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DISSERTATIO GRADUALIS, OBSERVATIONES QUASDAM CIRCA REDUCTIONEM ANGULORUM AD HORIZONTEM CONTINENS.

QUAM

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PRÆSIDE

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§. I.

Inæqualitates in ſuperficie telluris obviæ maxime impediunt, quo minus immediata menſuratione explorari queant diſtantiæ locorum longiori intervallo disjunctorum. Commodiſſima igitur ad hujusmodi diſtantias emetiendas inventa eſt ratio Trigonometrica, qua per ſeriem triangulorum loca iſta debite connectuntur. Angulis enim ſingulorum horum triangulorum menſuratis, ex dimenſione unici lateris totius ſeriei reliqua omnia calculo trigonometrico inveniuntur, unde diſtantia extremorum terminorum colligitur. Quum vero non tantum hæc ipſa triangula, ſed etjam anguli obſervati ſinguli in diverſis atque diverſimode inclinatis plerumque conſtituantur planis, neceſſe eſt ut prius omnes anguli menſurati in quovis triangulo ad Horizontem ſeu planum aliquod commune reducantur. Ex obſervatis ſcilicet utriusque lateris anguli cujusvis dati ad hoc planum inclinationibus, inveſtigandus eſt angulus horizontalis ſeu mutua inclinatio planorum, per latera dati anguli transeuntium atque ad commune iſtud planum perpendicularium.

§. II.

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Inter diversas a Mathematicis detectas methodos computandi reductionem angulorum ad Horizontem, simplicissima sine dubio censenda est atque rigore geometrico præ reliquis omnibus semet præcipue commendans illa, quæ ex principiis Trigonometriæ Sphæricæ deducta, a plerisque jam adhiberi solet. Posita nimirum Sphæra, radio quocunque descripta, cujus centrum hæreat in vertice anguli reducendi, ductis planis binis per latera hujus anguli ad Horizontem normalibus, & tertio plano in quo idem angulus jacet, ex horum sectione in superficie Sphæræ oritur triangulum Sphæricum, cujus dantur duo latera verticalia ex cognitis objectorum observatorum elevationibus supra vel depressionibus infra Horizontem: tertium vero latus est mensura anguli reducendi, & angulus huic lateri oppositus est ipse angulus reductus qui quæritur. Eo igitur redit problema propositum, ut ex datis tribus trianguli Sphærici lateribus unus angulorum inveniatur, cujus adeo facillima est solutio.

Sit enim angulus observatus sive reducendus, rectis a dato puncto ad bina quævis objecta ductis comprehensus $= v$, angulus vero ille ad Horizontem reductus $= v + x$, dicanturque horum objectorum supra planum horizontale per verticem anguli v ductum altitudines apparentes a & b (quæ quidem ne-

gativæ ſumendæ ſunt pro objectis infra hoc planum depreſſis): poſito Sinu Toto $= 1$, ex datis in triangulo Sphærico tribus lateribus v, $90° - a$ & $90° - b$ invenitur (Elem. Trigon. Sphær.) lateri v oppoſitus angulus quæſitus $v + x$ ſecundum utramvis harum formularum:

$$Sin\ \tfrac{1}{2}(v+x) = \sqrt{\frac{Sin\ \tfrac{1}{2}(v-a+b)\ Sin\ \tfrac{1}{2}(v+a-b)}{Cos\ a\ Cos\ b}}\ (I) \text{ vel}$$

$$Cos\ \tfrac{1}{2}(v+x) = \sqrt{\frac{Cos\ \tfrac{1}{2}(v+a+b)\ Cos\ \tfrac{1}{2}(v-a-b)}{Cos\ a\ Cos\ b}}\ (II).$$

Quæ formulæ, poſita altitudinum apparentium ſemiſumma $\tfrac{1}{2}(a+b) = m$ earumque ſemidifferentia $\tfrac{1}{2}(a-b) = n$, transformantur in has:

$$Sin\ \tfrac{1}{2}(v+x) = \sqrt{\frac{Sin\ (\tfrac{1}{2}v - n)\ Sin\ (\tfrac{1}{2}v + n)}{Cos\ (m+n)\ Cos\ (m-n)}}\ (I.)$$

$$Cos\ \tfrac{1}{2}(v+x) = \sqrt{\frac{Cos\ (\tfrac{1}{2}v + m)\ Cos\ (\tfrac{1}{2}v - m)}{Cos\ (m+n)\ Cos\ (m-n)}}\ (II)$$

Exempl. I. Dn. MAUPERTUIS ad terminum auſtralem (B) baſeos menſuratæ in fluvio Tornoënſi obſervavit angulum comprehenſum rectis ex B ad cacumen montis *Avaſaxa* (A) atque ſignum (y) inter hunc & montem *Cuitaperi* poſitum ductis $= 61° \ 30' \ 5'', 4 = v$, altitudines vero ipſorum y & A invenit reſpective $1° \ 23' \ 30''$ ($= a = m + n$) & $40' \ 30''$ ($= b = m - n$) cfr. Ejus *Figure de la Terre determinée par les obſervations faites au cercle Polaire.*

Si

Si jam quærendus fit angulus reductus $v + x$ fecundum form. I, calculus ita inftituetur:

$a = m + n =$	$1^\circ 23' 30''$	$L\,Cos\,a$	$= 0.0001281$
$b = m - n =$	$40' 30''$	$L\,Cos\,b$	$= 0.0000301$
$a - b = 2n =$	$43' \; 0''$	$L\,Sin(\frac{1}{2}v + n) =$	$\bar{1}.7132124$
$\frac{1}{2}(a - b) = n =$	$21' 30''$	$L\,Sin(\frac{1}{2}v - n) =$	$\bar{1}.7040815$
$\frac{1}{2}v$	$= 30^\circ 45' 2'',7$	2)	$\bar{1}.4174521$
$\frac{1}{2}v + n$	$= 31^\circ \; 6' 32'',7$	$L\,Sin\,\frac{1}{2}(v + x) =$	$\bar{1}.7087260$
$\frac{1}{2}v - n$	$= 30^\circ 23' 32''7$	$\frac{1}{2}(v + x)$	$= 30^\circ 45' 15'',85$
$v + x$	$= 61^\circ 30' 31'',7$	x	$= 26'',3.$

Exempl. 2. Idem Auctor inter cacumina montium *Pullingi* (*P*) & *Kittis* (*Q*) obfervavit in *Niemi* (*N*) angulum $51^\circ \; 53' \; 13'',7 = v$, atque deprehendit figni *P* elevationem apparentem $= 18' \; 30'' = (a = m + n)$, alterius vero *Q* depreffionem $= 14'$ $(= - b = - m + n)$. Ex his datis fecundum form. II. angulus Horizontalis $v + x$ ita invenietur:

$a = m + n =$	$18' 30''$	$-\,L\,Cos\,a$	$= 0.0000063$
$b = m - n =$	$- 14' \; 0''$	$-\,L\,Cos\,b$	$= 0.0000036$
$a + b = 2m =$	$4' 30''$	$L\,Cos(\frac{1}{2}v + m) =$	$\bar{1}.9537301$
m	$= 2' \; 15''$	$L\,Cos(\frac{1}{2}v - m) =$	$\bar{1}.9540067$
$\frac{1}{2}v$	$= 25^\circ 56' 36'',85$	2)	$\bar{1}.9077467$
$\frac{1}{2}v + m$	$= 25^\circ 58' 51'',85$	$L\,Cos\,\frac{1}{2}(v + x) =$	$\bar{1}.9538733$
$\frac{1}{2}v - m$	$= 25^\circ 54' 21'',85$	$\frac{1}{2}(v + x)$	$= 25^\circ 56' 32'',15$
$v + x$	$= 51^\circ 53' \; 4'',3$	x	$= - 9''.4.$

§. III.

Allata (§. II.) methodus reducendi angulos ad horizontem quamvis ſimpliciſſima videatur, eamque præ ceteris omnibus commendandam judicemus, quoties valde magnæ ſunt altitudines illæ apparentes a, b; vel nimis obliquus angulus obſervatus v; eadem tamen in reliquis caſibus, qui quidem maxime obvii ſunt, ſatis commode adhiberi nequit. Quum enim manifeſtum ſit, reductionis inveniendæ quantitatem x, quoties admodum exigui fuerint nominati iſti anguli, totam determinari per ultimas logarithmorum cyphras; ſequitur primo, ad eandem inveniendam, ſi ad ſcrupulos usque ſecundos horumque partes decimales exacta deſideretur, vulgares logarithmorum tabulas non ſemper ſufficere. Deinde ſi vel unius minuti ſecundi error tolerari poſſet, incommoda tamen fiet hæc methodus ob partes iſtas proportionales, quæ pro ſingulorum Sinuum vel coſinuum Logarithmis in hac methodo diligentius computandæ ſunt. His enim neglectis, error calculi major ſæpe orietur quam tota reductio inveſtiganda. Sic ſi in exemplo I. (§. II.) aſſumatur $\frac{1}{2} v + n = 31^\circ\ 7'$ & $\frac{1}{2} v - n = 30^\circ\ 24'$, foret $x = 1'\ 20''$ adeoque plus quam triplo major valore vero ipſius x ſupra invento.

§. IV.

Quum in dimenſionibus diſtantiarum, quæ ope triangulorum inter ſe connexorum inſtituuntur, rariſſimi

ſint

ſint caſus, in quibus objectorum obſervatorum altitudines vel depreſſiones apparentes 2 aut 3 gradus excedant, ſtationesque ſemper quantum fieri poſſit tales eligantur, ut anguli nimis obliqui evitentur; patet totam illam reductionem, quæ quæritur, paucis plerumque ſcrupulis abſolvi.

Hoc vero quoties locum obtinet, commodiorem cenſemus, immo exactiorem (§. III.) methodum, quæ ſequenti analyſi detegitur. Poſitis iisdem ac in §. II, quum ex form. I. ſequatur eſſe: $Sin\ \frac{1}{2}(v + x)^2\ Cos\ (m + n)\ Cos\ m - n) = Sin\ (\frac{1}{2}v + n).\ Sin\ (\frac{1}{2}v - n)$; ſed (Princ. Trigon) $Cos\ (m + n)\ Cos\ (m - n) = Cos\ m^2 - Sin\ n^2 = 1 - Sin\ m^2 - Sin\ n^2$ atque $Sin\ (\frac{1}{2}v + n)\ Sin\ (\frac{1}{2}v - n) = Sin\ \frac{1}{2}v^2 - Sin\ n^2$: his valoribus ſubſtitutis, facta multiplicatione atque terminorum debita transpoſitione [ob $1 - Sin\ \frac{1}{2}(v + x)^2 = Cos\ \frac{1}{2}(v + x)^2$ & $Sin\ \frac{1}{2}(v + x)^2 - Sin\ \frac{1}{2}v^2 = Sin\ \frac{1}{2}x\ Sin\ (v + \frac{1}{2}x)$] erit $Sin\ \frac{1}{2}x\ Sin\ (v + \frac{1}{2}x) = Sin\ m^2\ Sin\ \frac{1}{2}(v + x)^2 - Sin\ n^2\ Cos\ \frac{1}{2}(v + x)^2$.

Quoniam vero exigui ſupponuntur m & n adeoque etiam x, patet pro $Sin\ m$, $Sin\ n$, & $Sin\ \frac{1}{2}x$ ſubſtitui poſſe m, n & $\frac{1}{2}x$ reſpective, nec non v tam pro $v + \frac{1}{2}x$ quam pro $v + x$, unde facta diviſione per $\frac{1}{2}\ Sin\ v = Sin\ \frac{1}{2}v.\ Cos\ \frac{1}{2}v$, obtinetur: $x = m^2\ Tg\ \frac{1}{2}v - n^2\ Cotg\ \frac{1}{2}v$. In hac autem formula quum aſſumtus ſit radius $= 1$, dividantur porro m^2 & n^2 per numerum minutorum ſecundorum, quæ continet arcus circuli radio

dio æqualis, qui numerus (= 206264, 8) dicatur r: quo facto invenitur reductio quæsita in scrupulis secundis:

$$x = \frac{m^2}{r} Tg \tfrac{1}{2} v - \frac{n^2}{r} Cotg \tfrac{1}{2} v.$$

Illustratio: In Exemplo 1 (§. II) est $\frac{1}{2} v = 30° 45' 2'', 7$, $\frac{1}{2} a = 41' 45'' = 2505''$ & $\frac{1}{2} b = 20' 15'' = 1215''$, adeoque $m = \frac{1}{2}(a + b) = 3720''$ & $n = \frac{1}{2}(a - b) = 1290''$. Ex his datis ita investigatur x:

$Log \frac{1}{r}$	$= \bar{6}.\ 6855749$	$Log \frac{1}{r}$	$= \bar{6}.\ 6855749$
$2\ L\ 3720$	$= 7.\ 1410858$	$2\ L\ 1290$	$= 6.\ 2211794$
$L\ Tg \frac{1}{2} v$	$= \bar{1}.\ 7744842$	$L\ Cotg \frac{1}{2} v$	$= 0.\ 2255158$
$L\ 39,916$	$= 1.\ 6011449$	$L\ 13,560$	$= 1.\ 1322701.$

$$x = 39'', 916 - 13'', 560 = 26'', 356.$$

Exempl. II. $\frac{1}{2} v = 25° 56' 36'', 85$; $\frac{1}{2} a = 9' 15'' = 555''$; $\frac{1}{2} b = -7' = -420''$; $m = \frac{1}{2}(a + b) = 135''$; $n = \frac{1}{2}(a - b) = 975''$.

$Log \frac{1}{r}$	$= \bar{6}.\ 6855749$	$Log \frac{1}{r}$	$= \bar{6}.\ 6855749$
$2\ L\ 135$	$= 4.\ 2606676$	$2\ L\ 975$	$= 5.\ 9780092$
$L\ Tg \frac{1}{2} v$	$= \bar{1}.\ 6870953$	$L\ Cotg \frac{1}{2} v$	$= 0.\ 3129047$
$L\ 0,043$	$= \bar{2}.\ 6333378$	$L\ 9.\ 473$	$= 0.\ 9764888.$

$$x = 0'', 043 - 9'', 473 = -9'', 43.$$

Si neglectis scrupulis secundis in exemplo 1:o sumatur $\frac{1}{2} v = 30° 45'$ & in ex. 2:o $\frac{1}{2} v = 25° 56'$, iidem omnino prodeunt ipsorum x valores, adeo ut hinc ne quidem differentia $= 0'',01$ oriatur.