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ON CALCULATING WELFARE LOSSES OF TAXATION AND PUBLIC PROVISION

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ABSTRACT: In the paper an index number measuring inefficiency of taxation is introduced. It is based on an economy-wide generalisation of the input distance function and is related to the Debreu's coefficient of resource utilization. Related indices are an input based quantity index, the index of real endowment, and a productivity index indicating effectiveness of the public sector and the society welfare index which offer a convenient summary way of analyzing the extent and nature of government intervention from the perspective of welfare and standard-of-living in the society. These indices should have wide applicability since they are based on a reinterpretation of similar concepts used in production theory. Hedonic methods seem particularly promising in offering a practical way of measuring the extent of intervention through the change in the marginal rates of substitution of the private goods. The public intervention considered may in principle range through widely differing spheres of economic activity. One may consider provision of public infrastructure, eg. construction of roads or a bridge, or public services. Alternatively, one may examine monitoring and regulation of the private sector, or the introduction of legal restrictions. In the paper special attention has been given to the empirical applicability of the methods presented. In this respect some convenient formulas are presented to facilitate calculations in special classes of preferences or using a flexible econometric representation of consumption technology.
KEY WORDS: Welfare loss of taxation, Debreu's coefficient of resource utilization, Malmqvist indices, Welfare index, Effectiveness of government, Hedonic methods.

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TIIVISTELMÄ: Tutkimuksessa esitetään verotuksen hyvinvointitappioiden indeksi. Tämä perustuu talouden panoskäytön etäisyysfunktioon ja on Debreun resurssien käyttökertoimen johdannainen. Lisäksi esitellään panoskäytön volyymi-indeksi, ns. reaalivarantojen indeksi, ja tuottavuusindeksi kuvaamaan julkisen sektorin tuloksellisuutta sekä hyvinvointi-indeksi, jotka antavat yleiskuvan julkisten toimien luonteesta ja laajuudesta talouden hyvinvoinnin kannalta. Indeksien käyttökelpoisuutta puoltaa se, että ne perustuvat tuotantoteorian käsitteiden uudelleen tulkintaan. Erityisesti hedoninen menetelmä soveltuu julkisen sektorin toimien arviointiin, sillä vaikutuksia voidaan mitata epäsuorasti hyödykkeiden rajasubstituutiosuhteiden muutoksina. Tarkasteltavat toimet voivat periaatteessa vaihdella perusrakenteen, esim. tiet ja sillat, tai julkisten palveluiden järjestämisestä aina yksityisen sektorin kontrolli- ja sääntelytoimiin tai jopa lainsäädäntöön asti. Tutkimuksessa kiinnitetään erityistä huomiota menetelmien sovellettavuuteen. Tätä varten esitetään kaavoja, jotka helpottavat laskelmia joissakin preferenssien erityisluokissa ja käytettäessä ekonometrian joustavia preferenssiesityksiä.
ASIASANAT: Verotuksen hyvinvointitappio, Debreun resurssien käyttökerroin, Malmqvistin indeksi, Hyvinvointi-indeksi, Julkissektorin tuloksellisuus, Hedoninen menetelmä.

Contents

1 Introduction

Tax policy, public provision and other forms of public intervention are concerned with choices between far from perfect instruments. The familiar social excess burden arises from the inability to employ optimal lump-sum taxes in pursuit of both revenue and distributional objectives. Calculation of deadweight losses has progressed from traditional constructs of consumer and producer surplus to the Hicksian concepts of equivalent and compensated variation.[1] The latter concepts are based on compensated demand functions holding utility level constant.

In the case of a single consumer the equivalent variation resulting from the transition from state 0, with prices and income p, m_0 to state 1, with prices and income q, m_1 is defined as the increase

$$EV(p, m_0, q, m_1) = e(p, u_1) - e(p, u_0), \tag{1}$$

where e is the expenditure function for the consumer and u_1 is the ex post level of utility $u_1 = V(q, m_1)$ and u_0 is the ex ante level of utility $u_0 = V(p, m_0)$. On the other hand (1) may be seen as a change in the money-metric utility, with the reference price vector p.

One can write (1) in an alternative form,

$$EV(p, m_0, q, m_1) = (m_1 - m_0) - (e(q, u_1) - e(p, u_1)). \tag{2}$$

Therefore equivalent variation is equal to the change in nominal income corrected for the corresponding change in the cost of living while prices change from p to q. The cost of living is defined as the expenditure needed to obtain the ex post utility level.

Consider a formula where the components on the R.H.S. of (1) are logarithms of the original terms. Similarly, one may go on to form a logarithmic counterpart to (2). Therefore, one finds that (2) which is based on absolute changes is in direct analogy with an expression formulated in terms of relative changes

$$\triangle(\log Q) = \triangle(\log m) - \triangle(\log P), \tag{3}$$

where $\triangle(\log m)$ is the relative increase in nominal income, P denotes for a Konüs cost-of-living index taken at the ex post utility level and Q is the corresponding implicit volume index.

Alternatively, one may arrive at a continuous (differential) formulation of (3) by a simply differentiating the logarithm of the budget condition, with the Divisia price

[1] There is little reason to accept what may be a poor approximation when the proper calculation of Hicksian concepts is scarcely more difficult, see eg. [Vartia (1983)].

index $d(\log P) = \sum_k \hat{w}_k d(\log p_k)$ and the implicit Divisia volume index $d(\log Q) = \sum_k \hat{w}_k d(\log x_k)$, where w_k are the budget shares, $w_k = \partial e(p,u)/\partial \log p_k$.

One substitutes for discrete approximation to arrive at (3) with the Törnqvist price index in differential form, $\Delta(\log P) = \sum_k \hat{w}_k \Delta(\log p_k)$, and the corresponding Törnqvist volume index, $\Delta(\log Q) = \sum_k \hat{w}_k \Delta(\log x_k)$, where $\hat{w}_k = 0.5(w_{k0} + w_{k1})$.

Consider a tax structure which is defined by the difference between consumer and producer price $q - p$. A partial equilibrium measure of the deadweight loss of such a tax structure can be defined as the negative of the equivalent variation from the first-best state 0 which is characterized by prices p and lump-sum taxes which are set equal to the tax revenue $(q-p)x(q,u_1)$ collected in the after-tax state.

In this case $m_1 - m_0 = (q-p)x(q,u_1)$, and

$$-EV(p, m_0, q, m_1) = (e(q,u_1) - e(p,u_1)) - (q-p)x(q,u_1), \tag{4}$$

which corresponds to the triangle of loss under the compensated demand curve. Alternatively, it gives directly the smallest amount that the income in the first-best situation could be reduced to achieve at the same utility level as the second-best offers.

It has been noted in literature that generally welfare indicators based on weighted sums of equivalent variation across consumers are the preferred choice as they allow one to make consistent assessments of welfare comparisons as opposed to the similar concepts that are based on compensated variation, see eg. [Ebert (1985)] and [Ebert (1987)].

The above type of indicators can generally be seen as belonging to an input-based (Debreu-Allais) approach to measure inefficiency as opposed to belonging to an output-based (Hicks-Boiteux) approach [Diewert (1981)]. The latter focuses on answering to the question: "By how much could output be increased?" and the former on the question "By how much could input be reduced?" In tax problems the former approach has more appeal because the "output" is measured in terms of utility levels which have no natural metric while the inputs are endowed with the canonical metric of the physical commodity space [Kay & Keen (1988)]. Note, that selecting an output-based approach is tantamount to specifying a set of distributional weights in the form of the need to selecting a reference price vector at which to calculate the equivalent gains (see [King (1983)] and [Roberts (1980)]).

Kay and Keen (1988) present a unified approach to the calculation of welfare losses in a many-person, general equilibrium framework. Their approach has wide applicability since they are utilizing the input-based method, *Debreu's coefficient of resource utilization* [Debreu (1951)] which identifies the input vector with the aggregate endowment of the economy. Furthermore, they show that this measure simultaneously generalizes formula (4) and gives it an exact general equilibrium interpretation.[2] Section 2 of this paper gives a terse treatment of their approach and introduces the relevant concepts and some notation.

In the following sections of the paper a complementary approach to [Kay & Keen (1988)] is introduced and its connections with previous work explored. Specifically, an alternative input-based measure of inefficiency is presented in form of an index number which provides equivalent amount of information on inefficiency in a general equilibrium framework. In the construction one utilizes the analogy of (3) with (2). As opposed to *Debreu's coefficient of resource utilization* the index formula turns out to correspond to the *endowment distance function* of the economy, a generalization of (input) distance function introduced by [Shephard (1953)] in production theory and by [Malmquist (1953)] in consumer theory.

Similarly as with [Kay & Keen (1988)] the output is measured in terms of the utility levels that are achieved in the after-tax situation. In the section 3 some formulas are presented for the endowment distance function that are applicable either as exact expressions or as providing useful approximations using *society's cost-of-living index* which is defined as a weighted sum of individual cost-of-living indices. Some restrictive classes of preference structures which have been widely used in the applied work provide examples where the calculations are simplified to a significant degree.

Section 4 considers a functional form for an expenditure function which has particular appeal in the analysis of micro data. It is a quadratic extension of the functional form originally introduced as an "Almost Ideal Demand System" (AIDS) by [Deaton & Muellbauer (1980)]. All formulas necessary for the calculation of the inefficiency index are presented for this special case.

In the section 5 the connections between the approach of the present paper and index number theory are explored further. The production interpretation of our input-based measure of inefficiency is exploited to present the Malmquist index of real endowment, a society level generalization of the Malmquist *standard-of-living index* in consumer theory and the Malmquist index of real input in production theory. Similarly, one may form the Malmquist input-based productivity indices, and *society's welfare index*, to evaluate two states of the economy. The productivity index compares differences in minimum endowment requirements conditional on the attainment of a given utility profile. The index is greater than one if the *efficiency of the public sector* is increased when the second best taxes underlying the utility producing technology are employed. Since the above indices build directly on the input distance function they offer potentially useful additional tools of assessing the multitude of the effects that public intervention may produce. The approach may be seen as a index number based generalization of the

[2]The connection between equivalent variation and Debreu's work had been noted also by [Ebert (1985)].

traditional CBA. It provides economy-wide input based measures to evaluate the change in welfare resulting from the transition from state 0 characterized with prices and income and level of public provision to state 1 with change in all the above attributes of the economy. Considering a case of intervention which has more far-reaching effects than just reforming an existing tax-subsidy system has hopefully more relevance and appeal to application.

The section 6 takes up an empirically implementable special case of the general situation of public intervention. Here, a "nonseparable" government intervention affects the marginal rates of substitution (or transformation) in the economy. As a concrete example one may consider eg. the provision of public goods and services. In this case the effects are in principle estimable from the data by applying the well-known hedonic approach. Explicit expressions and calculations are presented to assist in the application of the index number approach in this case. In particular it turns out that the change in welfare as measured by the *society's welfare index* may be decomposed into a product two components with the first component, *society's standard-of-living index* taking account of the decrease in virtual resources in the economy needed to finance the public intervention and the other component the productivity index measuring the change in the "effectiveness" of the economy after a benevolent public sector intervention. The calculations involved are illustrated by an example in section 7.

2 Input based measures of efficiency

The production process of the economy concerns with transformation of inputs $z \in \Re^k_+$ into outputs $x \in \Re^n_+$ which is modelled by a closed and convex input correspondence $Z(x) = \{z : (-z, x) \in Y\}$, where Y is the (net) production possibility set. Free disposal of net output is assumed for simplicity. Let α be a boundary point, $\alpha \in \partial Z(x)$, by the support theorem for convex sets, $\exists\ p(\alpha) \in \Re^k, p(\alpha) > 0$ such that $z \in Z(x) \Longrightarrow p(\alpha)(z - \alpha) \geq 0$, see (fig. 1).

Consider a pair $(x, \bar{z})$, $\bar{z} \in Z(x)$, and define a measure of distance of $\bar{z}$ from the boundary of $Z(x)$ in form of a price based index of value (see [Kay & Keen (1988), p. 260–262]).

$$D(x, \bar{z}) = \inf\{p(\alpha)(\bar{z} - \alpha)/p(\alpha)\bar{z} : \alpha \in \partial Z(x)\}. \tag{5}$$

Furthermore $D(x, \bar{z}) = 1 - \gamma$ where the infimum of (5) is obtained in $\tilde{\alpha} = \gamma\bar{z}$. Above γ is the celebrated Debreu's *coefficient of resource utilization* [Debreu (1951)]. Kay & Keen (1988) refer to $D(x, \bar{z})$ as the *Debreu measure of inefficiency* as it gives directly the amount of input wasted in comparison to an efficient combination of inputs.[3]

[3] In figure 1, $D(x, z) = 1 - \gamma$ is given by the ratio BA/BO.

Under standard conditions $\gamma = 1/\lambda(x, \bar{z})$, where $\lambda(x, \bar{z})$ is the *input distance function* introduced independently by [Shephard (1953)] and [Malmquist (1953)].

Assume in the following k commodities and a competitive market in each. Consumer and producer prices are denoted by q and p, respectively.[4] There are F firms in the economy that behave as price takers with net production vectors $y_f \in Y_f$, and profits $\pi_f = py_f$. There are no externalities between the firms so that the total production set of the economy $Y = \sum_f Y_f$.

There are n households in the economy. Their preferences are characterized by expenditure functions $e_h(q, u_h)$. The household h faces a budget constraint

$$qx_h = e_h(q, u_h) = b_h + p\omega_h + \theta_h\pi, \tag{6}$$

where b_h is the lump-sum grant from the government ω_h is the factor endowment of the household[5] and θ_h is the household's vector of shares, $\sum_h \theta_h = \mathbf{1}$ in the profits of firms.

The total endowment of the economy $\omega = \sum_h \omega_h$ corresponds to the input vector $\bar{z}$. The output vector of the economy is the utility profile of the economy in the taxed state $\mathbf{u} = (u_1, \cdots, u_n)$. Following [Kay & Keen (1988)] define

$$\Psi(\mathbf{u}) = \sum_h \Psi_h(u_h), \tag{7}$$

where $\Psi_h(u_h) = \{x_h : u_h(x_h) \geq u_h\}$. Above $\Psi(\mathbf{u})$ is the set of aggregate consumption bundles that can be distributed among the consumers so that no one is less well-off than in the taxed state. Furthermore,

$$Z(\mathbf{u}) = \Psi(\mathbf{u}) - \mathbf{Y}, \tag{8}$$

corresponds to the input requirement set Z(x).

Define the aggregate expenditure function

$$E(q, \mathbf{u}) = \sum_h e_h(q, u_h), \tag{9}$$

[4]Introduction of nonlinear consumer prices (eg. tax schedules w.r.t. the consumption of some portion of the endowment of leisure) presents no unsurmountable difficulties, in principle. One must, however, be careful in defining (and proving the existence) of the corresponding equilibrium concepts in the relevant markets. Furthermore, the structure of tax equilibria may be of complicated nature, see [Fuchs & Guesnerie (1983)].

[5]Kay and Keen make an important distinction as actually measuring ω as traded units. The motivation is thatif true endowments were observable one could directly introduce first-best taxes. Note, however, that since commodity taxes are levied only on transactions one should introduce lump-sum terms corresponding to the consumption of that part which is not transacted in markets, eg. leisure and labour input used in household production. These points are illustrated with the help of an example in section 7.

for the utility vector $\mathbf{u} = (u_1, \cdots, u_n)$ and similarly, aggregate compensated demand function

$$X(q, \mathbf{u}) = \sum_h x_h(q, u_h). \tag{10}$$

Equilibrium condition for the economy is

$$X(q, \mathbf{u}) - \omega = y + g, \tag{11}$$

where $y = \sum_f y_f$ and g is the vector of government net production.

Summing over the households budget constraint (6) one finds by the Walras' law that

$$(q - p)X(q, \mathbf{u}) = \sum_h b_h + pg. \tag{12}$$

so that the government's budget constraint is satisfied in an equilibrium. Assume below for simplicity that $g = 0$.[6]

Kay and Keen show that under standard regularity conditions (see eg. [Debreu (1959), p. 83–88]) the Debreu measure of inefficiency due to introduction of a tax system and lump-sum subsidies gives precise and general formulations of the familiar concepts of consumer and producer surplus.

$$\begin{aligned} D(\mathbf{u}, \omega) &= \tilde{p}X(q, \mathbf{u}) - E(\tilde{p}, \mathbf{u}) + \tilde{p}(\tilde{y} - y) \\ &= E(q, \mathbf{u}) - E(\tilde{p}, \mathbf{u}) - (q - p)X(q, \mathbf{u}) \\ &+ \tilde{\pi} + \tilde{p}\omega - (\pi + p\omega). \end{aligned} \tag{13}$$

The second equation is obtained by adding the L.H.S. and subtracting the R.H.S. of (11) multiplied with prices p.[7]

The expression on the second row of (13) is the sum of negative equivalent variations over the households, see (4) and corresponds to the triangle of loss under the compensated aggregate demand curve. The bottom row corresponds to the general equilibrium measure of lost producer surplus. It consists of the distortion of profit income and change in the value of aggregate endowment where the latter is interpreted as the rental payment earned by the factors of production in fixed supply.[8]

[6] See, however, section 6 below. Possible costs and distortions associated with the existing second best transfer system, the case considered by [Wildasin (1984)], could be subsumed under g.

[7] In addition [Kay & Keen (1988)] define $\tilde{p}$ using a convenient normalization of the first-best prices. The index number approach that is developed below is not dependent on a particular normalization.

[8] In principle one could also allow for productive inefficiency at the initial state in (13), i.e. y is not necessarily profit maximizing at prices p.

3 Distance function as an index of efficiency

Debreu showed that his measure of inefficiency has the following properties which should according to [Kay & Keen (1988)] be considered as necessary conditions on any candidate for measure of inefficiency

P1: $D(\mathbf{u}, \omega) \geq 0$.

P2: $D(\mathbf{u}, \omega) = 0 \Longleftrightarrow \mathbf{u}$ is a Pareto efficient utility profile.

P3: $\mathbf{u_a} > \mathbf{u_b} \Longrightarrow D(\mathbf{u_b}, \omega) > D(\mathbf{u_a}, \omega)$.

Note that output oriented efficiency measures generally suffer from failure to satisfy P3. It may happen that for two initial equilibria of the economy which generate the same utility profiles the associated measures of inefficiency are not equal. Since the Debreu measure is input oriented it identifies increase in efficiency as an endowment reducing tax change rather than Pareto improving tax change as the output oriented approach does.

The condition P3 implies that for two aggregate endowments ω_a, ω_b

$\omega_a > \omega_b \Longrightarrow D(\mathbf{u}, \omega_a) > D(\mathbf{u}, \omega_b)$.

The converse to P3 is generally not valid [Kay & Keen (1988)]. If output is, however, one-dimensional the converse statement holds. In the above framework this is achieved by introducing a specific concave social welfare function which has as arguments the utility profile $\mathbf{u} = (u_1, \cdots, u_n)$, for details see [Kay & Keen (1988)].

Recall that $D(\mathbf{u}, \omega) = 1 - \gamma$ where the utility profile $\mathbf{u}$ is under the first-best obtainable by the endowment $\tilde{\alpha} = \gamma\omega$. It is immediately obvious that any monotone transformation of $1-\gamma$ say ψ that is normalized in the sense that $\psi(0) = 0$ satisfies the properties P1-P3. Consider for example the function

$$\psi : (1 - \gamma) \longmapsto \log(1/\gamma) = \log \lambda(\mathbf{u}, \omega), \tag{14}$$

where λ is the Malmquist-Shephard (input) distance function, an index number measuring inefficiency that will be explored in more detail in the rest of the paper.

Assume below for simplicity that the vector of government production $g = 0$. In analogy with ([Kay & Keen (1988)]) our inefficiency index can now be expressed as

$$\frac{1}{\gamma} = \lambda(\mathbf{u}, \omega) = \frac{E(q, \mathbf{u}) - (q - p)X(q, \mathbf{u}) - \pi + (\tilde{p} - p)\omega}{E(\tilde{p}, \mathbf{u}) - \tilde{\pi}}. \tag{15}$$

The above analysis shows that the assumption of constant producer prices is in general unnecessary for the measurement of inefficiency. It is, however, a common simplification in practical empirical work particularly in the case of micro data.[9] If for simplicity the total profits are zero then rental payments earned by the factors in fixed supply are equal to total expenditures valued at producer prices $E(\tilde{p}, \mathbf{u}) = \tilde{p}\omega$, and if $\tilde{p} = p$

$$\begin{aligned} \frac{1}{\gamma} = \lambda(\mathbf{u}, \omega) &= \frac{E(q, \mathbf{u}) - (q - \tilde{p})X(q, \mathbf{u})}{E(\tilde{p}, \mathbf{u})} \\ &= \left(1 - \frac{(q - \tilde{p})X(q, \mathbf{u})}{E(q, \mathbf{u})}\right) \frac{E(q, \mathbf{u})}{E(\tilde{p}, \mathbf{u})}. \end{aligned} \tag{16}$$

Therefore the input distance function can be expressed as a product of two numbers, where the first is the value share of goods and services net of taxes out of total value and the latter is *society's cost-of-living index* $E(q, \mathbf{u})/E(\tilde{p}, \mathbf{u})$, a generalized cost-of-living index. Recall that γ is the largest radial contraction of the aggregate endowment that could be done without making anybody worse off than in the taxed state.

Write the society expenditure function in the form $E(q, \mathbf{u}) = \sum_h \exp\{\log e_h(q, u_h)\}$, and introduce budget shares using Shephard's lemma

$$w_{hk}(q, u_h) = \frac{q_k x_{hk}(q, u_h)}{e_h(q, u_h)} = \frac{\partial \log e_h(q, u_h)}{\partial \log q_k}. \tag{17}$$

Now one may write R.H.S. of (15) in the form of a weighted sum

$$\begin{aligned} \lambda(\mathbf{u}, \omega) &= \sum_h s_h \left(1 - \sum_k \frac{(q_k - p_k)}{q_k} w_{hk}(q, u_h)\right) e^{\log e_h(q, u_h) - \log e_h(p, u_h)} \\ &= \sum_h s_h \left(\sum_k \frac{1}{1 + \tau_k} \frac{\partial \log e_h(q, u_h)}{\partial \log q_k}\right) e^{\log e_h(q, u_h) - \log e_h(p, u_h)} \\ &= \sum_k \frac{1}{1 + \tau_k} \frac{\sum_h s_h \partial(e_h(q, u_h)/e_h(p, u_h))}{\partial \log q_k}. \end{aligned} \tag{18}$$

where the weight function s_h is the individual's h share of the society's total expenditure at producer prices $s_h = e_h(p, u_h)/E(p, \mathbf{u})$, and implicit tax rates τ_k are defined in terms of producer's prices $q_k = (1 + \tau_k)p_k$. The last member on the R.H.S. of (18) involves $e_h(q, u_h)/e_h(p, u_h)$ which is the individual cost-of-living index evaluated at utility level u_h.[10]

[9] See, however, section 7.

[10] The formula (18) is perhaps best interpreted in conjunction with the existence of at least one nontaxable good, leisure, and not in accordance of the Kay and Keen interpretation given above in the footnote 4.

Note that the above inefficiency index is in fact based on a covariance between a vector of producer's shares index. $1/(1+\tau_k)$ and the price gradient of the society's cost-of-living index. The latter index is obtained by a weighted sum of invidually defined cost-of-living indices. Therefore, inefficiency is decreased as the angle between these two positive vectors is increased. In order to minimize inefficiency one should have low producer's shares in those commodities that have large weight in the change of the society's cost-of-living index, a version of the Ramsey rule. On the other hand inefficiency index is maximized if the vector of producer's shares and the gradient of the society's cost-of-living index are collinear.

An additional interesting feature is that since the above measure of inefficiency is calculated as a index number it can easily be written in a form using budget shares and tax-rates and not unit taxes and compensated demands. This has some appeal in empirical applications where some data are typically available in the form of index numbers with no natural units of measurement. In addition, numerical calculation may work better since value shares are usually more slowly changing variates than quantities, see [Vartia (1983)].

Similarly as in the case of considering *Debreu measure of inefficiency* one may derive the necessary conditions for the inefficiency index to be minimized and to arrive at the well-known necessary first-order conditions on the optimal commodity taxes. This avenue is, however, not pursued in the present paper.

Consider instead two interesting special preference structures

1: homothetic preferences.

In this case the expenditure function has the form $e_h(q, u_h) = \beta(q)u_h$. Therefore utility is produced under constant returns. Furthermore, $w_{hk}(q, u_h) = \beta_k^*(q)$ independent of the utility level u_h or real expenditure. Furthermore,

$$\begin{aligned} \lambda(\mathbf{u}, \omega) &= \frac{\beta(q)}{\beta(p)} \sum_k \frac{w_k(q)}{1+\tau_k} \\ &= \frac{\beta(q)}{\beta(p)} \left(1 - \sum_k \frac{\tau_k w_k(q)}{1+\tau_k}\right), \end{aligned} \tag{19}$$

and is independent of any distributional effects.[11] The coefficient in the front of the sum is simply the true cost-of-living index. In the last expression the sum is

[11] In applied work homothetic functional forms are often employed, eg. CES-forms and linear expenditure systems in consumer analysis. These have linear income expansion paths, i.e. Engel curves and satisfy the Gorman condition for exact aggregation. Thus their use in numerical general equilibrium modelling work inhibits the examination of proper distributional effects other than those arising between broad consumer groups with different β parameters in (19).

taken over the taxable commodities and the tax rates $\tau_k/(1+\tau_k)$ are now calculated taking the after tax situation (ad valorem) as the reference point.

2: budget shares are independent of relative prices.
The example refers to a generalization of a Stone-Geary utility function with the share parameters smoothly deforming the indifference curves as the utility level changes. In this case the expenditure function must be of the form $\log e_h(q,u_h) = \sum_k \alpha_{hk} \log q_k$, giving $w_h(q,u_h) = \alpha_{hk}$, where $\sum_k \alpha_{hk} = 1$. Constant budget shares α_{hk} depend only on utility levels, i.e. on index h.[12] Similarly,

$$\begin{aligned} \lambda(\mathbf{u},\omega) &= \sum_h s_h \exp\{\sum_i \alpha_{hi} \log(1+\tau_i)\} \sum_k \frac{\alpha_{hk}}{1+\tau_k} \\ &= \sum_h s_h \frac{\sum_k \partial\left(\prod_i (1+\tau_i)^{\alpha_{hi}}\right)}{\partial \tau_k}. \end{aligned} \tag{20}$$

To arrive at an interesting interpretation of the inefficiency index define a probability distribution where a random vector α takes values α_h on unit simplex, $\sum_k \alpha_{hk} = 1$ with point probabilities s_h. Consider the following nonstandard form of the probability generating function $\mathcal{P}(\nu) = \mathcal{E}_\alpha \prod_i (1+\nu_i)^{\alpha_i}$, which is defined by a multidimensional power series with non-integer powers. Then (20) can be expressed as the sum of the components of the gradient of the above nonstandard probability generating function at the point $\nu_k = \tau_k$. A Second order Taylor approximation at the point $\nu = (0,0,\cdots,0)$ gives

$$\begin{aligned} \left[\frac{\partial \mathcal{E}_\alpha \left(\prod_i (1+\nu_i)^{\alpha_{hi}}\right)}{\partial \nu_k}\right]_\tau &\approx \mathcal{E}_\alpha \alpha_k - \mathcal{E}_\alpha \alpha_k \tau_k + \sum_i Cov(\alpha_k,\alpha_i)\tau_i + \mathcal{E}_\alpha \alpha_k \sum_i \mathcal{E}_\alpha \alpha_i \tau_i \\ &+ \frac{1}{2}\left(2\mathcal{E}_\alpha \alpha_k \tau_k^2 - 3\mathcal{E}_\alpha \alpha_k \sum_i \alpha_i \tau_k \tau_i + \mathcal{E}_\alpha \alpha_k \sum_i \sum_j \alpha_i \alpha_j \tau_i \tau_j\right). \end{aligned}$$

Summing over k gives

$$\begin{aligned} \lambda(\mathbf{u},\omega) &\approx 1 + \sum_k \sum_i Cov(\alpha_k,\alpha_i)\tau_i + \sum_k \mathcal{E}_\alpha \alpha_k \tau_k^2 - \sum_i \sum_j \mathcal{E}_\alpha \alpha_i \alpha_j \tau_i \tau_j \\ &= 1 + \sum_k \mathcal{E}_\alpha \alpha_k \tau_k^2 - \sum_k \sum_{i \neq k} \left(Cov(\alpha_k,\alpha_i) + \mathcal{E}_\alpha \alpha_k \mathcal{E}_\alpha \alpha_i\right) \tau_k \tau_i. \end{aligned} \tag{21}$$

The last formula is due to, $\sum_i \alpha_i = 1 \implies \sum_i Cov(\alpha_k,\alpha_i) = 0$.

[12] This dependence is assumed to be smooth, with expenditure an increasing function of the utility level. The example is included here because some microsimulation work of tax reforms that neglect a proper modelling of demand responses customarily use a convention of keeping individual budget shares constant to maintain consistent aggregates of the after reform figures.

Return to the general case and invoke the mean value theorem to expand the price gradient of the cost-of-living indices. Now one can write (18) in the form

$$
\begin{aligned}
\lambda(\mathbf{u},\omega) &= \sum_h s_h \left(\sum_k \frac{1}{1+\tau_k} \frac{\partial \log e_h(p,u_h)}{\partial \log q_k} \right) \\
&- \sum_h s_h \exp\{\log e_h(q^*,u_h) - \log e_h(p,u_h)\} \\
&\times \sum_k \frac{1}{1+\tau_k} \sum_l \left(\frac{\partial^2 \log e_h(q^*,u_h)}{\partial \log q_k \partial \log q_l} + \frac{\partial \log e_h(q^*,u_h)}{\partial \log q_k} \frac{\partial \log e_h(q^*,u_h)}{\partial \log q_l} \right) \frac{\tau_l}{1+\tau_l} \\
&= \sum_h s_h \left(\sum_k \frac{w_{hk}(p,u_h)}{1+\tau_k} \right) \qquad (22) \\
&- \sum_h s_h \left(\frac{e_h(q^*,u_h)}{e_h(p,u_h)} \right) \sum_{k,l} \left(\frac{\partial w_{hk}(q^*,u_h)}{\partial \log q_l} + w_{hk}(q^*,u_h) w_{hl}(q^*,u_h) \right) \frac{\tau_l}{(1+\tau_l)(1+\tau_k)},
\end{aligned}
$$

where $q^* = p + \lambda(q-p)$, i.e. $q_k^* = (1+\lambda\tau_k)p_k$, with the scalar $\lambda \in [0,1]$.

The above formula provides an alternative approximation to those previously appearing in the literature. A well-known result due to Diewert shows that the Törnqvist index provides an second-order approximation to the true cost-of-living index and thus the use of it provides a first-order approximation to the change in (18). Furthermore, the second-order approximation is exact if the corresponding linearly homogeneous expenditure function is translog. Similarly, the index by Vartia can be shown to be exact for a CES-aggregator function, see [Diewert (1976)] and [Sato (1981)].

In contrast to using index numbers as a tool of approximation the formula (13) due to [Kay & Keen (1988)] and index form (18) are exact expressions. In the next section we illustrate the use of distance function (18) in the case of a popular functional form used in the econometric analysis of micro expenditure data to provide an alternative approach to empirical application.

4 Cost of living index and a flexible representation of preferences

Introduce the following quadratic extension of the logarithmic polar form [13]

$$\log e_h(q,u_h) = \log \alpha(q) - \frac{1}{\beta(q)u + \delta(q)}, \tag{23}$$

where

$$\begin{aligned}
\log \alpha(q) &= \sum_k a_k \log q_k + \frac{1}{2}\sum_k \sum_l \gamma_{kl} \log q_k \log q_l, \\
\log \beta(q) &= \sum_k \beta_k \log q_k, \quad \log \delta(q) = \sum_k \delta_k \log q_k.
\end{aligned}$$

Homogeneity properties of the expenditure function imply that α is a degree one homogenous function in prices q and β and δ are homogenous of degree zero. This implies that the following restrictions hold for the parameters

$$\begin{aligned}
\sum_k a_k &= 1, \\
\sum_l \gamma_{kl} &= 0, \quad \forall k = 1, \cdots, m \\
\sum_k \beta_k &= 0 = \sum_k \delta_k.
\end{aligned}$$

In the special case of $\delta(q) = \beta(q)$ the formula reduces to the AIDS-form of the expenditure function as originally introduced by [Deaton & Muellbauer (1980)]. In applications the parameters a_k and β_k may depend on household characteristics, see [Blundell, et.al. (1993)]. By Shephard's lemma

$$\begin{aligned}
w_k(q,u) &= a_k + \sum_l \gamma_{kl} \log q_l + \frac{\beta_k}{\beta(q)u + \delta(q)} + \frac{(\delta_k - \beta_k)\delta(q)}{(\beta(q)u + \delta(q))^2}, &(24)\\
w_k(q,\bar{m}) &= a_k + \sum_l \gamma_{kl} \log q_l + \beta_k \bar{m} + (\delta_k - \beta_k)\delta(q)\bar{m}^2, &(25)
\end{aligned}$$

where $\bar{m} = \log m - \log \alpha(q)$, a kind of real expenditure term deflated by the function $\alpha(q)$.

[13]The logarithmic polar form was originally defined by Gorman, the present quadratic extension is slightly different (more flexible) from the one introduced by [Blundell, et.al. (1993)].

The logarithmic cost-of-living index is given by

$$\log\left(\frac{e_h(q,u_h)}{e_h(p,u_h)}\right) = \sum_k a_k\tau_k + \frac{1}{2}\sum_k\sum_l \gamma_{kl}(2\log q_k - \tau_k)\tau_l + (m_h - \log\alpha(q)) \qquad (26)$$
$$- \frac{\exp\{\sum_k \beta_k\tau_k\}(m_h - \log\alpha(q))}{1 + (m_h - \log\alpha(q))\exp\{\sum_k \delta_k \log q_k\}(1 - \exp\{\sum_k(\beta_k - \delta_k)\tau_k\})},$$

where $\tau_k = \log q_k - \log p_k$, and m_h is the total consumption outlay of consumer h after introduction of second best taxes, given by (6).

Now (25) and (26) gives the necessary components of (18).

Final point in this section concerns the modifications that are needed in empirical calculations to guarantee that the distributional features of the original data survive the smoothing implicit in the econometric estimation of the parameters of model (23). [King (1983)] recommends that the residuals of estimated budget share equations, eg. $w_{hk} - \hat{w}_{hk} = \hat{\varepsilon}_{hk}$ in (25) are taken to represent the individual characteristics of the micro level households reflecting eg. differences in preferences or in household production. These residuals are then taken as such to represent differences in the α_k terms in (23). A preferable method would be to endow the "residuals" with the distributional features of the data and characterize them with some distributional assumptions, eg. $\varepsilon_k \sim N(0,\sigma_k^2)$[14] in agreement with the data and *not to condition on the particular sample available.*[15]

5 Malmquist index and evaluation of public intervention

In the following the analogies with the approach of the present paper to calculate welfare losses of taxation and index number theory are explored further. The production interpretation of our input-based measure of inefficiency is exploited to present the Malmquist index of real endowment, a society level generalization of the Malmquist *standard-of-living index* in consumer theory and the Malmquist index of real input in production theory. Similarly, one may form the Malmquist input-based productivity index, and *society's welfare index.* Since these indices build directly on the input distance function they offer potentially useful additional tools of assessing the multitude of the effects that public intervention may produce.

[14]The variance component may be heteroscedastic if necessary.

[15]Because the above measure of inefficiency is calculated as a index number involving budget shares and tax-rates, calculation of its confidence limits may work better since value shares and parameters in (23) are usually better behaved than corresponding quantities, cf. the remarks after (18).

Consider comparing two states of the economy, $(\mathbf{u^j}, \omega^j)$ j = 1,2, characterized by the utility profiles and total endowments of the economy. The Malmquist index of real endowment with respect to the utility profile $\mathbf{u^1}$ is given by[16]

$$Q^1(\omega^2, \omega^1) = \frac{\lambda^1(\mathbf{u^1}, \omega^2)}{\lambda^1(\mathbf{u^1}, \omega^1)}. \tag{27}$$

In consumer theory the index (27) is the standard-of-living index introduced by [Malmquist (1953)] which is dual to the Konüs cost-of-living

index. Note that an increase in the aggregate endowment $\omega^2 > \omega^1 \Longrightarrow Q^1(\omega^2, \omega^1) > 1$. On the other hand $Q^1(\omega^2, \omega^1) > 1$ implies that endowment ω^2 is greater than ω^1 from the point of view of attaining the utility profile $\mathbf{u^1}$.

Suppose $\mathbf{u^1}$ is associated with consumer prices q^1, and lump-sum transfers as in (6). In analogy with (15) one can produce the formula

$$Q^1(\omega^2, \omega^1) = \frac{\left(E(q^1, \mathbf{u^1}) - (q^1 - p^2)X(q^1, \mathbf{u^1}) - \pi^2 + (\tilde{p}^2 - p^2)\omega^2\right)\left(E(\tilde{p}^1, \mathbf{u^1}) - \tilde{\pi}^1\right)}{\left(E(\tilde{p}^2, \mathbf{u^1}) - \tilde{\pi}^2\right)\left(E(q^1, \mathbf{u^1}) - (q^1 - p^1)X(q^1, \mathbf{u^1}) - \pi^1 + (\tilde{p}^1 - p^1)\omega^1\right)}. \tag{28}$$

Above p^i is the vector of producer prices associated with the equilibrium $(q^1, \mathbf{u^1}, \omega^i)$, and similarly $\tilde{p}^i$ supports the efficient outcome, $i = 1, 2$. Assume for simplicity that profits are zero. Now the formula (15) corresponds to

$$Q^1(\omega^2, \omega^1) = \frac{\sum_h s_{2h} \frac{e_h(q^1, u_h^1)}{e_h(\tilde{p}^2, u_h^1)} \sum_k \frac{w_{hk}(q^1, u_h^1)}{1+\tau_{2k}} + \left(1 - \frac{p^2\omega^2}{\tilde{p}^2\omega^2}\right)}{\sum_h s_{1h} \frac{e_h(q^1, u_h^1)}{e_h(\tilde{p}^1, u_h^1)} \sum_k \frac{w_{hk}(q^1, u_h^1)}{1+\tau_{1k}} + \left(1 - \frac{p^1\omega^1}{\tilde{p}^1\omega^1}\right)}, \tag{29}$$

where the weight function s_{ih} is the individual's h share of the society's total expenditure at producer prices $\tilde{p}^i$, $s_{ih} = e_h(\tilde{p}^i, u_h^1)/E(\tilde{p}^i, \mathbf{u^1})$, and implicit tax rates are given by $1 + \tau_{ik} = q_k^i/p_k^i$, $i = 1, 2$. Notice that (29) is a ratio where both the denominator and the nominator are formed by summing the covariance between the producer's shares and the price gradient of the society's cost-of-living index together with the relative increase in the earnings to the factors of production in fixed supply with respect to that in the corresponding first-best.

In the special case of homothetic preferences

$$Q^1(\omega^2, \omega^1) = \frac{\frac{\beta(q^1)}{\beta(\tilde{p}^2)} \sum_k \frac{w_k(q^1)}{1+\tau_{2k}} + \left(1 - \frac{p^2\omega^2}{\tilde{p}^2\omega^2}\right)}{\frac{\beta(q^1)}{\beta(\tilde{p}^1)} \sum_k \frac{w_k(q^1)}{1+\tau_{1k}} + \left(1 - \frac{p^1\omega^1}{\tilde{p}^1\omega^1}\right)} \tag{30}$$

[16]The notation is adapted from [Caves, et.al. (1982)].

$$= \frac{\beta(\tilde{p}^1)\left(\sum_k \frac{w_k(q^1)}{1+\tau_{2k}} + \frac{(\tilde{p}^2-p^2)\omega^2}{\sum_h b_h^2+p^2\omega^2}\right)}{\beta(\tilde{p}^2)\left(\sum_k \frac{w_k(q^1)}{1+\tau_{1k}} + \frac{(\tilde{p}^1-p^1)\omega^1}{\sum_h b_h^1+p^1\omega^1}\right)}. \tag{31}$$

The above formula is a ratio of price indices taken at the prices supporting the corresponding first-best multiplied with a second term which is also a ratio. The latter ratio is taken between expressions that are sums of two terms where the first is the by now familiar covariance between second-best producer's shares and the budget shares and the second is the relative change in the income earned by fixed factors (relative to the total income) corresponding to the change from the second-best to the first-best.

Similarly, there exists a Malmquist index of real endowment which has the utility profile $\mathbf{u}^2$ as a point of reference,

$$Q^2(\omega^2,\omega^1) = \frac{\lambda^2(\mathbf{u}^2,\omega^2)}{\lambda^2(\mathbf{u}^2,\omega^1)}. \tag{32}$$

Following [Caves, et.al. (1982)] one may form the geometric mean of the indices (27) and (32).[17]

$$[Q^1(\omega^2,\omega^1)Q^2(\omega^2,\omega^1)]^{\frac{1}{2}} = \left(\frac{\lambda^1(\mathbf{u}^1,\omega^2)}{\lambda^1(\mathbf{u}^1,\omega^1)}\frac{\lambda^2(\mathbf{u}^2,\omega^2)}{\lambda^2(\mathbf{u}^2,\omega^1)}\right)^{\frac{1}{2}}. \tag{33}$$

Taking logarithms one may express (33) in terms of the corresponding *Debreu's coefficients of resource utilization* using some self-explaining notation[18]

$$\frac{1}{2}[\log Q^1(\omega^2,\omega^1) + \log Q^2(\omega^2,\omega^1)] = \frac{1}{2}[\log\gamma_1^1 - \log\gamma_2^1] + \frac{1}{2}[\log\gamma_1^2 - \log\gamma_2^2]. \tag{34}$$

Furthermore, one may go on to construct the input based Malmquist productivity index

[17]In the case of a translog distance function the Törnqvist index is consistent with flexible representation of technology. In addition, if λ^1 and λ^2 are translog and have identical second order coefficients it is exact for this geometric mean, if the corresponding society level utility profile used as a reference is the mean of original profiles, see the analogous result by [Caves, et.al. (1982)]. On the other hand [Blackorby, et.al. (1978)] note that once separability is imposed (i.e. additive separability over the members of the society in the "utility production" framework considered in this paper) translog function is no longer capable of providing a second order approximation to an arbitrary unknown separable distance function. Therefore, in a many person economy with nonhomothetic preferences the translog function must be interpreted as an exact functional form and not as an approximation.

[18]Compare with (16) to find the close correspondence of Malmquist index with the Konüs cost-of-living indices.

using the implicit utility technology in state 1 (for details, see [Caves, et.al. (1982)])

$$M^1(\omega^2,\omega^1,\mathbf{u}^2,\mathbf{u}^1)=\frac{\lambda^1(\mathbf{u}^1,\omega^1)}{\lambda^1(\mathbf{u}^2,\omega^2)}. \tag{35}$$

The index compares differences in minimum endowment requirements conditional on the attainment of a given utility profile. The index is greater than one if ω^2 is more productive than ω^1, i.e. the efficiency is increased when simultaneously the second best taxes underlying the utility technology in state 1 are employed.

Similarly, productivity index using the implicit utility technology in state 2

$$M^2(\omega^2,\omega^1,\mathbf{u}^2,\mathbf{u}^1)=\frac{\lambda^2(\mathbf{u}^1,\omega^1)}{\lambda^2(\mathbf{u}^2,\omega^2)}. \tag{36}$$

The above equation offers a convenient tool to evaluate the effectiveness of government intervention. Specifically the index calculates the relative change in the inefficiency measure of taxation induced by the intervention. Government intervention may in this formulation affect both initial endowments and utility profiles in the economy. The change in the former may happen through the provision of some public goods and public services, say free of charge, and the latter in the form of introducing second-best taxes and lump-sum transfers.

The following index provides a society level generalisation of the original Malmquist welfare index which has the aggregate endowment level, say ω^2 as a point of reference,

$$U^2(\mathbf{u}^2,\mathbf{u}^1)=\frac{\lambda^2(\mathbf{u}^1,\omega^2)}{\lambda^2(\mathbf{u}^2,\omega^2)}. \tag{37}$$

Similarly, one may define $U^1(\mathbf{u}^2,\mathbf{u}^1)$. By the condition P3, Pareto improvement in the economy implies that the above welfare index is increased, but the converse need not hold.

Consider circumstances where government intervention has a nonseparable effect on the utility levels of consumers, i.e. the marginal rates of substitution (MRS) are not invariant but change after the intervention. Now one may in principle measure the effect through the apparent and visible effect in the consumption patterns of households. In the alternative case where the marginal rates of substitution are invariant, the possible efficiency increase cannot be estimated from the observed behaviour of the MRS. In this case possible economies or diseconomies created by the the intervention are impossible to identify even if they exist. Similar observations hold with respect to production technology. Since output is in this case directly observable one has correspondingly less strict conditions for identifiability.

A specific example of public intervention could be setting emission standards to limit air pollution. Standards operate as restrictions in production technology eg. in power generation. On the other hand polluting may hurt production in other sectors of the economy. Laundry business offers a classic example. Therefore, after setting some environmental standards it may be possible to use less inputs to produce the same output as before in some sectors of the economy and possibly the converse holds in some other sectors. In the following section of the paper public intervention is examined from this perspective and the well-known hedonic approach is employed to the above purpose.

6 Hedonic approach in uncovering the effects of public provision

The hedonic approach was originally motivated by the need to adjust cost of living indices for quality improvements (the classic reference is [Fisher & Shell (1972)]). This approach views that the quality of a good or a service is related to measurable characteristics such as performance ratings. In the process of estimating a demand function of the consumer or the production technology of the producer the shadow prices of the quality characteristics of a commodity are also estimated. The existence of a quality-adjusted price index implies the existence of a corresponding quality-adjusted quantity index. Multiplication of the two adjusted indices yields the same value index as would the unadjusted indices. The standard results of neoclassical theory, especially those related to various forms of duality (see above), hold in the quality-adjusted price and quantity space once we base the analysis on quality-adjusted indices.[19]

The hedonic approach offers a convenient way to conceptually separate the effects of taxes and subsidies from public intervention of a more general nature. Furthermore, the introduction of public goods and publicly provided non-transferable services may in several instances be seen as changing the marginal rates of substitution between private goods. Similar effects may occur as public intervention of a more general nature takes place.

To be more specific, consider a public facility of size g. Assume that after its introduction consumers may use ordinary commodities more effectively, say x_k units of the commodity k in the new situation are equally effective as $\theta_k(g)x_k$ units before the introduction of the facility. Equivalently, one may think of this innovation as corresponding to the price of the commodity k, q_k being lowered to $q_k^\theta = q_k/\theta_k(g)$, in the initial state. Below one

[19]The hedonic approach has been used in numerous cases other than mere adjustment of price indices for quality change, eg. in the construction of equivalence scales to adjust the consumption levels of households for differences in household composition, in several industry-level production functions and in evaluating the effects of a major new urban transit system or the siting of a waste dump.

considers transformed quantities and prices

$$\begin{aligned} x^\theta &= \Theta(g)x, \;\; y^\theta = \Theta(g)y, \;\; q^\theta = \Theta(g)^{-1}q, \;\; p^\theta = \Theta(g)^{-1}p \\ e_h(q^\theta, u_h^\theta) &= q^\theta x^\theta(q^\theta, u_h^\theta) = qx^*(q, u_h^\theta) = e_h^\theta(q, u_h^\theta) \\ p^\theta y^\theta(p^\theta) &= py^*(p), \end{aligned} \tag{38}$$

where $\Theta(g)$ is the diagonal matrix with constants $\theta_k(g)$ on the diagonal, $\Theta(g) = diag(\theta_1, \cdots, \theta_n)$.[20]

In order to guarantee a well-defined formulation one sets the following condition.

The production possibility set of the economy $Y = \sum_f Y_f$ *is such that* $\Theta y^* \in Y$, *and* $\Theta^{-1}y^* \in Y$ *for all those production plans* y^*, *which are associated with the equilibria of the economy involved in the definition of the Debreu measure of inefficiency, see* (13).[21]

Note that the above condition will simplify the model as effectively allowing the production process to accommodate frictionlessly the enhancement of consumer goods through Θ.[22] Furthermore, implicitly the condition assumes that the enhancement of typical outputs of the production process are accompanied by the enhancement of some inputs, say labour, to guarantee a feasible production plan in the initial situation with say, increased total endowment.

Note that the utility level of the consumer is changed to u_h^θ, $u_h^\theta(x_h) = u_h(x_h^\theta) = u_h(\Theta x_h)$, after the introduction of the public facility. See (fig. 2) where initial choice of the consumer facing prices $p = (1, 1)$ is given by the point x corresponding to the utility level U. Consumer goods are enhanced with $\Theta = diag(2, 0)$ and effectively the price of commodity 1 is lowered by 50 per cent. The budget line (in efficiency units) is subsequently shifted to the right giving a choice x^θ with a higher utility level U^θ. In original (physical) units of measurement x^θ corresponds to x^* and the indifference curve indexed by U^θ is deformed to the curve U^* drawn with a dotted curve in figure 2.

[20] Gorman in his 1976 article proposes an immediate extension of the simple structure in (38) by allowing $\Theta(g)$ to be an arbitrary positive matrix to provide a change of coordinates (with presumably a positive inverse, Gorman leaves to the reader to work out details of the model) in conjunction with a translation of coordinates. The extension may be interpreted as allowing spillover effects in other sectors of the economy. Most of the results given below can be extended to cover this more general affine case.

[21] Therefore, the total production possibility set is locally a cone. A global condition that Y is closed under the linear map Θ, $\Theta Y \subset Y \subset \Theta^{-1}Y$ gives a specific example of holotheticity in production, considered by [Sato (1981)]. In the case of holothecity, (technical) progress is by definition equivalent to the availability of extra inputs (eg. input augmenting case of technical progress) in the original point of reference.

[22] A similar condition should also hold for consumption possibility sets of the consumers and the consumption bundles of the consumers, in a more general case (note here the handling of leisure endowment) than the simple case of this paper where all consumption possibility sets are simply $\Re^k_+$.

Consider the equilibrium of the economy after the introduction of lump-sum transfers b_h, consumer prices q, and a facility with a hedonic effect characterized by Θ, with the corresponding attainable utility profile $\mathbf{u}^\theta$. The facility is "financed" by resource cost $-g$ in (11) which is now interpreted as the government net demand for final goods rather than government production as in the earlier case. The individual (6), and the government budget condition (12) are modified accordingly with the lump-sum transfers adjusted for the total costs of the facility, $-pg$, valued at producer prices.

Start with the equilibrium condition (11) in the final state. By (38) the expenditure functions corresponding to the initial state $e_h(q^\theta, u_h^\theta)$, and the final state $e_h^\theta(q, u_h^\theta)$ are identical. Therefore one may write

$$y(p) + \omega - g = X(q, u^\theta) = \Theta^{-1} X^\theta(q^\theta, u^\theta) = \Theta^{-1}(y^\theta(p) + \Theta(\omega - g)). \qquad (39)$$

Next multiply (39) from the left by the matrix Θ. The aggregate supply $y^\theta(p) + \Theta(\omega - g)$ corresponds to an equilibrium if y_f maximizes the profits py_f in Y_f is equivalent with the statement y_f^θ maximizes the profits if the prices are p^θ. This follows from the following sequence of inequalities which holds both for each firm, $f \in F$, and subsequently for $y = \sum y_f$.[23]

$$py(p) = p^\theta(\Theta y(p)) \leq p^\theta y(p^\theta) = p(\Theta^{-1} y(p^\theta)) \leq py(p). \qquad (40)$$

Furthermore, the reference point $p(\alpha)$ corresponding to the utility profile $\mathbf{u}^\theta$ in the final state changes to the supporting hyperplane characterized by $\Theta^{-1}p(\alpha)$ corresponding to the utility profile $\mathbf{u}^\theta$ in the initial state.

Therefore, it has been proved that the "after tax and facility" equilibrium with prices p and q and resources ω corresponds to an equilibrium in the initial state with prices p^θ and q^θ and where the resources available to the economy are now changed to the amount $\omega^\theta - g^\theta = \Theta(g)(\omega - g)$, where the total endowment has to be adjusted for the resource costs $-g$.

The welfare cost of the subsidy and tax system $q^\theta - p^\theta$ in the initial state associated with the last equilibrium corresponds to $\lambda(\mathbf{u}^\theta, \omega^\theta)$. This is equal to $\lambda^\theta(\mathbf{u}^\theta, \omega)$, the welfare cost of the tax system $q - p$ in the final state.

The Malmquist indices of real endowment (29), or better the society standard-of-living indices, with respect to the utility profile $\mathbf{u}^\theta$, at the final state and $\mathbf{u}$ at the initial state, respectively are given by

$$Q^\theta(\omega - g, \omega) = \frac{\lambda^\theta(\mathbf{u}^\theta, \omega - g)}{\lambda^\theta(\mathbf{u}^\theta, \omega)} = \frac{\lambda(\mathbf{u}^\theta, \Theta(\omega - g))}{\lambda(\mathbf{u}^\theta, \Theta\omega)}, \qquad (41)$$

[23] The inequalities follow from profit maximizing behaviour.

$$Q(\omega - g, \omega) = \frac{\lambda(\mathbf{u}, \omega - g)}{\lambda(\mathbf{u}, \omega)} = \frac{\lambda^\theta(\mathbf{u}, \Theta^{-1}(\omega - g))}{\lambda^\theta(\mathbf{u}, \Theta^{-1}\omega)}. \tag{42}$$

These indices measure the volume of total resources available after deducting for the input requirements due to providing the public facility. If $g = \nu\omega$, i.e. the share of these resource costs in the value of total resources is independent of the producer prices used as the reference price vector in the economy, then

$$Q^\theta(\omega - g, \omega) = 1 - \nu = Q(\omega - g, \omega), \tag{43}$$

i.e. the indices of real input are in this case independent on the choice of the reference utility profile.

The endowment-based Malmquist productivity index (35) with respect to the second best tax technology available after intervention, is given by

$$\begin{aligned} M^\theta(\omega - g, \omega, \mathbf{u}^\theta, \mathbf{u}) &= \frac{\lambda^\theta(\mathbf{u}, \omega)}{\lambda^\theta(\mathbf{u}^\theta, \omega - g)} = \frac{\lambda(\mathbf{u}, \Theta\omega)}{\lambda(\mathbf{u}^\theta, \Theta(\omega - g))} \\ &= M(\Theta(\omega - g), \Theta\omega, \mathbf{u}^\theta, \mathbf{u}). \end{aligned} \tag{44}$$

Consider initial state of the economy. In the first-best the economy can achieve utility profile $\mathbf{u}^\theta$ with total endowment $\alpha_\theta = \gamma_\theta \Theta(\omega - g)$. Similarly, the economy can achieve utility profile $\mathbf{u}$ with total endowment $\alpha = \gamma\omega$.

A sufficient condition for $\mathbf{u}^\theta$ to be *a potential Pareto improvement* P3 relative to the initial state $\mathbf{u}$ is that $\alpha_\theta \geq \alpha$. This condition is equivalent to (note P3 and (15), $\gamma = 1/\lambda$)

$$\frac{\lambda(\mathbf{u}, \omega)}{\lambda(\mathbf{u}^\theta, \Theta(\omega - g))} \Theta(\omega - g) \geq \omega. \tag{45}$$

By (42) and (44)

$$\frac{\lambda(\mathbf{u}, \omega)}{\lambda(\mathbf{u}^\theta, \Theta(\omega - g))} = \frac{M(\Theta(\omega - g), \Theta\omega, \mathbf{u}^\theta, \mathbf{u})}{Q(\Theta\omega, \omega)}. \tag{46}$$

The gain from the introduction of the public facility consists of the impact in the "effectiveness" of the public sector as measured by the productivity index M and on the other hand on the volume of total resources available after deducting for the real resource costs of providing the public facility. The latter vector, in efficiency units $\Theta(\omega - g)$, is in (45) deflated by the society's standard-of-living index Q. In the special case where $g = \nu\omega$, i.e. resource costs are independent of the reference producer price vector, the inequality (45) is transformed into

$$\frac{M(\Theta(\omega - g), \Theta\omega, \mathbf{u}^\theta, \mathbf{u}) \times Q(\Theta(\omega - g), \Theta\omega)}{Q(\Theta\omega, \omega)} \Theta\omega \geq \omega. \tag{47}$$

This inequality has an immediate interpretation as a straightforward decomposition of the various effects involved in the process of "producing the utility profiles of the consumers". Let the scalar $\mu = M(\Theta(\omega-g), \Theta\omega, \mathbf{u}^\theta, \mathbf{u}) \times Q(\Theta(\omega-g), \Theta\omega)\ /Q(\Theta\omega, \omega)$. Assume that the linear transformation Θ is decomposable as a direct sum of mutually orthogonal projections P_j, $\Theta = \sum_j \mu_j(\Theta)P_j$.[24] If the projections span the endowment space, the total endowment ω is presentable by a set of orthogonal base vectors $\omega = \sum_j \mu_j(\omega)\xi_j$, with the vectors ξ_j in the linear subspace spanned by the column space of P_j. Collecting terms implies that the condition (47) is transposed into

$$\mu\Theta\omega = \mu \sum_j \mu_j(\Theta)\mu_j(\omega)\xi_j \geq \sum_j \mu_j(\omega)\xi_j. \tag{48}$$

This holds if $M(\Theta(\omega - g), \Theta\omega, \mathbf{u}^\theta, \mathbf{u}) \times Q(\Theta(\omega - g), \Theta\omega) \times \min_j\{\mu_j(\Theta)\} \geq Q(\Theta\omega, \omega)$.

Note that there is a natural upper bound

$$Q(\Theta\omega, \omega) = \frac{\lambda(\mathbf{u}, \Theta\omega)}{\lambda(\mathbf{u}, \omega)} \leq \max\{\mu_1(\Theta), \cdots, \mu_n(\Theta)\}. \tag{49}$$

Because, if all resources are multiplied by the R.H.S. of (49) then one easily can guarantee utility levels $\mathbf{u}$.

The society welfare index with the endowment $\omega^\theta - g^\theta$ as a point of reference is given by

$$\begin{aligned} U^\theta(\mathbf{u}^\theta, \mathbf{u}) &= \frac{\lambda^\theta(\mathbf{u}, \omega - g)}{\lambda^\theta(\mathbf{u}^\theta, \omega - g)} = \frac{\lambda(\mathbf{u}, \Theta(\omega - g))}{\lambda(\mathbf{u}^\theta, \Theta(\omega - g))} \\ &= \frac{\lambda(\mathbf{u}, \Theta\omega)/\lambda(\mathbf{u}^\theta, \Theta(\omega - g))}{\lambda(\mathbf{u}, \Theta\omega)/\lambda(\mathbf{u}, \Theta(\omega - g))} \\ &= M(\Theta(\omega - g), \Theta\omega, \mathbf{u}^\theta, \mathbf{u}) \times Q(\Theta(\omega - g), \Theta\omega). \end{aligned} \tag{50}$$

Note that $U^\theta > 1$ does not necessarily imply a Pareto improvement in the economy. The welfare index is however a product of two indices with the second one, society's standard-of-living index, Q measuring the real resource cost of providing the public facility taking simultaneously account of the potential increase in the virtual endowment in the economy, ω if it is provided. The productivity index, M measures the change in the "effectiveness" of the economy after a benevolent public sector intervention and potential net change in the welfare loss of taxation which is due to the existing tax-subsidy system.

An interesting ratio of the inefficiency indices in before and after intervention situations (46) is representable using the welfare index as

$$\frac{\lambda(\mathbf{u}, \omega)}{\lambda^\theta(\mathbf{u}^\theta, \omega - g)} = \frac{M(\Theta(\omega - g), \Theta\omega, \mathbf{u}^\theta, \mathbf{u})}{Q(\Theta\omega, \omega)}$$

[24] The assumption is trivial with a diagonal Θ with $\mu_j(\Theta) = \theta_j$, but a more general matrix could also be considered, cf. footnote (19).

$$= \frac{U^\theta(\mathbf{u}^\theta, \mathbf{u})}{Q(\Theta(\omega - g), \Theta\omega)Q(\Theta\omega, \omega)}. \quad (51)$$

To get an example of the calculations involved suppose for simplicity that profits are zero.[25] Furthermore, assume the case of homothetic preferences to further simplify the calculations, cf. (31). In this case, one gets

$$Q^\theta(\omega - g, \omega) = \frac{\beta(\tilde{p_\theta}^0)\left(\sum_k \frac{w_k(q_\theta^g)}{1+\tau_k^g} + \frac{(\tilde{p_\theta}^g - p_\theta^g)(\omega^\theta - g^\theta)}{\sum_h b_h^g + p_\theta^g(\omega^\theta - g^\theta)}\right)}{\beta(\tilde{p_\theta}^g)\left(\sum_k \frac{w_k(q_\theta^0)}{1+\tau_k^0} + \frac{(\tilde{p_\theta}^0 - p_\theta^0)\omega^\theta}{\sum_h b_h^0 + p_\theta^0\omega^\theta}\right)}, \quad (52)$$

and

$$M(\Theta(\omega - g), \Theta\omega, \mathbf{u}^\theta, \mathbf{u}) = \frac{\lambda(\mathbf{u}, \Theta\omega)}{\lambda(\mathbf{u}^\theta, \Theta(\omega - g))}$$

$$= \frac{\beta(\tilde{p}_\theta^g)\left(\sum_k \frac{w_k(q^0)}{1+\bar{\tau}_k} + \frac{(\bar{p}^0 - p^0)\Theta\omega}{\sum_h b_h^0 + p^0\Theta\omega}\right)}{\beta(\bar{p}^0)\left(\sum_k \frac{w_k(q_\theta^g)}{1+\tau_k^g} + \frac{(\tilde{p_\theta}^g - p_\theta^g)(\omega^\theta - g^\theta)}{\sum_h b_h^g + p_\theta^g(\omega^\theta - g^\theta)}\right)} \quad (53)$$

Note above the use of superscript g to indicate instances where the tax system has to finance the facility with a resource cost $-p^g g$, (in physical units of measurement) and use of efficiency units $\tilde{p_\theta}^0 = \Theta^{-1}\tilde{p}^0$, $\omega^\theta = \Theta\omega$. There are now changes both in the total endowment of the economy and in the effectiveness of the public sector. Since in general the equilibrium prices are changed, the owners of resources with a "virtual increase" in endowment may be additionally affected by a decrease in the producer price and their total earnings may go even down giving respectively less scope for taxation of owners. These somewhat unexpected effects are illustrated by the following parametric example.

7 Examples

Example 1.[26] Consider an exchange economy with two commodities, leisure and production, so that $Y = \{(0,0)\}$. The traders, l and c initially hold $(1,0)$ and $(0,1)$, respectively. Both traders have identical preferences which are homothetic. The indifference curve has slope -1, at $(1,1)$ and on the line x = 2y it has slope $-1/8$ (see fig. 3).

Since the preferences are homothetic, the budget line corresponding to equilibrium prices is tangent to the indifference curve at the total endowment, i.e. it has slope -1 at $\omega = (1,1)$. Therefore, the equilibrium prices are in the ratio 1:1 and the competitive

[25] In fact this is a rather mild additional assumption, since one assumes above that the production possibility set is locally a cone.

[26] Analytical details are available on request.

bundles for both traders lie on the line $x = y$, and are equal to $x = (1/2, 1/2)$ (see fig. 3).

Introduce government to the economy. Government provides education which doubles the leisure endowment of trader l as measured in efficiency units $\omega^\theta = \Theta\omega$, with $\Theta = diag(2, 0)$. The total resource cost of providing education g^θ is $1/5$ of the endowment. The society standard of living indices, see (41) and (42), are independent of the reference utility level and are both equal to $4/5$.

Consider first the first-best case where education is financed by a tax on the *consumption* of the second commodity, production. The tax rate is set to $\tau_c^\theta = (q_2^\theta - p_2^\theta)/p_2^\theta = 1/3$.[27]

By Walras' law the government's budget condition is always satisfied in an equilibrium in the commodity markets, see (12). The endowment available after reduction due to the resource costs of providing education is $\omega^\theta - g^\theta = (8/5, 4/5)$.[28] By homotheticity, the budget line associated with equilibrium consumer prices, q^θ, is supporting the indifference curve at the above point. The prices are $q^\theta = (1, 8)$, giving producer income 2 to trader l and 6 to trader c, as measured in units of leisure. The equilibrium bundles lie on the line $2y = x$, and are $x_l^\theta = (2/5, 1/5)$, $x_c^\theta = (6/5, 3/5)$ (see fig. 3). The tax burden is proportional to income $T_l = 2/5$, and $T_c = 6/5$. In this case the Malmquist productivity index (53) which indicates the effectiveness of the public sector is given by

$$M^\theta(\omega - g, \omega, \mathbf{u}^\theta, \mathbf{u}) = \frac{\lambda(\mathbf{u}, \Theta\omega)}{\lambda(\mathbf{u}^\theta, \Theta(\omega - g))} = \frac{\|\omega^\theta\|/\|\omega^\theta - g^\theta\|}{1} = \frac{5}{4}. \tag{54}$$

On the other hand the society welfare index

$$U^\theta(\mathbf{u}^\theta, \mathbf{u}) = \frac{\lambda(\mathbf{u}, \Theta(\omega - g))}{\lambda(\mathbf{u}^\theta, \Theta(\omega - g))} = 1, \tag{55}$$

and shows no change by construction of the example. Note that after introducing education the trader l is considerably worse-off than before whereas the trader c has a substantial increase in utility.

In the above case the government was able to *tax the consumption* of the second commodity and no inefficiencies were created by taxation. Consider next the second-best case where the government can only set a *tax on market transactions* of the commodity, production. The example is otherwise the same as before but the resource cost due to the provision of education is changed to $g^\theta = (4/5, 1/5)$, and the tax rate is correspondingly raised to $\tau_c^\theta = (q_2^\theta - p_2^\theta)/p_2^\theta = 3$ (see fig. 4). The equilibrium is obtained at the

[27] All calculations are made in terms of the initial state, in efficiency units $(x^\theta, y^\theta) = (\Theta x, \Theta y)$, and $(p^\theta, q^\theta) = (\Theta^{-1}p, \Theta^{-1}q)$, cf. fig. 2.

[28] The example is constructed to keep the points ω and $\omega^\theta - g^\theta$ on the same indifference curve.

same consumer prices $q^\theta = (1,8)$, as before with bundle on the line $2y = x$, for trader l, $x_l^\theta = (2/5, 1/5)$, and income of one unit of leisure. Since tax is levied only on transactions the budget line for trader c is defined by *producer prices* which are at the equilibrium $p^\theta = (1,2)$, giving a bundle $x_c^\theta = (4/5, 3/5)$, with income of two units of leisure (see fig. 4). Therefore, the entire tax burden is shifted on the trader l, with $T_l = 6/5$, and $T_c = 0$. The first best equilibrium of the economy corresponding to the endowment $\omega^\theta - g^\theta$ is supported by prices which correspond to allocations on the line from the origo to $\omega^\theta - g^\theta$. The utility levels corresponding to the second best allocations, $\{x_l^\theta, x_c^\theta\}$ are obtainable by an endowment ω^* lying just below the point $\omega^\theta - g^\theta$ (see fig. 4). In the first best case the tax burden is proportional to the income of traders as in the case considered above. The inefficiency index corresponding to the use of second best taxes relative to the first best is equal to

$$\lambda^\theta(\mathbf{u}^\theta, \omega - g) = \frac{\|\omega^\theta - g^\theta\|}{\|\omega^*\|}. \tag{56}$$

The society real resource cost of providing education is measured by the standard-of-living indices with respect to the after intervention utility profiles $\mathbf{u}^\theta$ and the initial utility profiles $\mathbf{u}$, respectively. These are given by

$$\begin{aligned} Q^\theta(\omega - g, \omega) &= \frac{\lambda^\theta(\mathbf{u}^\theta, \omega - g)}{\lambda^\theta(\mathbf{u}^\theta, \omega)} = \frac{\|\omega^\theta - g^\theta\|/\|\omega^*\|}{\|\omega^\theta\|/\|\alpha^\theta\|} < 1 \\ Q(\omega - g, \omega) &= \frac{\lambda(\mathbf{u}, \omega - g)}{\lambda(\mathbf{u}, \omega)} = \frac{\|\omega - g\|/\|\beta\|}{\|\omega\|/\|\omega\|} < 1. \end{aligned} \tag{57}$$

The dual concepts that correspond to those above are the society welfare indices

$$\begin{aligned} U^\theta(u^\theta, u) &= \frac{\lambda^\theta(\mathbf{u}, \omega - g)}{\lambda^\theta(\mathbf{u}^\theta, \omega - g)} = \frac{\|\omega^\theta - g^\theta\|/\|\mu(\omega^\theta - g^\theta)\|}{\|\omega^\theta - g^\theta\|/\|\omega^*\|} = \frac{\|\omega^*\|}{\mu\|\omega^\theta - g^\theta\|} < 1 \\ U(u^\theta, u) &= \frac{\lambda(\mathbf{u}, \omega - g)}{\lambda(\mathbf{u}^\theta, \omega - g)} = \frac{\|\omega - g\|/\|\beta\|}{\|\omega - g\|/(\|\omega^*\|\|\beta\|/(\mu\|\omega^\theta - g^\theta\|))} = U^\theta(u^\theta, u), \end{aligned} \tag{58}$$

where μ is the radial expansion of the vector $\omega^\theta - g^\theta$ needed in order to just reach the indifference curve corresponding to the initial endowment ω (dotted curve in fig. 4). Similarly, $\|\omega^*\|/(\mu\|\omega^\theta - g^\theta\|)$ gives the largest radial contraction of the vector β needed in order to just reach the indifference curve corresponding to the endowment ω^*.[29]

Now the society welfare index is decreased. This effect involves both an improvement in the situation of the trader c and a substantial worsening in the situation of the trader l.

[29]To see this, note that homothecity implies that with no loss of generality utility function may be assumed to be linearly homogenous. Then $U(\beta) = u_\omega \Longrightarrow U((1/\mu)\beta) = (1/\mu)u_\omega$.

In this case the Malmquist productivity indices (35) (53) that correspond to the effectiveness public sector intervention are given by

$$
\begin{aligned}
M^\theta(\omega - g, \omega, \mathbf{u}^\theta, \mathbf{u}) &= \frac{\lambda(\mathbf{u}, \Theta\omega)}{\lambda(\mathbf{u}^\theta, \Theta(\omega - g))} = \frac{\|\omega^\theta\|/\|\alpha\|}{\|\omega^\theta - g^\theta\|/\|\omega^*\|} > 1 \\
M(\omega - g, \omega, \mathbf{u}^\theta, \mathbf{u}) &= \frac{\lambda(\mathbf{u}, \omega)}{\lambda(\mathbf{u}^\theta, \omega - g)} = \frac{\|\omega\|/\|\omega\|}{\|\omega - g\|/(\|\omega^*\|\|\beta\|/(\mu\|\omega^\theta - g^\theta\|))} \\
&= \frac{\|\omega^*\|/\|\beta\|}{\mu\|\omega - g\|\|\omega^\theta - g^\theta\|} > 1. \qquad (59)
\end{aligned}
$$

They both show an increase in the effectiveness of the public sector. On the other hand this is more than counterbalanced by the heavy cost of providing education, see (58).

Example 2. To present a parametric example for use in econometric estimation of the hedonic effects consider the quadratic extension of the AIDS-form (23) in section 4 equipped with hedonic price indices. The hedonic indices depend on the size of the facility through the function, say θ_k, defined by a suitable, eg. linear parametrization. The parameters present in θ_k may be estimated from the equation system

$$
\begin{aligned}
w_k^\theta(q^\theta, u^\theta) &= \frac{\partial \log e_h^\theta(q^\theta, u_h^\theta)}{\partial \log q_k^\theta} \\
&= a_k + \sum_l \gamma_{kl} \log q_l^\theta + \beta_k \bar{m}^\theta + (\delta_k - \beta_k)\delta(q^\theta)(\bar{m}^\theta)^2 \\
&= a_k + \sum_l \gamma_{kl}(\log q_l + \log \theta_l) + \beta_k \bar{m}^\theta + (\delta_k - \beta_k)\delta(q^\theta)(\bar{m}^\theta)^2 \qquad (60)
\end{aligned}
$$

where $\bar{m}^\theta = \log m - \log \alpha(q^\theta)$, and

$$
\log \alpha(q^\theta) = \sum_k a_k(\log q_k + \log \theta_k) + \frac{1}{2}\sum_k \sum_l \gamma_{kl}(\log q_k + \log \theta_k)(\log q_l + \log \theta_l). \qquad (61)
$$

In general, one is only able to identify relative hedonic scales, i.e. one has to set one of the $\theta_k' s$ equal to one.[30]

[30]Some authors have not adhered to the restrictions implied by the hedonic approach to the proper specification of cost or expenditure functions. Conditions for identification of meaningful hedonic scales are the same as the corresponding assumptions concerning the equivalence scales of consumption and are considered in a great detail by Dickens, et. al (1993).

8 Conclusion

In the present paper an inefficiency index which builds directly on an economy-wide generalisation of the input distance function has been introduced. Similarly as with the Debreu's coefficient of resource utilization it offers a measure of inefficiency that has wide applicability. The paper surveys through some important recent literature and presents economy-wide input based quantity index, the real endowment index, and productivity index indicating the effectiveness of the public sector. Together with the society welfare index these indices offer a convenient summary way of analyzing the nature and extent of government intervention from the perspective of society welfare and standard-of-living. The applicability of the method has been demonstrated by presenting some empirically applicable examples.

Hedonic methods seem particularly promising in offering a chance of incorporating the above analysis into the calculation of the society's welfare and standard-of-living indices. The public intervention considered by the above method may in principle range through widely differing spheres of economic activity. One may consider provision of public infrastructure, eg. construction of roads, a bridge, railway link, port facilities, or airport. Alternatively, one may examine public monitoring and regulation of privately provided services, or the introduction of legal restrictions and passing of laws for the protection of the consumer, eg. cases of setting minimum standards on the provision of information on safety and hazards related to the use of certain products. One may consider the effects of setting up posts of ombudsmen, and reforms in the functioning of the public administration.

The hedonic model concentrates on influences in virtual endowment and consumption of commodities as effectively allowing the production process to accommodate frictionlessly the enhancement of consumer goods through Θ. There are cases in which it may seem preferable to measure the hedonic effect involved solely in terms of producer prices and producer technology as opposed to a global effect considered above. First, the scope of the government intervention may be limited to a specific geographic area or to a special activity. Second, the introduction of public facilities such as infrastructure often have effects directly on the producer sector of the economy and the shadow price is best measured in that sector.[31] If the public intervention considered has effects only on the production side of the economy it could be dealt in an analogous way as an increase in productivity that is factor augmenting. A particularly important example concerns spending on education and labour training both of which enhance human capital. In empirical studies one could apply a specification of industry cost functions where the effects concern the input requirements or equivalently, their hedonic factor costs.

[31]It may be practical over-simplification to assume that the hedonic effects are uniform across consumers and production processes.

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Figure 1.

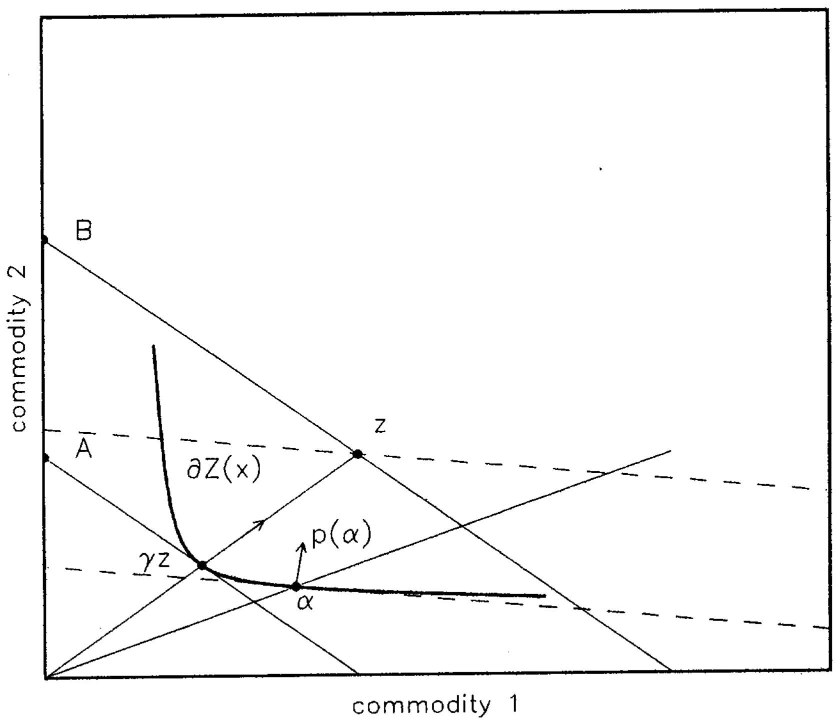

Figure 2.

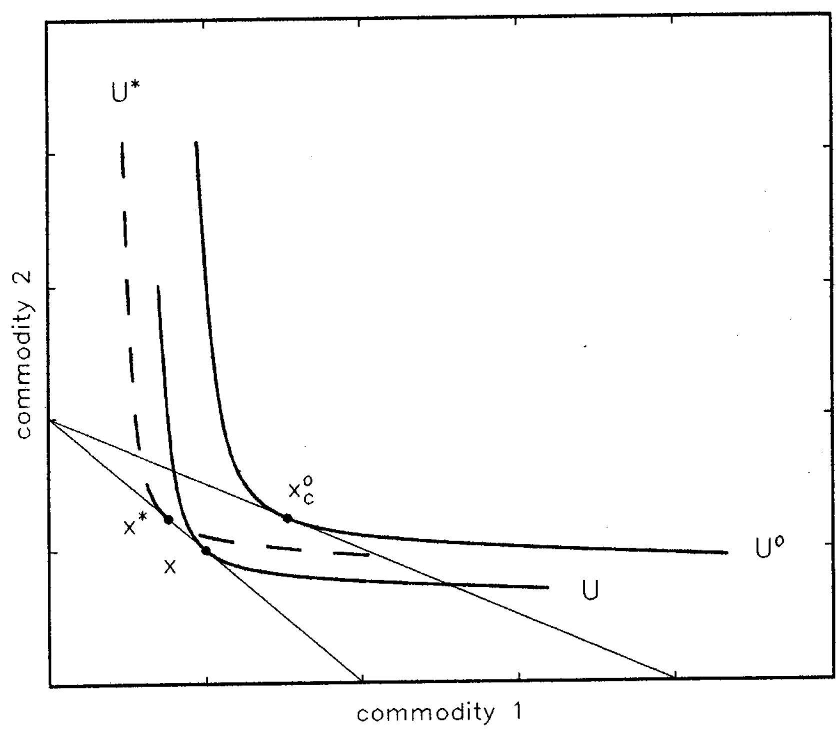

Figure 3.

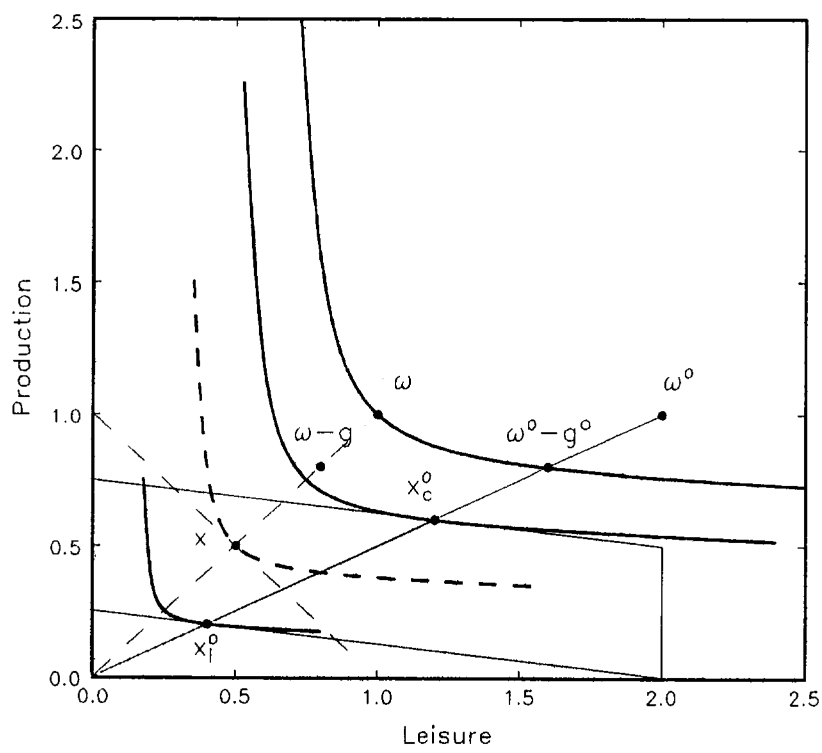

Figure 4.