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99

TAX INCIDENCE AND OPTIMAL FOREST TAXATION UNDER STOCHASTIC DEMAND**

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ABSTRACT: The paper studies the incidence and welfare effects of forest taxation in the presence of competitive roundwood markets when future demand for timber is uncertain thus making future timber price stochastic. The nature of risk is important. Under idiosyncratic risk the land site tax is like a pure profit tax and is fully borne by forest owners, while the burden of the yield tax -- though levied on forest owners -- is generally shared by both sides of the market. Incidence of the yield tax does not, however, matter qualitatively for the optimal structure of taxation. It is desirable to introduce a yield tax when the land site tax has been set to the optimal level. The optimal yield tax is generally distortionary, less than 100% and determined by the trade-off between its insurance and incentive properties. In the case of aggregate risk, government budget constraint is stochastic and the land site tax does not matter at all. The optimal yield tax is non-distortionary -- fully borne by forest owners -- and its level depends on the risk attitudes towards variability of private income vis-a-vis public consumption. The neutrality of the optimal yield tax is due to the fact that the yield tax has no insurance role in the presence of aggregate risk.

Keywords: risk, tax incidence, optimal forest taxation
JEL Classification: D81, H22, Q23

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TIIVISTELMÄ: Paperissa tarkastellaan metsäverotuksen kohtaantoa ja yhteiskunnan kannalta optimaalista metsäverotusta, kun raakapuumarkkinoilla vallitsee puhdas kilpailu ja puun tulevaan kysyntään liittyy epävarmuuta, mistä johtuen raakapuun tuleva hinta on stokastinen. Kysyntään liittyvä epävarmuus voi olla luonteeltaan joko idiosynkraattista, vain yritystasolla ilmenevää tai suhdanneriskityyppistä, aggregaattitasolla ilmenevää. Idiosynkraattisen riskin tapauksessa pinta-alavero on puhtaan voittoveron kaltainen ja sen rasoitus jää kokonaisuudessaan metsänomistajille. Sitä vastoin myyntituloveron rasitus jakaantuu yleensä markkinoiden molempien osapuolten kannettavaksi. Myyntituloveron kohtaannolla ei kuitenkaan ole kvalitatiivista vaikutusta metsäverotuksen optimaaliseen rakenteeseen. Annettuna optimaalinen pinta-alavero, on haluttavaa ottaa käyttöön myyntitulovero, joka pienentää puun tulevaan hintaan liittyvää yksilötason riskiä. Myyntitulovero on optimissa vääristävä ja sen taso riippuu negatiivisesti kannustinvaikutuksesta ja positiivisesti vakuutusominaisuudesta. Jos riski on aggregatiivista, valtion budjettirajoitus on stokastinen ja pinta-alaveroa ei tarvita lainkaan. Optimaalinen myyntitulovero on ei-vääristävä - metsänomistajat kantavat rasituksen täysin - ja sen taso riippuu siitä, kuinka metsänomistajat suhtautuvat epävarmuuteen yksityisissä tuloissa versus verotuksella rahoitetuissa julkisissa menoissa. Aggregaattiriskin tapauksessa myyntituloveron neutraalisuus optimissa on seurausta veron vakuutusominaisuuden puuttumisesta.

Asiasanat: riski, veron kohtaanto, optimaalinen metsäverotus
JEL-luokitus: D81, H22, Q23

CONTENTS

1. INTRODUCTION

The effects of forest taxation on timber supply has been studied extensively since 1970s, mostly using the so-called rotation framework under the circumstances of perfect capital markets and certainty (see e.g. Chang (1982), (1983)). The main focus of interest has been in two broad classes of forest taxes, namely annual property taxes (levied on the value of timber and/or land) and yield taxes (levied on the stumpage income). The general flavour of results has been that yield taxes tend to increase rotation periods, while property taxes tend to shorten or leave rotation periods unchanged. More specifically, it has been shown that a lump sum type land site tax has no effect on the rotation period. Relaxing the assumptions of perfect capital markets and certainty leads to some qualifications of these results. Under uncertainty, the timber supply effect of the land site tax is sensitive to what is assumed about absolute risk-aversion (see Koskela (1989a), (1989b)).

All these results of taxation are, however, derived under three restrictive assumptions. First, it has been assumed that forest owners fully bear the incidence of taxes.[1] This is not necessarily the case, however. In fact, the standard theory of tax incidence implies that forest owners bear forest taxes fully only if either timber supply is totally insensitive to timber price and/or demand for timber is infinitely elastic with respect to timber price (see Kotlikoff and Summers (1984)). These extreme cases are hard to defend; most empirical studies show that both the demand for and the supply of timber are sensitive to timber prices, but not infinitely so (see, e.g. Johansson & Löfgren (1985)). Analyzing only the supply side effects of forest taxes cannot give due account of other potentially important channels of influence of taxes. A general purpose of this paper is to rectify this omission in the literature of forest taxation by analyzing forest taxes in a simple partial equilibrium framework, where forest taxes affect not only timber supply, but also timber prices and thereby indirectly income of forest owners and profits of firms in the forest industry.

Second, using a market equilibrium framework enables us to relax the assumption of the exogenous future timber price uncertainty. Allowing for an endogenous determination of timber prices means that the timber price uncertainty is also determined as a part of market equilibrium. This necessitates some assumptions about the underlying cause of uncertainty, which gives rise to timber price uncertainty at the market level. Naturally, there are various

[1] As for the earlier literature on incidence of forest taxation, one should mention Johansen and Löfgren (1985), who suggested a static framework for analysis, but did not develop the incidence implications of forest taxes.

possibilities here; e.g. uncertainty may enter the roundwood markets through the demand side as a random shock in the demand for final product and/or in the production technology and/or through the supply side as a random shock in forest growth. It is assumed in what follows that the source of uncertainty is a technological shock in the future production function, which shows up the stochastic future demand for timber thus making the future timber price random.

Third, and finally, the earlier literature has analyzed the comparative static effects of forest taxes. While this is an important ingredient for the the aim of analyzing tax reforms, it does not come to grips with welfare aspects of forest taxation when governments face a budget constraint[2].

The purpose of this paper is to extend the existing literature in all e respects: By analyzing forest taxation in a market equilibrium context under uncertainty with endogenous prices, by deriving the optimal structure of forest taxes, when the social welfare function consists of the expected utility of forest owners and the expected profits of the firm in the forest industry. The forest taxes to be compared are land site tax and gross yield tax. The land site tax has no effect on relative prices and is thus non-distortionary so that it is natural to regard it as a benchmark. The gross yield tax, on the other hand, is a tax levied on the timber selling revenue and is commonly used in various countries.

As for the general features of the framework, there is a stochastic component in the future production function, which makes future demand for timber stochastic. Firms in the forest industry are assumed to be risk-neutral and maximize their expected profits, while private forest owners are risk-averse and make harvesting decisions so as to maximize the expected utility of the present value of harvest revenue. The agents on both sides of the market are assumed to have rational expectations over uncertain future timber price. The analysis of market equilibrium under uncertainty easily becomes intractable so that some simplifying assumptions are needed. We use two such assumptions. First, the production function is assumed to be quadratic in terms of timber input. This gives rise to linear demands for timber. Second, the forest growth function is assumed to be linear. Though restrictive, the linear growth function is very convenient and carries the message. Since the nature of risk turns out to be important, we analyze separately two cases; idiosyncratic risk -- when risk are independent across forest owners and law of large numbers quarantees a deterministic government tax revenue -- and aggregate risk -- common to all forest owners -- when government tax revenues are stochastic and private agents ultimately bear all the risk.

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[2] The optimal design of forest taxes when government faces a budget constraint in setting tax rates has been analyzed in Amacher and Brazee (1995) and in Koskela and Ollikainen (1995). Both papers abstract from tax incidence issues however.

To anticipate results, it it shown that under idiosyncratic risk land site tax is like a pure profit tax and is fully borne by forest owners, while the burden of the yield tax -- though levied on forest owners -- is generally shared by both sides of the market. As for the welfare aspects, the incidence of the yield does not matter qualitatively for the optimal structure of taxation. It is desirable to introduce the yield tax at the margin though the land site tax has been set to the optimal level. The optimal yield tax is generally distortionary, less than 100% and determined by the trade-off between its insurance and incentive properties. Finally, in the presence of aggregate risk, the land site tax does not matter at all and the optimal yield tax is non-distortionary -- fully borne by forest owners --and depends solely on risk attitudes of forest owners towards the variability of private income vis-a-vis the variability of public consumption. It has nothing to do with either insurance or incentive considerations.

The paper is organized as follows: Section 2 presents a simple market equilibrium model of harvesting decisions and equilibrium determination of timber prices under the circumstances, where there is uncertainty about the future production technology. The optimal forest taxation with and without endogenous timber prices and with idiosyncratic and aggregate risk is analyzed in section 3. Finally there is a brief concluding section.

2. A MODEL OF HARVESTING DECISIONS WITH STOCHASTIC DEMAND AND ENDOGENOUS TIMBER PRICES

2.1 Demand for timber under stochastic production function

Firms in the forest industry produce final product (pulp, paper) by using roundwood as an input over two periods, now and future. Current and future timber inputs are denoted by x and z respectively. Production functions are assumed to be identical for both periods and quadratic in terms of roundwood input. The current production function is known for certain, while the future production function is subject to a technological shock, which affects the demand for timber additively. These assumptions can be written as $Q_1 = f(x) = [ba - (1/2)bx]x$ for current and $Q_2 = f(z) = [b\hat{a} - (1/2)bz]z$ for future production respectively, where $\hat{a}$ is assumed to be normally distributed by expectation $\bar{a}$ and variance σ_a^2 so that $\bar{a} = a$[3]. The forest industry firm pays p_1 for the current roundwood and p_2 for the future one. Normalizing the price of final product to one the decision problem of the risk-neutral firms is to choose x and z so as to maximize the present value of its

[3] To facilitate the analysis of market equilibrium the parameters of the production functions have been chosen so that b will describe the price sensitivity and a the shift parameter of demand for timber function (the demand functions are solved in equation [2]).

expected profits as in [1].

[1] $$\underset{\{x,z\}}{Max}\ \pi=[ba-(1/2)bx]x-p_1x+R^{-1}\{[b\hat{a}-(1/2)bz]z-p_2z\}$$

where $R=1+r$ is the interest rate factor in the capital market. This yields the current and future demand for timber as functions of the parameters of the production function and the timber prices as follows

[2]

a) $x^d=a-\frac{p_1}{b}$

b) $z^d=\hat{a}-\frac{p_2}{b}$

A notable feature of the demand functions which depend negatively on (expected) timber prices, is their separability. This results from a lack of interrelatedness between production functions. Not surprisingly, timber demands depend negatively on (expected) timber prices.

2.2 Timber Supply Behavior under Stochastic Timber Price

Forest owners are assumed to be risk-averse and maximize the expected utility of the present value of harvest revenue. The decision problem is to determine how much to cut today and how much to allocate timber for the future, given current timber price and the probability distribution of the future timber price. The harvesting possibilities - which determine the biological trade-off between current and future harvesting - are given in equation [3]. According to it one can harvest in future an amount z that is left from the current harvesting plus the growth of the remaining stock, $(1+f)[Q-x]$, where Q is the original stock of timber and f is the growth rate, assumed to be constant for convenience

[3] $$z=(1+f)[Q-x]$$

Harvesting timber is costly, c and proportional to the amount of fellings. Future timber price $\hat{p}_2$, which is solved later on as a part of market equilibrium, is uncertain and normally distributed by $\hat{p}_2 \approx N(\overline{p}_2,\sigma_p^2)$. Government levies a land site tax T, which the forest owner has to pay regardless cutting or silvicultural activites so that T is a lump sum tax. The yield tax τ is a proportional tax and levied on the gross timber selling revenue. In what follows we denote the after-tax timber prices by $p_i^*=p_i(1-\tau)$, i = 1,2.

These assumptions lead to the following present value of harvest revenue V.

[4] $$V=(p_1^*-c)x+R^{-1}(p_2^*-c)z-(1+R^{-1})T$$

In what follows, the partial derivatives are denoted by primes for functions with one argument and by subcsripts for functions with many arguments.To sharpen the analytics the preferences are desribed by an exponential utility function $u(V)=-\exp(-AV)$, where $A=-u''(V)/u'(V)$ is the Arrow-Pratt measure of constant absolute risk-aversion (see e.g. Hirschleifer and Riley (1992)). Now the forest owner's expected utility maximizing problem can be formulated as choosing x (and z via the growth function [3]) so as to maximize $EU=-\exp(v)$, where $v=-A\overline{V}+(1/2)A^2R^{-2}(1-\tau)^2z^2\sigma_p^2$. This is equivalent to

[5] $$\underset{\{x\}}{Max}\ M=\overline{V}-\frac{1}{2}AR^{-2}(1-\tau)^2z^2\sigma_p^2.$$

and leads to the following harvesting rule [6].

[6] $$Rp_1-\overline{p}_2(1+f)+zA(1+f)R^{-1}(1-\tau)\sigma_p^2-(1-\tau)^{-1}\psi=0$$

where $\psi=(r-f)c$.

Harvesting rule [6] includes many interesting features, particularly in terms of the role of harvesting cost c. Under price certainty and zero harvesting cost one gets the familiar benchmark case $Rp_1-p_2(1+f)=0$, according to which the marginal return of harvesting is equated to the opportunity cost of harvesting at the margin. Under these circumstances the yield tax works like a pure profit tax and has no effect on harvesting. Under certainty, but with positive harvesting costs, the cutting rule reduces to $Rp_1-p_2(1+f)-(1-\tau)^{-1}\psi=0$ so that now the gross yield tax matters unless $r=f$. In particular, the comparative statics of the gross yield tax in terms of timber supply gives $x_\tau^s\prec 0$ as $r\geq f$ and ambiguous otherwise. One should mention that the condition for dynamic efficiency in OLG models is $r\succ n$, where n is the growth rate[4]. Allowing for timber price uncertainty with risk-aversion has the effect of increasing current harvesting[5].

Given the constant growth function, the current and future supply of roundwood can be solved explicitly from the first-order condition [6]. Utilizing the growth function [3] makes

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[4] Löfgren (1991) has studied harvesting decision in the conventional OLG model augmented with a simple forest technology and has shown that market equilibrium may be dynamically inefficient.

[5] One should notice that, under certainty, the net yield tax, i.e., a yield tax levied on the net timber price p_i-c, i=1,2, has no effect on harvesting regardless of values of f and r.

it possible to express current and future harvesting respectively as

[7]

$$\text{a) } x^s = Q + \frac{C}{B},$$

$$\text{b) } z^s = Q(1+f) - x^s(1+f),$$

where $B = A(1+f)^2 R^{-1}(1-\tau)\sigma_p^2 \succ 0$ and $C = Rp_1 - \overline{p}_2(1+f) - (1-\tau)^{-1}\psi \prec 0$.

Comparative statics of current harvesting is straightforward and given by

[8]

$$\text{a) } x^s_{p_1} = \frac{R}{B} \succ 0$$

$$\text{b) } x^s_{\overline{p}_2} = -\frac{(1+f)}{B} \prec 0$$

$$\text{c) } x^s_{\sigma_p^2} = \frac{z}{(1+f)\sigma_p^2} \succ 0$$

$$\text{d) } x^s_A = \frac{\sigma_p^2}{A} x_{\sigma_p^2} \succ 0$$

$$\text{e) } x^s_T = 0$$

$$\text{f) } x^s_\tau = \frac{C - \psi(1-\tau)^{-1}}{B(1-\tau)} \prec 0 \text{ as } r \geq f$$

Current (expected future) timber price affects current harvesting positively (negatively). Increases in timber price risk and risk-aversion boost current harvesting. As for forest taxes, the land site tax has no effect on the timing of harvesting. The gross yield tax has an a priori ambiguous effect on current harvesting; it depends on the relationship between the interest rate r and the growth rate of forest *f*. A sufficient but not a necessary condition for $x^s_\tau \prec 0$ is that $r \geq f$. Note also that comparative statics of future timber supply is simply $-(1+f)$ times the comparative statics of current supply in terms of variables other than Q.

2.3. Market Equilibrium under Rational Expectations

After developing the properties of the demand for and supply of timber we consider the market equilibrium in the roundwood markets and the role of forests taxes in its determination. Here we stick to the assumtion of competitive roundwood markets and assume that the equilibrium prices are determined by equality of demand and supply when private agents are price-takers.[6]

By setting $x^d = x^s$ and utilizing equations [2a] and [7a] yield the current price equation in terms of various underlying parameters, expected future timber price and timber price risk (associated with the term B), $p_1 = \frac{b\{(a-Q)B + \bar{p}_2(1+f) - (1-\tau)^{-1}\psi\}}{B+bR}$.

One should note that current timber price is deterministic, though risk affects it. The stochastic future equilibrium timber price p_2 is obtained in a similar way by setting $z^d = z^s$ and utilizing equations [2b] and [7b]. This leads to the following equation for future timber price $\hat{p}_2 = \frac{b\{\hat{a}B + (1+f)[Rp_1 + (1-\tau)^{-1}\psi]\}}{B+b(1+f)}$.

Due to uncertain technological shock future timber price p_2 is uncertain as well. To get the expectation and the variance of the future price, one has to take a stand on how expectations are formed. We make use of the rational expectations hypothesis, which can be defined as a situation in which agents do not make systematic mistakes in forecasting. Slightly more precisely, under rational expectations, agents' subjective beliefs about probability distributions correspond to the objective probability distributions so that expectations are assumed to be the same as the conditional expectations of the model used to describe the behaviour of agents.[7] The next step is to solve for p_1 and $\hat{p}_2$ and develop the properties of a simultaneous equilibrium. This gives for $\bar{p}_2$ and σ_p^2 (for details see appendix)

[9]

$$\text{a)}\ \bar{p}_2 = \frac{b\{\bar{a}[B+bR]B + bR(1+f)(a-Q) + (1+f)(1-\tau)^{-1}\psi\}}{B+bR+b(1+f)^2}$$

$$\text{b)}\ \sigma_p^2 = \frac{b^2(B+bR)^2}{\left[B+bR+b(1+f)^2\right]}\sigma_a^2$$

The expected future timber price is expressed in terms of underlying parameters, the original

[6] It would be an interesting area for research to analyze how forest taxes influence under the imperfectly competitive roundwood markets, where roundwood prices are subject to bargaining by both sides of the markets. An equally interesting area of research would to try to sort out which explanation performs best empirically. See Johansson and Löfgren (1985) for some preliminary analyses of roundwood market imperfections.

[7] See e.g. Bray (1985) for an introduction to the issues involved when modelling rational expectations equilibrium under uncertainty.

amount of timber, its growth function, interest rate and and timber price risk according to [9a]. Timber price risk in turn is determined both by uncertainty associated with future technology of firms and by the parameters of the model including the yield tax according to [9b].

2.4. Incidence of Forest Taxes and Their Underlying Determinants

The final step is to develop the properties of the resulting simultaneous equilibrium. This is a straightforward exercise (see appendix for the details). Before deriving the comparative statics of forest taxes it is useful to develop results for other important parameters of the model.

As shown in appendix, risk-aversion, the shift parameter reflecting a general level of demand, price sensitivity and timber price risk all affect positively both current and expected timber price. A rise in risk-aversion tends to decrease (increase) the equilibrium current (expected future) timber price. This is due to the fact that, the demand for timber remaining unchanged, higher risk-aversion leads to higher (lower) current (future) harvesting and thus 'lower' current (higher expected future) timber prices . Changes in the pure risk -- given by the timber price variance -- affects up to a scale factor like changes in risk-aversion. In fact, the relationship can be expressed as follows: $p_{i\sigma_p^2} = \frac{\sigma_p^2}{A} p_{iA}$ i = 1,2.

As for the effects of forest taxes, it is obvious that the land site tax T has has no price effects because it works like a pure non-distortionary tax so that $p_{1T} = \bar{p}_{2T} = 0$. The land site tax is thus fully borne by forest owners.

An increase in the gross yield tax usually raises the current timber price and decreases expected future timber price, as equations [10] and [11] indicate

[10] $$p_{1\tau} = \Delta^{-1}\left[z_{p_2}^d x_\tau^s\right] = -\frac{b\{[Rp_1 - \bar{p}_2(1+f)] - 2\psi(1-\tau)^{-1}\}}{(1-\tau)\left[B + bR + b(1+f)^2\right]} \succ 0 \text{ as } r \geq f$$

[11] $$\bar{p}_{2\tau} = -(1+f)p_{1\tau}$$

where $\Delta \succ 0$ is the determinant defined in appendix and $z_{p_2}^d, x_\tau^s$ refer to partial derivatives of z^d and x^s, respectively. In the case of $r \geq f$ the current timber supply is decreased by a rise in τ so that p_1 increases and $\bar{p}_2$ decreases. Otherwise the price effect is ambiguous.

Before discussing special cases we define the current demand elasticity $\eta = \frac{x^d_{p_1} p_1}{x^d}$, and the current supply elasticity $\varepsilon = \frac{x^s_{p_1} p_1}{x^s}$ using the equations [2a] and [7a] as follows

[12]

a) $\eta = \left[\frac{ab}{p_1} - 1 \right]^{-1} \succ 0$

b) $\varepsilon = \frac{Rp_1}{BQ + C} \succ 0$

In the extreme case of $b = 0$, the production function is linear and we have the infinite price elasticity of the demand for timber. Then $p_{1\tau} = \overline{p}_{2\tau} = 0$ from the equations [10] and [11] so that the gross yield tax has no effect on timber prices; tax burden is fully borne by forest owners. On the other hand, if B increases - e.g. due to a rise in risk-aversion or in the timber price risk reflecting increased variablility of timber demand - then the price elasticity of timber supply decreases. In the extreme case one gets again $p_{i\tau} = 0$ for i =1,2.

On the other hand, if the price insensitivity parameter b increases - the production function becomes more concave - then, by applying the L'Hospital's rule, the expression [10] reduces in the limit to

[10']

$$p_{1\tau|b=\infty} = -\frac{Rp_1 - \overline{p}_2(1+f) - 2\psi(1-\tau)^{-1}}{(1-\tau)\left[R + (1+f)^2\right]} \succ 0 \text{ as } r \geq f$$

when the price elasticity of timber demand is zero. Finally, when risk-aversion and/or timber price risk fall, B and C approach zero in [12b] and price elasticity of supply approaches infinity; one again ends up with the equation [10']. In both of these cases ($b \to \infty$, or $B, C \to 0$) the net price $p_{1\tau}(1-\tau)$ remains unchanged, and forest owners fully shift the yield tax burden onto firms in the forest industry.

One can now summarize the findings in

Proposition 1: In terms of incidence, (a) land site tax is like a pure profits tax and is fully beared by forest owners, while (b) the burden of the yield tax is generally shared by both sides of the market. More specifically, (c) forest owners pay a high share of the yield tax if the production function is close to linear and/or risk-aversion of forest owners and/or volatility of timber demand are very high, while (d) the yield tax is largely passed onto firms in the forest industry if production function is very concave and/or risk-aversion or volatility of timber demand are very low.

An intuitive explanation goes as follows: If volatility of timber demand is high and/or forest owners are very risk-averse, then timber supply tends to be relatively inelastic. On the other hand, if production function is close to linear, the demand elasticity tends to be high. In both of these cases, forest owners bear a major part of the burden of the yield tax, because yield tax passes onto firms via higher prices only to "a small extent". Vice versa happens if uncertainty is low, forest owners are not very risk-averse and timber demand is relatively inelastic. Under these circumstances timber prices faced by firms are "strongly" affected by the gross yield tax.

As we noted earlier, timber price risk in equilibrium depends also on the parameters of the model. As equation [9b] reveals, one gets for the yield tax

[13] $$\sigma^2_{p\tau} = 2\sigma^2_p \frac{b^2 B_\tau r(1+f)^2}{(B+bR-b(1+f)^2)^4} \prec 0$$

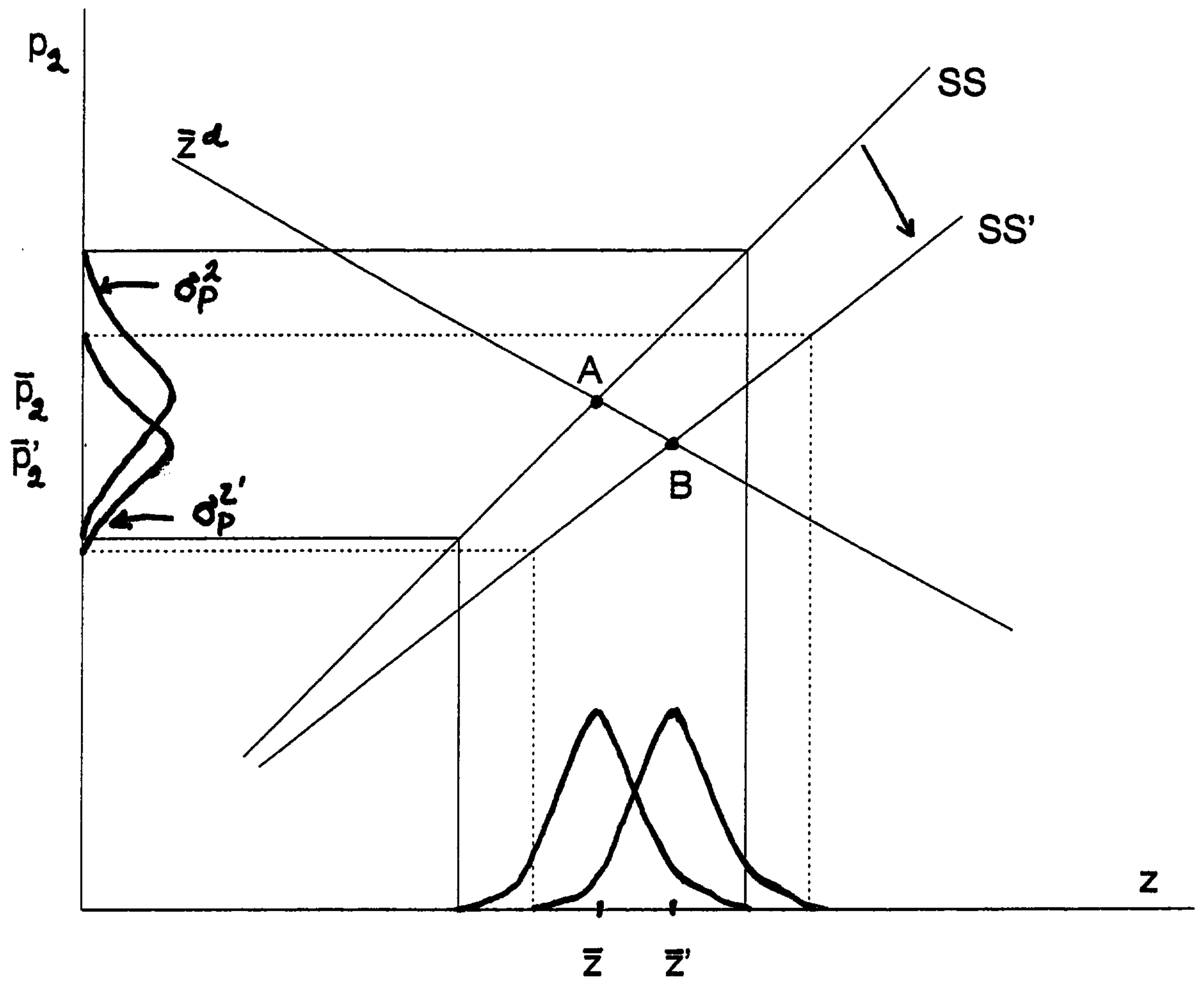
where $B_\tau = -A(1+f)^2 R^{-1}\sigma^2_p \prec 0$.

Thus we have from [9b] and [13]

Corollary 1: Under idiosyncratic stochastic demand, the equilibrium timber price risk is independent of the land site tax but is a decreasing function of the yield tax.

Figure 1 provides a simple geometric illustration the effect of yield tax on expected timber price and timber price variance (see also Newberry and Stiglitz (1981), p. 134). A downward sloping future demand function is stochastic and fluctuates according to the density function σ^2_a the expected demand being $\overline{z}^d$. An upward sloping future supply function is SS and expected equilibrium price is determined by the intersection of demand and supply as $\overline{p}_2$ and the density function for prices is $\sigma^2_p(A)$. A rise in the yield tax will increase future timber supply so that supply function SS shifts downwards. On the other hand, the timber price variance decreases to $\sigma^{2'}_p$ due to the fact that the SS curve becomes flatter. In the new equilibrium the supply curve isS'S', the equilibrium expected timber price $\overline{p}'_2$ and the equilibrium amount of timber used $\overline{z}'$.

FIGURE 1: A rise in yield tax , expected timber price and timber price risk



3. OPTIMAL FOREST TAXATION UNDER IDIOSYNCRATIC AND AGGREGATE RISK

3.1 Idiosyncratic risk and endogenous prices

The above analysis of roundwood market equilibrium and its comparative static properties provides a basis to consider the issue of optimal forest taxation from the viewpoint of society. Before doing this one has to clear up few things. First, in the line with the optimal taxation literature it is assumed that forest taxes are chosen so as to keep the government tax revenue given[8]. This requirement is taken here as exogenously given. Second, some assumptions concerning the nature of technological uncertainty has to be made. We assume

[8] See e.g. Atkinson and Stiglitz (1980).

first that the risk is idiosyncratic, i.e., that it is identically and independently distributed among individual forest owners. Due to this assumption government tax revenue requirement remains deterministic by the law of large numbers. The present value of government forest tax revenues can then be written as[9]

[14] $$G=(1+R^{-1})T+\tau[p_1x+R^{-1}\bar{p}_2z].$$

The social planner's problem - acting as a "benevolent dictator" - is to choose the land site tax T and yield tax rate τ so as to maximize the social welfare function subject to both the government budget constraint [14] and behavioral and market constraints analyzed in section 2.

The social welfare function consists of the sum of the expected indirect objective function M^* of the representative forest owner and the indirect profit function of the representative firm in the forest industry $\pi^*(\cdot)$ in terms of tax parameters and timber prices[10] so that

[15] $$W=M^*(T,\tau,p_1,\bar{p}_2,\sigma_p^2)+\pi^*(p_1,\bar{p}_2).$$

Before any private decisions are made, government is assumed to announce a tax policy and commits to it. The first-order conditions for the social welfare maximization under the binding tax revenue requirement in a Stackelberg equilbrium with the government as the dominant player can be obtained by setting the partial derivatives of the Lagrangian function $\Omega=W+\lambda G$ with respect to T and τ zero.[11]

As the land site tax does not affect timber supply and thereby equilibrium prices, its

[9] The idiosyncratic risk case has been analyzed in an optimal income taxation framework e.g. by Varian (1980). If risk is aggregative, then the government tax revenues are stochastic and in one way or another private agents must ultimately bear all the risk whether through random taxes, random government expenditures or random government deficits. We come back to this later on.

[10] M^* and π^* are defined as the maximum expected utility and the maximum expected profit respectively and can be obtained by substituting the equations [2] and [7] for x and z in the objective functions of forest owners and firms.

[11] In the literature on dynamic games this kind of equilibrium would be described as "open-loop equilibrium", see, e.g. Basar & Olsder (1982). If government does not want - or is not able to - enter into binding commitment, but instead reoptimizes at the beginning of each period, then one ends up in a Nash equilibrium without commitment. Persson and Tabellini (1990) contains an excellent overview of main recent developments in this area.

optimum is expressed in [16].

[16] $$\Omega_T = -(1+R^{-1}) + \lambda(1+R^{-1}) = 0 \Leftrightarrow \lambda = 1$$

At the optimum land site tax is chosen so that the the loss of marginal utility ($-(1+R^{-1})$) is equal to the increase in the tax revenues at the value of the Lagrangian multiplier $\lambda \succ 0$ ($\lambda(1+R^{-1})$.

The optimal condition for the gross yield tax is much more complicated as equation [17] suggests. The yield tax rate affects the welfare of the forest owners and firms in the forest industry not only directly, but also indirectly by changing the equilibrium prices. Finally, one has also to account for the timber supply and timber price effects of the yield tax via the government budget constraint [14]. By allowing for these effects gives

[17] $$\Omega_\tau = M_\tau^* + \lambda G_\tau + \left[M_{p_1}^* + \pi_{p_1}^* + \lambda G_{p_1}\right] p_{1\tau} + \left[M_{\bar{p}_2}^* + \pi_{\bar{p}_2}^* + \lambda G_{\bar{p}_2}\right] \bar{p}_{2\tau} + M_{\sigma_p^2}^* \sigma_{p\tau}^2 = 0$$

which implicitly defines the optimal value of the gross yield tax τ^*. When the land site tax has been chosen optimally, the equation [17] reduces to

[18] $$\Omega_\tau(T = T^*) = AR^{-2}(1-\tau)z^2\sigma_p^2 + \tau\left[p_1 - \bar{p}_2 R^{-1}(1+f)\right]x_\tau^s + \left[M_{p_1}^* + \pi_{p_1}^* + G_{p_1}\right] p_{1\tau}$$

$$+\left[M_{\bar{p}_2}^* + \pi_{\bar{p}_2}^* + G_{\bar{p}_2}\right] \bar{p}_{2\tau} + M_{\sigma_p^2}^* \sigma_{p\tau}^2 = 0,$$

where we have utilized the fact that $M_\tau^* = (p_1 x + R^{-1}\bar{p}_2 z)(1+R^{-1})^{-1} M_T^* + AR^{-2}(1-\tau)z^2\sigma_p^2$ and $G_\tau = (p_1 x + R^{-1}\bar{p}_2 z)(1+R^{-1})^{-1} G_T + \tau\left[p_1 - \bar{p}_2 R^{-1}(1+f)\right]x_\tau^s$.

Utilizing the envelope theorem[12] for the partial derivatives of M^* and π^*, developing the expressions $G_{p_1} = \tau x + \tau\left[p_1 x_{p_1} + \bar{p}_2 R^{-1} z_{p_1}\right] \succ 0$, $G_{\bar{p}_2} = \tau R^{-1} z + \tau\left[p_1 x_{\bar{p}_2} + \bar{p}_2 R^{-1} z_{\bar{p}_2}\right] \succ 0$ and

12 The envelope theorem states that the change in the objective function (here the expected indirect utility and profit functions) with respect to an exogenous parameter is the same both in the case when endogenous variables are adjusted to the optimum and in the case they are not adjusted. Thus we get in terms of prices the following: $M_{p_1}^* = (1-\tau)x \succ 0$, $M_{\bar{p}_2}^* = (1-\tau)zR^{-1} \succ 0$ and, $\pi_{p_1}^* = -x \prec 0$ and $\pi_{\bar{p}_2}^* = -R^{-1}z \prec 0$. The forest owners are gained and the firms in the forest industry are lost respectively by the timber price increases.

using the facts that $\bar{p}_{2\tau} = -(1+f)p_{1\tau}$, $z^s_{p_1} = -(1+f)x^s_{p_1}$ and $z^s_{\bar{p}_2} = -(1+f)x^s_{\bar{p}_2}$ make it possible to write the equation [18] as follows

$$[19] \qquad \Omega_\tau(T = T^*) = AR^{-2}(1-\tau)z^2(\sigma_p^2 - \frac{1}{2}(1-\tau)\sigma_{p\tau}^2) + \tau\left[p_1 - \bar{p}_2 R^{-1}(1+f)\right]x^s_\tau$$

$$-p_{1\tau}x_{p_1} f R^{-1}\tau\left[Rp_1 - \bar{p}_2(1+f)\right],$$

where $M^*_{\sigma_p^2} = -AR^{-2}(1-\tau)^2 z^2 \prec 0$ has been used.

In order to see whether the yield tax is needed at all one needs the evaluation of the partial derivative of the Lagrangian at the margin, when yield tax is zero. This yields

$$[20] \qquad \Omega_\tau(T = T^*, \tau = 0) = AR^{-2}z^2(\sigma_p^2 - \frac{1}{2}\sigma_{p\tau}^2) \succ 0$$

Under idiosyncratic uncertainty, it is welfare-increasing to introduce yield tax at the margin. It decreases timber price risk due to the volatility of the future demand for timber, which is beneficial for risk-averse forest owners. Yield tax has a negative effect on the risk on the one hand (the term σ_p^2) and an indirect negative effect by decreasing σ_p^2 on the other (the term $-\frac{1}{2}\sigma_{p\tau}^2$). If there is no uncertainty or forest owners are risk-neutral, then [20] is zero and τ is not needed once $T = T^*$. How far should one go of increasing the yield tax rate as a risk-sharing device? The partial derivative of the Lagrangian [19] at the margin, when yield tax can be expressed as

$$[21] \qquad \Omega_\tau(T = T^*, \tau = 1) = \left[Rp_1 - \bar{p}_2(1+f)\right]R^{-1}x^s_\tau(1+\varepsilon),$$

where $\varepsilon = \frac{f z^d_{p_2} x^s_{p_1}}{\Delta} \succ 0$ is the correction factor due to the endogeneity of timber prices.[13]

As for the sign of [21] one should distinguish between two cases. First, when $r = f$, the harvesting rule [6] implies $Rp_1 - \bar{p}_2(1+f) = 0$ at $\tau = 1$. Then $\Omega_\tau(T = T^*, \tau = 1) = 0$ so that

---

[13] The formula [10] for $p_{1\tau} = \Delta^{-1}\left[z^d_{p_2} x^s_\tau\right]$ has been used to rewrite [18] into [20].

100% yield tax is optimum. Second, when $r \succ f$, one gets $Rp_1 - \overline{p}_2(1+f) \succ 0$ from [6] and $x_\tau^s \prec 0$ from [8f] at $\tau = 1$. In the case $r \prec f$, we have $Rp_1 - \overline{p}_2(1+f) \prec 0$ from [6] and $x_\tau^s \succ 0$ from [8f] at $\tau = 1$. Thus $\Omega_\tau(T = T^*, \tau = 1) \prec 0$ with $r \neq f$ so that the optimal yield tax is less than 100% regardless of the sign of the yield tax. The optimal yield tax rate $0 < \tau^* < 1$ can be solved from equation [19] so as to give

[22] $$\tau^* = \frac{AR^{-1}z^2(\sigma_p^2 - \frac{1}{2}(1-\tau)\sigma_{p\tau}^2)}{AR^{-1}z^2(\sigma_p^2 - \frac{1}{2}(1-\tau)\sigma_{p\tau}^2) - \left[Rp_1 - \overline{p}_2(1+f)\right]R^{-1}x_\tau^s(1+\varepsilon)} \succ 0$$

Thus when $r \neq f$ the optimal yield tax is affected by its insurance role -- the terms $AR^{-1}z^2(\sigma_p^2 - \frac{1}{2}(1-\tau)\sigma_{p\tau}^2)$ -- by its incentive role -- the term $\left[Rp_1 - \overline{p}_2(1+f)\right]R^{-1}x_\tau^s$ -- and by the incidence factors -- the term $\varepsilon \succ 0$.

Thus we have

Proposition 2: If the land site tax has been set to the optimal level under idiosyncratic uncertainty, then regardless of the incidence of the yield tax (a) it is desirable to introduce the yield tax at the margin, (b) the optimal yield tax is generally distortionary, less than 100 % and is determined by the trade-off between its the insurance and incentive properties, (c) the optimal yield tax is 100% if the interest rate is equal to the growth rate of forest and zero if there is no uncertainty or forest owners are risk-neutral.

As for the role of the incidence of forest taxes, one gets from [22]

Corollary 2: Allowing for the endogenous timber prices under idiosyncratic risk has no qualitative effect on the optimal structure of forest taxation.

The intuition is the following. The optimal yield tax is distortionary reflecting the trade- off between its insurance and incentive properties. The distortionary effect of the yield tax is higher so that the optimal yield tax is lower, ceteris paribus. But the yield tax also affects the timber price risk via supply behavior. This tends to increase its insurance property and lead to higher yield tax.

3.2 Aggregate risk and endogenous prices

Earlier we assumed that risk affecting future demand for timber was idiosyncratic and thus did not exist at the aggregate level. This made it possible to write the government tax revenue requirement [14] as deterministic. But to the extent that volatility of future timber demand is a business cycle phenomenon, the earlier results are not valid. This raises a question of the structure of forest taxation under aggregate risk, when government tax revenue is stochastic and private agents must after all bear all the risk in one way or another.[14]

The welfare implications of aggregate risk depend on substitutability between private income V and publicly provided consumption G on the one hand and the risk attitudes towards their variability on the other hand. If V and G are perfect substitutes and risk attitudes toward them are similar so that only their sum $V+G$ matters, then we have $V+G=(p_1-c)x+R^{-1}(\hat{p}_2-c)z$. Neither the land site tax T nor the gross yield tax τ enter the target function so that they do not matter at all. This is an example of the stochastic version of the Ricardian equivalence theorem (see e.g. Barro (1989)). Obviously these assumptions are extreme ones and should be relaxed. A simple way of postulating imperfect substitutability between private income and public consumption is to assume that the risk attitudes towards them differ.

To sharpen the analytics the preferences associated with stochastic tax revenues G -- distributed to forest owners as public consumption -- are described by an exponential utility function $u(G)=-\exp(-A_gG)$, where $A_g=-u''(G)/u'(G)$ is the Arrow-Pratt measure of constant absolute risk-aversion associated with G. Assuming that the future timber price, which will be solved as a part of equilibrium, is normally distributed enables to write the public consumption part of the expected utility as $EY=-\exp(y)$, where $y=-A_g\overline{G}+(1/2)A_g^2R^{-2}z^2\tau^2\sigma_p^2$.. We assume that the private income and public consumption enters utility function of forest owners in an additively separable way so that

[23] $$EU^0=-\exp(v)-\exp(y)$$

Maximizing [24] is equivalent to maximize

[24] $$W=M+N$$

where M is defined by [5] and

[14] The implications of aggregate risk has been analyzed in a different context e.g. by Gordon and Varian (1988).

[25] $$N=\overline{G}-\frac{1}{2}A_{g}R^{-2}\tau^{2}z^{2}\sigma_{p}^{2}$$

Accounting for [5] and [25] one gets

[26] $$W=(p_{1}-c)x+R^{-1}(\overline{p}_{2}-c)z-\frac{1}{2}R^{-2}z^{2}\sigma_{p}^{2}\Lambda$$

where $\Lambda=A^{2}(1-\tau)^{2}+A_{g}\tau^{2}$.

Choosing x optimally produces as the following harvesting rule [27], which differs slightly from the previously derived rule [6].

[27] $$Rp_{1}-\overline{p}_{2}(1+f)+R^{-1}z(1+f)\sigma_{p}^{2}\Lambda-\psi=0$$

The second-order condition is given in [28].

[28] $$W_{xx}=-R^{-1}(1+f)^{2}\sigma_{p}^{2}\Lambda\prec 0$$

Comparative statics of the aggregate risk model for current and future timber price, timber price risk, land site tax and risk-aversion associated with the owner's income are qualitatively similar to the idiosyncratic case. One also gets

[29] $$x_{A_{g}}=\frac{-[Rp_{1}-\overline{p}_{2}(1+f)-\psi]}{D\Lambda}\succ 0,$$

where $D=R^{-1}(1+f)^{2}\sigma_{p}^{2}\Lambda\succ 0$, so that risk aversion associated with government consumption affects current timber supply positively.

Finally, we have

[30] $$x_{\tau}=\frac{-[Rp_{1}-\overline{p}_{2}(1+f)-\psi]\Lambda_{\tau}}{D\Lambda}\geq(\prec)0,$$

where $\Lambda_{\tau}=-2[A(1-\tau)-A_{g}\tau]\geq(\prec)0$ as $\tau\geq(\prec)\frac{A}{A_{g}+A}$.

In the presence of aggregate risk the yield tax has an a priori ambiguous effect on current and thereby on future timber supply. This can be explained as follows: a rise in the yield tax decreases the variability of after-tax timber revenues on the one hand, which tends to decrease current (precautionary) timber supply. On the other hand, variability of government consumption becomes higher which has the opposite effect on timber supply. As for the incidence of the yield tax, its burden is shared by both sides of the market. In the important case of $\tau = \frac{A}{A_g + A}$ it is, however, non-distortionary and is fully borne by forest owners.

One can solve the expected future timber price and its variance in terms of exogenous parameters to obtain

[31]

$$\text{a)}\quad \overline{p}_2 = \frac{bR\{(a-Q)bD(1+f)+(D(1+f)+bR)(b\overline{a}D-b\psi\}}{D(D(1+f)+bR+b(1+f)^2)}$$

$$\text{b)}\quad \sigma_p^2 = \frac{[b(D(1+f)+bR)]^2}{[D(1+f)+bR+b(1+f)^2]^2}\sigma_a^2,$$

where $D = R^{-1}(1+f)\sigma_p^2\Lambda$.

As for the properties of simultaneous equilibrium in terms of forest taxes note first that the land site tax has no effects on timber prices ($p_{1T} = \overline{p}_{2T} = 0$). For the effects of yield tax on timber prices one gets that $p_{1\tau} \succ 0$ and $\overline{p}_{2\tau} = -(1+f)p_{1\tau}$, just as in the idiosyncratic case. The effect of yield tax on future timber price variance is

[32]
$$\sigma_{p\tau}^2 = 2\sigma_p^2 \frac{(1+f)b^2(1+f)^2}{[D(1+f)+bR+b(1+f)^2]^4} D_\tau \geq (\prec) 0 \text{ as } \Lambda_\tau \geq (\prec) 0$$

where $D_\tau = R^{-1}(1+f)\sigma_p^2\Lambda_\tau$.

Thus one has

Corollary 3: Under aggregate stochastic demand, the effect of the yield tax on the equilibrium timber price risk is ambiguous a priori.

Let us now turn to consider the optimal forest taxation. One should notice that the usual separation between the concern with the tax structure and the concern with the level of taxation is not complete any longer; varying tax rates will in general affect also the variability of government consumption in a way which has welfare implications. Hence, in contrast with the idiosyncratic risk, a simultaneous optimization of tax rates and the level of public consumption has to be carried out.

The social welfare function over forest owners indirect utility function and over public consumption and forest industry's indirect profit function is defined by the sum

[33] $$W = M^*(\tau, p_1, \bar{p}_2, \sigma_p^2) + N^*(\tau, p_1, \bar{p}_2, \sigma_p^2) + \pi^*(p_1, \bar{p}_2),$$

where π^* describes the indirect profit function of the firms in the forest industry.

The optimal level of the land site tax is given by $W_T = 0$, which always holds since [24] ($W = M^* + N^*$) does not include T. Thus it does not matter which is the level of the land site tax. This is understandable when V and G are perfect substitutes in the expected value sense; it does not matter for forest owners whether they get private income or public consumption.

As for the yield tax one gets

[34] $$W_\tau = M_\tau^* + N_\tau^* + \left\{(M_{p_1}^* + N_{p_1}^* + \pi_{p_1}^*) - (1+f)(M_{\bar{p}_2}^* + N_{\bar{p}_2}^* + \pi_{\bar{p}_2}^*\right\} p_{1\tau} + (M_{\sigma_p^2}^* + N_{\sigma_p^2}^*)\sigma_{p\tau}^2 = 0,$$

where the relationship $\bar{p}_{2\tau} = -(1+f)p_{1\tau}$ has been used.

Applying $\pi_{p_1}^* = -x \prec 0$, $\pi_{\bar{p}_2}^* = -R^{-1}z \prec 0$, $M_{p_1}^* + N_{p_1}^* = x$ and $M_{\bar{p}_2}^* + N_{\bar{p}_2}^* = R^{-1}z$ and $M_{\sigma_p^2}^* + N_{\sigma_p^2}^* = -\frac{1}{2}R^{-2}z^2\Lambda \prec 0$ makes it possible to reduce [34] to

[35] $$W_\tau = M_\tau^* + N_\tau^* - \frac{1}{2}R^{-2}z^2\Lambda\sigma_{p\tau}^2 = -\frac{1}{2}R^{-2}z^2(\sigma_p^2\Lambda_\tau + \sigma_{p\tau}^2\Lambda) = 0, \text{ as } \Lambda_\tau = 0$$

Now $\operatorname{sgn}\sigma_{p\tau}^2 = \operatorname{sgn}\Lambda_\tau$ from [32] so that $W_\tau = 0$ holds as $\Lambda_\tau = 0$, which gives

[36] $$\tau^{**} = \frac{A}{A + A_g} \succ 0.$$

Thus we have

<u>Proposition 3:</u> In the presence of aggregate risk when private income and public consumption are perfect substitutes in the expected value sense, the land site

tax does not matter. It is desirable to use the yield tax as a risk-shifting device between private income and public consumption. The optimal yield tax depends only on the risk attitudes of forest owners towards variability of private income vis-a-vis public consumption.

Interpretation goes as follows: If forest owners are more (less) averse to private income risk than to public consumption risk, the optimal yield tax is higher (less) than 50 %. This is natural; if agents are very worried about private income variability due to the volatility of future timber price, then the tax system which lowers private income risk at the expense of public consumption risk is to be preferred. A 'high' yield tax rate does precisely this. In the extreme case of risk neutrality towards public consumption ($A_g = 1$) one gets $\tau^{**} = 1$.

As for the incidence of the yield tax we have from [36] and [32]

Corollary 4: Allowing for endogenous timber prices under aggregate risk has no effect on the optimal yield tax. It is non-distortionary and is fully borne by forest owners. At the optimum timber price risk does not depend on the yield tax.

The yield tax is not beneficial in the insurance sense so that at the optimum it should neither be distortionary nor affect timber price risk..

4. CONCLUDING REMARKS

This paper has analyzed the incidence and optimal structure of forest taxes in a simple market equilibrium framework, where there is an underlying uncertainty about the future production technology, which gives rise to stochastic future demand for timber and to future timber price risk under rational expectations. This framework has made it possible to consider incidence of forest taxes and analyze the potential implications of endogenous timber prices for the optimal forest taxes. The forest taxes to be compared have been land site tax - which is non-distortionary - and the gross yield tax, which is a tax levied on the timber revenue.

As for the incidence, it has been shown that under idiosyncratic risk the land site tax is like a pure profits tax and is fully borne by forest owners, while the burden of the yield tax -- though levied on forest owners -- is generally shared by both sides of the market. More specifically, forest owners tend to pay a high (low) share of the yield tax if they are (not) very risk-averse and/or volatility of future demand of timber is high (low) or the production function of firms in the forest industry is close to linear (very concave).

In terms of the optimal structure of forest taxation the paper demonstrates that in the presence of idiosyncratic risk it is desirable to introduce the yield tax at the margin though the land tax has been set to the optimal level. Regardless of the incidence of the yield tax the optimal yield tax is generally distortionary and less than 100 % and affected by the insurance and incentive effects associated with timber supply.

If the risk is aggregative, then government budget constraint is stochastic and private agents must ultimately bear all the risk involved in one way or another. In this case, the land site tax does not matter at all, while it is desirable to use the yield tax rate as a risk-shifting device between private income and public consumption. The optimal yield tax is non-distortionary so that it is fully borne by forest owners. If forest owners are more (less) averse to private income risk than to public consumption risk, the optimal yield tax is higher (less) than 50 %.

This paper has analyzed the incidence and welfare effects of forest taxation with idiosyncratic and aggregate risk under the circumstances of competititve roundwood markets. But how do forest taxes influence and what are their welfare effects under imperfectly competive roundwood markets, where roundwood prices are subject to bargaining by both sides of the markets and timber demand is unilaterally determined by firms in the forest industry? This is undoubtedly an interesting area for research.

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APPENDIX : EQUILIBRIUM TIMBER PRICE, PRICE RISK AND COMPARATIVE STATICS OF ROUNDWOOD MARKET EQUILIBRIUM

To solve equilibrium timber current and future prices and its variance in terms of exogenous variables write the formulas for p_1 and $\hat{p}_2$ (p. 8) as follows.

(1) $$p_1 - \alpha_1 p_2 = \varepsilon_1$$

(2) $$\hat{p}_2 - \alpha_2 p_1 = \varepsilon_2,$$

where $\alpha_1 = \dfrac{b(1+f)}{B+bR}$, $\alpha_2 = \dfrac{b(1+f)R}{B+b(1+f)^2}$, $\varepsilon_1 = \dfrac{b\left[(a-Q)B-\psi(1-\tau)^{-1}\right]}{B+bR}$ and

$$\varepsilon_2 = \frac{b\left[(aB-(1+f)\psi(1-\tau)^{-1}\right]}{B+b(1+f)}.$$

This gives the following equation system

(3) $$\begin{bmatrix} 1 & -\alpha_1 \\ -\alpha_2 & 1 \end{bmatrix} \begin{bmatrix} p_1 \\ \hat{p}_2 \end{bmatrix} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix}$$

where determinant $D = 1 - \alpha_1\alpha_2 = \dfrac{B+bR+b(1+f)}{(B+bR)(B+b(1+f))} \succ 0$.

Solving (3) for $\hat{p}_2$ yields

(4) $$\hat{p}_2 = D^{-1}\left[\varepsilon_2 + \alpha_2\varepsilon_1\right] = \frac{b\left\{\hat{a}[B+bR]B+bR(1+f)(a-Q)+(1+f)(1-\tau)^{-1}\psi\right\}}{B+bR+b(1+f)^2}$$

and similarly for p_1.

Future timber price variance is given by $E\left[\hat{p}_2 - \bar{p}_2\right]^2$. Utilizing (4) gives the equation (9b) of the text.

The current and future equilibrium in the roundwood market under competitive conditions can be expressed alternatively as

$$
(5) \quad \begin{aligned} &\text{a) } x^d - x^s = 0 \\ &\text{b) } z^d - Q(1+f) + x^s(1+f) = 0 \end{aligned}
$$

To find the effects of forest taxes (T,τ) and other exogenous parameters (A,a,b) on equilibrium prices Cramer's rule is used. Denote any exogenous parameter by θ . Perturbating the equilibrium condition (5) in terms of timber prices and any exogenous variable θ gives

$$
(6) \quad \begin{bmatrix} x^d_{p_1} - x^s_{p_1} & -x^s_{p_2} \\ (1+f)x^s_{p_1} & z^d_{p_2} + x^s_{p_2}(1+f) \end{bmatrix} \begin{bmatrix} dp_1 \\ d\bar{p}_2 \end{bmatrix} = \begin{bmatrix} x^d_\theta - x^s_\theta & -x^s_\theta \\ (1+f)x^s_\theta & z^d_\theta + x^s_\theta(1+f) \end{bmatrix} [d\theta]
$$

This system is stable. The first matrix of the LHS of (6) can be solved for the Hessian determinant Δ

$$
(7) \quad \Delta = \frac{B + bR + b(1+f)^2}{b^2 B} \succ 0, \text{ where } B = A(1+f)^2 R^{-1}(1-\tau)\sigma_p^2 \succ 0.
$$

Substituting exogenous parameters for $d\theta$ gives comparative statics. For risk-aversion A one gets

$$
(8) \quad \frac{dp_1}{dA} = \Delta^{-1}\left[z^s_{p_2} x^s_A\right] \prec 0,
$$

because $z^s_{p_2} = -\frac{1}{b} \prec 0$ and $x^s_A \succ 0$ according to equation (8d). Utilizing these results gives the equation (12a) reported in the text. Respectively,

$$
(9) \quad \frac{d\bar{p}_2}{dA} = -\Delta^{-1}\left[z^d_{p_2} x^s_A (1+f)\right] = -(1+f)\frac{dp_1}{dA} \succ 0
$$

As for the effect of an increase in the demand shift parameter a on current and future timber price one gets.

$$
(10) \quad \frac{dp_1}{da} = -\Delta^{-1}\left[z^d_{p_2} + x^s_{p_2}((1+f)+1)\right] \succ 0
$$

$$
(11) \quad \frac{d\bar{p}_2}{da} = \Delta^{-1}\left[-z^d_{p_1} + x^s_{p_1}((1+f)+1)\right] \succ 0
$$

Utilizing comparative statics of x and the fact that $x^d_b = \frac{p_1}{b^2}$ and $z^d_b = \frac{p_2}{b^2}$ one obtains for the price sensitivity parameter b

(12) $$\frac{dp_1}{db} = -\Delta^{-1}\left[x_b^d z_{p_2}^d + x_b^d x_{p_2}^s (1+f) - z_b^d x_{p_2}^s\right] \succ 0$$

(13) $$\frac{d\overline{p}_2}{db} = \Delta^{-1}\left[-z_b^d x_{p_1}^d + z_b^d x_{p_1}^s + x_b^d x_{p_1}^s (1+f)\right] \succ 0 .$$

Finally, the effect of the gross yield tax on equilibrium prices yields

(14) $$\frac{dp_1}{d\tau} = \Delta^{-1}\left[z_{p_2}^d x_\tau^s\right] \succ 0 \text{ as } r \geq f$$

(15) $$\frac{d\overline{p}_2}{d\tau} = -\Delta^{-1}\left[x_{p_1}^d x_\tau^s (1+f)\right] = -(1+f)\frac{dp_1}{d\tau} \prec 0 \text{ as } r \geq f .$$

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