lambda_2 = (1-\tau_c)r\lambda_2 - \partial L/\partial B = (1-\tau_c)r\lambda_2 + (1-\tau_f)\rho\lambda_1 + (1-s)bq_1 - q_4 + q_5$$

$$(4.44f) \qquad \dot K = (1 - \tau_f)[f(K) - \rho B] + R - D$$

$$(4.44g) \qquad \dot B = R$$

$$(4.44h) \qquad \lambda_1(t_0) = - d\varphi_1(K)/dK = 1$$

$$(4.44i) \qquad \lambda_2(t_0) \le - d\varphi_2(B)/dB = -1, \; B_0 \ge 0, \; B_0[\lambda_2(t_0)+1] = 0$$

Conditions (4.44c) and (4,44h) imply that the initial time tranversality condition for debt is actually:

$$(4.45) \qquad \lambda_2(t_0) = -1.$$

We can rearrange some of these conditions into a more convenient form. By adding equation (4.44d) to equation (4.44e) and by using equation (4.44c) we obtain:

$$(4.46) \qquad \dot{\lambda_1} + \dot{\lambda_2} = (1-\tau_f)\rho\lambda_1 - (1-\tau_f)f'(K)\lambda_1 - q_4 + (1-a)q_5 = 0$$

and by solving this with respect to f':

$$(4.47) \qquad f'(K) = \rho - \frac{q_4}{(1-\tau_f)\lambda_1} + \frac{(1-a)q_5}{(1-\tau_f)\lambda_1}$$

By solving equation (4.44d) with respect to f' we obtain

$$(4.44d') \qquad f'(K) = \frac{1-\tau_e}{1-\tau_f}r - \frac{(1-s)q_1}{(1-\tau_f)\lambda_1}b - \frac{aq_5}{(1-\tau_f)\lambda_1} - \frac{\dot{\lambda_1}}{(1-\tau_f)\lambda_1}$$

We will use these two equations later in deriving the conditions for the feasible values of the firm's capital stock, K.

The constraints in (4.42d)-(4.42h) generate 32 different policies, which are listed in table 4.1 in section 4.2.2. Policies 1-4, 17-20 , 11-12, 15-16, 27-28 and 31-32 can be rejected on the same basis as in section 4.2. Policies 5, 7, 9 and 13 can be disregarded because in these the debt is simultaneously at its lower and upper limit. This is possible only if $K = 0$. This is in contradiction with our assumption set in chapter 2.

Policies 14 and 29 are feasible only when $\sigma < b$ and can be rejected on the basis of assumption (A4.2).

There are 10 remaining regimes that are feasible. We will rename regimes 25, 21 and 23, where the lower constraint on debt is binding ($B = 0$), as regimes A1', A3' and A4', retaining the references to the corresponding regimes of the basic model. Of the regimes where $B>0$ we use three symbol abbreviations starting with 'B'. Regimes 10, 6 and 8 are renamed regimes B1a, B3a and B4a, the letter 'a' indicating that the upper constraint on debt is binding ($B = aK$). Regimes 26, 14, 22 and 24 are called regimes B1b, B2b, B3b and B4b, the letter 'b' indicating that both constraints on debt are non-binding ($0 < B < aK$). The numerical part of the abbreviation indicates the firm's dividend policy in the same manner as above.

We will start the more detailed analysis with regimes A1', B1a and B1b, where $D=0$. This feature implies q_2, $q_3 \geq 0$, $q_1 = 0$. On the basis of equations (4.44a) and (4.44c) we obtain the following range for the co-state variables:

$$(4.48) \qquad \lambda_1 = -\lambda_2 = \frac{1-\tau_c}{1-s} + q_2 \geq \frac{1-\tau_c}{1-s}$$

Regime A1': $(D = D_x = B = 0;\ q_2, q_3, q_4 \geq 0,\ q_1 = q_5 = 0)$

From equation (4.47) we obtain:

$$(4.49) \qquad f'(K) = \rho - \frac{q_4}{(1-\tau_f)\lambda_1} \leq r \quad \Rightarrow \quad K \geq K^*$$

From equation (4.44f) we obtain:

$$(4.50) \qquad \dot{K} = I = (1-\tau_f)f(K) \quad \Rightarrow \quad \dot{K} > 0$$

The firm uses all of its internal financing for investments. Regime A1' is thus a growth regime like regime A1 in chapter 2. It is feasible only when $K \geq K^*$.

Regime B1a: $(D = D_x = 0,\ B > 0;\ q_2, q_3, q_5 \geq 0,\ q_1 = q_4 = 0)$

From equation (4.47) we obtain:

$$(4.51) \qquad f'(K) = \rho + \frac{(1-a)q_5}{(1-\tau_f)\lambda_1} \geq r \quad \Rightarrow \quad K \leq K^*$$

In regime B1a $B = aK \Rightarrow R = \dot{B} = a\dot{K}$. On the basis of (4.44f) we obtain:

$$(4.52) \qquad \dot{K} = \frac{1-\tau_f}{1-a}(f(K) - \rho aK)$$

Equation (4.52) and assumption (A4.2) imply that $\dot{K} > 0$.[70] The regime is thus a growth regime where the firm invests using retained earnings and debt as sources of finance.

Regime B1b: $(D = D_x = 0,\ 0 < B < aK;\ q_2, q_3 \geq 0,\ q_1 = q_4 = q_5 = 0)$

From equation (4.47) we obtain:

$$(4.53) \qquad f'(K) = \rho = r \qquad \Rightarrow K = K^*$$

This implies that $\dot{K} = 0$. The firm's budget equation is:

$$(4.54) \qquad R = -(1-\tau_f)[f(K)-\rho B]$$

---

[70] Note that assumption (A4.2), the property that $a < 1$ and the concavity of $f(K)$ imply that $\sigma(K) > b$ at all levels of K satisfying $0 \leq K \leq K_A^*$.

Equation (4.54) and assumption (A4.2) now imply that $R < 0$, meaning that the firm uses its internal financing to pay off debt.

Next we will move to regime B2b, where $0 < D < bN$.

Regime B2b: $(D_x = 0, 0 < D < bN, 0 < B < aK; q_1 = q_2 = q_4 = q_5 = 0, q_3 \geq 0)$

From equation (4.44) we obtain:

$$(4.55) \qquad \lambda_1 = -\lambda_2 = \frac{1 - \tau_c}{1 - s}$$

From equations (4.44d) and (4.44e) we obtain:

$$(4.56) \qquad f'(K) = \rho \quad \Rightarrow \quad K = K^*$$

The firm's budget equation is:

$$(4.57) \qquad R = -\{(1 - \tau_f)[f(K) - \rho B] - D\}$$

Assumption (A4.2) and the value of dividends, $D < (1-s)bN$, imply that $R < 0$, which means that the firm pays off debt as in regime B1b.

Next we will analyse the regimes where $D = (1-s)bN$. From equations (4.44a) - (4.44d) and the values of the shadow prices $q_1, q_3 \geq 0$, $q_2 = 0$ we obtain the following range for the co-state variables in these regimes:

$$(4.58) \qquad \frac{1 - \tau_e}{1 - s} \leq \lambda_1 = -\lambda_2 \leq \frac{1 - \tau_c}{1 - s}$$

Regime A3': $(D_x = B = 0, D = (1-s)bN; q_1, q_3, q_4 \geq 0, q_2 = q_5 = 0)$

We now obtain from equations (4.47) and (4.44d'):

$$(4.59) \qquad f'(K) = \rho - \frac{q_4}{(1 - \tau_f)\lambda_1} \leq r \Rightarrow K \geq K^*$$

On the basis of equation (4.44f) it holds that:

$$(4.60) \qquad \dot{K} = (1 - \tau_f)f(K) - (1 - s)bK$$

This, together with assumption (A4.2), implies that $\dot{K} > 0$, i.e. regime A3' is a growth regime.

Regime B3a: $(D_x = 0, D = (1-s)bN, B > 0; q_1, q_3, q_5 \geq 0, q_2 = q_4 = 0)$

From equation (4.47) we obtain:

$$(4.61) \qquad f'(K) = \rho + \frac{(1-a)q_5}{(1-\tau_f)\lambda_1} \geq r \quad \Rightarrow \quad K \leq K^*$$

By substituting $D = (1-s)bN$ and $R = \dot{B} = a\dot{K}$ into equation (4.44f) and rearranging the terms we obtain:

$$(4.62) \qquad \dot{K} = \frac{1}{1-a}\{(1-\tau_f)[f(K) - \rho B] - (1-s)bN\}$$

Assumption (A4.2) and properties of a and $f(K)$ imply again that $\dot{K} > 0$, i.e. regime B3a is a regime of real growth.

Regime B3b: $(D_x = 0, D = (1-s)bN, 0 < B < aK; q_1, q_3 \geq 0, q_2 = q_4 = q_5 = 0)$

From equation (4.47) we obtain:

$$(4.63) \qquad f'(K) = \rho = r \quad \Rightarrow K = K^*$$

By substituting $D = (1-s)bN$ and $\dot{K} = 0$ into equation (4.44f) we obtain:

$$(4.64) \qquad R = -\{(1-\tau_f)[f(K)-\rho B] - (1-s)bN\},$$

R is negative due to assumption (A4.2). The firm thus pays off its debt in this regime.

In the following we will go through the regimes where $D > (1-s)bN$. From equations (4.44a) - (4.44c) and the values of the shadow prices $q_1 \geq 0$, $q_2 = q_3 = 0$ we obtain:

$$(4.65) \qquad \lambda_1 = -\lambda_2 = \frac{1-\tau_e}{1-s} \quad \Rightarrow \dot{\lambda}_1 = \dot{\lambda}_2 = 0$$

From equation (4.44b) we obtain:

$$(4.66) \qquad q_1 = \frac{\tau_x}{1-s}$$

Regime A4': $(B = D_x = 0, D > bN; q_1, q_4 \geq 0, q_2 = q_3 = q_5 = 0)$

By using (4.65)-(4.66) and equation (4.44d') we obtain the following marginal condition:

$$(4.67) \qquad f'(K) = \frac{1-\tau_c}{1-\tau_f}r - \frac{(1-s)\tau_x}{(1-\tau_f)(1-\tau_e)}b$$

We will denote the constant capital stock implied by (4.67) as K_B*.[71] Because B is also constant, the firm is in a steady-state in this regime. For the Finnish tax parameters $(\tau_f=\tau_c=s)$ the marginal condition reduces to the same one as the corresponding condition, (2.26), of the basic model. This means that in the Finnish case $K_B* = K_A*$.

Equation (4.44e) gives us a feasibility condition for regime A4':

$$(4.68) \qquad \rho = \frac{1-\tau_c}{1-\tau_f}r - \frac{(1-s)\tau_x}{(1-\tau_f)(1-\tau_e)}b + \frac{(1-s)q_4}{(1-\tau_f)(1-\tau_e)} \quad \Rightarrow$$

$$\rho \geq \frac{1-\tau_c}{1-\tau_f}r - \frac{(1-s)\tau_x}{(1-\tau_f)(1-\tau_e)}b \quad (=p_E)$$

This condition is satisfied with a broad range of parameter values. For example, the stylised Nordic tax system, where $\tau_f = \tau_c$, as outlined in section 1.2, satisfies it easily. The condition may be violated in a system where τ_f is very high with respect to τ_c and s.

Regime B4a: $(B = aK, D_x = 0, D > bN;\ q_1, q_5 \geq 0, q_2 = q_3 = q_4 = 0)$

The marginal condition that defines the long run stationary capital stock can be derived in this case by substituting the co-state values from equation (4.65) into equation (4.44e), solving the resulting equation with respect to q_5 and substituting it and $q_4= 0$ into equation (4.47). We obtain:

$$(4.69) \qquad f'(K) = a\rho + (1-a)[\frac{1-\tau_c}{1-\tau_f}r - \frac{(1-s)\tau_x}{(1-\tau_f)(1-\tau_e)}b]$$

We will denote the capital stock defined in (4.69) by K_B^+. The firm's stock of debt is now $B = aK_B^+$. Both debt and capital are constants, which means that the firm is in a steady state in regime B4a.

[71] As in preceding sections, we assume that the rate of depreciation is high enough for there to be a unique value of K that satisfies (4.67). The same assumption also applies to equations (4.69) and (4.73).

Equation (4.69) expresses the firm's cost of capital as the weighted average of the costs of equity and debt, where the weights are the relative shares of debt and equity of the firm's capital stock.

We will call the expression in brackets in (4.69) the firm's effective cost of equity and depict it with p_E:

$$(4.70) \qquad p_E = \frac{1-\tau_c}{1-\tau_f}r - \frac{(1-s)\tau_x}{(1-\tau_f)(1-\tau_e)}b$$

We can conclude from this that the incentive generated by graduated dividend taxation (the second term on the right-hand side of (4.70)) only reduces the cost of equity capital.

By substituting $\rho = r$ and $\tau_f = \tau_c = s$ into (4.69) we obtain the marginal condition in the following form

$$(4.71) \qquad f'(K) = r - (1-a)\frac{\tau_x}{1-\tau_e}b$$

In this representation the second term is multiplied by the financing share of equity capital, $1-a$, which is less than one. When we compare equation (4.71) to equation (2.26) in chapter 2 we can conclude that in the present case where the firm uses debt the incentive generated by graduated taxation is smaller than in the case where the capital stock is financed completely with equity, i.e. $K_B^+ < K_A^*$.

The significance of these results is nevertheless lessened by the fact that regime B4a is feasible only on a fairly restrictive assumption. From equation (4.44e) we obtain

$$(4.72) \qquad \rho = \frac{1-\tau_c}{1-\tau_f}r - \frac{(1-s)\tau_x}{(1-\tau_f)(1-\tau_e)}b - \frac{(1-s)q_s}{(1-\tau_f)(1-\tau_e)} \quad \Rightarrow$$

$$\rho \le p_E < \frac{1-\tau_c}{1-\tau_f}r$$

The regime is thus feasible only if the firm's cost of debt is smaller than or equal to the effective cost of equity capital, p_E. This implies the condition $\tau_f > \tau_c$. Note that this is not satisfied in the stylised version of the Nordic tax system.

Regime B4b: $(D_x = 0, D > bN, 0 < B < aK;\ q_1 \geq 0, q_2 = q_3 = q_4 = q_5 = 0)$

From equations (4.47) and (4.44d') we obtain:

$$(4.73) \qquad f'(K) = \rho = \frac{1-\tau_c}{1-\tau_f}r - \frac{(1-s)\tau_x}{(1-\tau_f)(1-\tau_e)}b$$

Equation (4.73) implies the following feasibility condition for the regime

$$(4.74) \qquad \rho = p_E < \frac{1-\tau_c}{1-\tau_f}r \implies \tau_f > \tau_c$$

This requires the firm's cost of debt to be exactly equal to the effective cost of equity capital, $\rho = p_E$. This exemplifies a razor's edge case which can hold only under exceptional circumstances. This equality also implies that the regime can be effective only if the corporate tax rate is strictly higher than the tax rate on capital income. This does not occur in the stylised Nordic system.

Table 4.4 summarises the central features of the feasible regimes.

Table 4.4   Characteristics of the Feasible Regimes

Regime	K	B	D	$\dot{K}$	$\dot{B}$	Feasible in the Nordic case ($\tau_f=\tau_c$)
A1'	$\geq K^*$	$=0$	$=0$	>0	$=0$	yes
B1a	$\leq K^*$	$=aK$	$=0$	>0	>0	yes
B1b	$=K^*$	$0<B<aK$	$=0$	$=0$	<0	yes
B2b	$=K^*$	$0<B<aK$	$0<D<(1-s)bN$	$=0$	<0	yes
A3'	$\geq K^*$	$=0$	$=(1-s)bN$	>0	$=0$	yes
B3a	$\leq K^*$	$=aK$	$=(1-s)bN$	>0	>0	yes
B3b	$=K^*$	$0<B<aK$	$=(1-s)bN$	$=0$	<0	yes
A4'	$=K_B^*$	$=0$	$>(1-s)bN$	$=0$	$=0$	yes
B4a	$=K_B^+$	$=aK_B^+$	$>(1-s)bN$	$=0$	$=0$	no
B4b	$=K_B^*$	$0<B<aK$	$>(1-s)bN$	$=0$	$?$	no

4.3.4 Optimal Solution

In the following we will derive a solution for the model in the case of the stylised Nordic tax system where:

(A4.3) $\tau_f = \tau_c$

This assumption simplifies the solution procedure. In the more general case where the tax rates can differ, there are several different solutions depending on the relative sizes of the tax parameters. We allow the imputation credit to vary between $0 \leq s \leq \tau_f$.

Of the feasible regimes, A4' is the only one which is qualified to be the solution's final regime. Two other steady state regimes, B4a and B4b, do not satisfy the tax rate condition (A4.3). Furthermore, none of the other regimes can serve as the final regime since in all of them one state variable would increase or decrease infinitely and cross its upper or lower boundary (see table 4.4).

Using the requirement that the state and co-state variables must be continuous throughout the planning horizon, the regime chain defining the optimal policy of the firm is as follows (case: $s < \tau_f$):

Initial capital $\Rightarrow$ Regime B1a $\Rightarrow$ Regime B3a $\Rightarrow$ Regime B3b $\Rightarrow$

Regime A3' $\Rightarrow$ Regime A4'

In the case of full imputation, where $s = \tau_f$, the solution is the same except that regime B1a is absent. In both cases the firm begins to grow in the Ba regimes and finances the maximum amount of investment with debt. When the firm's capital stock achieves the level K^*, the firm will shift to regime B3b and begin to use internal financing to repay its debt. When the debt is fully paid off, the firm will begin a new phase of growth, now financing investments purely with internal financing (regime A3'). In the steady-state phase the firm is completely debt-free (regime A4'). The solution is illustrated in figure 4.3.

Next we will present the grounds for the composition of the regime chain beginning with the end of the chain.

Regime A3' is the predecessor of the final regime since it is the only feasible regime where the state and co-state variables can achieve the values prevailing in regime A4' ($K = K_B^*$, $B = 0$ and $\lambda_1 = -\lambda_2 = (1-\tau_e)/(1-s)$).

Regimes B1a and B3a are not suitable to be the predecessor of regime A3' owing to the continuity requirement for debt. The suitability of other feasible regimes can be analysed using the approach in appendix 2, i.e. by evaluating the first and

second time derivatives of the shadow price q_1 at the start of regime A3'. By taking the derivative of (4.44a) with respect to time and substituting $\dot{\lambda}_1$ in this and λ_1 in (4.44a) into (4.44d) and making use of (A4.3) we obtain:

(4.75) $\qquad \dot{q}_1 = (f'(K) - r)(1 - \tau_f)(\dfrac{1 - \tau_e}{1 - s} - q_1) + (1 - s)q_1 b$

By differentiating this again with respect to time we obtain

(4.76) $\quad \ddot{q}_1 = (1 - \tau_f)f''(K)\dot{K}(\dfrac{1 - \tau_e}{1 - s} - q_1) + [(1 - \tau_c)r + (1 - s)b - (1 - \tau_f)f'(K)]\dot{q}_1$

Regime A1' cannot be the predecessor of regime A3' for the following reason. We know from equation (4.49) that the switch between these regimes would have to happen when $K > K^*$, i.e. when $f' < r$.[72] We also know that $\lambda_1 = (1 - \tau_c)/(1 - s)$ and $q_1 = 0$ at this point. Using (4.75) it follows that $\dot{q}_1 < 0$ at the regime switch, implying that q_1 is becoming negative. This contradicts the requirement that $q_1 \geq 0$.

The suitability of regimes B1b and B2b as the predecessor of regime A3' can be excluded as follows. By equations (4.48), (4.58), (4.53) and (4.56) the regime switch should occur when $\lambda_1 = (1 - \tau_c)/(1 - s)$ and $f' = r$, the latter implying $K = K^*$. We also know that $q_1 = 0$ at this point. From equation (4.75) it now follows that $\dot{q}_1 = 0$. This and the following features

$$f'' < 0, \quad (1 - \tau_c)/(1 - s) - q_1 = \lambda_1 > 0 \text{ and } \dot{K} > 0$$

imply that $\ddot{q}_1 < 0$ at the regime switch. This, together with $q_1 = \dot{q}_1 = 0$, implies further that the shadow price q_1 becomes negative after the switch between regimes A1' and A3'. This is contrary to the requirement $q_1 \geq 0$ and means that neither regime B1b nor B2b can be the predecessor of regime A3'.

Regime B3b, on the other hand, is a suitable predecessor of regime A3'. The state and co-state variables are continuous and the optimality conditions are fulfilled on both sides of the regime switch.

Next let us assess the predecessor of regime B3b, where by (4.63) $K = K^*$. This means that its predecessor can only be one of the following regimes: B1a, B1b, B2b and B3a. The first three of these can be excluded using the same argument regarding the shadow price q_1 as above. Regime B3a, on the other hand, is a feasible predecessor of regime B3b.

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[72] In principle"=" is also possible, but in this case regime A1' would only apply temporarily.

In regime B3a it holds that $(1- \tau_e)/(1-s) \leq \lambda_1 = -\lambda_2 \leq (1- \tau_c)/(1-s)$. Denote the upper limit of the first co-state variable $\bar{\lambda}_1$. For this upper limit it holds that $\bar{\lambda}_1 = 1$ if $s = \tau_c$, and $\bar{\lambda}_1 < 1$ if $s < \tau_c = \tau_f$. Thus in the case of full imputation credit the co-state variables fulfil the initial time transversality conditions (4.44h) and (4.45). In this case regime B3a is the starting regime.

In the case of partial imputation the predecessor of regime B3a is regime B1a. This is the only feasible regime which fulfils the continuity requirements for the state and co-state variables. Furthermore, from equation (4.48) we see that regime B1a fulfils the initial time transversality conditions. Therefore it is the starting regime of the solution in the case of partial credit. Note that in both solutions the firm starts with a maximum amount of debt, i.e. the initial value of debt is $B_0 = aK_0$.
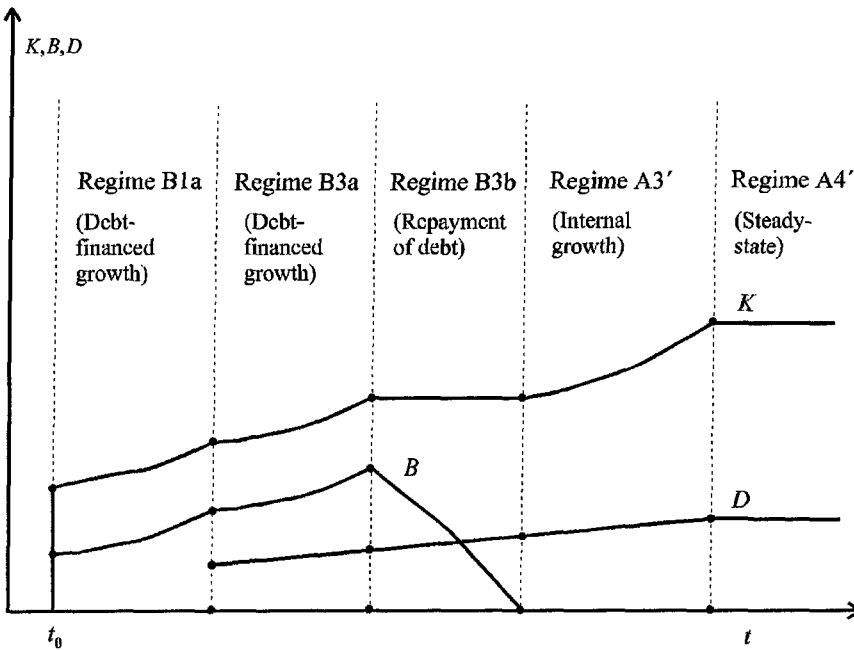

Figure 4.3 Optimal Paths of Variables K, B and D over Time[73]

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[73] Figure 4.3 aims to describe only the crude pattern of the time paths for the three variables. The heights of the trajectories are arbitrary. Note the different nature of K and B compared to D. Variables K and B describe stocks of capital and debt while D describes the flow of dividends.

4.3.5 Analysis of the Solution

According to the solution obtained above the firm is completely debt-free in the long-run equilibrium. In the case of uniform capital income taxation with full imputation, where $\tau_f = \tau_c = s$, the marginal condition defining the firm's steady-state capital stock is exactly the same as in the case of the basic model. This means that inclusion of debt in the model does not affect the special incentive effect of graduated dividend taxation.

In our solution the behaviour of the firm in various phases of the growth path is quite similar to the cases of van Schijndel (1988) and van Hilten et al. (1993). There it is assumed that the discount rate of the owner is lower than the firm's after-tax cost of debt. The authors distinguish three different phases in the firm's growth path:

     1. Maximum growth phase ($K < K^*$; $B = aK$ and $\dot{B} > 0$)

     2. Debt repayment phase ($K = K^*$; $0 < B < aK$ and $\dot{B} < 0$)

     3. Phase of internally financed growth ($K > K^*$; $B = 0$)

These three growth phases are also found in the solution to our model. As regards financing policy, both solutions are identical. However, in contrast to van Hilten et al. and van Schijndel the maximum growth phase in our study ends before the level of capital $K = K^*$ is reached, and is limited to regime B1a. In regime B3a, where capital is still $K < K^*$, the firm finances its investments with the maximum amount of debt, but distributes a dividend $D = bN$, and thus does not spend all its funds on growth.

van Hilten et al. and van Schijndel address the question of why the firm uses debt in the initial stage of the growth path, regardless of the fact that the cost of debt is higher than that of equity. They offer the explanation that by using the maximum amount of debt the firm can maximise the growth of its profits and equity. The maximisation of growth follows from the leverage effect of debt, according to which in this situation a debt-financed unit of capital in the case $f' > \rho$ boosts the firm's profits after taxes and interest expenses and thus increases the internal financing available for investments. When the firm's capital stock reaches the level where $f' = \rho$, this leverage effect vanishes, and it is no longer optimal for the firm to use debt.

But why is it optimal to pay off debt in the middle of the growth path? This can be explained on an intuitive level as follows. Both ways of using financing, investment and paying off debt, bring shareholder benefits in two ways: (a) by increasing the firm's after-tax profits and (b) by fostering tax savings via growth in capital base. Because the tax savings are the same in both alternatives, their rela-

tive advantage is determined according to their direct returns, i.e. the return on investments or interest savings from paying off debt. At a level of capital lower than K^* it holds that $f' > \rho$, so it pays the firm to use its profits for investments. At the level $K = K^*$ it holds that $f' = \rho$. Owing to the declining marginal returns on capital, additional investments offer a poorer return in this situation than paying off debt. So the firm's optimal choice is to use internal financing for the repayment of debt. When the debt is paid off completely, the firm uses its after-dividend internal financing for investments. This situation continues until the level of capital $K = K_B^*$ is reached, where the most beneficial way of using profits is to distribute them as a dividend.

4.3.6 Summary

This section analysed the firm's optimal debt policy under graduated dividend taxation. The basic model introduced in chapter 2 was supplemented with debt financing and the model's tax system was expanded by including corporate taxation, the imputation system and taxation of the owner's interest income. The framework did not allow for investment in financial assets. A solution to the model was derived for the Nordic case, where the corporate and capital income tax rates are equal.

We observed that graduated dividend taxation lowers the effective cost of the firm's equity capital, leaving the cost of debt untouched and thus discriminating against debt financing. This is reflected in the firm's policy so that in the final phase of growth and in the long-run equilibrium the firm finances its activities purely with equity. At its birth and in the early phases of growth the firm uses debt to maximise its profits and the growth of equity capital. At the level of the capital stock where the firm's marginal return on investment is equal to the marginal cost of borrowing, the firm starts paying off its debt.

Then, despite the inherent discrimination against debt in this framework, the firm nevertheless uses debt in the early phases of growth. This indicates that graduated dividend tax does not hamper the firm's debt-financed growth compared to a more neutral tax system that does not discriminate against debt.

4.4 Accelerated Depreciation and Dual Income Tax

4.4.1 Introduction

This section aims to assess whether a system of accelerated depreciation and the dualistic taxation of dividends interact in affecting a firm's behaviour. This problem is raised by the feature of the dual income tax systems of Norway and Finland where many assets, for example machinery and equipment, are included in the firm's capital base assessed at book value. With accelerated fiscal depreciation this value is lower than the replacement value of the assets. Because of the central role of the capital base in transmitting the effects of the graduated dividend taxation it seems evident that this deviation will have some consequences for the firm's policy.

We will use the same general approach applied above, but unlike the previous sections the emphasis here is more on studying the firm's policy in the long-run equilibrium than in the growth phase. It proved to be very difficult to reach an overall dynamic solution to the present problem. At the end of this section we, however, give some remarks concerning the dynamics.

We will assess two different systems of fiscal depreciation. In the first firms are always limited to deducting the fiscal depreciation allowed by the tax code. This system is applied in most of the OECD countries. In the second system firms have some freedom in choosing the level of fiscal depreciation. This is the system operated in the Nordic countries.[74]

4.4.2 The Model and the Optimality Conditions

We will model the system of fiscal depreciation using the approach by Boadway and Bruce (1979), Ylä-Liedenpohja (1984) and Kanniainen and Södersten (1995a). We introduce a new state variable, k, to describe the book value of capital and a new control variable, A, to depict fiscal depreciation allowances. The replacement value of the firm's capital stock is still depicted by K and it is assumed to depreciate at a constant exponential rate, δ. The maximum rate of fiscal depreciation allowed under tax law is denoted by ε and it is assumed to follow an exponential scheme and be strictly greater than the rate of economic depreciation, i.e. $\varepsilon > \delta$. We further assume that the (minimum) rate of depreciation required by accounting standards equals the rate of economic depreciation.

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[74] This distinction is made e.g. in Kanniainen and Södersten (1995a). The presence of the freedom to choose the level of the fiscal depreciation is also a central point in the study by Virolainen (1998).

We assume that uniform reporting is applied in the economy. In a uniform reporting system as used in most OECD countries outside the Anglo-Saxon world the balance sheet drawn up for the fiscal authorities must coincide with the commercial balance sheet. We further assume that in such a system the minimum depreciation required by the accounting standards forms a lower bound for the fiscal depreciation.[75]

Now we can define the constraints for fiscal depreciation. The assumptions stated above imply that in the Nordic system the amount of fiscal depreciation is constrained at every time point as follows:

(4.77) $\delta k \leq A \leq \varepsilon k$ (freedom in choosing A)

In the system where the fiscal depreciation is predetermined it holds that:

(4.78) $A = \varepsilon k$ (no freedom in choosing A)

The development of the variables K and k is described by the following equations:

(4.79) $\dot{K} = I - \delta K$

(4.80) $\dot{k} = I - A$

We assume as above that at the time of establishment an optimal amount of initial capital is invested in the firm. This initial investment increases the book and real values of capital by the same amount. This feature introduces into our model the following initial condition for the two state variables:

(4.81) $k(t_0) = K(t_0)$

Note that after the starting moment the book and real values may differ due to accelerated depreciation. If, on the other hand, the firm always follows the policy $A = \delta K$, then the equality $k(t) = K(t)$ holds throughout the firm's planning period.

We do not include debt financing here. Having debt in the model would not add much to the results but would complicate the analysis considerably. The framework in most studies assessing the effects of fiscal depreciation includes debt and

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[75] We apply these two assumptions to introduce a lower bound for fiscal depreciation in the Nordic case, where depreciation can vary within a range. These assumptions do not, however, correspond entirely with reality. Finland, for example, is classified as applying uniform reporting (see OECD (1987)) but according to the Finnish rules the fiscal rate chosen by the firm may be lower than the rate required by accounting standards, at least temporarily. It seems obvious, however, that there must be some lower bound for tax depreciation in a tax system. Even if this point proves to be important in the tax system studied here, the exact definition of this lower bound is not crucial for our arguments. To keep the model simple we will set the lower bound at δk, which is one of the obvious candidates for this role.

their particular aim is to study the interaction of the deductibility of the costs of debt and accelerated fiscal depreciation in a tax system that favours debt. As discussed in chapter 1, the form of the institutional constraints on borrowing has a significant impact on the incentive effects of the depreciation system. However, in a tax system like the one studied here, where debt is not favoured, the presence of debt and the institutional constraints become unimportant.

We assume the same tax system as in section 4.3. Thus, along with graduated dividend tax, there is firm-level taxation, an imputation system and also taxation of interest income. The firm's tax liability is $T = \tau_f[F(K) - A]$ where $F(K)$ is the firm's gross operating profit before depreciation. The firm's budget constraint is

$$(4.82) \qquad (1-\tau_f)F(K) = I + D - \tau_f A$$

We assume that the firm's capital stock, K, consists of depreciable capital that is included in the capital base, this being valued at its book value. Since we exclude debt the capital base is defined now as

$$(4.83) \qquad N = k$$

Again we make use of assumption (A4.2), meaning that we restrict the analysis to a case where the dividends paid out by the firm are subject to taxation on the margin as earned income in the steady state. The term σ is now defined as:

$$(4.84) \qquad \sigma \equiv \frac{1-\tau_f}{1-s} \frac{F(K) - \delta K}{k}$$

and can be interpreted as the tax-adjusted average rate of return on book capital after depreciation.[76]

The model is as follows[77]

$$(4.85a) \quad \max_{\{K_0,D,D_x,A\}} \int_{t_0}^{\infty} \left(\frac{1-\tau_c}{1-s} D - \frac{\tau_x}{1-s} D_x \right) e^{-(1-\tau_c)r(t-t_0)} dt + \varphi(K(t_0))$$

---

[76] Clearly the assumption (A4.2) is no more stringent here than before. This can be seen by examining the firm in the steady state, where $\dot{k} = \dot{K} = 0$. By combining (4.79) and (4.80) we see that $\alpha k = \delta K$ and that $k = (\delta/\alpha)K \leq K$, where α depicts the rate of fiscal depreciation chosen by the firm which satisfies the condition $\varepsilon \geq \alpha \geq \delta$. In the case $\alpha > \delta$ the book value is lower than the real value, $k < K$, and assumption (A4.2) is satisfied here albeit with a lower operating income (after depreciation) than in the preceding cases.

[77] In writing the model we retain as a starting point the Nordic system where firms have freedom in choosing the amount of fiscal depreciation. A system where fiscal depreciation is fixed can be regarded as a special case of this.

(4.85b) $\dot{K} = \left(1 - \tau_f\right)F(K) - D - \delta K + \tau_f A, \qquad K(t_0) = K_0$

(4.85c) $\dot{k} = \left(1 - \tau_f\right)F(K) - D - (1 - \tau_f)A, \qquad k(t_0) = k_0$

(4.85d) $h_1 = (1-s)bk + D_x - D \geq 0$

(4.85e) $h_2 = D \geq 0$

(4.85f) $h_3 = D_x \geq 0$

(4.85g) $h_4 = \varepsilon k - A \geq 0$

(4.85h) $h_5 = A - \delta k \geq 0$

(4.85i) $G(K,k) = K_0 - k_0 = 0, \quad \varphi(K) = -K$

The problem contains two state variables, K and k, three control variables, D, D_x and A and five control constraints. The regularity conditions discussed in chapter 2 are satisfied. As shown in Appendix 1 the sufficient conditions are also satisfied. Thus the solution that satisfies the first-order conditions is the optimal one.

A new feature in the model compared to the previous models is the initial value condition in (4.85i). Problems containing this kind of constraint are referred to in literature as initial-curve problems.[78] The initial-time transversality conditions have a special form in this case.[79]

The current-value Lagrangian of the problem is:

$$(4.86)\ L = \frac{1 - \tau_c}{1 - s}D - \frac{\tau_x}{1 - s}D_x + \lambda_1[(1-\tau_f)F(K)-D-\delta K+\tau_f A] + \lambda_2[(1-\tau_f)F(K)-D-(1-\tau_f)A]$$

$$+ \ q_1[(1-s)bk+D_x-D] + q_2 D + q_3 D_x + q_4(\varepsilon k-A) + q_5(A-\delta k)$$

The model's first-order conditions without complementarity conditions are:

(4.87a) $\partial L/\partial D = \dfrac{1 - \tau_c}{1 - s} - \lambda_1 - \lambda_2 - q_1 + q_2 = 0$

(4.87b) $\partial L/\partial D_x = -\dfrac{\tau_x}{1 - s} + q_1 + q_3 = 0$

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[78] Or a terminal-curve problem if there is a similar condition for the terminal-values of the states. See Chiang (1992) p. 11-12.

[79] See Seierstad and Sydsaeter (1987) Note 5, p. 186, or Salo (1994) p. 92.

(4.87c) $\partial L/\partial A = \tau_f\lambda_1 - (1-\tau_f)\lambda_2 - q_4 + q_5 = 0$

(4.87d) $\dot{\lambda}_1 = (1-\tau_c)r\lambda_1 - (1-\tau_f)F'(K)(\lambda_1+\lambda_2)+ \delta\lambda_1$

(4.87e) $\dot{\lambda}_2 = (1-\tau_c)r\lambda_2 - (1-s)bq_1 - q_4\varepsilon + q_5\delta$

(4.87f) $\dot{K} = \left(1-\tau_f\right)F(K) - D - \delta K + \tau_f A$

(4.87g) $\dot{k} = \left(1-\tau_f\right)F(K) - D - (1-\tau_f)A$

(4.87h) $\lambda_1(t_0) = \varphi'(K_0) + \mu\dfrac{\partial G}{\partial K} = 1 + \mu$, $\lambda_2(t_0) = \mu\dfrac{\partial G}{\partial k} = -\mu$, μ = constant

Note the form of the initial-time transversality conditions in (4.87h). By combining the two equations we obtain the condition as follows:

(4.88) $\lambda_1(t_0)+\lambda_2(t_0) = 1$

We will interpret and apply this condition later in section 4.4.4.

To ease later work we will combine equations (4.87e) and (4.87f) to obtain:

(4.89) $\dot{\lambda}_1 + \dot{\lambda}_2 = (1-\tau_c)r(\lambda_1+\lambda_2) - (1-\tau_f)F'(K)(\lambda_1+\lambda_2) - (1-s)bq_1 + \delta\lambda_1 - q_4\varepsilon + q_5\delta$

By substituting $\lambda_1+\lambda_2$ in (4.87a) into (4.87c) we obtain:

(4.90) $\lambda_2 = (\dfrac{1-\tau_c}{1-s} - q_1 + q_2)\tau_f - q_4 + q_5$

4.4.3 Firm's Optimal Policy in the Long-Run Equilibrium

4.4.3.1 Equilibrium Regimes

This section first identifies the feasible policies of the firm and afterwards examines which of these can serve as steady-state regimes. The two subsections that follow focus on analysing the firm's policy in the steady state.

The five constraints, (4.85d) - (4.85h), of our model generate 32 different policies, as listed in table 4.1 in section 4.2.2. Regimes 1-4, 17-20, 11-12, 15-16, 27-28, 31-32 and regimes 5, 7, 9 and 13 can be rejected as non-feasible on similar grounds as in section 4.3. Regimes 14, 29 and 30 are feasible only when $\sigma \leq b$

and can be omitted as being in contradiction with assumption (A4.2). Table 4.5 below lists the remaining 10 regimes.

Of the feasible regimes, we again use three symbol abbreviations, now beginning with D. The numerical part indicates the dividend policy and the final letter the firm's depreciation policy. The letter 'a' indicates that the upper constraint for A is binding, the letter 'b' that both constraints are non-binding and 'c' that the lower constraint is binding.

Table 4.5   *Basic Characteristics of the Feasible Regimes*

Regime	No. in table 4.1	Fiscal depr. A	Dividend, D	Shadow prices
D1a	10	$A = \varepsilon k$	$D = 0$	$q_1, q_5 = 0;\ q_2, q_3, q_4 > 0$
D1b	26	$\delta k < A < \varepsilon k$	$D = 0$	$q_1, q_4, q_5 = 0;\ q_2, q_3 > 0$
D1c	25	$A = \delta k$	$D = 0$	$q_1, q_4 = 0;\ q_2, q_3, q_5 > 0$
D3a	6	$A = \varepsilon k$	$D = (1-s)bN$	$q_2, q_5 = 0;\ q_1, q_3, q_4 > 0$
D3b	22	$\delta k < A < \varepsilon k$	$D = (1-s)bN$	$q_2, q_4, q_5 = 0;\ q_1, q_3 > 0$
D3c	21	$A = \delta k$	$D = (1-s)bN$	$q_2, q_4 = 0;\ q_1, q_3, q_5 > 0$
D4a	8	$A = \varepsilon k$	$D > (1-s)bN$	$q_2, q_3, q_5 = 0;\ q_1, q_4 > 0$
D4b	24	$\delta k < A < \varepsilon k$	$D > (1-s)bN$	$q_2, q_3, q_4, q_5 = 0;\ q_1 > 0$
D4c	23	$A = \delta k$	$D > (1-s)bN$	$q_2, q_3, q_4 = 0;\ q_1, q_5 > 0$

Clearly regimes D1a-D1c cannot serve as steady-state regimes. The firm does not distribute dividends, which cannot be the optimal policy in the final phase of the solution. Nor can regimes D3a-D3c be the equilibrium regimes of our model. In these regimes it holds in the steady state that $\sigma = b$, which contradicts assumption (A4.2). Regimes D4a-D4c, on the other hand, satisfy this assumption and are our candidates for steady-state regimes.

Before beginning the policy analysis it is necessary to assess the convergence properties of these three regimes. As will be seen later, the replacement value of the capital stock in regimes D4a-D4c is determined by a marginal condition closely related to equation (2.26) and is thus a constant. The accounting value of capital, on the other hand, can very well be non-constant in these regimes. Let us define $A = \alpha k$, where $\alpha = \varepsilon$ in regime D4a, $\delta < \alpha < \varepsilon$ in regime D4b and $\alpha = \delta$ in regime D4c, and assume that α is constant. Now, by using this definition for A and the equality $I = \delta K_D$' we may rewrite (4.80) as:

(4.91) $\dot{k}(t) = \delta K_D\text{'} - \alpha k(t),$

where K_D' depicts the steady-state value of the capital stock in these regimes. This differential equation can be solved as:[80]

(4.92) $$k(t) = e^{-\alpha t}(k_0 - \delta K_D'/\alpha) + \delta K_D'/\alpha$$

where k_0 is the starting value of k when the firm enters into the regime (time t'). The second term on the right-hand side of the equation and the expression in parenthesis are both constants. Because $-\alpha t < 0$ the first term vanishes when $t \to \infty$ and k converges to its equilibrium level $k = \delta K_D'/\alpha$ regardless of the starting value, k_0.

Next we check the convergence of the co-state variable λ_2. We can rewrite the differential equation for λ_2 in (4.87e) by using (4.90), (4.87b) and the values of the shadow prices, $q_2 = q_3 = 0$, as:

(4.93) $$\dot{\lambda_2} = [(1 - \tau_c)r + \eta]\lambda_2 - \tau_x b - \frac{1 - \tau_e}{1 - s}\tau_f \eta$$

Solving this we obtain:

(4.94) $$\lambda_2 = e^{[(1-\tau_c)r+\eta]t}\left\{\lambda_2(t_0) - \frac{\tau_x b + \dfrac{1 - \tau_e}{1 - s}\tau_f \eta}{(1 - \tau_c)r + \eta}\right\} + \frac{\tau_x b + \dfrac{1 - \tau_e}{1 - s}\tau_f \eta}{(1 - \tau_c)r + \eta},$$

where $\eta = \varepsilon$ in regime D4a, $\eta = 0$ in regime D4b and $\eta = \delta$ in regime D4c.

Since $(1-\tau_c)r + \eta > 0$, the path for λ_2 is divergent. If its starting value, $\lambda_2(t_0)$, is greater than the equilibrium value it will grow without any limit at the rate $(1-\tau_c)r + \eta$. If the starting value is smaller than the equilibrium value the co-state will decrease without limits. The paths of the state and co-state variables must satisfy the following transversality condition:

(4.95) $$\lim_{t \to \infty} e^{-(1-\tau_c)rt}\left[\lambda_1(K - \hat{K}) + \lambda_2(k - \hat{k})\right] \geq 0$$

where $\hat{k}$, $\hat{K}$, $\hat{\lambda_1}$ and $\hat{\lambda_2}$ are the paths of the variables that satisfy the first-order conditions and k and K are any feasible values of these two variables. As we saw in section 4.2, this condition requires a growth rate lower than the discount rate. Knowing that $\hat{k}$ and $\hat{K}$ converge and that the increase or decrease in $\hat{\lambda_2}$ occurs at a rate greater than the discount rate, $(1-\tau_c)r + \eta \geq (1-\tau_c)r$, we see that these paths do not satisfy this transversality condition. The only case that is in line with

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[80] The time point, t', at which the firm enters in the regime is assumed to be t'=0.

the transversality condition is where the starting value of λ_2 in these regimes is equal to the equilibrium value given by the second term on the right-hand side of (4.94). The transversality condition thus requires λ_2 to be a constant in regimes D4a - D4c, taking the value:

$$(4.96) \qquad \lambda_2 = \frac{1}{(1-\tau_c)r+\eta}[\tau_x b + \frac{1-\tau_e}{1-s}\tau_f \eta],$$

where η is as defined in (4.94).

Equations (4.87a) - (4.87c) and the value of q_3, together with the constancy of λ_2, imply that λ_1 is also constant in regimes D4a-D4c. Thus the optimal paths of the variables satisfy the conditions of a steady state in these regimes. This does not, however, necessarily occur in the early phases of these regimes because the accounting value of capital, k, can be time-dependent in them.

4.4.3.2 Optimal Policy under Exogenous Fiscal Depreciation

The purpose of this subsection is to study the firm's optimal policy in the long-run equilibrium when the firm is obliged to deduct the accelerated fiscal depreciation provided by tax law. In this case the firm follows the policy of regime D4a, where the upper constraint for A is binding:

$$(4.97) \qquad A = \varepsilon k, \; q_4 \geq 0 \text{ and } q_5 = 0$$

In this regime it also holds (as in regimes D4b-D4c):

$$(4.98) \qquad D \geq (1-s)bk, \; D_x \geq 0, \; q_1 = \frac{\tau_x}{1-s}, \; q_2 = 0, \; q_3 = 0$$

From (4.98) and (4.87a) we obtain:

$$(4.99) \qquad \lambda_1 + \lambda_2 = (1-\tau_e)/(1-s),$$

i.e. the sum of the co-state variables is a constant.

Adding and subtracting $q_4 \delta$ on the right-hand side of (4.89) and using this constancy result and (4.90) we obtain:

$$(4.100) \qquad \dot{\lambda}_1 + \dot{\lambda}_2 = 0 = (1-\tau_c)r(\lambda_1+\lambda_2) - (1-\tau_f)F'(K)(\lambda_1+\lambda_2) -$$

$$(1-s)bq_1 + \delta(\lambda_1 - q_4) + (\varepsilon - \delta)(\lambda_2 - \frac{1-\tau_e}{1-s}\tau_f)$$

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By using λ_2 in (4.96) and $\lambda_1+\lambda_2$ in (4.99), observing that according to (4.88) and (4.85a) $\lambda_1 - q_4 = (\lambda_1+\lambda_2)(1-\tau_f)$ and solving the equation for F', we obtain the following marginal condition determining the firm's steady-state capital stock[81]

$$(4.101) \quad F'(K) = \delta + \frac{1-\tau_e}{1-\tau_f}r\left[1 - \frac{\tau_f(\varepsilon-\delta)}{(1-\tau_e)r+\varepsilon}\right] - \frac{(1-s)\tau_x b}{(1-\tau_f)(1-\tau_e)}\left[1 - \frac{\varepsilon-\delta}{(1-\tau_e)r+\varepsilon}\right]$$

It is instructive to consider this condition first in the special case where $\varepsilon = \delta$, i.e. when fiscal and economic depreciation are equal. We apply this even though the case is actually in contradiction with our assumption $\varepsilon > \delta$. Equation (4.101) can be rewritten as follows

$$(4.101') \quad F'(K) - \delta = \frac{1-\tau_e}{1-\tau_f}r - \frac{(1-s)\tau_x}{(1-\tau_f)(1-\tau_e)}b$$

Observe that the right-hand sides of (4.101') and (4.67) are equal. Introducing depreciable capital into the model does not affect the incentive effects of graduated dividend taxation when fiscal depreciation corresponds to true economic depreciation. The left-hand side of (4.101') can be written $f'= F'- \delta$, where f' denotes the net operating profit of the firm. This observation gives support to the discussion concerning these concepts in chapter 2. However, now we observe that $f'= F'- \delta$ holds also more generally, e.g. in the presence of firm-level taxation and non-accelerated fiscal depreciation.

Let us return to condition (4.101) and the case $\varepsilon > \delta$. Using the terminology introduced in chapter 1, the expression $(\varepsilon-\delta)/((1-\tau_f)r+\varepsilon)$ in the second and third terms on the right-hand side of equation (4.101) gives the present value of the excess depreciation from a one-unit investment and the term $\tau_f(\varepsilon-\delta)/((1-\tau_f)r+\varepsilon)$ gives the present value of the tax saving, i.e. the amount of interest-free tax debt caused by this excess depreciation.

The effective cost of capital is expressed in (4.101) as the sum of three components. The first is the decrease in the market value of a one-unit investment, i.e. economic depreciation, the second is the before-tax cost of equity capital weighted by its share in financing and the third is the tax saving from a one-unit investment generated by graduated dividend taxation. Using the idea of Södersten (1982) and Kannianen and Södersten (1995a) we can imagine that investment is financed partly with equity and partly with tax debt, the respective shares being $[1-\tau_f(\varepsilon-\delta)/((1-\tau_f)r+\varepsilon)]$ and $\tau_f(\varepsilon-\delta)/((1-\tau_f)r+\varepsilon)$.[82] The weighted cost of tax debt is

[81] The right-hand side of (4.101) is assumed to be positive. This assumption, together with the second Inada condition ensures that there exists a unique value of K which satisfies equation (4.101).

[82] More precisely, the weight formulas describe the average proportions of the financing shares over the lifetime of the asset. See references in the next footnote.

not visible in (4.101) due to its zero unit cost. The weighted cost of equity is given by the long second term on the right-hand side of (4.101). Because the weight of equity is less than one, the effect of accelerated depreciation here is to lower the effective cost of equity. This was presented in chapter 1 and is of course a well known result in literature.

An especially interesting feature in (4.101) is the presence of the expression $(\varepsilon\text{-}\delta)/((1\text{-}\tau_f)r\text{+}\varepsilon)$ in the third term on the right-hand side. The bracketed part of the third term can be rewritten as $(\delta\text{+}(1\text{-}\tau_f)r)/(\varepsilon\text{+}(1\text{-}\tau_f)r)$ and interpreted to describe the average ratio between the accounting and replacement values of an investment over the lifetime of the asset.[83] With accelerated depreciation it is less than one, implying that the incentive effect of the graduated system is on average lower in this case than in the reference case in section 4.3 (and in equation (4.101')). An obvious explanation for this is that in a system that contains accelerated depreciation and graduated dividend taxation the contribution of an investment to the capital base is on average lower than its contribution to the replacement value of capital. Consequently the incentive effect of graduated taxation transmitted via the capital base is lower in this case than in the case with non-accelerated depreciation.

The two effects of accelerated depreciation on the cost of capital have opposite signs. Accelerated depreciation lowers the cost of equity (in the second term on the right-hand side of (4.101)) and reduces the tax saving from graduated taxation, increasing the effective cost of capital (in the third term). The net effect is determined as follows. The cost of capital

$$
(4.102) \qquad
\begin{Bmatrix} increases \\ does\,not\,change \\ decreases \end{Bmatrix}
\quad if \quad
\frac{(1-s)\tau_x}{(1-\tau_e)}b
\quad
\begin{Bmatrix} > \\ = \\ < \end{Bmatrix}
\quad
(1-\tau_c)r\tau_f
$$

We will also come across this condition later and study it more thoroughly. It suffices here to state that accelerated depreciation increases or decreases the cost of capital or leaves it unchanged, depending on the values of the parameters of the tax system and the interest rate.

It is worth summarising the results of this subsection. The main result is perhaps that pre-determined accelerated depreciation lowers the incentives of graduated dividend taxation. We also observed that the overall impact of accelerated depreciation on the firm's cost of capital may be very different in an environment of dual income taxation than in an ordinary tax system. This depreciation scheme

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[83] This can be shown using the approach presented in Kanniainen and Södersten (1995) p. 427 and Sörensen (1995) p. 442-443.

may decrease, leave unchanged or increase the cost of capital depending on the values of the parameters.

4.4.3.3 Optimal Policy when Fiscal Depreciation is Determined Endogenously

Next we will analyse the Nordic type of depreciation system, where the firm has some freedom in choosing the amount of fiscal depreciation. There are three different policies to be studied:

1. Regime D4a (maximum depreciation): $A = \varepsilon k$ (and $q_4 \geq 0$, $q_5 = 0$)

2. Regime D4b (indifference): $\delta k < A < \varepsilon k$ (and $q_4 = 0$, $q_5 = 0$)

3. Regime D4c (minimum depreciation): $A = \delta k$ ($q_4 = 0$, $q_5 \geq 0$)

We may obtain feasibility conditions for these policies by combining equations (4.90) and (4.96)

$$(4.103) \qquad \frac{1}{(1-\tau_c)r + \alpha}[\tau_x b + \frac{1-\tau_e}{1-s}\tau_f \alpha] - \frac{1-\tau_e}{1-s}\tau_f = q_5 - q_4$$

Using the values of the shadow prices we can obtain the following conditions for the three different policies (regimes). The firm will

$$(4.104) \qquad \begin{Bmatrix} \text{minimise depreciation (regime D4c)} \\ \text{be indifferent (regime D4b)} \\ \text{maximise depreciation (regime D4a)} \end{Bmatrix} \quad if \quad \frac{(1-s)\tau_x}{(1-\tau_e)}b \quad \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \quad (1-\tau_c)r\tau_f$$

This condition is closely related to condition (4.102). We can find a clear interpretation for it. The left-hand side tells us the loss in tax savings caused by graduated taxation from a one-unit increase in the present value of the excess depreciation. The right-hand side can be interpreted as giving the increased saving from a one-unit increase in the present value measured in terms of the after-tax return on tax debt. If the loss in tax saving cased by the graduated system (left-hand side) is greater than the increased return on tax debt (right-hand side), accelerated depreciation increases the firm's cost of capital and the firm's optimal policy is to choose minimum fiscal depreciation. If the inequality is reversed maximum depreciation is chosen. In the first case, where minimum fiscal depreciation is chosen, the firm 'buys' savings in the personal taxation of its shareholders by suffering losses in firm-level taxation.

In table 4.6 we present numerical calculations of the marginal tax rate on earned income for the indifference case, which is a watershed between the maximum

and minimum policies. Finnish parameter values are used. At levels of interest rates between 5.0 and 12.5 per cent the critical marginal tax rate, $\hat{\tau}_e$, varies between 34 and 42 per cent. This level is quite low considering that in the Finnish income tax schedule the top rate is 60 per cent and that the average marginal income tax rate is slightly over 50 %.[84]

Table 4.6   Critical Values of the Marginal Tax Rate on Earned Income

	Interest rate, r			
	5.0 %	7.5 %	10.0 %	12.5 %
Critical level of marginal tax rate, $\hat{\tau}_e$	34 %	37 %	39 %	42 %

Thus our model suggests that under typical parameter values for Finland a closely held company should minimise its fiscal depreciation. This is a remarkable result because it has been a widely held belief in theoretical literature as well as in practical discussion that it is profitable to maximise depreciation allowances. Sinn (1987) proves this result using a somewhat different approach than the one used here.[85] On the other hand, Kanniainen and Södersten (1994) construct a model where the firm fails to maximise the depreciation allowances. In their work the outcome is produced by an externality from increasing the debt/equity ratio which decreases the effective cost of borrowing. The mechanism in their work, however, is very different from that in our model.

Let us next consider the firm's cost of capital in the three different cases. In the case where the firm's optimal policy is to minimise fiscal depreciation (case: $A = \delta k$), the cost of capital is given by equation (4.101'). In this case accelerated fiscal depreciation provided by tax law has no effect on the firm's cost of capital. This means that the incentive term produced by graduated taxation is also unaffected. Note that this neutrality result is strongly dependent on the assumption concerning the lower limit of fiscal depreciation. We assumed that the rate at the lower boundary is determined by the accounting standards and equals the rate of true economic depreciation.

On the other hand, if it is optimal to maximise fiscal depreciation (case: $A = \varepsilon k$), which occurs when the marginal tax rate on earned income is low, the cost of capital is given by (4.101). Using equation (4.102) it can be shown that the cost

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[84] According to Viitamäki (1998) the average marginal tax rate on earned income was 52.0 per cent in 1996 and 50.8 in 1997.

[85] See also Kanniainen and Södersten (1994) and Sörensen (1995).

of capital now is lower and the steady-state capital stock is higher than in the case $A = \delta k$ and of section 4.3.[86]

The final case to discuss is the one where the firm is indifferent with respect to the level of fiscal depreciation. The explanation for this indifference is of course the equality between the costs and benefits from an increase or decrease in fiscal depreciation at given values of the parameters. The marginal condition determining the steady-state capital stock is again given by (4.101'). This means that even if the firm may deduct the fiscal depreciation using the accelerated rate, ε, the cost of capital is not affected by the rate of fiscal depreciation. The firm's investment policy in this case is neutral with respect to accelerated fiscal depreciation. This case is nevertheless a special case and holds only on a razor's edge condition. Table 4.7 summarises the central features of the three different policies.

Table 4.7   Characteristics of the Steady-State Regimes

Regime	Policy	Capital stock *	Condition
D4a	maximise A	$K_D^* > K_B^*$	$\dfrac{(1-s)\tau_x}{(1-\tau_e)}b < (1-\tau_c)r\tau_f$
D4b	indifference	$K_D^* = K_B^*$	$\dfrac{(1-s)\tau_x}{(1-\tau_e)}b = (1-\tau_c)r\tau_f$
D4c	minimise A	$K_D^* = K_B^*$	$\dfrac{(1-s)\tau_x}{(1-\tau_e)}b > (1-\tau_c)r\tau_f$

*) K_B^* is the steady state capital stock implied by (4.67).

To reiterate briefly, we showed that when the firm has freedom in determining fiscal depreciation in graduated dividend taxation the firm's policy can be to maximise or minimise fiscal depreciation or in a special case the firm can even be indifferent with respect to the size of the depreciation. A condition determining the optimal behaviour was given. On the parameter values typical in Finland the optimal policy of a wealth maximising closely held corporation is to minimise depreciation allowances. Assuming that the minimum depreciation allowed equals the true economic depreciation, the fiscal depreciation system is in this case neutral with respect to investment.

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[86] Assuming of course that f' in (4.67) is the marginal net operating income after depreciation.

4.4.4 Optimal Policy in the Growth Phase

In this subsection we turn to analyse the firm's policy during the growth path. We simplify the analysis by assuming unified capital income taxation with full imputation, $s = \tau_c = \tau_f$, as applied e.g. in Finland. We also concentrate on the case where the firm minimises its depreciation allowances, i.e. the case where $\tau_x b/(1-\tau_e) > \tau_f r$. It is not at all obvious in this case, however, that the firm will also apply the minimum depreciation policy during its growth path. Intuitively it is quite possible that the firm will maximise depreciation allowances and tax debt in the early phases of its growth cycle, perhaps the reason being to maximise growth of its equity.

We begin by discussing the initial-time transversality condition (4.85h) (and (4.86)), which provides a condition for the starting value of the firm's equity. In chapter 2 the corresponding condition was simply $\lambda_1(t_0)=1$, the left-hand side of this giving the marginal value of capital (at t_0), i.e. the size of an increase in the objective function, V, from a one-unit increase in K. Due to the additional initial value condition $K(t_0)=k(t_0)$, an increase in $K(t_0)$ in the present case also affects the objective function through $k(t_0)$. This means that an increase in K has a direct and an indirect effect on V.

Because $dK/dk = 1$ at time t_0, $\partial V/\partial K = \lambda_1$ and $\partial V/\partial k = \lambda_2$, we can now write

(4.105) $\qquad dV/dK = \partial V/\partial K + (\partial V/\partial k)(dk/dK) = \lambda_1+\lambda_2$,

Here λ_1 gives the direct effect and λ_2 the indirect effect of a change in K on V.

So, as given above in (4.88), the correct condition for the initial equity injection is expressed in the present case using both co-state variables. This observation is in line with the conceptual analysis by Kanniainen and Södersten (1995a). These authors claim that when the tax system contains accelerated fiscal depreciation a distinction has to be made between the shadow price of equity capital, given by $\lambda_1+\lambda_2$, and the shadow price of capital, the latter being defined as λ_1.

It is worth mentioning that the equation of motion for $\lambda_1+\lambda_2$ in (4.89) has many properties similar to those of the equation of motion for λ_1 in chapter 2. $\lambda_1+\lambda_2$ is decreasing during the adjustment phase and approaches its steady-state value smoothly (this value is $(1-\tau_e)/(1-s)$). The co-state variable λ_1, on the other hand, now behaves very differently compared to the same variable in the basic model.

The potential growth regimes of the solution are D1a-D1c and D3a-D3c. Regimes D1a-D1c, however, cannot be part of the solution. Equation (4.87a), the value of the shadow price, $q_2 > 0$, and the assumption concerning tax parameters, imply that $\lambda_1+\lambda_2 \geq 1$ in these regimes. Since according to the initial condition the

firm's capital stock is invested at time $t = t_0$ to the level where $\lambda_1 + \lambda_2 = 1$, it is obvious that the initial investment replaces regimes D1a-D1c. In regimes D3a-D3c it holds that $1 \geq \lambda_1 + \lambda_2 \geq (1-\tau_e)/(1-s)$ and the regimes can very well serve as growth regimes.

The predecessor of the final regime must fulfil the condition $\tau_x b/(1-\tau_e) > \tau_f r$ at the regime switch. Using equation (4.90), the value of λ_2 at this point given in (4.96), the value of q_1 given in (4.98) and the information on the shadow prices in table 4.5, we find that only regime D3c satisfies this condition. Hence regime D3c must be the immediate predecessor of the final regime D4c.

The total solution to the problem remains undetermined, however. We are not able to cover the gap between the initial investment and regime D3c. It seems possible that the internal growth occurs entirely in regime D3c but we cannot exclude regime D3a or regime D3b or both being included in the optimal solution. We know, nevertheless, with certainty that in the case assumed above the firm minimises its depreciation allowances in the steady state and also in the growth phase immediately before it.

4.4.5 Summary

The purpose of subsection 4.4 was to consider the effects of accelerated fiscal depreciation in the environment of dual income taxation. Two different systems of fiscal depreciation were studied. In the first the firm was always obliged to deduct the maximum depreciation allowed in the tax code. In the second the firm was able to choose the level of fiscal depreciation within the limits of maximum depreciation mentioned and a minimum level of depreciation defined by the accounting standards.

Our analysis showed that graduated income tax and accelerated depreciation tend to interact. When used by the firm, accelerated depreciation lowers the incentive effects of graduated dividend taxation. The total impact of accelerated depreciation on the cost of capital, however, includes in addition the effect transmitted via interest-free tax debt, which is of the opposite sign. The net effect on the firm's cost of capital can be positive, negative or zero, depending on the values of the parameters. As a result of the first effect the impact of accelerated depreciation on the firm's cost of capital is smaller under graduated dividend taxation than in a linear tax system.

If the net effect is negative, i.e. if the rise in the saving from interest-free tax debt is smaller than the decrease in the savings generated by graduated dividend taxation, the firm's optimal policy is to minimise depreciation allowances. It was shown that with the parameter values typical in Finland the value-maximising firm in our model framework minimises these allowances.

We were not able to derive a total dynamic solution to the problem, but we showed that the firm's policy is to minimise depreciation allowances, at least in the later part of the growth path.

5. Summary and Conclusions

This study examines the effects of certain features of Nordic dual income taxation (DIT) on the investment and financing behaviour of a corporation. Our approach was one of partial equilibrium analysis with a positive orientation.

Our focus was on the key element of DIT according to which the dividends received from a closely held corporation are divided into capital income and labour income, which are subject to different tax rate schedules. This division is made on an estimated basis and has raised some criticism because of its administrative difficulties and because it opens up tax planning opportunities.

The study examined this system from an incentive point of view, emphasising its potential violations of economic efficiency. A dynamic deterministic model in continuous time was used, closely following the approach in Sinn (1991). Unlike most related models in corporate tax literature, this approach allows us to study the influence of taxation not only in the long-run equilibrium, but also during the birth and growth phases of a firm.

We presented a specific way to model graduated dividend taxation which removes the non-linearity from the tax scheme and allows us to use standard tools from optimal control theory. This approach was used throughout the study. We assumed a stylised version of DIT in which the marginal tax rate on earned income is strictly higher than the flat rate on capital income and is constant. We also assumed that the capital base used in calculating the imputed capital income is defined as net capital (gross assets minus debt) assessed at book value.

Our strategy was to study the tax system with several slightly different model variants, each of which attempted to assess some specific feature of taxation or the firm's policy. The basic model, introduced and analysed in chapter 2, examined an equity-financed firm provided with two uses of funds, dividends and investment in homogenous capital earning decreasing returns. The shareholders were able to borrow and lend at the market interest rate, but the firm had no access to the capital markets. As in Sinn (1991), the tax system of the basic model contained only dividend taxation.

In chapter 3 we added to the model framework a more comprehensive tax system including firm-level taxation, capital gains taxation and an imputation system providing relief from the double taxation of dividends. The aim was to assess graduated dividend taxation in a more realistic and also more general tax environment.

Chapter 4 introduced three model variants, each adding some new policy alternatives for the firm. An important property of all these extensions was that they

affected the capital base, making the content of this concept more realistic. The first model allowed the firm access to the financial markets in the role of an investor. This model was first analysed assuming that the firm's capital base contains both assets and later assuming that only real capital (with decreasing returns) is included. In this setting we assessed the importance of the policy choice concerning inclusion of financial assets in the capital base, which was faced by the Nordic countries in their reforms in the early 1990s. As we described, Norway made a different choice from that of Finland.

The second model studied the firm's borrowing policy under graduated dividend taxation, while the third model focused on the firm's policy in an environment of depreciating capital and accelerated fiscal depreciation.

Our aim throughout the study was to emphasise the influence of the graduated tax scheme on the dynamic behaviour of the firm, i.e. to examine the firm's behaviour not only in long-run equilibrium but also in the birth and growth phases. This goal was achieved in most cases, but not in section 4.4, where a full dynamic solution was not reached.

Analysis of the basic model revealed that graduated taxation can be very non-neutral and induce behaviour that is very different than under linear dividend taxation. The firm's response to the introduction of the graduated scheme, however, was shown to depend on its profitability. A firm with a low average rate of return ($\sigma(K^*) < b$) behaves under graduated taxation exactly as under linear taxation. But the behaviour of a high-rate-of-return firm ($\sigma(K_A^*) > b$) is affected. Graduated taxation decreases the firm's cost of capital by including an additive term with a minus sign to it and consequently increases the firm's optimal steady-state capital stock. The firm's growth path is divided into two different growth regimes. An intermediate case ($\sigma(K^*)<b<\sigma(K_A^*)$) was also found where the steady-state capital stock is increased but less than in the high profitability case.

An economic interpretation for the results was given. The dividends of our low-rate-of-return firms are always subject to capital income taxation at the low proportional rate. In this situation the firm can ignore the graduated scheme and behave as if it were in linear dividend taxation. On the other hand, dividend payments by our high-rate-of-return firm are in danger of being subject to taxation on the margin as earned income. By increasing its capital stock the firm can reduce the portion of dividends subject to earned income taxation in the future and increase the portion that will be taxable as capital income. Hence, investments yield benefits to the owners of these firms in two forms: an increase in profits (and future dividends) and a tax saving from the decrease in the average tax rate of dividends. This tax saving lowers the firm's cost of capital and induces the firm to grow its capital stock to a higher steady-state level.

We showed that the new term in the cost of capital is sensitive to the earned income tax rate. Using numerical calculations we also showed that the cost of capital (excluding the rate of economic depreciation) can even be seen to be strongly negative in extreme cases.

The model variant in chapter 3, which included a broader tax system, showed that the investment incentive of the graduated system is not affected by capital gains taxation, but is enforced by corporate taxation if the imputation credit is incomplete. It was demonstrated that introduction of capital gains taxation and the imputation credit lessens the distortion against external equity compared to chapter 2 and consequently increases the initial investment and shortens the internal growth phase. However, capital gains taxation tends to increase the firm's cost of capital and decrease the steady-state capital stock. It was shown that the investment incentive generated by graduated taxation can serve to reduce this distortion against investment. However, a strong overcompensation is likely to occur.

The role of the two-asset model, where the firm can invest in financial and real capital, was to challenge the results of the extremely simple basic model. The question was whether the strong real distortions that we found are due to the isolation of our firm from the financial markets. In chapter 2 the firm was forced to use funds either for investment, yielding a strictly decreasing return, or for dividends potentially subject to a high marginal tax rate. The answer that we obtained in section 4.2 was yes and no. We found that introducing financial assets is not sufficient to abolish the real distortion of chapter 2, but if financial capital is included in the capital base the distortion is abolished. However, even in this case graduated taxation strongly affects the firm's behaviour, inducing the firm to avoid dividends and invest in the financial markets. We showed that the firm's policy depends in the long run on the relationship between the rate of return on financial investment, ρ, and the parameter b (imputed rate of return on capital). If ρ exceeds b, the firm is shown to grow infinitely. But if ρ is lower than b, it is sufficient for the firm to grow its financial capital to an equilibrium level, after which the firm can pay all of its profits as dividends. In either case dividends are never subject to earned income taxation.

On the other hand, if financial capital is excluded from the capital base the firm's behaviour is similar to that in the framework of the basic model and the real distortion prevails. These results suggested that in order to prevent real distortions financial capital should be included in the capital base.

In the analysis of the model including debt, our hypothesis concerning the distortion of graduated dividend taxation against debt financing was confirmed. We observed that the firm is fully debt-free in the long-run equilibrium and also in the growth phase immediately preceding it. However, we also found that the firm

maximises its use of debt in the early phase of its growth cycle, or, more precisely, maximises debt until its capital stock reaches the level K^*, the optimal steady-state capital stock in a world of linear dividend taxation. From this we drew the conclusion that the distortion against debt does not violate the growth of the firm compared to a linear dividend tax system.

The last model studied the effects of accelerated fiscal depreciation in an environment of graduated dividend taxation. It was observed that accelerated depreciation, if used by the firm, decreases the special investment incentive of graduated taxation. In the DIT environment the net incentive effect of accelerated depreciation can even be negative, i.e. increase the firm's cost of capital. In the Nordic-type tax system, where the firm can, within certain limits, choose its depreciation allowances, this latter result implies that the firm's optimal depreciation policy may be to minimise its fiscal depreciation allowances. We derived conditions for the different cases and showed using numerical calculations that with typical Finnish parameter values a closely held corporation maximises its equity value by minimising fiscal depreciation. We also discussed on conditions on which a system of accelerated fiscal depreciation is neutral with respect to investment.

Our study revealed that Nordic dual income taxation may have particularly strong effects which are manifested in a wide variety of ways in the behaviour of a firm. The firm's cost of capital in this system can fluctuate in a broad range and can even be negative due to the strong dependency of this concept on the circumstances of the shareholder. These results imply inefficiency and welfare losses.

On the other hand, our study presented several features that mitigate that picture. We observed that financial investment abolishes the real distortion when included in the capital base. It was also shown that the discrimination against debt does not violate the firm's growth in the important early growth phase and that under graduated taxation the firm's optimising behaviour can abolish distortions generated by accelerated depreciation.

Nevertheless, these latter features do not alter the fact that this tax system is very non-neutral, affecting the firm's behaviour in several ways. This is in fact a surprising observation bearing in mind the strong emphasis put on neutrality in preparations for the Nordic tax reforms in the late 1980s and early 1990s. Based on our analysis it is tempting to concur with Sörensen's (1998) words that the taxation of small enterprises is probably 'the Achilles heel of the dual income tax'.

This study has produced some tentative results concerning certain specific features of DIT. Our analysis in this area is of course far from comprehensive, leaving out several important themes, some of which we will list here. One interesting policy-oriented research subject would be to compare the incentive effects

of the Norwegian gross method and the Finnish-type net method considered in this study. Both systems probably contain strong non-neutralities but the exact nature of the distortions may nevertheless differ. The treatment of R&D investment under DIT also deserves some attention. As pointed out in section 1.2, the capital base concept in DIT usually only contains physical capital and not intangible assets, for example capitalised expenditure from R&D activities. This practice may lead to discrimination against investment in R&D capital, which can probably be seen as a less desirable feature of the tax system.

Recent tax research has reported that asymmetric and non-linear tax systems may have strong and surprising incentive effects on investment in an uncertain environment (see e.g. MacKie-Mason (1990)). Due to the non-linearity of the dividend tax schedule of DIT a further study using a stochastic framework could give additional information on the behavioural effects of dual income taxation.

A further tempting research subject, from a theoretical as well as from a policy point of view, is the potential interaction of graduated dividend taxation and certain details of the Finnish imputation system. This was discussed briefly in chapter 3. According to this system the so-called tax surpluses become obsolete after certain time limits. This feature of the Finnish imputation system may dampen the strong incentive effects of graduated dividend taxation found in this study.

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List of Symbols

A	Fiscal depreciation allowance
B	Debt
D	Dividend
D_x	Part of dividends subject to taxation as earned income
F	Stock of financial assets
G	Initial constraint function
H	Current-value Hamiltonian
I	Investment
K	Capital stock, replacement value of the capital stock
L	Current-value Lagrangian
N	Capital base
Q	New equity issued
R	New debt issued
S	Investment in financial assets
T	Firm's tax liability
V	Value of the firm
X	Subscript for 'unlisted firms'
Y	Subscript for 'listed firms'
a	Maximum debt to capital ratio
b	Presumed rate of return on capital invested in the firm
e	Share of the steady-state capital stock financed by external equity
f	Net operating income
h	Constraint function

k	Book value of the capital stock
q_i	Lagrange multiplier (shadow price of constraint i)
p	Cost of capital, pre-tax rate of return (after depreciation)
r	Shareholder's discount rate
s	Imputation rate
t	Time
w	Tax wedge; term describing the incentive generated by DIT
α	Weight; rate of fiscal depreciation applied by the firm ($\delta \leq \alpha \leq \varepsilon$)
δ	Rate of economic depreciation, minimum rate of fiscal depreciation
ε	Maximum rate of fiscal depreciation
φ	Initial-cost function
λ_i	Current-value co-state variable, shadow price of state variable i
ρ	Interest rate on debt and investment in financial assets
σ	Average rate of return on capital invested in the firm
τ_c	Tax rate on capital income (under dual income taxation)
τ_d	Tax rate on dividends (under ordinary linear dividend taxation)
τ_e	Tax rate on earned income (under dual income taxation)
τ_f	Corporate tax rate
τ_g	Effective accrued tax rate on capital gains
τ_p	Tax rate on interest income (under ordinary linear dividend taxation)
τ_x	Tax rate differential, $\tau_x = \tau_e - \tau_c$

Appendix 1

Sufficiency

We use the theorems of Leonard and Long (1993) to prove that the solutions derived in chapters 2-4 are optimal solutions. We present the conditions for the following more general problem:

(X1.1) $\max \int_0^\infty F(s,c,t)e^{-r't}dt + \sum \varphi_i(s_i(0))$

(X1.2) $\dot{s}_i = G_i(s,c,t),$ $i = 1,\dots,m$

(X1.3) $h_k(s,c) \geq 0,$ $k = 1,\dots,n$

(X1.4) $s_i(0) = s_{i0},$ $i = 1,\dots,m$

where s and c are vectors of the state and control variables, where $s = (s_1,\dots,s_m)$ and $c = (c_1,\dots,c_p)$. Function $\varphi_i(.)$ is an initial cost function for the state variable s_i, $i = 1,2,\dots,m$. The function h_k denotes the k:th constraint function of the control variables, $k=1,\dots,n$. r' denotes the after-tax discount rate. Below we denote with λ_i the co-state variable associated with the state variable s_i. We assume that h_k satisfies the control qualification and F, G_i, φ_i and h_k are continuous and continuously differentiable with respect to $s_1,\dots,s_m$ and $c_1,\dots,c_p$. Note that the problems in chapters 2- 4 satisfy both conditions.

In the following we denote with s, c, s_i, s_{i0}, λ_i the values of these variables that satisfy the necessary conditions and with s_i' the value of the i:th state variable on any feasible path. According to theorems 7.9.1 and 9.3.1 in Leonard and Long (1993), the necessary conditions for the problem (X1.1) - (X1.4) are also sufficient if the following three conditions (i) - (iii) are satisfied

(i)   the Lagrangean $L = F + \sum \lambda_i G_i + \sum q_k h_k$ is concave in (s,c)

(ii)   the initial value function $\varphi_i(s_{i0})$ is concave in s_{i0}

(iii) $\underline{\lim}_{t\to\infty} \sum \lambda_i(t)e^{-r't}[s_i'(t) - s_i(t)] \geq 0$

According to corollary 6.5.1 in Leonard and Long (1993), condition (i) is satisfied if F is concave in (s,c), each term $\lambda_i G_i$ is concave in (s,c) and each $q_k h_k$ is concave in (s,c). Note that $\lambda_i G_i$ is concave if G_i is concave and $\lambda_i \geq 0$ or G_i is

convex and $\lambda_i \leq 0$. Note also that the complementary slackness conditions require that $q_k \geq 0$. Thus, the concavity of $q_k h_k$ is implied by the concavity of h_k.

The basic model in chapter 2, where $m = 1$, satisfies condition (i) because F is linear, $\lambda_1 > 0$ and G_1 is concave due to the concavity of the operating profit function, each h_k is linear and thus concave. Condition (ii) is also satisfied since the initial value function is linear with respect to the state variable and thus concave. The basic model also satisfies condition (iii), because in each of the three different solutions the co-state and state variables are positive and constant in the steady-state phase and the feasible states are non-negative. Thus, the basic model satisfies the sufficient conditions. The model in chapter 3 has the same properties as the basic model and thus satisfies conditions (i)-(iii) on the grounds given above.

The model with financial capital in section 4.2, where $m=2$, satisfies condition (i) because F and each h_k are linear as in the basic model, $\lambda_1 = \lambda_2 \geq 0$, G_1 is concave and G_2 is linear. Condition (ii) is satisfied because φ_1 and φ_2 are linear in s_{10} and s_{20}. In section 4.2 we showed that condition (iii) is also satisfied. Thus the solutions that satisfy the necessary conditions are optimal.

In the model with debt, where again $m = 2$, condition (i) is satisfied because F is linear, h_k is linear, $\lambda_1 G_1$ is concave as above and $\lambda_2 G_2$ is concave because G_2 is linear. Conditions (ii) and (iii) are satisfied on the same grounds as in the basic model. Thus the debt model also fulfils the conditions of sufficiency.

In the model in section 4.4, as in the other models, F and each h_k satisfy the concavity requirement due to their linearity. The term $\sum \lambda_i G_i$ can be written using the notation of section 4.4 as follows:

(X1.5) $\sum \lambda_i G_i = (\lambda_1 + \lambda_2)[(1 - \tau_f)f(K) - D + \tau_f A] - \lambda_1 \delta K - \lambda_2 A$

All the three terms on the right-hand side of (X1.5) are concave. The second and third terms are concave whether or not λ_1 and λ_2 are positive, because δK and A are linear. The bracketed part of the first term is concave due to the concavity of $f(K)$. In section 4.4 we observed that in the solution satisfying the necessary conditions $\lambda_1 + \lambda_2 > 0$. From this it follows that the first term of the right-hand side of (X1.5) is concave and also that $\sum \lambda_i G_i$ is concave. Condition (i) is satisfied.

Condition (ii) is satisfied in the model in section 4.4 due to linearity and condition (iii), because the co-state and state variables converge in the final regime. Thus the sufficient conditions are satisfied.

Appendix 2

Proof that the switch from regime A1 to regime A3 occurs when $K < K^*$

Let us first assume that $K^{\#} \geq K^*$, where $K^{\#}$ denotes the firm's capital stock at the switch between regimes A1 and A3 (basic model) and K^* denotes the level of capital where $f'(K) = r$. This assumption implies that at the regime switch $f'(K^{\#}) \leq r$. Note that this point is also the initial point of regime A3. By differentiating (2.17a) with respect to time, substituting $\dot{\lambda}$ in this and λ in (2.17a) into equation (2.17h) we obtain

$$(X2.1) \qquad \dot{q}_1 = (f'(K) - r)(1 - \tau_p) + (r + b - f'(K))q_1$$

At the regime switch it holds that $\lambda = 1 - \tau_c$, $q_1 = 0$ [see equations (2.20) and (2.17a)] and, according to our assumption, that $f'(K^{\#}) \leq r$. Substituting these into equation (X2.1), we obtain $\dot{q}_1 \leq 0$. Let us differentiate (X2.1) again with respect to time and use (2.17a). We obtain:

$$(X2.2) \qquad \ddot{q}_1 = f''(K)\dot{K}\lambda + (r + b - f'(K))\dot{q}_1$$

From the following properties: $f'(K^{\#}) \leq r$, $f''(K) < 0$, $\dot{K} > 0$, $\lambda > 0$ and $\dot{q}_1 \leq 0$, we can conclude that $\ddot{q}_1 < 0$ at this point. This means that q_1 is declining after the switching point and becomes negative. The complementary condition (2.17d) nevertheless requires that $q_1 \geq 0$ in regime A3. From this contradiction we can conclude that the switch from regime A1 to regime A3 cannot happen at the point where $K \geq K^*$. So, it must hold that $K^{\#} < K^*$.

This result also gives us another way to show that regime A2 cannot be the predecessor of regime A3 in the solution to the basic model. If it were, then it would hold at the switching point that $\dot{q}_1 = 0$ and $\ddot{q}_1 < 0$, and the shadow price would become negative, which is against the complementary condition (2.17d). Thus the regime chain where regime A2 precedes regime A3 does not satisfy the first-order conditions and cannot be optimal.

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