Green supplier selection via Multiple Criteria Data Envelopment Analysis

By:
Abdollah Noorizadeh

Supervisor:
Professor Tuomo Kässi

Examiners:
Professor Tuomo Kässi
Associate Professor Pasi Luukka
Abstract

**Author:** Abdollah Noorizadeh  
**Title:** Green supplier selection via Multi Criteria Data Envelopment Analysis  
**Year:** 2014  
**Place:** Lappeenranta, Finland

**Type:** Master’s Thesis, Lappeenranta University of Technology (LUT)  
**Specification:** 163 pages, 6 figures, 9 tables and 4 articles

**Supervisor:** Professor Tuomo Kässi  
**Examiners:** Professor Tuomo Kässi, Associate Professor Pasi Luukka

**Keywords:** Multiple criteria Data Envelopment Analysis, Undesirable outputs, CO₂ emission, Sustainability, Green supplier selection

An appropriate supplier selection and its profound effects on increasing the competitive advantage of companies has been widely discussed in supply chain management (SCM) literature. By raising environmental awareness among companies and industries they attach more importance to sustainable and green activities in selection procedures of raw material providers.

The current thesis benefits from data envelopment analysis (DEA) technique to evaluate the relative efficiency of suppliers in the presence of carbon dioxide (CO₂) emission for green supplier selection. We incorporate the pollution of suppliers as an undesirable output into DEA. However, to do so, two conventional DEA model problems arise: the lack of the discrimination power among decision making units (DMUs) and flexibility of the inputs and outputs weights. To overcome these limitations, we use multiple criteria DEA (MCDEA) as one alternative. By applying MCDEA the number of suppliers which are identified as efficient will be decreased and will lead to a better ranking and selection of the suppliers. Besides, in order to compare the performance of the suppliers with an ideal supplier, a “virtual” best practice supplier is introduced. The presence of the ideal virtual supplier will also increase the discrimination power of the model for a better ranking of the suppliers. Therefore, a new MCDEA model is proposed to simultaneously handle undesirable outputs and virtual DMU. The developed model is applied for green supplier selection problem. A numerical example illustrates the applicability of the proposed model.
Acknowledgements

I believe that working on a particular topic for Master’s thesis by students own interest has two major merits compared to various predefined courses needed to be passed in Master studies. First, students have to study some of the assigned courses in each program, even though they do not have the same level of interest in all those subjects. Second, courses are planned for short-term teaching, which lack the time needed to study the subject deeply. This challenges one’s mind to think and compare this situation with huge motivation and hard work of a student to accomplish a thesis as good as possible. Also, completing a thesis project improves learning process via making mistakes and working with others.

After realizing, personally, the topic of interest for thesis, finding the supportive people which are ready to invest their time and energy on your work is not an easy task. Here, I found the opportunity to acknowledge some of those which did so. I am very grateful to my supervisor Professor Tuomo Kässi for all kind behavior and supports, before and during doing my thesis.

I wish to thank Dr. Abolfazl Keshvari, from Aalto University, School of Business, for all the advice and comments on my master’s thesis. I benefited from his knowledge about mathematical modeling and performance analysis. Besides, I am grateful to Professor Markku Kuula, from the same University and School, for prior consultation and recommendation regarding subject of pollution and supply chain performance. I also want to extend my thanks to Professor Reza Farzipoor Saen for coaching me during the Bachelor studies. I learned from him about data envelopment analysis (DEA) and supply chain management. In addition, the contribution of associate Professor Pasi Luukka in my thesis is appreciated.

I would like to express my gratitude to all those who have helped me when it was needed at Lappeenranta University of Technology. To mention some, Timo Alho, Janne Hokkanen and Ismo Vainikka working in Department of Business Economics and Law, International affairs, and academic library, respectively, as well as Professor Juha Väätänen, Suvi Tiainen, Riitta Salminen, and Petri Hautaniemi from Department of Industrial Engineering and Management.
I am greatly indebted to my friend, Mahdi Mahdiloo who introduced me to the world of DEA, and has taught me about writing scientific articles. I acted on Mahdi’s advice to solve some of the problems I faced through writing my thesis.

Finally, I wish words could help me to express my feelings to appreciate my parent’s for all the supports and the inherited instincts of planning, hardworking and persistence to achieve the goals.

Lappeenranta, Finland, December 10, 2014

Abdollah Noorizadeh
Table of Contents

Acknowledgements ........................................................................................................ ii

List of Abbreviations ........................................................................................................ v

List of Figures .................................................................................................................. vi

List of Tables ................................................................................................................... vii

List of Articles .................................................................................................................. viii

1. Introduction .................................................................................................................. 1
   1.1 Background and research gap .............................................................................. 1
   1.2 Research objectives and questions ..................................................................... 7
   1.3 Research methodology ....................................................................................... 8
      1.3.1 Theoretical framework ................................................................................ 8
      1.3.2 Data collection ............................................................................................ 10
      1.3.3 Structure of the study ................................................................................ 11

2. Literature review ......................................................................................................... 13
   2.1 Supply chain management ................................................................................... 13
   2.2 Sustainability ...................................................................................................... 16
   2.3 Green supplier selection ..................................................................................... 19

3. Data envelopment analysis .......................................................................................... 22
   3.1 Purpose of performance measurement ................................................................ 23
   3.2 Selection of inputs and outputs .......................................................................... 24
      3.2.1 Undesirable outputs .................................................................................. 24
      3.2.2 Non-discretionary variable ...................................................................... 25
      3.2.3 Dual-role factor ........................................................................................ 27
   3.3 Number of DMUs vs. number of inputs and outputs ......................................... 28
   3.4 Model orientation ............................................................................................... 29
   3.6 Unrealistic weighting schemes .......................................................................... 31

4. Proposed multiple criteria DEA model ........................................................................ 33

5. Numerical example ....................................................................................................... 40

6. Conclusions .................................................................................................................. 45
   6.1 Limitations and suggestions for further research .............................................. 45

References ....................................................................................................................... 47

Part II: Publications
## List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AHP</td>
<td>Analytic Hierarchy Process</td>
</tr>
<tr>
<td>ANP</td>
<td>Analytic Network Process</td>
</tr>
<tr>
<td>BSC</td>
<td>Balanced Scorecard</td>
</tr>
<tr>
<td>CBR</td>
<td>Case-Based Reasoning</td>
</tr>
<tr>
<td>CCR</td>
<td>Charnes, Cooper, and Rhodes</td>
</tr>
<tr>
<td>CO₂</td>
<td>Carbon dioxide</td>
</tr>
<tr>
<td>CRS</td>
<td>Constant Returns to Scale</td>
</tr>
<tr>
<td>DEA</td>
<td>Data Envelopment Analysis</td>
</tr>
<tr>
<td>DMU</td>
<td>Decision Making Unit</td>
</tr>
<tr>
<td>FST</td>
<td>Fuzzy Set Theory</td>
</tr>
<tr>
<td>GA</td>
<td>Genetic Algorithm</td>
</tr>
<tr>
<td>GSCM</td>
<td>Green Supply Chain Management</td>
</tr>
<tr>
<td>GSS</td>
<td>Green Supplier Selection</td>
</tr>
<tr>
<td>GST</td>
<td>Grey System Theory</td>
</tr>
<tr>
<td>MCDM</td>
<td>Multi-Criteria Decision Making</td>
</tr>
<tr>
<td>MOLP</td>
<td>Multiple Objective Linear Programming</td>
</tr>
<tr>
<td>NN</td>
<td>Neural Network</td>
</tr>
<tr>
<td>PPM</td>
<td>Parts Per Million</td>
</tr>
<tr>
<td>SCM</td>
<td>Supply Chain Management</td>
</tr>
<tr>
<td>TOPSIS</td>
<td>Technique for Order of Preference by Similarity to Ideal Solution</td>
</tr>
<tr>
<td>VRS</td>
<td>Variable Returns to Scale</td>
</tr>
</tbody>
</table>
List of Figures

Figure 1. Sustainability in SCM categories and different techniques for a green supplier selection .................................................. 10
Figure 2. Supply chain structure ................................................................................................................................. 14
Figure 3. Representation of sustainability concept ...................................................................................................... 17
Figure 4. Different kinds of non-discretionary factors ................................................................................................. 26
Figure 5. Different types of strategies for increasing the efficiency ............................................................................. 30
Figure 6. Comparison of results obtained by different objective functions of Model (6) ........................................... 44
List of Tables

Table 1. Summary of the applied approaches for supplier selection problem .......................... 5
Table 2. The minimum number of DMUs .................................................................................. 28
Table 3. The hypothetical data set and the results of evaluation ................................................. 29
Table 4. DEA weighting system ............................................................................................... 32
Table 5. The Criteria for evaluation of green suppliers’ performance ........................................ 40
Table 6. Data set for 18 suppliers ............................................................................................. 41
Table 7. Results of Models (2) and (3) ...................................................................................... 42
Table 8. Results of Model (5) .................................................................................................. 43
Table 9. Results of Model (6) .................................................................................................. 44
List of Articles


1. Introduction

This chapter explains the objectives and the research questions of the thesis, and describes the gap between previous works and this research. The research methodology and the structure of the study are also presented in this section.

1.1 Background and research gap

Companies use outsourcing as an appropriate approach for improving their performance and flexibility and also to diminish their operations costs (Solakivi et al. 2013). Outsourcing emphasizes that organizations should invest more on internal operations and activities in which consume the core competencies of the production system and outsource other tasks (Dolgui and Proth, 2013). Bettis et al. (1992) believe that competitiveness is the outcome of properly implementing and monitoring the strategy of outsourcing in entire organizations. Ballou (1992) finds that in the late 1980s American companies were involved with only 40% of the total production cost and the greater part was for suppliers by 60% (Gunasekaran et al. 2004). Furthermore, Ghodsypour and O’Brien (1998) argue that raw materials and component parts provided by suppliers have considerable portion of 70% in product final cost. All these evidences support the importance of the right suppliers’ selection in the implementation of a successful supply chain management (SCM).

Market globalization, fierce competition for more market share and placing great emphasis on customer orientation with the aim of increasing customer satisfaction are some other benefits of SCM for companies (Gunasekaran et al. 2001; Webster, 2002; Shepherd and Günter, 2006). Wang et al. (2011) emphasized that supply chain and logistics should be considered as among the most important economic activities in today’s industrialized lifestyle. Ting and Cho (2008) discussed about the importance of selecting right suppliers and how significantly it can lead to the increase/decrease of the cost, profitability and flexibility of the company. Weber et al. (1991) also believe that new trends and changes in SCM have been occurred and former business models should be revised. In a new situation, being successful in market by low-cost and high-quality products is not an achievable goal without access to proper suppliers. It is also important to note that, price, cost, quality, delivery, and flexibility are considered as central competitive
capabilities in organizations which can be transferred directly through supply chains and suppliers (Li et al. 2006).

Effective SCM is of great value in getting competitive advantage and improving performance of organizations (Li et al. 2006). In order to enjoy from these competences, the need for applying effective methods is essential. Appearing new economies, fast changing technology and increased competition in the global markets impose ever-increasing pressure to companies to use new technologies and mechanisms in their operations and supply chains. It is a general belief that the traditional tools for a new atmosphere are not efficient and it is imperative to apply new strategies and approaches to be competitive and profitable in an uncertain market.

In order to cut overall production costs and enjoy the benefits of optimized SCM, it is worth noting some features that should be taken into account when we are assessing the SCM. In the last decades, SCM were thought to be purely operational activities (Gattorna, 1998; Vilko, 2012, p. 16) and not enough attention is being paid to environmental issues. Therefore, the planet itself started to respond to human violent behaviors. Climate change and global warming caused by human activities in modern civilization show their irreversible changes to the environment by increasing the level of seas, ozone destruction, damaging the natural habitats, devastating floods, heat waves, intense wildfires, long droughts, and season creep. For instance, one can refer to Australia and Iceland as two most notable examples which are considerably impacted by a huge amount of wildfires and ice melting, respectively (Usatoday.com, 2014 and BBC News, 2014). For another example, National Aeronautics and Space Administration (NASA) (2014) reports that “the industrial activities have raised atmospheric carbon dioxide levels from 280 parts per million to 379 parts per million in the last 150 years”.

After appearing detrimental effects of climate change, governments, environmental organizations, non-governmental organizations (NGOs) and production plants have started to review and find the sources of these disasters. Meanwhile, by emerging the term “Sustainability” in all around the world, governors, authorities, researcher, and organizations showed great interest in this topic. They started to consider sustainability in social, economic and environmental aspects for current and future concerns. As a general understanding, sustainability
refers to using today’s resources without damaging them, while allowing future generations to also have the opportunity to enjoy those resources (Brundtland, 1987). This clearly indicates that we need to rethink and revise the way of doing things. To this end, many companies and organizations started to find more efficient and productive ways of using severely limited resources. One of the controversial topics is how to force enterprises to produce less industrial pollution at different sectors. Recently, one of the research interests of scholars is considering sustainability in SCM. According to our understanding, they use the terms “green”, “eco-friendly”, “environmentally friendly”, “carbon footprints impact”, “corporate social responsibility” and “low carbon” as alternative terms for measuring sustainability in SCM.

Therefore, in addition to considering a wide variety of different criteria in measuring the performance of SCM, green behavior in networks of manufacturing and distribution from upstream raw materials providers to distribution of final products or services to end customers also must be taken into account.

In order to take sustainability into account, we need to consider CO₂ emission of suppliers with other common supplier evaluation criteria. For a comprehensive study on suppliers’ evaluation and selection criteria, readers are referred to Dickson (1996). As mentioned earlier, the customers are much more demanding than they used to be, and better quality, competitive pricing, convenient availability, flexibility and product variety are essential elements expected from suppliers (Kruse and Bramham, 2003). Currently, customers are more serious about sustainability and eco-friendly effects of products or services on society and environment. Therefore, companies and manufacturers need to give more attention to pollutions and wastes as outcomes of their production process, and consider how using clean technologies and waste management systems can help cutting undesirable outputs of production. One of the solutions to this problem, which has recently attracted industries and researchers, is green supplier selection. It means, purchasing departments in companies have to consider green criteria in their buying process from suppliers and in entire SCM (Genovese et al. 2013). Brandenburg et al. (2014) state in traditional SCM, economic and financial business performance paly a main role, however, environmental objectives also should be considered in entire supply chain, ranging from supplier selection, reverse logistics, remanufacturing to product recovery. Moreover, Genovese et al. (2013) emphasized that greener supplier selection problem is a new version of supplier selection.
problem just by considering environmental factors in supplier selection process. After Dickson (1996) who presented 23 criteria for evaluating suppliers, Ha and Krishnan (2008) and Handfield et al. (2002) have also compiled their own 30 criteria and 50 criteria supplier selection criteria list. The environmental factors are also mentioned in these recent lists.

By looking at literature, we understand how collecting holistic set of attributes is important for GSS and it should be highly praised for the raise of the environmental awareness among managers and decision makers. Nevertheless, by increasing the number of criteria for evaluating and selecting suppliers, the complexity of the supplier selection process increases. In former decades the most important factor for selecting one vendor out of many was offering the lowest price. Increasing the number of suppliers’ selection criteria and also emerging new suppliers which are competing for every inch in the market, the task of selecting the best supplier has become more challenging. According to Barry Schwartz (2004), by increasing the number of choices which one can choose from a list, we are afraid of selecting one item and losing the rest. Consequently, selecting the right green supplier is a difficult decision which should be considered as a multi-criteria decision making (MCDM) problem with a list of qualitative and quantitative criteria.

Different analytical procedures ranging from simple weighted scoring to complex mathematical programming approaches have been used by different researchers to solve supplier selection problem (Mahdiloo et al. 2012). Table 1 shows some of the mostly applied techniques for supplier selection problem.
Table 1. Summary of the applied approaches for supplier selection problem

<table>
<thead>
<tr>
<th>Approach</th>
<th>Authors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS)</td>
<td>Singh et al. (2012), Toloie Eshlaghy and Kalantary (2011)</td>
</tr>
<tr>
<td>Balanced Scorecard (BSC)</td>
<td>Lee et al. (2014), Thanaraksakul and Phruksaphanrat (2009)</td>
</tr>
<tr>
<td>Statistical models</td>
<td>Ndubisi et al. (2005), Rezaei and Ortt (2012)</td>
</tr>
<tr>
<td>Case-Based Reasoning (CBR)</td>
<td>Choy et al. (2005), Zhao and Yu (2011)</td>
</tr>
<tr>
<td>Grey System Theory (GST)</td>
<td>Li et al. (2007), Huixia and Tao (2008)</td>
</tr>
<tr>
<td>Genetic Algorithm (GA)</td>
<td>Che (2010), Yang et al. (2011)</td>
</tr>
<tr>
<td>ELECTRE</td>
<td>Sevkli (2010), Vahdani et al. (2010)</td>
</tr>
<tr>
<td>Stochastic modeling</td>
<td>He et al. (2009), Ekhtiari, and Poursafary (2013)</td>
</tr>
<tr>
<td>Data Envelopment Analysis (DEA)</td>
<td>Weber (1996), Liu et al. (2000), Mahdiloo et al. (2014)</td>
</tr>
</tbody>
</table>

Data envelopment analysis (DEA) also belongs to the mathematical programming approaches and it is logical to classify under mathematical programming section. However, in order to highlight it, due to ever increasing model extensions and applications of DEA, we made an independent category for DEA.
Moreover, there are many papers in literature which have discussed the advantages of using a combination of different approaches. For example see Chai et al. (2013), Wu and Barnes (2011), and Ho et al. (2010). Several applications of these techniques demonstrate a great importance of this subject and the interest of scholars to find and use the best approaches for supplier selection process. Meanwhile, these tools find their own positions for selecting green suppliers by the advent of new criteria for evaluating environmental performance. A comprehensive summery of different approaches used for sustainability assessment of SCM and green supplier selection is provided by Brandenburg et al. (2014), Govindan et al. (2013) and Seuring (2013).

Among all the methods introduced in Table 1, DEA distinguishes itself in the following features (Wong and Wong, 2008; Paradi and Zhu, 2013):

- DEA is able to consider multiple inputs and outputs for efficiency measurement.
- The objectivity stemming from DEA weighting procedure frees the analysis from subjective estimates.
- DEA is nonparametric, i.e., it is free from an assumption related to functional forms of production, and enjoys greater flexibility compare to parametric methods (Bogetoft, 2012, p. 13).
- DEA is highly flexible and simple enough to model and integrate with other different multi criteria, weighing and optimizations methods.
- DEA utilizes the concept of efficient frontier as a measure for performance evaluation. It draws the best practice frontier proportional to performance of peers.
- DEA has the capacity to deal with qualitative and quantitative data simultaneously.
- For each inefficient DMU, DEA introduces benchmark units in order to identify the performance gaps and to evaluate improvement opportunities. And
- By using DEA we can measure performance of DMU over time periods.

DEA is suitable to be used as a tool for performance evaluation of suppliers in order to find and select the greener ones. However, in applying DEA, there are strong arguments for the lack of discrimination power and unrealistic weighting systems (for example, see Adler et al. 2002). Besides, defining a best practice unit as an attainable standard or benchmark for inefficient
suppliers is among great advantages of DEA. Moreover, incorporating the pollutions as a bad output of production is essential. This thesis explores how to construct a DEA approach which can be able to produce more accurate and reliable outcomes for green supplier selection.

1.2 Research objectives and questions

With the features and characteristics of DEA discussed in the previous subsection, DEA is justified to be used as an evaluation tool for green supplier selection. According to the importance of monitoring environmental issues in selecting suppliers which is mentioned earlier, the objective of this study is to apply multiple criteria data envelopment analysis (MCDEA) while considering negative impact of CO\textsubscript{2} emission (undesirable output) produced by suppliers on their efficiencies. In addition, we impose an extra restriction as virtual DMU to MCDEA model in order to compare the performance of suppliers with an ideal peer, which has the best performance in inputs and outputs. The idea is based on selecting the best values of each criterion from the existing suppliers’ data as a new virtual supplier called the ‘virtual best’ DMU. Consequently, the efficiency of each supplier is measured by its distance from the efficiency frontier estimated with the ‘virtual best’ supplier. It is worthwhile to note that by adding a virtual supplier to the set of suppliers, the discrimination power of models increases. This leads to better monitoring and recognizing the suppliers’ efficiencies and rankings (Wu et al. 2007).

By doing this, the new restriction acts as a standard for existing suppliers that their performance is compared with the virtual best supplier. This idea can be more useful in DEA when the intention is to consider the environmental efficiency of suppliers since it is always difficult to define a standard for the level of CO\textsubscript{2} emission. Besides, since there is always room for improvement in the performance of units, defining a target for further performance improvement of efficient units is important in DEA (Sowlati and Paradi, 2004). It means, in addition to finding the benchmark from efficient DMUs for inefficient ones, we can also recognize the benchmark for efficient DMUs.

From the outsourcing point of view, increasing the public and industrial awareness on global warming with serious harm to our environment and living style, the aim of this study is to build a
new MCDEA model to take into account amount of suppliers’ pollution to choose green suppliers. At this stage, the main research question can be defined as follow:

**Research question:** What is the appropriate approach for evaluating and selecting the green supplier in sustainable supply chain management framework?

This main question can be divided into three sub-questions in order to address the different specific aspects of the research problem.

**Sub-question 1:** How to improve the discriminating power of DEA and also weighting system of inputs and outputs?

**Sub-question 2:** What is the right way of modeling undesirable outputs in MCDEA?

**Sub-question 3:** How to define a proper reference DMU for target setting of inefficient DMUs?

The main research question aims to enhance understanding on selecting right tool for assessing the performance of the suppliers, synchronously monitoring their operational and environmental performance. Other three sub-questions also formulated to help researcher to build new DEA model with some plus points compare to traditional DEA models used in green supplier selection problem.

### 1.3 Research methodology

In this section, the theoretical framework of the research is presented first. This is followed by discussing the data collection of the study. Finally, the structure of the research is introduced.

#### 1.3.1 Theoretical framework

Companies and organizations in different sectors started to find most efficient and productive way of doing their business activities while considering environmental issues. SCM is one of the active sectors in this area in which tries to support the concept of sustainability through
incorporating different parts from upstream to downstream into this process. While in traditional SCM, economic performance was playing the main role, lately environmental and social performance became noteworthy features of SCM (Brandenburg et al. 2014). One finds great literature about the theory of green production, GSCM and applications of different performance measurement tools in this filed (e.g., Porter and van der Linde, 1995; Carter and Rogers, 2008; Brandenburg et al. 2014; Govindan et al. 2013; Seuring, 2013).

Widespread understanding of the climate change attracted the attention of the researchers and authorities. It seems that they understood that human activities, especially after industrial revolution along with technological breakthroughs and huge social welfare, are now showing the other side of the coin! A wide variety of different problems caused by global warming forced the people to rethink about how to avoid from aggravating the situation at first step, and improving the condition for next generation as a second concern.

As we discussed earlier, suppliers are part of companies’ networks and business partners in which their actions can directly affect performance and outcomes of companies. Thus, if corporations intend to be successful and increase their own market share, beyond a doubt, there are strong grounds towards better capturing the economic, environmental and social factors contributing to efficiency of suppliers and supply chains. As it is illustrated in Figure 1, we use DEA technique for measuring the efficiency of suppliers in the presence of economic and environmental indicators in GSCM context for GSS. Recent worthwhile studies have increasingly stressed the importance of integrating carbon emission produced by suppliers into evaluation criteria during the supplier assessment process (e.g., Min, H. and Galle, 2001; Genovese et al. 2013; Kannan et al. 2014).
To the best of our knowledge the first application of DEA for supplier selection problem is done by Weber (1996). After this work, many other researchers used DEA for selecting suppliers and new assumptions and variables are added to the problem. Application of DEA for GSS is one of these emerged topics in which researchers, practitioners and decision makers have shown a great interest in promoting and increasing environmental awareness using great merits of DEA.

As a result of integrating wastes and pollutions in supplier’s evaluation framework by buyers, at a rational sight, they should recognize the adverse impact of their bad outputs due to incorporating them into purchasing determinants by organizations. Consequently, suppliers will compete or collaborate to diminish the amount of undesirable factors to their lowest level in order to improve their own environmental performance.

1.3.2 Data collection

Data in research play a vital role to illustrate applicability of proposed concept, idea or model, specifically in quantitative studies. One, perhaps, can refer to data in research as metaphor for a person’s lawyer in the court who helps him/her to protect his/her rights; and a judge (reader) can rely on evidence to interpret what is mentioning and then make a right decision.
Since this study enjoys from mathematical programing as quantitative method for a green supplier selection, access to appropriate data set in order to demonstrate applicability of developed MCDEA model is very important. To this end, according to our research questions, objectives and reviewing the literature, we decided to use secondary cross-sectional data set derived from the latest literature (Kumar et al. 2014) of GSS problem published in prestigious and high ranked journal, OMEGA (The International Journal of Management Science). The Indian automobile spare parts manufacture is the original source of data used by this article.

Data collection from interviews, questionnaires and internal data bases of organizations is very time-consuming task while in most of the cases the validity of answers and data also should be taken into account. Whereas Saunders et al. (2009, p. 268) state that by using secondary data one can have huge saving in resources (e.g., time and money) as main advantage of using this method compare to primary data collection. In the meanwhile, authors believe that “secondary data must be viewed with the same caution as any primary data that is collected”. Furthermore, Stewart and Kamins (1993) advocate a secondary data approach because one already has access to data and there is no need to make trial and error regarding what kind of data set meet your research objectives and questions better. Thus, you can save your time by evaluating quality of data before any further step.

1.3.3 Structure of the study

This study consists of two main parts: it begins with part 1 which is comprised of six chapters divided to different sub-sections. This finishes with part 2 with four publications related to topic of this research. The introduction as the first chapter tries to express overview on the thesis and its purposes through background and research gap, research objectives and questions, research methodology (theoretical framework, data collection). Chapter 2 discusses the background of concepts of supply chain management with its various definitions, sustainability and why efficient production and consumption became more important than before, and the important role of companies by green supplier selection in environmental friendly behavior.

Chapter 3, after briefly explaining some basic concepts of DEA and how it works, aims to review DEA inputs and outputs selection. We will also introduce the concepts of non-discretionary,
undesirable and dual role factors in this section. Besides, the chapter concisely discusses the concepts of orientation, unrealistic weighting schemes and lack of discrimination power in traditional DEA models as some important streams within DEA community, researchers and practitioners. This is followed by proposed MCDEA model which is able to consider undesirable output and virtual best DMU as a benchmark peer in Chapter 4. Thereafter, Chapter 5 is a place to demonstrate applicability of the developed MCDEA model using secondary data derived from literature of GSS. And at final section of part 1, we present our conclusions with limitations and avenues for further research. In addition, part 2 of thesis provides the four related scientific publications of current thesis writer with applications of DEA under different circumstances for supplier selection problem.
2. Literature review

This chapter is structured as follows. Section 2.1 presents the definition and the literature of supply chain management. Section 2.2 briefly discusses the literature of sustainability and explains the importance of sustainability in supply chain management. Finally Section 2.3 reviews the literature on various methodologies applied on green supplier selection.

2.1 Supply chain management

By increasing the knowledge of customers on products and services which is mostly due to easy access to information, organizations endeavor to satisfy the customers in the presence of many competitors in the globalized markets. To this end, organizations have recognized that it is not possible to compete in global race and run thriving business without enjoying from decent SCM framework (Li et al. 2006).

Recently, companies are increasingly realizing that efficient supply chain is their most important asset which leads to providing proper products and services for their own customers. According to the literature, by having such a reliable supply chain, a company can obtain and increase the customer satisfaction and loyalty by offering reasonable price, good quality, precise delivery and product variety. In other words, SCM attempts to meet the objective of increasing productivity by reducing inventory and cycle time in the short run and increasing market share and profits for all members of the supply chain in the long run (Tan et al. 1998). Moreover, Li et al. (2006) believe that effective SCM system plays the main role in success of business by improving the performance of an individual organization, and the whole supply chain at the same time. More interestingly, a study by Sheridan in 1998 found that the organizations that make the best use of SCM benefit a 40% to 65% advantage in their cash-to-cash cycle time and carry 50% to 85% less inventory compared to rivals.

The term “supply chain management” appeared in the late 1980s and then started to become as a very useful concept in the 1990s. Before that, it was famous for its terms such as “logistics” and
“operations management” (Hugos, 2003, p. 2). In the literature, for supply chain management we can find the following definitions:

- “Supply chain management is the coordination of production, inventory, location, and transportation among the participants in a supply chain to achieve the best mix of responsiveness and efficiency for the market being served” (Hugos, 2003, p. 4).
- Chopra and Meindl (2001) declare that supply chain comprises of all the stages which satisfy customer’s desideratum directly or indirectly. These stages contain suppliers, manufacturers, transporters, warehouses, distributors, retailers and the customers.
- “A network of organizations that are involved, through upstream and downstream linkages in the different processes and activities that produce value in the form of products and services in the hand of the ultimate consumer” (Christopher 1998, p. 15; Agrell and Hatami-Marbini, 2013).

Chen and Paulraj (2004) drew a simple version of supply chain structure as illustrated in Figure 2. Based on this definition, “supply chain is a network of materials, information, and services processing links with the characteristics of supply, transformation, and demand”.

![Figure 2. Supply chain structure (Chen and Paulraj, 2004)](image)

In the past decades, a large number of studies investigate the importance and influence of SCM as an interdisciplinary field on enhancing competitive advantage of companies. This involves areas ranging from purchasing and supply, logistics and transportation, operations management, marketing, organizational theory, and management information systems to strategic management.
(Chen and Paulraj, 2004). Recently, environmental sustainability also became among interests of researchers and decision makers. To cite just a few examples, Li et al. (2006) considered a structure for recognizing the relationships among SCM practices (strategic supplier, partnership, customer relationship, level of information sharing, quality of information sharing, and postponement), competitive advantage (price, cost, quality, delivery dependability, product innovation, and time to market), and organizational performance (market performance and financial performance). Tsao and Lu (2012) addressed transportation cost discounts for unified facility location and inventory allocation in supply chain network design.

Gunasekaran and Ngai (2004) in a comprehensive study reviewed the literature of IT in SCM. They classified literature on IT in SCM according to a number of important factors; “(a) Strategic planning for IT in SCM, (b) Virtual enterprise and SCM, (c) E-commerce and SCM, (d) Infrastructure for IT in SCM, (e) Knowledge and IT management in SCM, (f) Implementation of IT in SCM”.

Beamon (1999) asserted the importance of choosing right supply chain performance measures for better analyzing and capturing the operations in supply chain structure. This paper, after reviewing and evaluating performance measures applied in supply chain, presents a new framework for performance measurement in supply chain. Miles and Snow (2007) stressed the importance of different organization theoretical perspectives in SCM. For one thing, it is about strategic choice by integrating exogenous resources into endogenous operations of organization and decision about what to do and what not to do. Also, based upon resources-based view there are possibilities of innovation and cost reduction for organization via supply chain partners by enjoying from their ideas and skills. Besides, in knowledge management perspective, organizations try to form collaborative networks with the aim of knowledge sharing. The main purpose of this stage is to boost overall network performance by learning from each other.
2.2 Sustainability

We can imagine the world as an apartment with a number of tenants in which different people with different life styles are living. Whereas they are independent from each other, they are responsible for their actions and behavior. If in one of the flats, for example, a fire starts by a cigarette, at a glance, the whole apartment will be burned. It is obvious that one’s mistake has a harmful impact on all. On the other hand, tenants sooner or later should move from apartment and new tenants will be replaced. It is not nice and fair, if new tenants have to pay for damages or services which the previous ones caused/used beforehand. We fervently believe that, unfortunately, it is the case with land and environment nowadays.

New technological breakthroughs, at a growing pace, are appearing in a modern life, while at the same time, constrains relating to damage to environment, lack of resources and experiencing inefficient use of these resources are growing pains. As mentioned earlier, negligence in devising a decent plan to deal with climate change and global warming leads to irreparable damage to the environment. This clearly indicates that we need to rethink and revise the way of doing things. To this end, most of companies and organizations started to find most efficient and productive ways of using resources. One of the controversial topics, in this area, is incorporating sustainability with environmental, economic and social perspectives into SCM. To pursue this interest, current thesis tries to benefit from mathematical modelling for considering sustainability in environmental and economic aspects of SCM.

It is worthwhile to emphasize that GSCM can play a fundamental role in competitive advantage of companies by fulfilling the customers benefit. Porter and Van der Linde (1995) declare that customers, directly or indirectly, have to pay the cost of inefficiently used resources and the pollution created, for example, tax for emission and disposal of wastes with no value for customers.

Sarkis et al. (2011) comprehensively reviewed organizational theories in GSCM literature. Their research investigates on nine theories that have been applied for a concept of being green in SCM. These theories are complexity theory, ecological modernization, information theory, institutional theory, resource based-view, resource dependence theory, social network theory,
stakeholder theory, and transaction cost economics. They also introduced diffusion of innovation, path dependency, social embeddedness, and structuration theories as another four organizational theories which are valuable for further investigation and study in GSCM.

Bras (2009) in a systemic and schematic way illustrated sustainability in a closed loop supply chain (Figure 3).

![Figure 3. Representation of sustainability concept (Bras, 2009)](image)

Giunipero et al. (2012) studied the drivers and barriers of implementing sustainable purchasing in SCM. They interviewed 21 leading supply management executives in order to rank derivers and barriers of GSCM. Given their findings, drivers can be ranked from high to low as follows:

- “Top management initiatives;
- Compliance with laws and regulations;
- Competitive differentiator;
- Cost savings;
- Increased resource utilization;
- Customer requirement;
- Competitors adopted;
- Reduce carbon footprint;
- ISO 14000; and
- Government Incentives”.
On the other hand, barriers are prioritized based on importance as follows:

- “Initial buyer and supplier investment;
- Economic uncertainty;
- Short vs. long term goals;
- Lack of regulations;
- Lack of standards;
- Additional burden on suppliers;
- Little top management support;
- Suppliers lack resources;
- External awareness; and
- Policy change difficult”.

Also, Govindan et al. (2014) comprehensively considered barriers of implementing GSCM in companies. They found problem in five main criteria ranging from outsourcing, technology, knowledge, financial, to involvement and support with 45 sub-criteria which are the source of failure in applying environmental concept.

Ageron et al. (2012) developed a framework for GSCM. According to them, enabling conditions and a number of factors need to be considered for success in GSCM; reasons influencing implementation of GSCM, defining performance criteria, greening supply chains, characteristics of suppliers, managerial approaches for GSCM, recognizing barriers to GSCM, as well as benefits and motivations behind GSCM.

Lintona et al. (2007) discussed the convergence of supply chains and sustainability. They believe that embedding sustainability into entire supply chain is useful for a broad range of reasons, for example:

- Considering environmental impact in product design;
- Manufacturing by-products through clean and lean production techniques;
- Using by-products produced during the production to support sale of original product;
- Extending the life of product, end-of-life concept of product by appropriate initial product design; and finally,
- Decent recovery processes at end-of-life concerning variety of products.
According to different regulations on climate change and global warming, which all try to improve the current and future livability of the world, Plambeck (2012) emphasized that companies have to find new solutions for reducing their own direct emissions as well as monitoring and imposing some environmental standards for their suppliers and customers. They can promote environmental awareness of suppliers and customers by expanding their knowledge about the consequences of the climate change and providing them special intensives. Plambeck (2012) considered Walmart as the highest ranked corporation in the world regarding revenue with an effective SCM structure. He believes that by means of emission reduction, Walmart enjoys from reducing costs via energy efficiency, increasing revenues due to consumers attention to green products, enhancing public relations by its efforts for greenhouse gas emissions reduction, increasing employees motivation and having the chance to participate at climate policy makers committees, forcing suppliers to measure and report their emissions according to defined targets, scrutinizing in entire supply chain for all of its 6000 private brand products with the intention of any possible costs and emission reduction, long term purchasing contract to motivate suppliers to invest in equipment to enhance environmental performance, and finally, collaboration with third parties, e.g., nonprofit organizations, academics, suppliers and any other committees whose concern is reducing emissions.

2.3 Green supplier selection

By relying more on suppliers companies have recognized the importance of right supplier selection in increasing product quality, decreasing final product costs, more product variety, on time delivery to markets. Now, in addition to former factors for evaluating suppliers, due to promoting awareness among general public and to be in competition by other organizations, environmental standards and regulators also must be considered. For example, recently, European Union (EU) has set a target of cutting CO₂ emission to near zero by 2050 (Bartocci and Pisani, 2013).

Based upon Matthews et al. (2008), companies produce only 14% of the emissions in entire supply chain before using and disposal of goods. This implies the huge impact of suppliers and other parties in supply chain on producing the rest of the pollution. Therefore, beyond doubt,
suppliers are among high influencer of environmental performance of organizations and decision makers need to take serious actions to raise the ecological efficiency of the supply chains.

Noci (1997) addressed a three phase approach for selecting green suppliers. In the first phase, corporate green strategies of suppliers with regard to re-active and pro-active strategies are reviewed. In the second phase, four main criteria including green competencies, current environmental efficiency, supplier’s green image, and net life cycle cost are recognized. Afterwards, in the final phase, in order to implement supplier selection decision, AHP, as a rating system, is used.

Büyüközkan and Çifçi (2011) applied a fuzzy ANP framework for sustainable supplier selection in Turkish white goods industry. In their approach after assessing the criteria by experts in fuzzy environment, missing values for comparison of criteria are estimated using incomplete preference relations. Then, fuzzy ANP is applied for appraisal of alternatives, and finally, ranking of suppliers are obtained by the optimal solutions of the model.

Awasthi et al. (2010) used fuzzy TOPSIS for assessing environmental performance of suppliers. Their approach entails three steps; i) recognizing “usage of environment friendly technology, environment friendly materials, green market share, partnership with green organizations, management commitment to green practices, adherence to environmental policies, involvement in green projects, staff training, lean process planning, design for environment, environmental certification, and pollution control initiatives” as suitable factors which can be represented environmental performance of suppliers; ii) comparison of the criteria and then aggregating them in order to get overall performance score for each supplier and iii) conducting sensitivity analysis to illustrate the role of each criterion in environmental performance position of suppliers.

For green partner selection in electronic industry in Taiwan, Yeh and Chuang (2011) utilized two multi-objective genetic algorithm in the presence of four objectives (i) production cost and transportation cost should be minimized (ii) production time and transportation time should be minimized (iii) product quality should be maximized, and (iv) the green evaluation scores should
be maximized. After that to help decision makers to select right suppliers, they applied the weighted sum approach to obtain the set of Pareto-optimal solutions.

Kuo et al. (2010) used Delphi technique to find six main dimensions in their green supplier selection procedure, which are quality, cost, delivery, service, environment, and corporate social responsibility. After identifying main and sub-criteria, they combined ANN and ANP into DEA in order to deal with missing values and also to improve discrimination power problem of the model.

Hsu and Hu (2009) discussed the importance of hazardous substance management (HSM) issue in Taiwanese assembly manufacturer of computer products for GSS. They categorized HSM in five dimensions; procurement management, R&D management, process management, incoming quality control, and management system with different number of criteria for each one. They then applied ANP to find the best green supplier.

Mirhedayatian et al. (2014), modeled network data envelopment analysis (NDEA) in the presence of undesirable outputs, dual-role factors, and fuzzy data for assessing 10 soft drinks providers in Iran while taking into account environmental concerns. They considered defective parts per million (PPM) and CO₂ emission as undesirable outputs produced by companies.
3. Data envelopment analysis

DEA was first introduced by Charnes, Cooper, and Rhodes (CCR) in 1978 and it is a linear-programming-based methodology for measuring relative efficiency of decision making units (DMUs) which consume inputs to produce outputs (Charnes et al. 1987).

By using DEA, performance of each DMU is measured relative to other DMUs. Efficiency score is measured as the ratio of the weighted summation of outputs to the weighted summation of inputs. Based on the theory of DEA, the efficiency score of each particular DMU is measured by finding the most optimal weights of inputs and outputs for that particular DMU. However, the efficiency ratio of all DMUs by applying the optimal weights of this particular DMU should be less than or equal to one. Two restrictions should be applied: the first restriction is for imposing the non-negativity among the weights, and the second one is to obtain ratios of not greater than one for each DMU according to the weighting scheme which is applied for all the DMUs in the sample. Consequently, the score for efficient and inefficient DMUs are one and less than one, respectively. This allows to have a weighting scheme to compare DMUs with each other, which cause them to try their best to get the most favorable weights (Anderson et al. 2002). This means DMUs can freely choose the light weights for input and heavy weights for output in a way that maximize their efficiencies, provided that this system of weights be accessible for all the peers. This freedom of choice helps the DMU to be in the best possible situation. By the free imputation of input-output values, inefficient DMUs (suppliers) can be recognized from efficient DMUs in a more logical way. Because all of them are free to choose their own value system and even if after one cannot get good ranking among others, it proves that DMU under evaluation, indubitably, is not efficient (Noorizadeh et al. 2012; Farzipoor Saen, 2010a).

Cook et al. (2014) believe that before applying DEA we need to answer following questions:

- What kinds of favorable outcomes researchers and practitioners are looking for in their performance measurement and analysis?
- What are the subject of assessment and features of DMUs? To what extent can the chosen inputs and outputs describe the performance of those DMUs (Non-discretionary, Undesirable and Dual role factors)?
Should there be any balance between number of DMUs, and the allocated number of inputs and outputs?

Why does one need to choose the right model of orientation based on the strategy of the research (focus on inputs or outputs, or both at the same time)\

We also can add,

What is the highlighted problem of the traditional DEA models with unrealistic weighting schemes?

At this juncture, below, this study tries to answer abovementioned concerns based upon the literature of DEA.

3.1 Purpose of performance measurement

No one can cast on the fact that dramatic changes and progression in different aspects of technology like computers and software, transportation modes and medicine are considerably affected by performance measurement systems to fulfill endless intention and motivation to improve.

According to Bogetoft (2012, p. xi): “Measuring and managing performance is important to anyone, individuals, firms, and organizations. No matter how good we think we are, we can always be better. It requires, however, that we measure performance appropriately and understand what drives performance. In this way, we can learn better practices, make better decisions, and motivate improved performance”.

In this fast moving global business, continuous improvement is the tool to prevent from falling behind the competitors. SCM (GSCM) is one of the areas which should be taken into account for measuring performance during the time, due to its vital role in success or failure of each company (Gunasekaran et al. 2004). Besides, Akyuz and Erkan (2010) discussed several motives for performance measurement in SCM:
• Assessment of success history;
• Considering customer expectation relative to offered products and services;
• Enhanced analyzing for deep discovering of processes;
• Highlighting bottlenecks, waste, problems and potential solutions for them;
• Providing appropriate data and information for decisions;
• Tracking progress; and
• Promoting transparency for clear communication and co-operation.

Therefore, designing and using the efficient and practical structures and tools are necessary. This study aims to achieve this goal by measuring the economic and environmental performance of suppliers in order to find efficient and inefficient ones for further decisions and selection.

3.2 Selection of inputs and outputs

Based upon the purpose of the performance measurement, the inputs and outputs should be carefully selected. The criteria under study must be able to reflect the right outcomes and fulfill the defined purpose of research and not mislead the decision makers. Kleinsorge et al. (1992) believe that “selecting relevant resource inputs and performance outputs and the way they are measured is the most important part of any measurement system”.

In DEA, one needs to consider factors with the nature of cost as input and elements with the nature of benefit as output. In another words, the smaller value of inputs and larger value of outputs represent better performance of DMUs compared to their peers (Cook et al. 2014). There are, however, situations in which we cannot classify inputs and outputs just by the conventional definition above.

3.2.1 Undesirable outputs

In former decades there was not that much attention to undesirable outputs of processes for suppliers’ evaluation problem. However, by increasing knowledge of researchers and decision
makers about lack of resources for future generations and global environmental conservation awareness, bad outputs should be recognized and forced to be decreased. To this end, interest of DEA researchers for modelling undesirable outputs has recently increased.

Based on the assumptions in the conventional DEA models, producing more outputs while consuming fewer inputs is a representative of better performance. However, if there are undesirable outputs in the efficiency assessment, it is logical to select DMUs with more good (desirable) outputs and less bad (undesirable) outputs (Cooper et al. 2007, p. 367). However, how to incorporate and analyze undesirable outputs in DEA in order to fully reflect its impact on the efficiency of the DMUs is a challenge.


In the supplier selection context, Farzipoor Saen (2010b) applied a DEA model for supplier selection in the presence of both undesirable outputs and imprecise data. He considered defective PPM of suppliers as an undesirable output. Noorizadeh et al. (2012) modeled undesirable outputs in the cross-efficiency formulation to provide a complete ranking of suppliers. Mirhedayatian et al. (2014) proposed a network DEA model for performance evaluation of suppliers in food industry. They also considered PPM and CO₂ emission as undesirable outputs of the production.

3.2.2 Non-discretionary variable

In measuring performance of DMUs, it is easy to say that one DMU is less or more efficient than peers just by relying on their apparent inputs and outputs. However, there are non-discretionary (exogenous) variables which can influence the performance of DMUs while they are not under control of DMUs and their management. Traditional studies, in most cases are interested to build the models using controllable factors which steer the decision makers to suppose that the
inefficiencies of DMUs are due to the bad management of DMUs. In this situation, the influence of uncontrollable factors is neglected and outcomes of the models are not reliable to make a right decision (Yang and Pollitt, 2009).

One can find pioneer study considering impact of non-discretionary variable on the efficiency of DMUs by Banker and Morey (1986). They proved how these factors can affect the performance of 60 chain restaurants in the fast food industry. After their study, Ruggiero (1996), Ruggiero (1998), Ruggiero (2004), Syrjänen (2004), Muñiz et al. (2006), and Yang and Pollitt (2009) came up with different style of formulating non-discretionary factors in DEA.

Liu et al. (2000) used non-discretionary factors among their criteria for measuring performance of suppliers. Distance and supply variety are generally considered as non-discretionary factors to select suppliers. Also, Farzipoor Saen (2009), interpreted distance of suppliers from the buyers as a non-discretionary input. Azadi et al. (2012) combined chance-constrained DEA and stochastic data with non-discretionary factors for appropriate supplier selection. Besides, Noorizadeh et al. (2014) modelled supply variety of suppliers as a non-discretionary output. In addition, they categorized the exogenous variables, as is seen in Figure 4. In this clustering, non-discretionary factors first divided into temporary and permanent ones. And then temporary factors intersected to short term and long term.

![Figure 4. Different kinds of non-discretionary factors (Noorizadeh et al. 2014)](image)
3.2.3 Dual-role factor

In applying DEA, determining variables as inputs or outputs is a serious point of measuring the efficiency of DMUs. There are, however, certain circumstances which are not easy for decision makers to determine the role of variables as an input or output. This means that, they let them play the role of inputs and outputs (dual-role) simultaneously, and it is the task of the DEA model to conclude which factor plays the role of an input or an output.

Beasley (1990, 1995) provided the first DEA model to capture the effect of dual-role factors in assessing the efficiency of Chemistry and Physics Departments at 50 UK universities. In these studies, ‘research income’ were treated as both inputs and outputs. Nevertheless, the model proposed by Beasley (1990, 1995) had some problems regarding possibility of obtaining 100% efficiency score with each DMU due to lack of constraints on the multipliers. The dual-role factor also is not treated in a same way on the input and the output sides (Noorizadeh et al. 2012). To overcome the above-mentioned limitations, Cook et al. (2006) applied a new model and presented the advantages of their proposed model with the same data set used by Beasley (1990, 1995). Furthermore, Cook and Zhu (2007) developed a standard constant returns to scale (CRS) DEA model to deal with dual-role (flexible) factors both for an individual DMU case and also for the aggregate efficiency evaluation of the collection of DMUs. However, Toloo (2009) pointed out that the model proposed by Cook and Zhu (2007), because of a computational problem may produce incorrect efficiency scores and introduced a new model to overcome the problem. Amirteimoori and Emrouznejad (2012) believe that the model developed by Toloo (2009) overestimates the efficiency scores and might be infeasible in many real situations. Noorizadeh et al. (2012) proposed a new way of modelling flexible factors in DEA which does not have the problems of the previous studies.

In supply chain setting, Farzipoor Saen (2010a) extended the model proposed by Cook et al. (2006) for considering multiple dual-role factors. In his study, ratings for service-quality experience (EXP) and service quality credence (CRE) of suppliers are treated as dual-role factors. Mahdiloo et al. (2013) and Mirhedayatian et al. (2014) took into account research and
development (R&D) cost of suppliers as a dual-role variable. In addition, carbon foot-print of suppliers also considered as flexible criterion by Kumar et al. (2014).

3.3 Number of DMUs vs. number of inputs and outputs

It is well proved that the discrimination power of DEA models decrease when the number of inputs and outputs increase. However, Cook et al. (2014) believe that it is not always a good idea to decrease the number of efficiency evaluation factors to make a balance with the number of DMUs, since all the criteria might carry valuable data for the analysis. In general, there is a “rule of thumb” that the number of DMUs should be three times more than the number of inputs ($m$) and outputs ($s$). Table 2 shows different authors' perspectives that determine minimum number of DMUs.

<table>
<thead>
<tr>
<th>Authors</th>
<th>Number of DMUs ($n$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boussofiane et al. (1991)</td>
<td>$n &gt; (m \times s)$</td>
</tr>
<tr>
<td>Golany and Roll (1989)</td>
<td>$n &gt; 2(m + s)$</td>
</tr>
<tr>
<td>Bowlin (1998), Sinuany-Stern and Friedman (1998)</td>
<td>$n &gt; 3(m + s)$</td>
</tr>
</tbody>
</table>

For example, with three inputs and three outputs, Boussofiane et al. (1991) recommend having at least nine DMUs.

Here, using a simple numerical example, we show that by increasing the number of inputs and outputs the discrimination power of DEA models can be decreased. Table 3 presents the data set for eight hypothetical DMUs. We calculated the CCR (Model 2) efficiency scores ($\theta$) of eight DMUs twice; first by considering two inputs ($x_1, x_2$) and two outputs ($y_1, y_2$), second by considering three inputs ($x_1, x_2, x_3$) and three outputs ($y_1, y_2, y_3$). The CCR efficiency scores obtained by first setting ($\theta_1$) shows better discrimination among DMUs than CCR efficiency scores by second setting ($\theta_2$).
Table 3. The hypothetical data set and the results of evaluation

<table>
<thead>
<tr>
<th>DMUs</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$y_1$</th>
<th>$y_2$</th>
<th>$y_3$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>4</td>
<td>8</td>
<td>7</td>
<td>8</td>
<td>6</td>
<td>0.9794</td>
<td>0.9794</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>9</td>
<td>7</td>
<td>5</td>
<td>2</td>
<td>8</td>
<td>0.4166</td>
<td>1.0000</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>2</td>
<td>6</td>
<td>9</td>
<td>4</td>
<td>5</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>7</td>
<td>9</td>
<td>0.7859</td>
<td>1.0000</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>9</td>
<td>4</td>
<td>6</td>
<td>9</td>
<td>8</td>
<td>0.5182</td>
<td>1.0000</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>6</td>
<td>0.4500</td>
<td>0.9303</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>4.5</td>
<td>5</td>
<td>4</td>
<td>8</td>
<td>4</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>2</td>
<td>4</td>
<td>7.5</td>
<td>6</td>
<td>7</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

It is worth noting that in order to augment the discrimination power of DEA models, different approaches like super-efficiency analysis (Anderson and Petersen, 1993), MAJ (Mehrabian, Alirezaee and Jahanshahloo, 1999), super slack-based measure (Tone, 2002), cross-efficiency (Sexton et al. 1986) and common set of weights (CSW) (Roll et al. 1991) are developed in the literature.

Farzipoor Saen (2008a) applied super-efficiency analysis for ranking suppliers while they offer discounts for large volumes of purchases. Noorizadeh et al. (2013) considered the efficiency score of 18 suppliers with modified CCR and modified cross-efficiency evaluation models in the presence of non-discretionary inputs. They illustrated how suppliers can be completely ranked by the cross-efficiency matrix without having any tie among efficient suppliers. Furthermore, Bafrooei et al. (2014) benefited from common set of weights for analyzing suppliers in petrochemical industry.

3.4 Model orientation

In most of the circumstances, the main goal of running DEA models is to improve the performance of organizations by identifying reference targets for the inefficient DMUs. Inefficient DMUs can improve their efficiency by maximizing outputs with given inputs (output-oriented) or minimizing inputs with fixed outputs (input-oriented). In order for being successful in implementing each of these scenarios, one needs to know priorities and capabilities of DMUs to emphasize on inputs reduction or outputs expansion. Further, there is the possibility of
focusing on perfect utilizing of inputs and outputs concurrently, if both have the same importance for management (Cook et al. 2014). In the former case, for example, CCR input-oriented or CCR output-oriented can determine the direction of improvement. In the later situation, Charnes et al. (1985), Tone (2001), Färe and Lovell (1978), and Cooper et al. (1999) introduced the models such as: additive, slacks-based measure (SBM), Russell measure and range adjusted measure (RAM), respectively to combine both orientations. Figure 5 uses $x_1$ and $y_1$ from Table 4 to depict orientation concept.

Figure 5. Different types of strategies for increasing the efficiency

It is important to note that in both cases of input and output orientations, the efficiency scores are the same and just benchmark peers due to stress on input or output might change (Cook et al. 2014).

Liu et al. (2000) benefited from CCR input-oriented to evaluate the performance of about 400 suppliers which categorized into 18 groups based on their supplied materials. They incorporated price index, delivery performance and distance factors as inputs and supply variety and quality as outputs into their model.

Weber (1996) applied additive model for measuring vendor efficiency in six baby food manufacturers. Mahdiloo et al. (2014a) used SBM model for supplier benchmarking in the
presence of flexible factors. They provided a comparison and advantages of proposed SBM model with pervious DEA models which consider the flexible criteria. Azadi et al. (2015) suggested an enhanced Russell measure (ERM) model in fuzzy setting for selecting the most sustainable suppliers for a resin company, among 26 raw material providers. This study covered all three dimensions of economic, environmental and social aspect in supplier selection process.

3.6 Unrealistic weighting schemes

According to DEA, each DMU is free in choosing its own inputs and outputs weights. Due to this weight flexibility, sometimes they might allocate very small weights (nearly zero) to some of their inputs and outputs (Cooper et al. 2011). Bal et al. (2008) mentioned that weight restriction techniques are good solution to deal with this problem which can also improve discrimination power among DMUs. In order to take a serious action regarding freedom of inputs and outputs for weights allocation, and permitting for incorporating the decision makers opinions about the priorities of criteria into DEA models, Thompson et al. (1986) suggested to constraint the feasible region of the weights and introduced the assurance region (AR) model. Charnes et al. (1990) presented cone ratio envelopment idea for restricting the weights for bank performances assessment.

Furthermore, some other alternatives purposed to solve unrealistic weighting system. Sexton et al. (1986) introduced cross-efficiency evaluation for peer-appraisal of DMUs instead of self-appraisal. In this model, each DMU should choose same weighting system which all other DMUs also used to obtain their efficiencies (Mahdiloo et al. 2011). Roll et al. (1991) proposed CSW for finding the common average weights for different variables by running a DEA model, and then calculating the efficiency score of all DMUs (Mahdiloo et al. 2014b). In addition to two former solutions, to obtain more rational input and output weights, Li and Reeves (1999) formulated a multiple criteria DEA approach without a priori information about weights provided by decision makers.

To depict the problem of DEA with allocating zero weights to some of inputs and outputs, Table 4 shows the optimal weights \((v_1, v_2, u_1, u_2)\) obtained from data set \((x_1, x_2, y_1, y_2)\) previously
represented in Table 3. Consider for example DMU#7, has determined optimal values of zero to its second input and first output. These optimal weights are the best possible weights to maximize the efficiency score of this DMU.

Table 4. DEA weighting system

<table>
<thead>
<tr>
<th>DMUs</th>
<th>$v_1$</th>
<th>$v_2$</th>
<th>$u_1$</th>
<th>$u_2$</th>
<th>$\theta_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.04509284</td>
<td>0.1936340</td>
<td>0.00596817</td>
<td>0.1172082</td>
<td>0.9794</td>
</tr>
<tr>
<td>2</td>
<td>0.2500000</td>
<td>0.000000</td>
<td>0.08333333</td>
<td>0.000000</td>
<td>0.4166</td>
</tr>
<tr>
<td>3</td>
<td>0.000000</td>
<td>0.5000000</td>
<td>0.08333333</td>
<td>0.062500</td>
<td>1.0000</td>
</tr>
<tr>
<td>4</td>
<td>0.03859649</td>
<td>0.1824561</td>
<td>0.000000</td>
<td>0.1122807</td>
<td>0.7859</td>
</tr>
<tr>
<td>5</td>
<td>0.02143758</td>
<td>0.09205549</td>
<td>0.002837327</td>
<td>0.0557219</td>
<td>0.5182</td>
</tr>
<tr>
<td>6</td>
<td>0.2000000</td>
<td>0.000000</td>
<td>0.05714286</td>
<td>0.0214285</td>
<td>0.4500</td>
</tr>
<tr>
<td>7</td>
<td>0.5000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.1250000</td>
<td>1.0000</td>
</tr>
<tr>
<td>8</td>
<td>0.000000</td>
<td>0.5000000</td>
<td>0.08333333</td>
<td>0.062500</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

In application of DEA for supplier selection, Farzipoor Saen (2008b) believe that the absence of decision makers attitude regarding priority of inputs and outputs increases the chance of making a biased decision on selecting the right suppliers. He imposed weight restrictions into imprecise DEA model for selecting the best suppliers.

Falagario et al. (2012) evaluated 45 suppliers in Italian public procurement. Cross-efficiency evaluation is applied for ranking different bids. To validate the proposed approach, they compared their findings with fuzzy AHP and Linear Weighting method.
4. Proposed multiple criteria DEA model

In this study we use multiple criteria DEA (MCDEA) model to analyze the efficiency of suppliers while monitoring their produced CO₂ emission. The proposed MCDEA model can handle the issue of discrimination power and irrational input and output weighing system. The model is able to take undesirable output of firms and also one additional ideal virtual DMU into account. The main reason to add the virtual best supplier is to highlight the importance of CO₂ emission reduction. Besides, the new restriction will increase discrimination power of the MCDEA model once again.

In DEA, efficiency score, $E_o$, is defined as the ratio of summation of weighted outputs to the summation of weighted inputs. Given $k$ outputs and $m$ inputs the efficiency ratio can be defined as:

$$E_o = \frac{\text{Weighted sum of outputs}}{\text{Weighted sum of inputs}} = \frac{\sum_{r=1}^{k} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}$$

Model (1) is called CCR (Charnes et al. 1978) model and is used to measure the efficiency score of the DMU under assessment (DMU₀) in a sample of $n$ DMUs:

$$\max h_o = \frac{\sum_{r=1}^{k} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}$$

subject to:

$$\sum_{r=1}^{k} u_r y_{rj} / \sum_{i=1}^{m} v_i x_{ij} \leq 1, \quad j = 1, 2, ..., n,$$

$$u_r \geq \varepsilon, \quad r = 1, 2, ..., k,$$

$$v_i \geq \varepsilon, \quad i = 1, 2, ..., m.$$  

Model (1) is not a linear model, since it has a fractional objective function. By applying a standard technique (for more information, see, e.g., Charnes and Cooper, 1962, and Charnes et al. 1978), Model (1) can be converted to a liner model (Model 2).
\[ \text{max } h_o = \sum_{r=1}^{k} u_r y_{ro} \]

subject to:

\[ \sum_{i=1}^{m} v_i x_{io} = 1, \]  \hspace{1cm} (2)

\[ \sum_{r=1}^{k} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \quad j = 1, 2, ..., n, \]

\[ u_r \geq \varepsilon, \quad r = 1, 2, ..., k, \]

\[ v_i \geq \varepsilon, \quad i = 1, 2, ..., m. \]

In this form, assumption is to maximize the weighted sum of outputs with assumed weighted sum of inputs equal to one (input-oriented), whereas if the objective function is minimizing the weighted sum of inputs, weighted sum of given outputs have to be equal to one (output-oriented) (Hollingsworth and Smith, 2003; and Ramanathan, 2003, p. 43). Where Charnes et al. (1981) proposed \( \varepsilon \) as a non-Archimedean constant to insure allocation of the lowest possible positive weights instead of zero to inputs and outputs. The nomenclatures used in this study are:

**DMU\(_o\):** the decision making unit under investigation

**n:** the set of DMUs (suppliers)

**j = 1, ..., n**  \hspace{1cm} collection of DMUs

**r = 1, ..., k**  \hspace{1cm} the set of desirable outputs

**i = 1, ..., m**  \hspace{1cm} the set of inputs

**s = k+1, ..., p**  \hspace{1cm} the set of undesirable outputs

\( y_{ro}^g \): \( r \)th desirable output of the DMU\(_o\)

\( x_{io} \): \( i \)th input of the DMU\(_o\)

\( y_{so}^b \): \( s \)th undesirable output of the DMU\(_o\)

\( \mu_r^g \): the weight for \( r \)th desirable output

\( v_i \): the weight for \( i \)th input

\( \mu_s^b \): the weight for \( s \)th undesirable output

\( y_{rj}^g \): the \( r \)th desirable output of DMU\(_j\)
\( x_{ij} \): the \( i \)th input of DMU\(_j\)

\( y_{sj}^b \): the \( s \)th undesirable output of DMU\(_j\)

\( y_{rg}^g \): the \( r \)th desirable output of ‘virtual best’ DMU

\( y_{sv}^b \): the \( s \)th undesirable output of ‘virtual best’ DMU

\( x_{iv} \): the \( i \)th input of ‘virtual best’ DMU

At this juncture, in order to enable the model to consider undesirable output of production for evaluation of green suppliers, we need to incorporate this new variable into Model (2). Let us suppose that there are \( n \) homogeneous suppliers which consume \( i = 1, 2, \ldots, m \) inputs and produce \( r = 1, 2, \ldots, k \) desirable (good) outputs and \( s = k+1, k+2, \ldots, p \) undesirable (bad) outputs. And let \( y_j \) be decomposed into two parts and, correspondingly, tag them as \( y_j^g \) and \( y_j^b \) for good and bad outputs of DMU\(_j\), and recognize \( x_j \) as inputs consumed by DMU\(_j\), and \( x_{ij} \) the quantity of inputs \( i \) consumed by DMU\(_j\).

To evaluate the performance of the suppliers, in this study, CO\(_2\) emission produced by suppliers is considered as an undesirable output. Now, following Korhonen and Luptacik (2004), Yang and Pollitt (2009), Mahdiloo et al. (2011), and Noorizadeh et al. (2014) undesirable outputs will be treated like inputs in Model (3).

\[
\begin{align*}
\max h_o &= \sum_{r=1}^{k} u_r^g y_{ro}^g \\
\text{subject to:} \\
\sum_{i=1}^{m} v_i x_{io} + \sum_{s=k+1}^{p} u_s^b y_{so}^b &= 1, \\
\sum_{r=1}^{k} u_r^g y_{rg}^g - \sum_{s=k+1}^{p} u_s^b y_{sj}^b - \sum_{i=1}^{m} v_i x_{ij} &\leq 0, \quad j = 1, 2, \ldots, n, \\
u_r^g &\geq \varepsilon, \quad r = 1, 2, \ldots, k, \\
u_s^b &\geq \varepsilon, \quad s = k+1, \ldots, p, \\
v_i &\geq \varepsilon, \quad i = 1, 2, \ldots, m.
\end{align*}
\]
At this time, Model (3) is able to treat undesirable outputs in the assessment process. However, the discrimination power of this model is poor for ranking DMUs, and flexibility of weights which are devoted to inputs and outputs are high. To overcome these limitations, Li and Reeves (1999) proposed MCDEA. Therefore, at this stage, the study enjoys from advantages of MCDEA with more restricted comparison of peers with the aim of increasing discrimination power among them, and more reasonable weights of inputs and outputs. San Cristóbal (2011) addressed three plus points of MCDEA; i) in this scenario, the linear programming using variety of devoted weights to obtain optimal solutions can be compared to an efficient DMU in traditional DEA with non-unique optimal solution; ii) using MOLP, model can generate optimal weights that exclusively seek to define best possible solutions for each objective; however, other non-dominated solutions under any given inputs/outputs are also valuable to provide alternative options for a DMU under evaluation. In certain circumstances, some of these non-dominated solutions can be more beneficial in contrast with those optimizing individual objectives; iii) one can analyze and estimate the possibility of variability/stability of a DMU’s efficiency score relative to the changes in inputs/outputs via calculating the non-dominated solutions associated with a DMU.

Li and Reeves (1999) benefited from multiple objective linear programming (MOLP) structure for modelling a multiple criteria DEA approach in which three different objective functions are defined under the same constraints. Corresponding deviation variable form of proposed MCDEA, Model (4), can be defined as follows:

\[
\begin{align*}
\min \quad & d_o \\
\text{subject to:} \quad & \sum_{i=1}^{m} v_i x_{io} + \sum_{s=k+1}^{p} u_s^b y_{so} = 1, \\
& \sum_{r=1}^{k} u_r^g y_{rj} - \sum_{s=k+1}^{p} u_s^g y_{sj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0 \quad j = 1, 2, ..., n, \\
& u_r^g \geq \varepsilon, \quad r = 1, 2, ..., k, \\
& u_s^g \geq \varepsilon, \quad s = k+1, ..., p, \\
& v_i \geq \varepsilon, \quad i = 1, 2, ..., m, \\
& d_j \geq 0, \quad j = 1, 2, ..., n.
\end{align*}
\]
Where $d_o$ is the deviation variable for DMU$_o$ and $d_j$ is the deviation variable for the $j^{th}$ DMU. Unit under evaluation, in Model (4), is efficient if and only if $d_o = 0$. The $d_o$ can be interpreted as the inefficiency score of the DMU under assessment. Efficiency score of the DMU can be calculated as $h_o = 1$ (Li and Reeves 1999). Model (5) is another form of Model (4), minimizing $d_o$, with two new objective functions, minimizing the maximum deviation and minimizing the sum of the deviations.

$$
\text{min} \quad d_o
$$

$$
\text{min} \quad M,
$$

$$
\text{min} \quad \sum_{j=1}^{n} d_j
$$

subject to:

$$
\sum_{i=1}^{m} v_i x_{io} + \sum_{s=k+1}^{p} u_s^b y_{so} = 1, \quad (5)
$$

$$
\sum_{r=1}^{k} u_r^a y_{rj} - \sum_{s=k+1}^{p} u_s^b y_{sj} - \sum_{i=1}^{m} v_l x_{ij} + d_j = 0 \quad j = l, 2, \ldots, n,
$$

$$
M - d_j \geq 0
$$

$$
u_r^a \geq \epsilon, \quad r = l, 2, \ldots, k,
$$

$$
u_s^b \geq \epsilon, \quad s = k+1, \ldots, p,
$$

$$
v_l \geq \epsilon, \quad i = l, 2, \ldots, m,
$$

$$
d_j \geq 0, \quad j = l, 2, \ldots, n.
$$

Where the first objective function is the same as the objective function in Model (4), $M$ as the second objective function shows the maximum quantity among all deviation variables $d_j \ (j = l, 2, \ldots, n)$, and third one is representing sum of the deviations of all DMUs. Li and Reeves (1999) emphasized that the feasible region for decision variables in MCDEA is the same as traditional DEA models and the designation of added constrains, $M - d_j \geq 0$, is to make $M$ the maximum deviation.

In Model (5) DMU$_o$ is efficient if and only if $d_o = 0$ for optimizing the first objective function. DMU$_o$ is minimax efficient if and only if $d_o = 0$ for minimizing the second objective function.
In a same way, DMU\(_o\) is *minsum* efficient if and only if \(d_o = 0\) for minimizing the third objective function. It should be noticed that in this DEA model, efficiency score is \(1 - d_o\) for a unit under assessment, regardless of being efficient or not (Li and Reeves, 1999). Moreover, they believe that *minsum* and *minmax* scenario will identify fewer efficient DMUs compared to the classical DEA models.

However, Model (5) might not also be able to give a complete ranking of the DMUs. To this end, in the next step, a “virtual best” DMU as a new restriction is incorporated into Model (5) with the aim of evaluating DMUs (suppliers) not just only with real counterparts but also with a virtual supplier which has an ideal performance. Virtual DMU is made by choosing the best possible inputs and outputs that is consumed and produced with real suppliers. This technique changes the existing efficiency frontier and compares the suppliers with a new frontier (Wu and Blackhurst, 2009). As far as we know, the idea of creating virtual DMU (unobserved) was first proposed by Thanassoulis and Allen (1998) for an alternative approach to weights restrictions. Thereafter, Appalla (2003) applied the concept of virtual best DMU as an augmented DEA for enhancing discrimination power among efficient suppliers. Sowlati and Paradi (2004) introduced practical DEA model by adding artificial DMUs into a set of observed units.

Therefore, Model (6) is an augmented MCDEA model with the capability of considering undesirable outputs.

\[
\begin{align*}
\text{min} & \quad d_o \\
\text{min} & \quad M \\
\text{min} & \quad \sum_{j=1}^{n} d_j + d_v, \\
\text{subject to:} & \\
\sum_{i=1}^{m} v_i x_{io} + \sum_{s=k+1}^{p} u_s^a y_{so}^a = 1, \\
\sum_{r=1}^{m} u_r^g y_{rj}^g - \sum_{s=k+1}^{p} u_s^b y_{sj}^b - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0
\end{align*}
\]
\[
\sum_{r=1}^{k} u_r^g y_{rv}^g - \sum_{s=k+1}^{p} u_s^b y_{sv}^b - \sum_{i=1}^{m} v_i x_{iv} + d_v = 0 \quad j = 1, 2, \ldots, n,
\]

\[
M - d_j \geq 0, \quad j = 1, 2, \ldots, n,
\]

\[
M - d_v \geq 0,
\]

\[
u_r^g \geq \varepsilon, \quad r = 1, 2, \ldots, k,
\]

\[
u_s^b \geq \varepsilon, \quad s = k+1, \ldots, p,
\]

\[
v_i \geq \varepsilon, \quad i = 1, 2, \ldots, m,
\]

\[
d_j \geq 0, \quad j = 1, 2, \ldots, n,
\]

\[
d_v \geq 0,
\]

The aim of creating a virtual standard in the current research is to prove that each supplier can perform in a best scenario in terms of its inputs, outputs, and especially undesirable outputs. In another words, if supplier A (standard DMU), for example, is able to consume fewer \(x\) to produce more \(y^g\) and less \(y^b\) compare to peers, other suppliers also should be able to do so. We can define a standard virtual DMU in the following way (Noorizadeh et al. 2012).

\[
y_{rv}^g = \max (y_{rj}), \quad r = 1, 2, \ldots, k, \quad j = 1, 2, \ldots, n,
\]

\[
y_{sv}^b = \min (y_{sj}), \quad s = k+1, \ldots, p, \quad j = 1, 2, \ldots, n, \quad (7)
\]

\[
x_{iv} = \min (x_{ij}), \quad i = 1+2, \ldots, m, \quad j = 1, 2, \ldots, n.
\]

\[
d_v = \text{deviation variable}
\]

As mentioned earlier, the virtual best DMU improves the discriminating power of Model (6) and yields complete ranking of suppliers. Albeit, the main aim of adding this extra constrain into model is to having a DMU as a standard which is using resources to produce products in an efficient way. In other words, in the eco-efficiency perspective “doing more with less, or producing economic output with minimal natural resources and environmental degradation” (Kuosmanen, 2005).
5. Numerical example

In order to illustrate the applicability of the developed model, the problem of green supplier selection is introduced. The dataset for this study is taken from Kumar et al. (2014). Tables 5 and 6 depict the definitions of the criteria and the dataset for 18 suppliers, respectively.

Table 5. The Criteria for evaluation of green suppliers’ performance (Adapted from Kumar et al. 2014)

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$: Net Price (Indian Rupees per kilogram)</td>
<td>the offered net price by supplier.</td>
</tr>
<tr>
<td>$x_2$: Distance (Km)</td>
<td>the supplier distance from the manufacture.</td>
</tr>
<tr>
<td>$x_3$: Shelf Life (Months)</td>
<td>longevity of product supplied.</td>
</tr>
<tr>
<td>$x_4$: Lead Time (Days)</td>
<td>average delivery time taken by the supplier to deliver an order.</td>
</tr>
<tr>
<td>$y_1^g$: Industry Position and Rating (Likert 1-5)</td>
<td>the position in industry (production leadership, reputation, financial position and credit rating of each supplier).</td>
</tr>
<tr>
<td>$y_2^g$: Grade and Finishing (Likert 1–5)</td>
<td>the grade for steel tubes supplied; finish quality and the ability to meet quality specifications consistently.</td>
</tr>
<tr>
<td>$y_3^g$: Past Business and communication (Likert 1–5)</td>
<td>the number of past business transactions which has been done with each supplier.</td>
</tr>
<tr>
<td>$y_4^g$: Performance (Likert1–5)</td>
<td>the performance history of each supplier (profit achieved per unit net price).</td>
</tr>
<tr>
<td>$y_1^b$: CO$_2$ emissions (Metric Tonnes)</td>
<td>the overall carbon footprint produced by each supplier.</td>
</tr>
</tbody>
</table>
Table 6. Data set for 18 suppliers

<table>
<thead>
<tr>
<th>Suppliers (DMUs)</th>
<th>Inputs</th>
<th>Outputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x_{1j}$</td>
<td>$x_{2j}$</td>
<td>$x_{3j}$</td>
</tr>
<tr>
<td>1</td>
<td>83</td>
<td>1659</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>76</td>
<td>1499</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>77</td>
<td>1355</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>85</td>
<td>1440</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>84</td>
<td>459</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>84</td>
<td>1749</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>77</td>
<td>146</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>75</td>
<td>1371</td>
<td>12</td>
</tr>
<tr>
<td>9</td>
<td>77</td>
<td>637</td>
<td>5</td>
</tr>
<tr>
<td>10</td>
<td>81</td>
<td>266</td>
<td>8</td>
</tr>
<tr>
<td>11</td>
<td>82</td>
<td>1774</td>
<td>7</td>
</tr>
<tr>
<td>12</td>
<td>82</td>
<td>1408</td>
<td>12</td>
</tr>
<tr>
<td>13</td>
<td>76</td>
<td>138</td>
<td>5</td>
</tr>
<tr>
<td>14</td>
<td>77</td>
<td>607</td>
<td>10</td>
</tr>
<tr>
<td>15</td>
<td>75</td>
<td>1545</td>
<td>10</td>
</tr>
<tr>
<td>16</td>
<td>85</td>
<td>212</td>
<td>6</td>
</tr>
<tr>
<td>17</td>
<td>77</td>
<td>53</td>
<td>5</td>
</tr>
<tr>
<td>18</td>
<td>75</td>
<td>456</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 7, displays the efficiency scores obtained by Models (2) and (3). We calculated the efficiency of suppliers without and with taking undesirable output into account. As is seen, suppliers 1, 4, 7, 10, 13, and 17 received the efficiency score of 1 in both cases. Also, the efficiency score of 11 suppliers decreased in the second scenario. However, except suppliers 2, 6, 9, and 15, rankings of the other suppliers remained the same. Therefore, we need to have a model that does not suffer from lack of discrimination power and be able to distribute the inputs and outputs weights more evenly. To do so, Model (5) is applied for better ranking and monitoring the negative impact of CO₂ emission on performance of suppliers.
Table 7. Results of Models (2) and (3)

<table>
<thead>
<tr>
<th>DMUs</th>
<th>Efficiency scores obtained by Model (2)</th>
<th>Rank</th>
<th>Efficiency scores obtained by Model (3)</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.863</td>
<td>10</td>
<td>0.863</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>0.702</td>
<td>16</td>
<td>0.680</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>0.773</td>
<td>12</td>
<td>0.757</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>0.742</td>
<td>14</td>
<td>0.739</td>
<td>13</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>0.928</td>
<td>7</td>
<td>0.921</td>
<td>7</td>
</tr>
<tr>
<td>9</td>
<td>0.759</td>
<td>13</td>
<td>0.737</td>
<td>14</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>0.620</td>
<td>17</td>
<td>0.609</td>
<td>17</td>
</tr>
<tr>
<td>12</td>
<td>0.733</td>
<td>15</td>
<td>0.713</td>
<td>15</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>14</td>
<td>0.592</td>
<td>18</td>
<td>0.586</td>
<td>18</td>
</tr>
<tr>
<td>15</td>
<td>0.870</td>
<td>9</td>
<td>0.847</td>
<td>10</td>
</tr>
<tr>
<td>16</td>
<td>0.903</td>
<td>8</td>
<td>0.893</td>
<td>8</td>
</tr>
<tr>
<td>17</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>18</td>
<td>0.814</td>
<td>11</td>
<td>0.808</td>
<td>11</td>
</tr>
</tbody>
</table>

Table 8 shows the results of Model (5). The efficiency scores of the suppliers obtained by Model (5) when the objective is minimizing $d_o$ is identical to results of Model (3) in Table (7). Nevertheless, the weights in MCDEA model (Table 8) are distributed more equally throughout the inputs and outputs than those in CCR model (Table 7) (Li and Reeves 1999). Then, Model (5) is run with the two other objective functions of minisum and minimax. Now the number of the efficient suppliers have significantly decreased to two efficient suppliers (suppliers 7 and 13) in minisum and only supplier 13 in minimax scenario. Besides, the contrast between inefficient suppliers can be observed better by changing their ranking. For example, see the ranking of the suppliers 11 and 18 obtained with three different objective functions of Model (5). Supplier 11 is ranked as 17, 14, and 11 in minimizing $d_o$, minisum and minimax objective functions, respectively. Also supplier 18 is ranked 11, 7, and 2 with the same objective functions.

Based on the results of the objective function of minimizing $d_o$, there are 6 suppliers with the efficiency score of 1. Also with the minisum function still two suppliers 7 and 13 have the efficiency score of 1 and are not ranked. Therefore, the objective functions of minimizing $d_o$ and
minisum could not give a complete ranking of suppliers. Minimax objective function identified only one supplier as efficient. However, there is no guarantee that minimax objective function can always provide a complete ranking. As a result, the MCDEA model with the virtual best supplier is applied (Model 6) and the results are displayed in Table 9.

Table 8. Results of Model (5)

<table>
<thead>
<tr>
<th>DMUs</th>
<th>Minimizing $d_o$</th>
<th>Rank</th>
<th>Minisum</th>
<th>Rank</th>
<th>Minimax</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>1</td>
<td>0.743</td>
<td>6</td>
<td>0.479</td>
<td>17</td>
</tr>
<tr>
<td>2</td>
<td>0.863</td>
<td>9</td>
<td>0.790</td>
<td>4</td>
<td>0.584</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>0.680</td>
<td>16</td>
<td>0.462</td>
<td>12</td>
<td>0.492</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>1.000</td>
<td>1</td>
<td>0.471</td>
<td>11</td>
<td>0.580</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>0.757</td>
<td>12</td>
<td>0.384</td>
<td>16</td>
<td>0.593</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>0.739</td>
<td>13</td>
<td>0.299</td>
<td>18</td>
<td>0.492</td>
<td>15</td>
</tr>
<tr>
<td>7</td>
<td>1.000</td>
<td>1</td>
<td>1.000</td>
<td>1</td>
<td>0.562</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>0.921</td>
<td>7</td>
<td>0.390</td>
<td>15</td>
<td>0.737</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>0.737</td>
<td>14</td>
<td>0.364</td>
<td>17</td>
<td>0.490</td>
<td>16</td>
</tr>
<tr>
<td>10</td>
<td>1.000</td>
<td>1</td>
<td>0.908</td>
<td>3</td>
<td>0.501</td>
<td>12</td>
</tr>
<tr>
<td>11</td>
<td>0.609</td>
<td>17</td>
<td>0.457</td>
<td>14</td>
<td>0.521</td>
<td>11</td>
</tr>
<tr>
<td>12</td>
<td>0.713</td>
<td>15</td>
<td>0.457</td>
<td>13</td>
<td>0.495</td>
<td>13</td>
</tr>
<tr>
<td>13</td>
<td>1.000</td>
<td>1</td>
<td>1.000</td>
<td>1</td>
<td>1.000</td>
<td>1</td>
</tr>
<tr>
<td>14</td>
<td>0.586</td>
<td>18</td>
<td>0.479</td>
<td>10</td>
<td>0.524</td>
<td>10</td>
</tr>
<tr>
<td>15</td>
<td>0.847</td>
<td>10</td>
<td>0.664</td>
<td>8</td>
<td>0.537</td>
<td>9</td>
</tr>
<tr>
<td>16</td>
<td>0.893</td>
<td>8</td>
<td>0.746</td>
<td>5</td>
<td>0.462</td>
<td>18</td>
</tr>
<tr>
<td>17</td>
<td>1.000</td>
<td>1</td>
<td>0.641</td>
<td>9</td>
<td>0.557</td>
<td>8</td>
</tr>
<tr>
<td>18</td>
<td>0.808</td>
<td>11</td>
<td>0.711</td>
<td>7</td>
<td>0.751</td>
<td>2</td>
</tr>
</tbody>
</table>

As you can see, this model can fully rank all suppliers with three different objective functions of MCDEA. With different objective functions, supplier 13 receives the highest efficiency scores, and is the first candidate for selection. In our example, minimax objective function is more restrictive than the other two objective functions. Therefore, for the final assessment and selection of the suppliers, minimax objective function is used. Figure 6 depicts changes of the suppliers ranking by three different objective functions of Model (6). As is shown in this figure, suppliers 4, 5, 8, 14 and 18 significantly improved their rankings when minimax creation is applied; however, supplier 9 and 16 experienced considerable reduction in their ranking in minimax setting compared to the other two objective functions.
Table 9. Results of Model (6)

<table>
<thead>
<tr>
<th>DMUs</th>
<th>Minimizing $d_o$</th>
<th>Rank</th>
<th>Minisum</th>
<th>Rank</th>
<th>Minimax</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.650</td>
<td>13</td>
<td>0.591</td>
<td>10</td>
<td>0.465</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>0.842</td>
<td>4</td>
<td>0.842</td>
<td>3</td>
<td>0.516</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>0.507</td>
<td>18</td>
<td>0.330</td>
<td>15</td>
<td>0.439</td>
<td>17</td>
</tr>
<tr>
<td>4</td>
<td>0.685</td>
<td>11</td>
<td>0.305</td>
<td>16</td>
<td>0.525</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>0.672</td>
<td>12</td>
<td>0.672</td>
<td>9</td>
<td>0.547</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>0.614</td>
<td>15</td>
<td>0.297</td>
<td>17</td>
<td>0.461</td>
<td>14</td>
</tr>
<tr>
<td>7</td>
<td>0.757</td>
<td>8</td>
<td>0.757</td>
<td>6</td>
<td>0.483</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>0.694</td>
<td>10</td>
<td>0.520</td>
<td>12</td>
<td>0.694</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>0.711</td>
<td>9</td>
<td>0.711</td>
<td>8</td>
<td>0.456</td>
<td>15</td>
</tr>
<tr>
<td>10</td>
<td>0.897</td>
<td>2</td>
<td>0.718</td>
<td>7</td>
<td>0.472</td>
<td>12</td>
</tr>
<tr>
<td>11</td>
<td>0.593</td>
<td>16</td>
<td>0.297</td>
<td>17</td>
<td>0.474</td>
<td>11</td>
</tr>
<tr>
<td>12</td>
<td>0.617</td>
<td>14</td>
<td>0.463</td>
<td>13</td>
<td>0.447</td>
<td>16</td>
</tr>
<tr>
<td>13</td>
<td>0.977</td>
<td>1</td>
<td>0.977</td>
<td>1</td>
<td>0.902</td>
<td>1</td>
</tr>
<tr>
<td>14</td>
<td>0.544</td>
<td>17</td>
<td>0.363</td>
<td>14</td>
<td>0.488</td>
<td>8</td>
</tr>
<tr>
<td>15</td>
<td>0.826</td>
<td>5</td>
<td>0.826</td>
<td>4</td>
<td>0.503</td>
<td>7</td>
</tr>
<tr>
<td>16</td>
<td>0.857</td>
<td>3</td>
<td>0.857</td>
<td>2</td>
<td>0.406</td>
<td>18</td>
</tr>
<tr>
<td>17</td>
<td>0.784</td>
<td>6</td>
<td>0.764</td>
<td>5</td>
<td>0.487</td>
<td>9</td>
</tr>
<tr>
<td>18</td>
<td>0.761</td>
<td>7</td>
<td>0.571</td>
<td>11</td>
<td>0.753</td>
<td>2</td>
</tr>
</tbody>
</table>

Figure 6. Comparison of results obtained by different objective functions of Model (6)
6. Conclusions

Developing new and complex products and services, growing specialization and ever increasing customers’ expectations highlighted the importance of allocating the tasks and jobs to different professional suppliers, as external capabilities (Dolgui and Proth, 2013). Selecting the right suppliers significantly influence the cost, profitability and flexibility of the companies (Ting and Cho, 2008). Weber et al. (1991) emphasized that being successful in the global market by low-cost and high-quality products is not an achievable goal without access to proper suppliers. However, previously environmental and social factors were ignored in evaluation and selection of the suppliers. Negligence in devising effective measurement systems for controlling the environmental impacts of companies has caused climate change and irreversible damages to the environment. Moreover, managers and decision makers have widely acknowledged the importance of sustainable actions of their suppliers’ on the cost of the final products and also their social image. Therefore, measuring the efficiency of suppliers with considering their environmental impacts is an important task.

In this study, we used multiple criteria DEA (MCDEA) model to evaluate and select the green suppliers. The MCDEA model can handle the issue of discrimination power and irrational input and output weighing system. In MCDEA, three different objective functions of minimizing $d_o$, minimizing the maximum deviation and minimizing the sum of the deviations operate under MOLP structure. (San Cristóbal, 2011). The amount of CO$_2$ emission of suppliers is incorporated into MCDEA model as an undesirable output. Finally, to provide a complete ranking among suppliers, a virtual best supplier, which is made by the best inputs and outputs values in the data, is applied. By applying the virtual best supplier, the efficiency frontier is changed and DMUs are compared with a better reference target.

6.1 Limitations and suggestions for further research

We applied MCDEA model under CRS assumption. However, in real world applications, the economic scale of companies should also be considered for efficiency measurement. To this end, the performance of the suppliers should be analyzed under the variable returns to scale (VRS) assumption (Banker et al. 1984). By solving the models with CRS and VRS technologies, the
MCDEA efficiency scores can be decomposed into technical and scale efficiencies and help suppliers to identify the source of their inefficiencies for further improvement. Also most productive scale size of units can be recognized where suppliers are able to get the most outputs per input (Bogetoft, 2012, p.100). Besides, given the widespread use of DEA for green supplier selection problem, it is important to apply other efficiency evaluation tools on the same data set. We suggest to apply stochastic non-smooth envelopment of data (StoNED) and stochastic frontier analysis (SFA) and use their results to validate the results obtained by DEA.
References


Part II: Publications
Article I


© 2014 Inderscience Enterprises Ltd. retains the copyright of the paper.
A new model for ranking suppliers in the presence of both undesirable and non-discretionary outputs

Abdollah Noorizadeh
Department of Industrial Management,
Lappeenranta University of Technology,
P.O. Box 20, 53851 Lappeenranta, Finland
E-mail: ab.noorizadeh@gmail.com

Mahdi Mahdiloo
Department of International Business and Asian Studies,
Griffith Business School, Gold Coast Campus,
Griffith University,
QLD 4222, Australia
E-mail: mahdi.mahdiloo@griffithuni.edu.au

Reza Farzipoor Saen*
Department of Industrial Management,
School of Management and Accounting, Karaj Branch,
Islamic Azad University,
Karaj, P.O. Box 31485-313, Iran
Fax: 0098 (26) 34418156
E-mail: farzipour@yahoo.com
*Corresponding author

Abstract: Data envelopment analysis (DEA) can be used for supplier selection problem due to its multiple criteria nature. In suppliers’ evaluation, there might be some factors, which are beyond the control of their management, that are needed to be modelled in an appropriate way. Also, there are some situations in which some factors are undesirable and they are favourable to be decreased. The aim of this paper is to propose a model for evaluation of suppliers’ performance in the presence of both undesirable and non-discretionary outputs. This model can rank efficient suppliers by a super-efficiency DEA model. A numerical example has sought to demonstrate that the proposed model is actually applicable.

Keywords: supplier selection; undesirable output; non-discretionary output; super-efficiency.

1 Introduction

Supply chain management (SCM) is widely used by pioneer companies for improvement of their competitiveness strength. Chopra and Meindl (2001) declare that supply chain comprises all the stages which satisfy customer’s desideratum whether directly or indirectly. These stages include suppliers, manufacturers, transporters, warehouses, distributors, retailers and the customers. Furthermore, among all the activities performed to manage the supply chain, suppliers’ evaluation plays a crucially important role. Every decision made in the supply chain is directly affected by the evaluation and selection of the suppliers. To increase competitive advantage, improve end-user satisfaction by high-quality products, reduce purchasing costs and in general, enhance the efficiency of supply chain, it is essential to select right suppliers (Kumar et al., 2004; Ordoobadi and Wang, 2011; Noorizadeh et al., 2013; Choudhary and Shankar, 2013). Since equal up to
70% of the product cost is composed by the raw materials and component parts (Ghodsypour and O’Brien, 1998), purchasing managers play a key role in reducing the final products costs by selecting good suppliers.

On the other hand, benchmarking is a managerial tool that can be used in the process of suppliers’ evaluation. Evaluation of suppliers enables companies to distinguish efficient and inefficient suppliers in comparison with each other. After recognition of inefficient suppliers, overall efficiency of supply chain can be increased by benchmarking from efficient suppliers. Benchmarking provides a means of determining how well a business unit or organisation is performing in comparison to similar units. This provides a broader perspective for the use of performance measure as well as a measure of ‘best practice’ (Parker, 2000).

The rest of this paper proceeds as follows. In Section 2, literature review is presented. Section 3, introduces the model which selects the suppliers. Numerical example and concluding remarks are discussed in Sections 4 and 5, respectively.

2 Literature review

To select the vendors, Weber and Current (1993) used a multi-objective programming problem. Weber (1996) used data envelopment analysis (DEA) to evaluate vendors and to find benchmark values for each inefficient vendor. Mohammady Garfamy (2006) using the data for a hypothetical firm, applied DEA and total cost of ownership (TCO) concept to compare and select the suppliers. To evaluate distribution centres performance trends, Ross and Droge (2002) applied windows analysis using four years data. To select the best suppliers, Vokurka et al. (1996) integrated expert system technology with a decision-support framework. They also incorporated subjective judgements of purchasing experts into their expert system. Using a scoring method and fuzzy expert systems approach, Kwong et al. (2002) carried out suppliers’ assessment. To select suppliers and assign the optimal amount order quantities, which should be bought from each supplier, Özgen et al. (2008) proposed a combination of the analytic hierarchy process (AHP) and a multi-objective possibilistic linear programming (MOPLP). Choudhary and Shankar (2013) proposed an integer linear programming approach for joint decision-making of multi-period procurement lot-sizing, supplier selection, and carrier selection problem. They believe that proposed model is able to simultaneously determine the timings of procurement, lot-sizes, suppliers and carriers in an appropriate way.

Ertay et al. (2011) used an integrated method based on fuzzy AHP and ELECTRE III to build a decision support system for supplier evaluation and selection in the presence of quantitative and qualitative criteria. Labib (2011) compared fuzzy logic and AHP to support the decision of the selection of the appropriate supplier. Mishra et al. (2012) suggested a combination of the multi-attribute decision-making (MADM), fuzzy sets theory and VIKOR method to select suppliers. Azadi et al. (2013) applied a goal directed benchmarking theory for benchmarking and selecting suppliers in an uncertain environment and in the presence of fuzzy data.

Lasch and Janker (2005) used multivariate analysis for suppliers rating purpose. Ndubisi et al. (2005) applied a multiple regression model for supplier selection. Considering the assumptions that the suppliers cannot supply perfect quality items, the capacity of suppliers is limited, the amount of demand is predicted, and the buyer has a maximum storage capacity in each period, Rezaei and Davoodi (2008) solved the
A new model for ranking suppliers

supplier selection problem. Ustun and Demirtas (2008) considered time horizon and a number of tangible and intangible criteria in their proposed two-stage method for supplier selection problem. They also determined suppliers optimum order allocations using the proposed approach.

Noorizadeh et al. (2013) applied DEA cross-efficiency evaluation for suppliers ranking in the presence of non-discretionary inputs in order to complete ranking of suppliers and avoiding from unrealistic weighting schemes. Mahdiloo et al. (2012) developed an algorithm for ranking suppliers in the presence of volume discount offers in terms of multiple criteria in the context of cross-efficiency evaluation. To select the suppliers, Farzipoor Saen (2010) incorporated both undesirable outputs and imprecise data into a single DEA model. However, he did not consider non-discretionary outputs in his model.

In the case of undesirable outputs, examples from different areas can be found in Yaisawarng and Klein (1994), Färe et al. (1989, 1996), Pittman (1983), Korhonen and Luptacik (2004), Mahdiloo et al. (2011) and Barros et al. (2012). Yang and Pollitt (2009) incorporated undesirable outputs as well as non-discretionary inputs simultaneously into a DEA model and analysed the performance of Chinese coal-fired power plants. Although they took into account undesirable outputs and non-discretionary inputs, their model cannot give a complete ranking among all decision-making units (DMUs) and there may exist lack of discrimination among efficient DMUs. Golany and Roll (1989), and Bowlin (1998) argued that the lack of discrimination power occurs when there are insufficient DMUs or the number of inputs and outputs is too high relative to the number of DMUs. Therefore, our paper differentiates itself from Yang and Pollitt (2009) from two aspects. Firstly, we consider undesirable outputs as well as non-discretionary ones in supplier selection context. Secondly, our model can rank all efficient DMUs.

To the best of knowledge of authors, there is no paper to evaluate suppliers in the presence of both undesirable and non-discretionary outputs. The proposed model ranks all DMUs applying super-efficiency concept. The objective of this paper is to propose a model dealing with both undesirable and non-discretionary outputs via super-efficiency DEA model for ranking the suppliers. The contributions of this paper are as below:

1 Proposed model considers undesirable outputs. In many cases, there are situations in which some outputs are allowed to be decreased. Take for instance, defective parts detected by buyer, which are undesirable outputs, are favourable to decrease.

2 Proposed model considers non-discretionary outputs.

3 Proposed model discusses the evaluation of suppliers’ performance in the presence of both undesirable and non-discretionary outputs and also can rank efficient suppliers by applying super-efficiency DEA model. Therefore, proposed model does not suffer from lack of discrimination power.

3 Proposed model

DEA was first developed by Charnes et al. (1978) as a non-parametric programming technique to evaluate the relative efficiency of homogenous DMUs. The weighted sum of outputs divided by the weighted sum of inputs is defined as the efficiency score of each DMU (Liu et al., 2000). In DEA, producing more outputs and consuming fewer inputs is
generally considered as a measure of efficiency. However, when some outputs are undesirable, DMUs with more desirable outputs relative to less undesirable outputs and inputs are supposed to be efficient (Cooper et al., 2007). To select suppliers, in this paper, defective parts per million (PPM) is considered as an undesirable output.

### Table 1  Nomenclatures

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DMU&lt;sub&gt;o&lt;/sub&gt;</td>
<td>The decision-making unit under investigation</td>
</tr>
<tr>
<td>j = 1, ..., n</td>
<td>Collection of DMUs</td>
</tr>
<tr>
<td>r = 1, ..., k</td>
<td>The set of desirable outputs</td>
</tr>
<tr>
<td>i = 1, ..., m</td>
<td>The set of inputs</td>
</tr>
<tr>
<td>s = k + 1, ..., p</td>
<td>The set of undesirable outputs</td>
</tr>
<tr>
<td>y&lt;sub&gt;r&lt;/sub&gt;&lt;sup&gt;o&lt;/sup&gt;</td>
<td>The r&lt;sup&gt;th&lt;/sup&gt; desirable output of the DMU&lt;sub&gt;o&lt;/sub&gt;</td>
</tr>
<tr>
<td>x&lt;sub&gt;i&lt;/sub&gt;&lt;sup&gt;o&lt;/sup&gt;</td>
<td>The i&lt;sup&gt;th&lt;/sup&gt; input of the DMU&lt;sub&gt;o&lt;/sub&gt;</td>
</tr>
<tr>
<td>y&lt;sub&gt;s&lt;/sub&gt;&lt;sup&gt;o&lt;/sup&gt;</td>
<td>The s&lt;sup&gt;th&lt;/sup&gt; undesirable output of the DMU&lt;sub&gt;o&lt;/sub&gt;</td>
</tr>
<tr>
<td>μ&lt;sub&gt;r&lt;/sub&gt;</td>
<td>The weight for r&lt;sup&gt;th&lt;/sup&gt; desirable output</td>
</tr>
<tr>
<td>v&lt;sub&gt;i&lt;/sub&gt;</td>
<td>The weight for i&lt;sup&gt;th&lt;/sup&gt; input</td>
</tr>
<tr>
<td>μ&lt;sub&gt;s&lt;/sub&gt;</td>
<td>The weight for s&lt;sup&gt;th&lt;/sup&gt; undesirable output</td>
</tr>
<tr>
<td>μ&lt;sub&gt;rD&lt;/sub&gt;</td>
<td>The weight for r&lt;sup&gt;th&lt;/sup&gt; desirable and discretionary output</td>
</tr>
<tr>
<td>μ&lt;sub&gt;rF&lt;/sub&gt;</td>
<td>The weight for r&lt;sup&gt;th&lt;/sup&gt; desirable and non-discretionary output</td>
</tr>
<tr>
<td>y&lt;sub&gt;r&lt;/sub&gt;&lt;sup&gt;j&lt;/sup&gt;</td>
<td>The r&lt;sup&gt;th&lt;/sup&gt; desirable output of DMU&lt;sub&gt;j&lt;/sub&gt;</td>
</tr>
<tr>
<td>x&lt;sub&gt;i&lt;/sub&gt;&lt;sup&gt;j&lt;/sup&gt;</td>
<td>The i&lt;sup&gt;th&lt;/sup&gt; input of DMU&lt;sub&gt;j&lt;/sub&gt;</td>
</tr>
<tr>
<td>y&lt;sub&gt;s&lt;/sub&gt;&lt;sup&gt;j&lt;/sup&gt;</td>
<td>The s&lt;sup&gt;th&lt;/sup&gt; undesirable output of DMU&lt;sub&gt;j&lt;/sub&gt;</td>
</tr>
<tr>
<td>θ</td>
<td>Efficiency measure for DMU&lt;sub&gt;o&lt;/sub&gt;</td>
</tr>
<tr>
<td>s&lt;sub&gt;r&lt;/sub&gt;</td>
<td>Shortages in r&lt;sup&gt;th&lt;/sup&gt; desirable output</td>
</tr>
<tr>
<td>s&lt;sub&gt;i&lt;/sub&gt;</td>
<td>Excesses in i&lt;sup&gt;th&lt;/sup&gt; input</td>
</tr>
<tr>
<td>s&lt;sub&gt;s&lt;/sub&gt;</td>
<td>Excesses in s&lt;sup&gt;th&lt;/sup&gt; undesirable output</td>
</tr>
<tr>
<td>λ&lt;sub&gt;j&lt;/sub&gt;</td>
<td>Reference weights associated with DMU&lt;sub&gt;j&lt;/sub&gt;</td>
</tr>
<tr>
<td>O&lt;sub&gt;D&lt;/sub&gt;</td>
<td>The set of discretionary and desirable outputs</td>
</tr>
<tr>
<td>O&lt;sub&gt;F&lt;/sub&gt;</td>
<td>The set of non-discretionary and desirable outputs</td>
</tr>
<tr>
<td>ε</td>
<td>Defined as an infinitesimal constant (a non-Archimedean quantity)</td>
</tr>
</tbody>
</table>

Model (1) is based on fractional Charnes, Cooper, and Rhodes (CCR) (Charnes et al., 1978) model. Following Korhonen and Luptacik (2004) and also Yang and Pollitt (2009), undesirable outputs are incorporated into Model (1) like inputs. Briefly, the applied notations have been addressed in the nomenclature (Table 1).
A new model for ranking suppliers

\[
\begin{align*}
\max h_A &= \sum_{r=1}^{k} \mu_r \bar{y}_r \varepsilon \\
\sum_{i=1}^{m} v_i x_i + \sum_{s=k+1}^{p} \mu_s y_{so} \leq 1, & \quad j = 1, 2, \ldots, n, \\
\sum_{r=1}^{k} \mu_r \bar{y}_r \varepsilon \\
\sum_{i=1}^{m} v_i x_i y_{ij} + \sum_{s=k+1}^{p} \mu_s y_{ij} \leq 1, & \quad j = 1, 2, \ldots, n,
\end{align*}
\]

(1)

\[
\mu_r \geq \varepsilon, \quad r = 1, 2, \ldots, k,
\]

\[
v_i \geq \varepsilon, \quad i = 1, 2, \ldots, m,
\]

\[
\mu_s \geq \varepsilon, \quad s = k + 1, \ldots, p
\]

\[\varepsilon > 0 \text{ (non-Archimedean)}.
\]

Using a standard technique proposed by Charnes et al. (1978), Model (1) can be converted into a linear programming problem as follows:

\[
\begin{align*}
\max h_B &= \sum_{r=1}^{k} \mu_r \bar{y}_r \\
\sum_{i=1}^{m} v_i x_i + \sum_{s=k+1}^{p} \mu_s y_{so} &= 1, \\
\sum_{r=1}^{k} \mu_r \bar{y}_r - \sum_{s=k+1}^{p} \mu_s y_{so} - \sum_{i=1}^{m} v_i x_i &\leq 0, & \quad j = 1, 2, \ldots, n,
\end{align*}
\]

(2)

\[
\mu_r \geq \varepsilon, \quad r = 1, 2, \ldots, k,
\]

\[
v_i \geq \varepsilon, \quad i = 1, 2, \ldots, m,
\]

\[
\mu_s \geq \varepsilon, \quad s = k + 1, \ldots, p
\]

\[\varepsilon > 0 \text{ (non-Archimedean)}.
\]

Therefore, Model (2) is a linear multiplicative CCR model which can treat undesirable outputs. Model (3) is the dual (envelopment) form of Model (2). This model suggests improvement targets for inefficient DMUs to become efficient.

\[
\begin{align*}
\min h_C &= \theta - \varepsilon \bigg( \sum_{r=1}^{k} s_r^p + \sum_{i=1}^{m} s_i^h + \sum_{s=k+1}^{p} s_s^h \bigg) \\
\sum_{r=1}^{n} y_r^p \lambda_j - s_r^p &= \bar{y}_r^p, & \quad r = 1, 2, \ldots, k,
\end{align*}
\]

(3)

\[
\sum_{i=1}^{m} x_i \lambda_j + s_i^h - \theta x_i = 0, & \quad i = 1, 2, \ldots, m,
\]

\[
\sum_{s=k+1}^{p} y_s^h \lambda_j - \theta y_s^h + s_s^h = 0, & \quad s = k + 1, \ldots, p
\]

\[\lambda_j > 0, \quad j = 1, 2, \ldots, n,
\]

\[s_i^h \geq 0, \quad i = 1, 2, \ldots, m,
\]

\[s_r^p \geq 0, \quad r = 1, 2, \ldots, k,
\]

\[s_s^h \geq 0, \quad s = k + 1, \ldots, p
\]

\[\varepsilon > 0 \text{ (non-Archimedean)}.
\]

Supply variety is one of the criteria used in this paper for supplier selection problem. Liu et al. (2000) considered this factor as a non-discretionary output. It might be asked why
this factor should be considered as a non-discretionary factor? In order to clarify the issue, Figure 1 depicts the different kinds of non-discretionary factors including temporary and permanent factors. The temporary factors refer to those factors that can be controlled by the DMU after a short period of time. For example, despite the fact that suppliers can increase supply variety by spending too much expenses, it is impossible for them to increase it in short-term. Therefore, we call these kinds of factors as temporary and short-term non-discretionary factors. Moreover, there are some other factors that are not under control of managers in short-term and it takes long time of the suppliers to change these types of factors. The suppliers’ distance from the buyer is an explicit example for this kind of temporary and long-term non-discretionary factor. Permanent non-discretionary factors refer to those which by no means can be controlled by the DMU. For instance, in the efficiency evaluation of farming lands, amount of rain as an input, is out of the control permanently.

Figure 1 Different kinds of non-discretionary factors

Therefore, to incorporate undesirable output (PPM) and non-discretionary output (supply variety) into a single model simultaneously, Model (4) is developed that is based on Banker and Morey’s (1986) idea for the inclusion of non-discretionary outputs in DEA models.

\[
\begin{align*}
\text{min } & h_D = \theta - e \left( \sum_{r=1}^{k} s^p_r + \sum_{i=1}^{m} s^p_i + \sum_{s=k+1}^{p} s^p_s \right) \\
\text{s.t. } & \quad \sum_{j=1}^{n} y^p_j \lambda_j - s^p = y^p_0, \quad r = 1, 2, ..., k, \quad r \in O^p \cup O^y, \\
& \quad \sum_{j=1}^{n} x_i \lambda_j - \theta x_i + s_i = 0, \quad i = 1, 2, ..., m, \\
& \quad \sum_{j=1}^{n} y^p_j \lambda_j - \theta y^p_0 + s^p = 0, \quad s = k + 1, ..., p, \\
\lambda_j & \geq 0, \quad j = 1, 2, ..., n, \\
\lambda_i & \geq 0, \quad i = 1, 2, ..., m, \\
s^p_i & \geq 0, \quad i = 1, 2, ..., m, \\
s^p_r & \geq 0, \quad r = 1, 2, ..., k, \quad r \in O^p \cup O^y, \\
s^p_s & \geq 0, \quad s = k + 1, ..., p, \\
e & > 0, \text{ (non-Archimedean).}
\end{align*}
\]

(4)
A new model for ranking suppliers

where \( \lambda \) is intensity vector, determining ‘best practice’ for the DMU. The variable \( s^g \) addresses shortages in desirable outputs. \( s^g \) and \( s^b \) correspond to excesses in inputs and undesirable outputs, respectively. The DMU is efficient in the presence of both undesirable outputs and non-discretionary outputs, if and only if \( h_d = 1 \), i.e., \( \theta = 1 \), \( s^g = 0 \), \( s^b = 0 \). Notice that the slack \( s^{g}, r \in O^g \) are omitted from the objective function. Since the levels of non-discretionary outputs are not subject to managerial control, these have nothing to do with minimising the efficiency score of DMU by the entire output vector’s slacks. Such a minimisation should be determined only with respect to the slacks which are composed of discretionary outputs. From another point of view, to ensure that no priority is given to any slack associated with non-discretionary outputs, these slacks are eliminated from the objective function. This property can be shown in the Model (5) which is the dual form of Model (4).

\[
\text{max } h_E = \sum_{r \in O^g} \mu^g_{Dr} y^g_{r0} + \sum_{r \in O^b} \mu^b_{Fr} y^b_{r0}
\]

s.t.

\[
\begin{align*}
\sum_{i=1}^m v_i x_{i0} + \sum_{r=k+1}^p \mu^g_{Fr} y^g_{r0} & = 1 \\
\left( \sum_{r \in O^g} \mu^g_{Dr} y^g_{r0} + \sum_{r \in O^b} \mu^b_{Fr} y^b_{r0} \right) & - \left( \sum_{i=1}^m v_i x_{i0} + \sum_{r=k+1}^p \mu^g_{Fr} y^g_{r0} \right) & \leq 0, \quad j = 1, \ldots, n
\end{align*}
\]

\( v_i \geq \varepsilon \quad i = 1, \ldots, m \)

\( \mu^g_{Fr} \geq \varepsilon \quad s = k + 1, \ldots, p \)

\( \mu^D_{Dr} \geq \varepsilon \quad r = 1, \ldots, k \)

\( \mu^b_{Fr} \geq 0 \quad r = 1, \ldots, k \)

As is seen above, in the case the slacks associated with the non-discretionary outputs would not be omitted from the objective function of Model (4), \( \mu^D_{Fr} \) will be greater than or equal to \( \varepsilon \) instead of \( \mu^D_{Fr} \geq 0 \) in Model (5).

Hence, these non-discretionary outputs do not enter directly into the efficiency measures being optimised in the objective function of Model (4). They can, nevertheless, affect the efficiency evaluations by virtue of their presence in the constraints. Outcome of Model (4) is an efficiency score equal to one to efficient DMUs and less than one to inefficient DMUs.

Although Models (4) and (5) can give a complete ranking of inefficient DMUs, they are not able to rank efficient DMUs thoroughly. Lack of discrimination among the efficient suppliers is a problem that might be occurred when DEA method is employed to select the suppliers. In particular, this problem happens when there are not sufficient suppliers or the number of inputs and outputs is too high relative to the number of suppliers. The model proposed by Anderson and Petersen (1993) has the advantages of the basic DEA models and also allows differentiating among the efficient units. In this model, to construct the new efficiency frontier, the dataset related to DMU is excluded from the reference set. The exclusion of an efficient DMU might change the efficiency frontier. Now each efficient DMU has a super-efficiency score greater than or equal to
100%, which is derived by the distance of efficient DMU due to the new frontier. This technique has been termed ‘super-efficiency analysis’.

At this juncture, in order to derive the complete ranking of suppliers, we incorporate both undesirable and non-discretionary outputs in a super-efficiency DEA model.

\[
\begin{align*}
\min h_F &= \theta - \epsilon \left( \sum_{r=1}^{k} s_r^g + \sum_{i=1}^{m} s_i^g + \sum_{s=k+1}^{p} s_s^b \right) \\
\text{s.t.} & \quad \sum_{j=1}^{n} y_{ij}^p \lambda_j - s_j^g = y_{ij}^g, \quad r = 1, 2, ..., k, \quad r \in O_{ij}^g \cup O_{ij}^b, \\
& \quad \sum_{j=1}^{n} x_{ij} \lambda_j - \theta x_{ij} + s_i^g = 0, \quad i = 1, 2, ..., m, \\
& \quad \sum_{j=1}^{n} y_{ij}^b \lambda_j - \theta y_{ij}^b + s_s^b = 0, \quad s = k + 1, ..., p, \\
& \quad \lambda_j \geq 0, \quad j = 1, 2, ..., n, \\
& \quad s_j^g \geq 0, \quad i = 1, 2, ..., m, \\
& \quad s_j^g \geq 0, \quad r = 1, 2, ..., k, \quad r \in O_{ij}^g \cup O_{ij}^b, \\
& \quad s_s^b \geq 0, \quad s = k + 1, ..., p \\
& \quad \epsilon > 0 \text{ (non-Archimedean)}. 
\end{align*}
\]

Notice that the efficiency scores from this model are obtained through eliminating the DMU from the reference set. Thus, the efficient DMUs have super-efficiency score greater than or equal to 1. Since the exclusion of inefficient DMUs cannot affect the efficiency frontier, their super-efficiency score would be the same as their simple efficiency score. In the next section, a numerical example is presented.

4 Numerical example

For illustrative purposes, the problem of supplier selection is introduced. The dataset for this example is partially taken from Liu et al. (2000). Tables 2 and 3 depict the definition of the criteria and the dataset for 18 suppliers where price has been used as an input for selecting suppliers. And the outputs been utilised in this study are supply variety, delivery performance and PPM which are non-discretionary desirable output, discretionary desirable output and undesirable output, respectively.

Table 2  The criteria for evaluation of suppliers performance

<table>
<thead>
<tr>
<th>(x_1): Price;</th>
<th>(y_i^s): Supply variety; the number of parts that a supplier supplies is considered as an output and is known as a non-discretionary output variable.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y_i^d): Delivery performance; the delivery performance is represented by the percentage of purchase orders delivered within the delivery window according to the purchase orders.</td>
<td></td>
</tr>
<tr>
<td>(y_i^f): PPM; defective parts per million (PPM) detected by the buyer.</td>
<td></td>
</tr>
</tbody>
</table>
A new model for ranking suppliers

Table 3  Dataset of input and outputs for 18 suppliers

<table>
<thead>
<tr>
<th>Supplier</th>
<th>$x_1$</th>
<th>$y_1^g$</th>
<th>$y_2^g$</th>
<th>$y_3^g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>2</td>
<td>90</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>13</td>
<td>80</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>3</td>
<td>90</td>
<td>21.3</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>3</td>
<td>90</td>
<td>30</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
<td>24</td>
<td>90</td>
<td>13.8</td>
</tr>
<tr>
<td>6</td>
<td>100</td>
<td>28</td>
<td>90</td>
<td>18.6</td>
</tr>
<tr>
<td>7</td>
<td>100</td>
<td>1</td>
<td>85</td>
<td>30</td>
</tr>
<tr>
<td>8</td>
<td>100</td>
<td>24</td>
<td>97</td>
<td>26.4</td>
</tr>
<tr>
<td>9</td>
<td>100</td>
<td>11</td>
<td>90</td>
<td>25.8</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
<td>53</td>
<td>100</td>
<td>25.8</td>
</tr>
<tr>
<td>11</td>
<td>100</td>
<td>10</td>
<td>95</td>
<td>21.9</td>
</tr>
<tr>
<td>12</td>
<td>100</td>
<td>7</td>
<td>98</td>
<td>14.7</td>
</tr>
<tr>
<td>13</td>
<td>100</td>
<td>19</td>
<td>90</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>100</td>
<td>12</td>
<td>90</td>
<td>6.3</td>
</tr>
<tr>
<td>15</td>
<td>80</td>
<td>33</td>
<td>95</td>
<td>0</td>
</tr>
<tr>
<td>16</td>
<td>100</td>
<td>2</td>
<td>95</td>
<td>15.9</td>
</tr>
<tr>
<td>17</td>
<td>80</td>
<td>34</td>
<td>95</td>
<td>0</td>
</tr>
<tr>
<td>18</td>
<td>100</td>
<td>9</td>
<td>85</td>
<td>30</td>
</tr>
</tbody>
</table>

Table 4  Results of evaluation by Model (4)

<table>
<thead>
<tr>
<th>Supplier</th>
<th>Efficiency scores</th>
<th>Reference set</th>
<th>$s_i$</th>
<th>$s_i^g$</th>
<th>$s_i^f$</th>
<th>$s_i^h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.7389</td>
<td>$\lambda_{15} = 0.947$</td>
<td>0</td>
<td>29.26</td>
<td>0</td>
<td>18.95</td>
</tr>
<tr>
<td>2</td>
<td>0.6535</td>
<td>$\lambda_{15} = 0.842$</td>
<td>0</td>
<td>14.79</td>
<td>0</td>
<td>20.21</td>
</tr>
<tr>
<td>3</td>
<td>0.7418</td>
<td>$\lambda_{15} = 0.947$</td>
<td>0</td>
<td>28.26</td>
<td>0</td>
<td>16.14</td>
</tr>
<tr>
<td>4</td>
<td>0.7352</td>
<td>$\lambda_{15} = 0.947$</td>
<td>0</td>
<td>28.26</td>
<td>0</td>
<td>22.74</td>
</tr>
<tr>
<td>5</td>
<td>0.7474</td>
<td>$\lambda_{15} = 0.947$</td>
<td>0</td>
<td>7.26</td>
<td>0</td>
<td>10.46</td>
</tr>
<tr>
<td>6</td>
<td>0.7438</td>
<td>$\lambda_{15} = 0.947$</td>
<td>0</td>
<td>3.26</td>
<td>0</td>
<td>14.09</td>
</tr>
<tr>
<td>7</td>
<td>0.6943</td>
<td>$\lambda_{15} = 0.895$</td>
<td>0</td>
<td>28.53</td>
<td>0</td>
<td>21.47</td>
</tr>
<tr>
<td>8</td>
<td>0.7953</td>
<td>$\lambda_{15} = 1.021$</td>
<td>0</td>
<td>9.69</td>
<td>0</td>
<td>21.56</td>
</tr>
<tr>
<td>9</td>
<td>0.7383</td>
<td>$\lambda_{15} = 0.947$</td>
<td>0</td>
<td>20.26</td>
<td>0</td>
<td>19.55</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>$\lambda_{10} = 1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>0.7824</td>
<td>$\lambda_{15} = 1$</td>
<td>0</td>
<td>23</td>
<td>0</td>
<td>17.52</td>
</tr>
<tr>
<td>12</td>
<td>0.8131</td>
<td>$\lambda_{15} = 1.032$</td>
<td>0</td>
<td>27.04</td>
<td>0</td>
<td>12.13</td>
</tr>
<tr>
<td>13</td>
<td>0.7579</td>
<td>$\lambda_{15} = 0.947$</td>
<td>0</td>
<td>12.26</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>0.7531</td>
<td>$\lambda_{15} = 0.947$</td>
<td>0</td>
<td>19.26</td>
<td>0</td>
<td>4.77</td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>$\lambda_{15} = 1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>16</td>
<td>0.7873</td>
<td>$\lambda_{15} = 1$</td>
<td>0</td>
<td>31</td>
<td>0</td>
<td>12.72</td>
</tr>
<tr>
<td>17</td>
<td>1</td>
<td>$\lambda_{17} = 1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>18</td>
<td>0.6943</td>
<td>$\lambda_{15} = 0.895$</td>
<td>0</td>
<td>20.53</td>
<td>0</td>
<td>21.47</td>
</tr>
</tbody>
</table>
Table 4 shows the results of evaluation using Model (4). Outcome of Model (4) is efficiency score of one for the efficient DMUs and less than one for the inefficient DMUs. Therefore, suppliers 10, 15, and 17 are efficient and other suppliers are inefficient. Inefficient suppliers can use these results from a marketing perspective. If a particular supplier is poorly performing, then the supplier can use the analysis results for benchmarking purposes. This result may be interpreted so that the supplier should reduce the input as well as undesirable output and also provide better performance on desirable outputs. For instance, supplier 16 is an inefficient supplier; thus, supplier 15 is chosen as the benchmark supplier for supplier 16 ($\lambda_{15} = 1$). Since $s_{i}^e = 31$, supplier 16 must increase its own supply variety to 33 in long-term. And $s_{i}^b = 12.72$ means that supplier 16 should reduce PPM to 3.18.

It is obvious that for the inefficient suppliers a complete ranking is given; however, efficient suppliers are not ranked. Consequently, the next step is to select the best supplier among those three efficient suppliers applying the developed super-efficiency model [Model (6)].

Table 5 displays the super-efficiency and ranking results obtained by using Model (6). The suppliers have been ranked based on their objective values in descending order.

As Table 5 implies, supplier 10, by objective value of 1.2149, received the highest value and suppliers 17 and 15 were introduced as second and third candidates for selection.

Table 5  Results of evaluation by Model (6)

<table>
<thead>
<tr>
<th>Supplier rank</th>
<th>Supplier no. (DMU)</th>
<th>Objective value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>1.2149</td>
</tr>
<tr>
<td>2</td>
<td>17</td>
<td>1.0303</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>1</td>
</tr>
</tbody>
</table>

5  Concluding remarks

Considering the recent widely spread economic crisis, for companies to survive, it is vital to apply various tools and methods to reduce costs. Based on the fact that in manufacturing industries, the raw materials and component parts comprise up to 70% of total product cost, selecting efficient suppliers is one of the most important roles of decision-makers (Ghodsypour and O’Brien, 1998). Consequently, it is essential to select the suppliers possess a lower price index, lower PPM rate, more supply variety, and better delivery performance. Among the aforementioned criteria, delivery performance is considered as desirable output and price is incorporated as input. Moreover, PPM and supply variety are considered as undesirable output and non-discretionary output, respectively.

This paper proposed an innovative method facilitating the supplier selection problem by super-efficiency technique that takes into account both undesirable and non-discretionary outputs. Furthermore, in order to improve the performance of poorly performing suppliers, improvement targets for inefficient suppliers are determined. Using
A new model for ranking suppliers

A numerical example, we demonstrated that decision-makers should use super-efficiency model if they are interested in a complete ranking of suppliers.

Further researches can be done based on the results of this paper. For instance, the developed model can be extended to consider dual-role factors. The behaviour of these factors as inputs or outputs is not known and can be determined after running the DEA model (Farzipoor Saen, 2011). In addition, DEA is suitable for supplier performance analysis over time. Malmquist (1953) index can be used to measure the growth in the efficiency of suppliers which have cooperation with the company. The results of this paper might be extended to a DEA-based Malmquist index to measure the efficiency growth over time. Noteworthy as well, using this tool, company can recognise the poor performing suppliers to cut collaboration with them.

Acknowledgements

The authors wish to thank the anonymous reviewer for the valuable suggestions and comments which improved the quality of this paper. The authors also would like to express thanks to Mr. Afshin Abrishamkar for proof reading of the manuscript.

References


Article II


This article is © 2014 Emerald Group Publishing and permission has been granted for this version to appear here (http://dx.doi.org/10.1108/BIJ-10-2012-0068). Emerald does not grant permission for this article to be further copied/distributed or hosted elsewhere without the express permission from Emerald Group Publishing Limited.
Benchmarking suppliers’ performance when some factors play the role of both inputs and outputs: A new development to the slacks-based measure of efficiency

Mahdi Mahdiloo
Department of International Business and Asian Studies, Griffith Business School, Gold Coast Campus, Griffith University, Brisbane, Australia

Abdollah Noorizadeh
Department of Industrial Management, Lappeenranta University of Technology, Lappeenranta, Finland

Reza Farzipoor Saen
Department of Industrial Management, Karaj Branch, Islamic Azad University, Karaj, Iran

Abstract
Purpose – The purpose of this paper is to develop a slack-based measure (SBM) model in the presence of dual-role factor. Then it is applied in supplier selection problem.

Design/methodology/approach – The developed model in this paper is based on data envelopment analysis (DEA) technique.

Findings – The proposed evaluation platform is capable of identifying ill-performing suppliers which seek to future improvement. The findings provide valuable insights for practitioners, as well as academicians, policy makers and also integrate selection criteria under the supply chain.

Originality/value – This is the first time that a non-radial DEA model considers dual-role factors. The proposed model does not deal with dual-role factor as a non-discretionary factor. The proposed model considers dual-role factors on both the input and output sides in a similar manner. The proposed model can fully measure the inefficiency of suppliers. The proposed model can give a complete ranking of suppliers.

Keywords Benchmarking, Data envelopment analysis

Paper type Research paper

1. Introduction
Supply chain refers to an integrated value system of suppliers, manufacturers, subcontractors, distributors and retailers working together with the prime purpose of creating value to the end-
In today’s competitive business environment, identifying and developing the activities that can help improve the relationship between the buyer and the supplier is quite essential. Selecting the good suppliers is one of the most important activities that can help sustaining the relationship between the buyer and the supplier. In the current competitive environment, suppliers are important resources for manufacturers. They have a large and direct impact on the costs and the quality of the products (Handfield et al., 1999). Talluri and Narasimhan (2004) emphasize that selecting and managing suppliers for strategic and long term partnerships is a key ingredient for the success of a supply chain. In order to launch new products, meet the demand of consumers and make the new products competitive in terms of quality and costs, manufacturers should focus on selecting good suppliers and try to develop the relationship with them. An efficient purchase department can continuously save the operating costs of the enterprise and consequently affect the overall profitability of the enterprise (Che and Wang, 2008). For an appropriate selection of suppliers, lots of works can be found in the literature. They are ranging from analytic hierarchy process (AHP), fuzzy set theory, case-based reasoning (CBR), artificial neural network (ANN) to data envelopment analysis (DEA). Table 1 categorizes some of these works. Also, strengths and weaknesses of different methods are discussed in this table.

Table 1. A summary of methods for suppliers selection

<table>
<thead>
<tr>
<th>Technique</th>
<th>Authors</th>
<th>Strengths</th>
<th>Weaknesses</th>
</tr>
</thead>
</table>
| AHP       | Barbarosoglu and Yazgac (1997), Chan (2003), Hou and Su (2007), Özgen et al. (2008), Liao and Kao (2010), Yang et al. (2010). | ● It is a practical and easily understandable way of transferring subjective judgments into weights of attributes and score of suppliers.  
● It can use the benefits of group decision-making.  
● It is well supported by the user-friendly software of Expert Choice. This software can show the problem in a hierarchy with three levels, i.e. the purpose of solving the problem, the criteria of solving the problem, and different decision making alternatives. Sensitivity analysis is also available in the software which allows decision makers to know the effects of changes of the weights of criteria and alternatives on the final decision.  
● The analyses on the AHP are based on the pairwise comparison, which helps people judge better. | ● The comparison between two criteria is represented by two different scales (Stewart, 1992).  
● Since the analyses are conducted based on the subjective judgments of a particular expert or group of experts, the analyst should re-analyze the problem when some of the experts are altered by others or even when a new expert’s opinion is to be included in the decision-making process.  
● As the number of criteria and suppliers increases, the experts might make more inconsistent judgments.  
● If the number of criteria and suppliers is high, constructing pairwise comparison matrices will be very time-consuming.  
● In a group decision-making process, when different weights, based on the |
The validity evaluation of the AHP results can be carried out by measuring the inconsistency of responses in the decision making process. If the opinion of different experts are obtained, the analyst uses their average to finalize the decision. However, due to the variances, this weight can not reflect all the experts’ opinions (Mahdiloo et al., 2011).

<table>
<thead>
<tr>
<th>Method</th>
<th>Authors</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuzzy set theory</td>
<td>Li et al. (1997), Holt (1998), Lin and Chen (2004), Chang et al. (2006), Jain et al. (2004), Hsu et al. (2010), Kuo et al. (2010a), Vinodh et al. (2011)</td>
<td>- It can deal with the uncertainty of decision makers, uncertain data, and qualitative data.</td>
<td>- It is difficult for practitioners with poor mathematical background to understand the fuzzy sets.</td>
</tr>
<tr>
<td>CBR</td>
<td>Choy et al. (2004a), Choy et al. (2004b), Choy et al. (2005), Lau et al. (2005), Faez et al. (2009), Zhaoa and Yu (2011)</td>
<td>- It captures all ‘memory’ of human being without losing them due to lapse of time and carelessness (El-Sawalhi et al., 2007).</td>
<td>- In cases where there is no similar or approximate solution, the system will give incorrect solution (El-Sawalhi et al., 2007).</td>
</tr>
<tr>
<td></td>
<td>Kuo et al. (2010b), Qian (2009), Luo et al. (2009), Wei et al. (1997)</td>
<td>- Both quantitative and qualitative criteria can be used simultaneously.</td>
<td>- To obtain weights of items, some supplemental methods need to be combined with the CBR method (Kwon and Kim, 2004).</td>
</tr>
<tr>
<td></td>
<td>Weber (1996), Talluri and Narasimhan (2003), Mohammady Garfamy (2006), Farzipoor Saen (2008), Wu (2009), Kang</td>
<td>- Since ANN is designed to be more like human judgment functioning methods, it can cope with complexity, conflicts and uncertainty under different situations. (Zarandi et al., 2008). - ANN can produce both linear and nonlinear relationships among variables and is able to learn these relationships directly from the data being modeled (Aksoy and Öztürk, 2011). - There is no need to know the statistical distribution of the dataset.</td>
<td>- In ANN, unlike the other methods, there is no way to validate the results, other than on the basis of a track record. Statistical models, for example, provide significance tests and various performance measures to evaluate results, while the ANN does not use significance tests (Zahavi and Levin, 1997). - Since ANN is a black box that sheds little light on what is going on inside the model (Ha et al., 2005), the managers cannot obtain an explanation as to why a supplier is (not) selected. - ANN needs to find the best topology of the network and the search grid, which requires an extensive amount of explorations results (Zahavi and Levin, 1997). - The NN requires a large amount of historical data for training (Tran, 2002).</td>
</tr>
<tr>
<td>DEA</td>
<td>Weber (1996), Talluri and Narasimhan (2003), Mohammady Garfamy (2006), Farzipoor Saen (2008), Wu (2009), Kang</td>
<td>- It can deal with quantitative and qualitative data.</td>
<td>- Where suppliers use the best possible set of weights for their inputs and outputs, DEA can suffer from unrealistic weighting scheme for some suppliers. These suppliers try to disregard those criteria on which they have weakly performed by giving them a zero weight. It follows that these suppliers might be identified as efficient, while their efficiency scores are only due to the type of weights that they have used.</td>
</tr>
</tbody>
</table>

ANN: Artificial Neural Network
CBR: Case-based Reasoning
DEA: Data Envelopment Analysis
Among all the methods introduced in Table 1, DEA distinguishes itself in the following features (Wong and Wong, 2008):

- The objectivity stemming from DEA weighting variables during the optimization procedure, frees the analysis from subjective estimates. On the contrary, other methods introduced in Table 1 are subjective as they are very dependent on the weightings provided by the decision maker.
- DEA is highly flexible and able to mold with other analytical methods to create a more meaningful and efficient way of evaluating performances.
- DEA utilizes the concept of efficient frontier as a measure for performance evaluation, i.e. against regression-type analyses which reflect average or central tendency behavior of the observations, DEA deals with the best performance and evaluates all performances by deviations from the best frontier line. These two points of view can result in major differences when used as methods of evaluation. The efficient frontier used in DEA serves appropriately as an empirical standard of excellence.
- DEA inherits the feature that allows the inclusion of qualitative data in performance analysis as well as quantitative measures.

DEA was first introduced by Charnes, Cooper, and Rhodes (CCR) in 1978 and it is a linear-programming-based methodology that uses multiple inputs and outputs to calculate efficiency measure. The efficiency score for each decision making unit (DMU) is defined as a weighted sum of outputs divided by a weighted sum of inputs, where all efficiencies are restricted to a range from 0 to 1. To avoid the potential difficulty in assigning these weights among various DMUs, a DEA model computes weights that give the highest possible relative

---

2 There are two streams among DEA specialists to tackle these two drawbacks. In the first stream, the researchers impose additional information on the weights of criteria to restrict feasible region of the weights. By doing this, suppliers are not quite free to choose their own weights. This stream includes methods which are known as weight restriction methods. In the second stream, researchers try to increase discrimination power of DEA models. This stream includes models which are called super-efficiency models.
efficiency score to a DMU while keeping the efficiency scores of all DMUs less than or equal to one under the same set of weights (Liu et al., 2000).

Weber (1996) was a pioneer researcher in applying DEA for supplier selection problem. Weber discussed the advantages of DEA in supplier selection problem. Mohammady Garfamy (2006) presented the methodology of applying DEA to compare overall supplier performances based on total cost of ownership (TCO) concept and demonstrated this application through a study for a hypothetical firm. Talluri and Narasimhan (2003) developed a max-min DEA model for supplier selection problem. Wu (2009) presented a hybrid model using DEA, decision trees (DT) and neural networks (NNs) to assess supplier performance. Farzipoor Saen (2008) proposed an algorithm for ranking suppliers in the presence of volume discount offers. Kang and Lee (2010) suggested a supplier performance evaluation model based upon AHP and DEA methods. In their study, DEA was applied first to evaluate quantitative factors, and then the results were transformed into pairwise comparison values for AHP analysis.

In the very recent applications of DEA in supplier evaluation and selection, Falagario et al. (2012) proposed the use of cross-efficiency evaluation in public procurement. In cross-efficiency evaluation, the efficiency score of each supplier is evaluated based on the most favorable weights for the supplier itself (self-evaluation) and other suppliers (peer-evaluation). Therefore, suppliers do not get unrealistic efficiency scores due to the unrealistic weights attached to their input and output factors. The cross-efficiency also has the capability to provide a complete ranking among all the suppliers. This capability is known as discrimination power in DEA literature. Hosseinzadeh Zoroufchi et al. (2012) discussed four different possibilities of arbitrary, benevolent, aggressive and neutral in cross-efficiency evaluation and modeled the neutral version of it in non-radial SBM model. They also considered undesirable outputs and applied the developed model for selecting suppliers. Zeydan et al. (2011) solved the same problem in three steps; finding the weights of the factors by AHP, integrating and transforming qualitative measures into one quantitative data by the technique for order preference by similarity to ideal solution (TOPSIS) and ranking suppliers by DEA. They modeled the problem in an uncertain environment and the first two steps are done by fuzzy data in fuzzy AHP and fuzzy TOPSIS methods. Toloo and Nalchigar (2011) considered both cardinal and ordinal data in DEA and used a mixed integer linear programming (MILP) to provide a complete ranking for all the suppliers. Hadi-Vencheh and Niazi-Motlagh (2011) proposed the use of an integrated voting
AHP-DEA to solve supplier selection problem and found the weights of the factors in a new way compared to the conventional pairwise comparison of AHP.

However, sometimes, in applying DEA, there are some factors which simultaneously play the role of both inputs and outputs and are famous as dual-role factors. Beasley (1990, 1995), in a study of the efficiency of university departments, treated research funding as both input and output. However, as Cook et al. (2006) addressed, the model proposed by Beasley (1990, 1995) has two limitations; in the absence of assurance region or cone-ratio constraints on the multipliers, each DMU may be 100% efficient and the dual-role factor is considered differently on the input than on the output side. Cook et al. (2006) developed a new model that does not have these limitations. In the case of supplier evaluation, Farzipoor Saen (2010a) was the first author who discussed dual-role factors. He proposed a model to consider multiple dual-role factors and used it for selecting third-party reverse logistics providers. In this study the ratings for service-quality experience and service-quality credence were considered as dual-role factors. Furthermore, Farzipoor Saen (2010b) proposed a method for selecting suppliers in the presence of dual-role factor and weight restrictions. In this paper, the research and development cost was considered as both input and output. Recently, Noorizadeh et al. (2011) developed a model to select suppliers and incorporated dual-role factors, non-discretionary inputs and weight restrictions into a single model.

However, the aforementioned references which consider dual-role factor suffer from the following limitations:

1. They are all based upon radial DEA models. However, as Morita et al. (2005) argue, in DEA, non-zero input and output slacks are very likely to present themselves after the radial efficiency score improvement. The presence of these slacks will lead to a decrement in the efficiency score of inefficient DMUs. Therefore, the radial DEA models which do not penalize DMUs for the presence of positive slacks cannot fully measure the inefficiency of DMUs.

2. Cook et al. (2006), Farzipoor Saen (2010a), Farzipoor Saen (2010b), and Noorizadeh et al. (2011) considered dual-role factor as a nondiscretionary factor. (In Section 2, we will discuss the reason). However, in real world applications,
there might be some dual-role factors which are under control of decision maker and should be considered as discretionary factors.  

3. The aforementioned references determine an efficiency score of 1 to the efficient DMUs and less than 1 for inefficient ones. In other words, they classify the DMUs into two groups; those that are efficient and construct efficiency frontier and those that are inefficient. These models provide no information regarding the relative ranking of the efficient DMUs. In the case of applying DEA with dual-role factors to select appropriate suppliers, managers and decision makers are not interested in dichotomous classification of suppliers. They expect to use methods which can completely rank suppliers.

These discussions motivated us to model dual-role factors in a new way which does not have the problems discussed above. Table 2 represents a comparison among conventional models which consider dual-role factors and the proposed model in this paper.

<table>
<thead>
<tr>
<th>Table 2. Characteristics of models which consider the dual-role factors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Radial/Non-radial</strong></td>
</tr>
<tr>
<td>Beasley (1990, 1995)</td>
</tr>
<tr>
<td>Cook et al. (2006)</td>
</tr>
<tr>
<td>Farzipoor Saen (2010a) and Farzipoor Saen (2010b)</td>
</tr>
<tr>
<td>Proposed model</td>
</tr>
</tbody>
</table>

3 Searching among references of DEA with dual-role factors implies that most, perhaps all, of the dual-role factors used in the literature are discretionary. Therefore, considering dual-role factors as nondiscretionary seems to be incorrect.
Based on Table 2, the model presented in this paper has the following distinctive contributions:

- This is the first time that a non-radial DEA model considers dual-role factors.
- The proposed model does not deal with dual-role factor as a nondiscretionary factor.
- The proposed model considers dual-role factors on both the input and output sides in a similar manner.
- The proposed model can fully measure the inefficiency of suppliers.
- The proposed model can give a complete ranking of suppliers.

This paper proceeds as follows. In Section 2, proposed model for selecting suppliers is presented. Numerical example and concluding remarks are discussed in Sections 3 and 4, respectively.

2. Proposed model

Model (1) is the model proposed by Beasley (1990, 1995), where members $k$ of a set of $K$ DMUs are to be evaluated in terms of $R$ outputs $Y_k = (y_{rk})_{r=1}^{R}$ and $I$ inputs $X_k = (x_{ik})_{i=1}^{I}$. In addition, assume that a particular factor is held by each DMU in the amount $w_k$, and serves as both input and output factor. The used nomenclatures in this paper are summarized as follows:

- $\text{DMU}_o$: the decision making unit under investigation
- $k=1,...,K$: collection of DMUs (suppliers)
- $r=1,...,R$: the set of outputs
- $i=1,...,I$: the set of inputs
- $f=1, ..., F$: the set of dual-role factors
- $x_{ik}$: the $i$th input of DMU$_k$
- $x_{io}$: the $i$th input of the DMU$_o$
- $v_i$: the weight for $i$th input
- $y_{rk}$: the $r$th output of DMU$_k$
- $y_{ro}$: the $r$th output of DMU$_o$
- $u_r$: the weight for $r$th output
- $w_{fk}$: the $f$th dual-role factor of DMU$_k$
- $w_{fo}$: the $f$th dual-role factor of DMU$_o$
- $\gamma_f$: the weight for $f$th dual-role factor when it is treated on the output side
- $\beta_f$: the weight for $f$th dual-role factor when it is treated on the input side
- $\theta$: the efficiency score of DMU$_o$
\( s_r^+ \): shortages in \( r \)th output
\( s_i^- \): excesses in \( i \)th input
\( s_f^+ \): shortages in \( f \)th dual-role factor when it is treated on the output side
\( s_i^- \): excesses in dual-role factor when it is treated on the input side
\( \lambda_k \): reference weights associated with DMU\(_k\)

\[
\max e = \sum_{r=1}^{R} u_r y_{ro} + \gamma w_o
\]

s.t.
\[
\begin{align*}
\sum_{i=1}^{I} v_i x_{io} + \beta w_o &= 1, \\
\sum_{r=1}^{R} u_r y_{rk} + \gamma w_k - \sum_{i=1}^{I} v_i x_{ik} - \beta w_k &\leq 0, \quad k = 1, 2, \ldots, K, \\
u_r, v_i, \gamma, \beta &\geq 0.
\end{align*}
\]

Apart from the efficiency score of each DMU, one of the purposes of Model (1) is to determine the behavior of \( w \). We want to know in which situation the DMU can benefit more; where \( w \) is an input or an output. We expect the model to judge this issue fairly. However, it cannot do it in practice. To elaborate this issue, the linear input-oriented Model (2) which is the dual of Model (1) is considered.
min $\theta$

s.t.

$$\sum_{k=1}^{K} x_{ik} \lambda_k \leq \theta x_{io}, \quad i = 1, \ldots, I,$$

$$\sum_{k=1}^{K} w_k \lambda_k \leq \theta w_o,$$

$$\sum_{k=1}^{K} y_{rk} \lambda_k \geq y_{ro}, \quad r = 1, \ldots, R,$$

$$\sum_{k=1}^{K} w_k \lambda_k \geq w_o,$$

$$\lambda_k \geq 0,$$

$\theta$ free.

In Model (2), where a DMU has an efficiency score of $\theta$, all discretionary inputs, including $w_o$, are reduced by $1 - \theta$. This factor is also included on the output side where we assume $w_o$ will not be reduced. Thus, Models (1) and (2) treat $w_o$ differently on the input from the output side. To overcome this problem, Cook et al. (2006) recommend treating $w_o$ as being nondiscretionary on the input side. Since, on the output side, variables generally remain constant in the optimization process of an input-oriented model, $w_o$ can be viewed as nondiscretionary as well. This leads to the development of Model (3):

$$\max e = \frac{\left( \sum_{r=1}^{R} u_r y_{ro} + \gamma w_o - \beta w_o \right)}{\left( \sum_{i=1}^{I} v_i x_{io} \right)}$$

s.t.

$$\frac{\left( \sum_{r=1}^{R} u_r y_{rk} + \gamma w_k - \beta w_k \right)}{\left( \sum_{i=1}^{I} v_i x_{ik} \right)} \leq 1, \quad k = 1, \ldots, K,$$

$$u_r, v_i, \gamma, \beta \geq 0.$$

It should be noted that, since this model is not proposed for considering multiple dual role factors, the "f" subscript which is related to the set of dual-role factors, does not appear in the
model. Using a standard technique (see, e.g. Charnes et al., 1978) to transform the above fractional Model (3) into a linear one, there will be the following linear programming model.

\[
\max e = \sum_{r=1}^{R} u_r y_{ro} + \gamma w_o - \beta w_o
\]

\[s.t.
\sum_{i=1}^{I} v_i x_{io} = 1,
\sum_{r=1}^{R} u_r y_{rk} + \gamma w_k - \beta w_k - \sum_{i=1}^{I} v_i x_{ik} \leq 0, \quad k = 1, \ldots, K,
\]

\[
u_r, v_i, \gamma, \beta \geq 0.
\]

To consider multiple dual-role factors in DEA, Farzipoor Saen (2010a) proposed Model (5). Assume that some factors are held by each DMU in the amount \( w_{fk} \) \( (f = 1, \ldots, F) \), and serve as both input and output factors. The proposed model for considering multiple dual-role factors is as follows:

\[
\max e = \sum_{r=1}^{R} u_r y_{ro} + \sum_{f=1}^{F} \gamma_f w_{fo} - \sum_{j=1}^{F} \beta_f w_{fo}
\]

\[s.t.
\sum_{i=1}^{I} v_i x_{io} = 1,
\sum_{r=1}^{R} u_r y_{rk} + \sum_{f=1}^{F} \gamma_f w_{fk} - \sum_{j=1}^{F} \beta_f w_{fk} - \sum_{i=1}^{I} v_i x_{ik} \leq 0, \quad k = 1, \ldots, K,
\]

\[
u_r, v_i, \gamma_f, \beta_f \geq 0.
\]

However, Models (4) and (5) have two limitations: (1) they treat dual-role factors on the input side as nondiscretionary inputs. This idea is derived by moving dual-role factors on the input side to the output side but with the opposite value and (2) they are radial and do not enjoy the advantage of non-radial models in the full measurement of the inefficiency of the DMUs.
Developing a slack-based measure (SBM) of efficiency (Tone, 2001) to consider dual-role factors can be a solution to these problems.

Therefore, we define the production possibility set of the model as:

\[
P = \left\{ \left( x_{io}, y_{ro}, w_{fo} \right) | x_{io} \geq \sum_{k=1}^{K} \lambda_k x_{ik}, w_{fo} \geq \sum_{k=1}^{K} \lambda_k w_{jk}, y_{ro} \right\}
\]

\[
\leq \sum_{k=1}^{K} \lambda_k y_{rk}, w_{fo} \leq \sum_{k=1}^{K} \lambda_k w_{jk}, \lambda_k \geq 0 \right\}
\]

(6)

We consider an expression to describe a certain DMU \((x_{io}, y_{ro}, w_{fo})\) as:

\[
x_{io} = \sum_{i=1}^{I} \lambda_k x_{ik} + s_i^-, \quad y_{ro} = \sum_{r=1}^{R} \lambda_k y_{rk} - s_r^+, \quad w_{fo} = \sum_{f=1}^{F} \lambda_k w_{fk} + s_f^-
\]

and

\[
w_{fo} = \sum_{f=1}^{F} \lambda_k w_{fk} - s_f^+, \quad k = 1, \ldots, K
\]

where \(s_i^-\) and \(s_f^-\) indicate the inputs excesses. Also, \(s_r^+\) and \(s_f^+\) relate to the outputs shortfalls, and are called slacks. Using \(s_i^-\), \(s_f^-\), \(s_r^+\) and \(s_f^+\) an index \(\rho\) is defined as follows:

\[
\rho = \frac{1 - \frac{1}{T+P} \left( \sum_{i=1}^{I} \frac{s_i^-}{x_{io}} + \sum_{f=1}^{F} \frac{s_f^-}{w_{fo}} \right)}{1 + \frac{1}{R+P} \left( \sum_{r=1}^{R} \frac{s_r^+}{y_{ro}} + \sum_{f=1}^{F} \frac{s_f^+}{w_{fo}} \right)}
\]

(7)

From the conditions \(x_{ik} \geq 0\), \(w_{fk} \geq 0\) and \(\lambda_k \geq 0\), it holds \(x_{io} \geq s_i^-\) and \(w_{fo} \geq s_f^-\). Also from \(x_{io} \geq s_i^-\) and \(w_{fo} \geq s_f^-\), we have \(0 \leq \rho \leq 1\). The SBM efficiency, when some factors have dual-role, is obtained from the following fractional program.
\[
\min \rho = \frac{1 - \frac{1}{I+F} \left( \sum_{i=1}^{I} \frac{s_{i}^{-}}{x_{io}} + \sum_{f=1}^{F} \frac{s_{f}^{+}}{w_{fo}} \right)}{1 + \frac{1}{R+F} \left( \sum_{r=1}^{R} \frac{s_{r}^{+}}{y_{ro}} + \sum_{f=1}^{F} \frac{s_{f}^{-}}{w_{fo}} \right)}
\]

s.t.

\[
\begin{align*}
\sum_{k=1}^{K} \lambda_k x_{ik} + s_{i}^{-} &= x_{io}, & i &= 1, \ldots, I, \\
\sum_{k=1}^{K} \lambda_k y_{rk} - s_{r}^{+} &= y_{ro}, & r &= 1, \ldots, R, \\
\sum_{k=1}^{K} \lambda_k w_{rk} + s_{f}^{-} &= w_{fo}, & f &= 1, \ldots, F, \\
\sum_{k=1}^{K} \lambda_k w_{rk} - s_{f}^{+} &= w_{fo}, & f &= 1, \ldots, F, \\
\lambda_k &\geq 0, s_{i}^{-} \geq 0, s_{r}^{+} \geq 0, s_{f}^{-} \geq 0, s_{f}^{+} \geq 0.
\end{align*}
\] (8)

Model (8) can be transformed into a linear program using the scale transformation in the similar way as the CCR model. Let us multiply a scalar variable \( t \ (> 0) \) to both the denominator and the numerator of (8). We adjust \( t \) so that the denominator becomes 1. Then this term is moved to constraints. The objective is to minimize the numerator. Thus, we have:

\[
\min \tau = t - \frac{1}{I+F} \left( \sum_{i=1}^{I} \frac{ts_{i}^{-}}{x_{io}} + \sum_{f=1}^{F} \frac{ts_{f}^{-}}{w_{fo}} \right)
\]

s.t.

\[
\begin{align*}
1 &= t + \frac{1}{R+F} \left( \sum_{r=1}^{R} \frac{ts_{r}^{+}}{y_{ro}} + \sum_{f=1}^{F} \frac{ts_{f}^{+}}{w_{fo}} \right), \\
\sum_{k=1}^{K} t\lambda_k x_{ik} + ts_{i}^{-} &= tx_{io}, & i &= 1, \ldots, I, \\
\sum_{k=1}^{K} t\lambda_k y_{rk} - ts_{r}^{+} &= ty_{ro}, & r &= 1, \ldots, R, \\
\sum_{k=1}^{K} t\lambda_k w_{rk} + ts_{f}^{-} &= tw_{fo}, & f &= 1, \ldots, F, \\
\sum_{k=1}^{K} t\lambda_k w_{rk} - ts_{f}^{+} &= tw_{fo}, & f &= 1, \ldots, F, \\
\lambda_k &\geq 0, s_{i}^{-} \geq 0, s_{r}^{+} \geq 0, s_{f}^{-} \geq 0, s_{f}^{+} \geq 0, t > 0.
\end{align*}
\] (9)
Model (9) is still a nonlinear programming problem since it contains the nonlinear terms. We can transform (9) into a linear program as follows. Let us define \( S_i^- = ts_i^- \), \( S_i^+ = ts_i^+ \), \( S_f^- = ts_f^- \), \( S_f^+ = ts_f^+ \) and \( \Lambda_k = t\lambda_k \). Then (9) becomes the following linear program in \( t, S_i^-, S_i^+, S_f^-, S_f^+, \) and \( \Lambda_k \).

\[
\begin{align*}
\min \tau &= t - \frac{1}{I+F} \left( \sum_{i=1}^I s_i^- \frac{x_{i0}}{x_{i0}} + \sum_{f=1}^F s_f^- \frac{w_{fo}}{w_{fo}} \right) \\
\text{s.t.} \\
1 &= t + \frac{1}{R+F} \left( \sum_{r=1}^R s_r^+ \frac{y_{ro}}{y_{ro}} + \sum_{f=1}^F s_f^+ \frac{w_{fo}}{w_{fo}} \right), \\
\sum_{k=1}^K \Lambda_k x_{ik} + S_i^- &= tx_{i0}, \quad i = 1, \ldots, I, \\
\sum_{k=1}^K \Lambda_k y_{rk} - S_r^+ &= ty_{ro}, \quad r = 1, \ldots, R, \\
\sum_{k=1}^K \Lambda_k w_{fk} + S_f^- &= tw_{fo}, \quad f = 1, \ldots, F, \\
\sum_{k=1}^K \Lambda_k w_{fk} - S_f^+ &= tw_{fo}, \quad f = 1, \ldots, F, \\
\Lambda_k \geq 0, S_i^- \geq 0, S_i^+ \geq 0, S_f^- \geq 0, S_f^+ \geq 0, t > 0.
\end{align*}
\]

Note that, since \( S_i^-, S_i^+, S_f^-, S_f^+ \) are all nonnegative, the proposed model can penalize any DMU which performs poorly in any input, output, and dual-role factor and can suggest improvement targets based on the amounts of \( S_i^-, S_i^+, S_f^-, S_f^+ \). This shows that the proposed model does not deal with dual-role factor as a non-discretionary factor. Consider the situation where we were supposed to follow the way proposed by Cook et al. (2006) and consider dual-role factor on the input side as a non-discretionary factor. To impose this assumption into the model, we should alter \( S_f^- \geq 0 \) by \( S_f^- = 0 \). (For more details on non-discretionary factors in SBM model, please see Farzipoor Saen, 2005). Also by multiplying \( t \) on \( w_{fo} \) in both the input and output sides simultaneously, Model (10) considers dual-role factors on both the input and output
sides in a similar manner. While we cannot see the same character in Model (2), the $\theta$ is multiplied only on $w_o$ when it is on the input side.

Now, let an optimal solution of (10) to be $(\tau^*, t^*, \Lambda_k^*, S_i^{-*}, S_f^{+*}, S_r^{-*}, S_f^{+*})$. Then we have an optimal solution of this model as defined by $\rho^* = \tau^*$, $\lambda^*_k = \Lambda^*_k / t^*$, $S_i^{-*} = S_i^{-*} / t^*$, $S_r^{+*} = S_r^{+*} / t^*$, $S_f^{+*} = S_f^{+*} / t^*$. The DMU is SBM efficient if and only if $\rho^* = 1$. To find the behavior of dual-role factors, the dual of (10) is as follows:

$$\max \tau = \theta$$

$$\text{s.t.}$$

$$\theta + \sum_{i=1}^{I} \nu_i x_{io} + \sum_{f=1}^{F} \beta_f w_{fo} - \left( \sum_{r=1}^{R} \nu_r y_{ro} + \sum_{f=1}^{F} \gamma_f w_{fo} \right) = 1,$$

$$\sum_{r=1}^{R} \nu_r y_{rk} + \sum_{f=1}^{F} \gamma_f w_{fk} - \left( \sum_{i=1}^{I} \nu_i x_{ik} + \sum_{f=1}^{F} \beta_f w_{fk} \right) + d_k = 0, \quad k = 1, \ldots, K,$$

$$v_i \geq \frac{1}{I + F} [1 / x_{io}], \quad i = 1, \ldots, I,$$

$$\beta_f \geq \frac{1}{I + F} [1 / w_{fo}], \quad f = 1, \ldots, F,$$

$$\nu_r \geq \frac{\theta}{R + F} [1 / y_{ro}], \quad r = 1, \ldots, R,$$

$$\gamma_f \geq \frac{\theta}{R + F} [1 / w_{fo}], \quad f = 1, \ldots, F.$$

Note that the second constraint in (11) was originally

$$\sum_{r=1}^{R} \nu_r y_{rk} + \sum_{f=1}^{F} \gamma_f w_{fk} - \left( \sum_{i=1}^{I} \nu_i x_{ik} + \sum_{f=1}^{F} \beta_f w_{fk} \right) \leq 0.$$ By changing inequality constraint to equality, $d_k$ gives us useful information. The $d_k$ is the inefficiency amount of DMU on the basis of the optimal weights calculated for DMU. DMU with $d_k = 0$ is recognized as the benchmark DMU for DMU, where the shadow price for $d_k$ shows the benchmark weight associated with the efficient DMU.

As Cook et al. (2006) explained, one of three possibilities exists in regard to the sign of $\hat{\gamma}_f - \hat{\beta}_f$, where $\hat{\gamma}_f$, $\hat{\beta}_f$ are the optimal values from Model (11).
Case 1: If $\hat{y}_f - \hat{\beta}_f < 0$, then the dual-role factor is “behaving like input”. Hence less of this factor is better, and would lead to an increase in efficiency.

Case 2: If $\hat{y}_f - \hat{\beta}_f > 0$, then the dual-role factor is “behaving like output”. Hence more of this factor is better, and would lead to an increase in efficiency.

Case 3: If $\hat{y}_f - \hat{\beta}_f = 0$, then dual-role factor is at equilibrium level.

So far, a SBM model with dual-role factors is developed. This model treats dual-role factor on both the input and output sides in a similar manner, fully measure the inefficiency of DMUs and do not consider dual-role factor as a nondiscretionary factor. However this model can suffer from the lack of discrimination among efficient DMUs, all of which have a score of unity. Hence, we need to develop a new index which can give a complete ranking of DMUs.

2.1. Balance index

Alirezaee and Afsharian (2007) called the restriction $\left[ \sum_{r=1}^{R} u_r y_{rk} - \sum_{i=1}^{I} v_i x_{ik} \right] \leq 0$, $k = 1, \ldots, K$, in the CCR model as the profit restriction for the $k$th DMU\(^4\). They employed profit restriction for describing a new index in addition to efficiency score for each DMU. They called it balance index. In other words, balance index for each DMU is the sum of quantities of profit restrictions of all DMUs. Therefore, DMU\(_1\) has a better rank than DMU\(_2\):  

- If DMU\(_1\) is efficient but, DMU\(_2\) is inefficient; or
- If the efficiency score of both DMUs are the same, and one of them obtains more negative quantity in balance index.

As Alirezaee and Afsharian (2007) addressed, since this method uses initial data of DEA models, it is a precise and logical method. In addition, it does not destroy competition among DMUs. Foremost, it is employed easily and with logical computation. Hence, it is an alphabetical method which can be useful in some other situations.

Now, using the second restriction in Model (11)

$$\sum_{r=1}^{R} u_r y_{rk} + \sum_{f=1}^{F} y_f w_{fk} - \left( \sum_{i=1}^{I} v_i x_{ik} + \sum_{f=1}^{F} \beta_f w_{fk} \right) \leq 0,$$

\(^4\) For details on CCR model, see Alirezaee and Afsharian (2007).
the balance index based on our proposed model is developed:

\[
BI_o = u_{ro} \sum_{r=1}^{R} y_{rk} + \gamma f_0 \sum_{f=1}^{F} w_{fk} - \left( v_{io} \sum_{i=1}^{I} x_{ik} + \beta_{fo} \sum_{f=1}^{F} w_{fk} \right), \quad k = 1, \ldots, K \quad (12)
\]

where

\[
\sum_{r=1}^{R} u_r y_{rk} + \sum_{f=1}^{F} \gamma_f w_{fk} \quad \text{and} \quad \sum_{i=1}^{I} v_i x_{ik} + \sum_{f=1}^{F} \beta_f w_{fk}
\]

are total revenue and total cost for kth DMU, respectively.

Formula (12) will be calculated for every DMU, separately. Fig. 1 depicts the process of the proposed approach.

Fig. 1. Process of the proposed method
3. Numerical example

For illustrative purposes, the problem of supplier selection is introduced. The data set for this study is partially taken from Farzipoor Saen (2008). The price ($x_1$), the number of shipments ($x_2$), and Research and Development cost (R&D) ($w_1$), were used as inputs. The number of bills received from the supplier without errors ($y_1$), the number of shipments to arrive on time ($y_2$), and R&D ($w_1$), were considered as outputs. R&D plays the role of both input and output. Table 3 shows the data set for 12 suppliers.

<table>
<thead>
<tr>
<th>Supplier</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$y_1$</th>
<th>$y_2$</th>
<th>$w_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>197</td>
<td>10</td>
<td>90</td>
<td>187</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>198</td>
<td>5</td>
<td>130</td>
<td>194</td>
<td>32</td>
</tr>
<tr>
<td>3</td>
<td>229</td>
<td>18</td>
<td>200</td>
<td>220</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>169</td>
<td>12</td>
<td>100</td>
<td>160</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>212</td>
<td>9</td>
<td>173</td>
<td>204</td>
<td>16</td>
</tr>
<tr>
<td>6</td>
<td>197</td>
<td>15</td>
<td>170</td>
<td>192</td>
<td>28</td>
</tr>
<tr>
<td>7</td>
<td>209</td>
<td>13</td>
<td>60</td>
<td>194</td>
<td>12</td>
</tr>
<tr>
<td>8</td>
<td>203</td>
<td>8</td>
<td>145</td>
<td>195</td>
<td>36</td>
</tr>
<tr>
<td>9</td>
<td>205</td>
<td>8</td>
<td>150</td>
<td>200</td>
<td>30</td>
</tr>
<tr>
<td>10</td>
<td>203</td>
<td>14</td>
<td>90</td>
<td>171</td>
<td>28</td>
</tr>
<tr>
<td>11</td>
<td>207</td>
<td>11</td>
<td>100</td>
<td>174</td>
<td>19</td>
</tr>
<tr>
<td>12</td>
<td>234</td>
<td>14</td>
<td>200</td>
<td>209</td>
<td>25</td>
</tr>
</tbody>
</table>

Table 4 reports the results of efficiency scores obtained by Models (3) and proposed Model (11). Also the behavior of dual-role factor for the 12 suppliers is depicted in this table. The computations have been conducted by Lingo software. The results of evaluation by proposed model show that suppliers 2, 3, 4, 5, 6, 7, 8, 9, and 12 are efficient with a relative efficiency score of 1, while the remaining 3 suppliers with relative efficiency scores of less than 1 are considered to be inefficient.
To validate the efficiency scores of the proposed Model (11), and to evaluate its association with Model (3), Spearman correlation analysis between efficiency scores of these two models is conducted. Since the correlation coefficient between the results of two approaches, at significant level of 0.01, is 0.883, there is a significant relationship between their results. Nevertheless, the results show that Model (3) is not able to fully measure the inefficiencies of inefficient DMUs. Comparing the efficiency scores of suppliers 1, 10, and 11, obtained by Models (3) and (11), confirm this issue, where their efficiencies have been decreased using the developed SBM model. A remarkable difference between the radial and non-radial efficiency score is found in supplier 1, whose its efficiency score decreased by 0.241. This is due to the fact that non-radial Model (11) can reflect the amount of non-zero slacks, when it is used as the efficiency measurement tool of suppliers. After supplier 1, supplier 10 and 11 stood second and third in terms of efficiency decrement when efficiency measurement tool is shifted from radial model to a non-radial one. One interesting point is found from our supplier selection case; the less inefficient supplier in a radial model, the more decline in efficiency score in a non-radial model is observed. The importance of this point is explained more, when the analyst wishes to find improvement targets for inefficient suppliers.

To improve the performance of inefficient suppliers, their inputs and outputs should be projected on the efficiency frontier. To this end, suppliers should use strategies to decrease their inputs and/or increase their outputs based upon the results of the radial or non-radial model.
When the outcome of the radial model is considered, inefficient suppliers have an easier way to reach the efficiency frontier. As the results of the non-radial model address, inefficient suppliers should do more changes on the level of their inputs and outputs. Consequently, radial models mislead inefficient suppliers. Table 6 shows the amount of $t$ and $\Lambda$ obtained by Model (10) which are used in $\lambda_k^* = \Lambda_k^*/t^*$ to find the weights of benchmark suppliers for inefficient suppliers 1, 10, and 11. This table also shows the projected inputs and outputs ($\hat{x}_1, \hat{x}_2, \hat{y}_1, \hat{y}_2$, and $\hat{\omega}_1$) for inefficient suppliers. For example, if supplier 1 decrease its second input to 7.338 and increase its $y_1$ and $y_2$ to 151.259 and 190.611 respectively, it will reach to the efficiency frontier.

Table 5. Benchmark values and projected inputs and outputs for inefficient suppliers

<table>
<thead>
<tr>
<th>Supplier</th>
<th>$T$</th>
<th>$A_2$</th>
<th>$A_5$</th>
<th>$\lambda_2$</th>
<th>$\lambda_5$</th>
<th>$\hat{x}_1$</th>
<th>$\hat{x}_2$</th>
<th>$\hat{y}_1$</th>
<th>$\hat{y}_2$</th>
<th>$\hat{\omega}_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.811</td>
<td>0.244</td>
<td>0.525</td>
<td>0.301</td>
<td>0.648</td>
<td>197.000</td>
<td>7.338</td>
<td>151.259</td>
<td>190.611</td>
<td>20</td>
</tr>
<tr>
<td>10</td>
<td>0.803</td>
<td>0.597</td>
<td>0.211</td>
<td>0.743</td>
<td>0.263</td>
<td>203.000</td>
<td>6.086</td>
<td>142.184</td>
<td>197.920</td>
<td>28.000</td>
</tr>
<tr>
<td>11</td>
<td>0.795</td>
<td>0.157</td>
<td>0.629</td>
<td>0.198</td>
<td>0.791</td>
<td>207.000</td>
<td>7.052</td>
<td>162.668</td>
<td>199.876</td>
<td>19.000</td>
</tr>
</tbody>
</table>

We can also use the multiplier Model (11) to determine the benchmark values of inefficient suppliers. The $d_{k1}, d_{k2},$ and $d_{k3}$ in Table 7 are the inefficiency scores of all suppliers with the optimal weights of inefficient suppliers 1, 10 and 11, respectively. When the optimal weights of suppliers 1, 10, and 11 are used, suppliers 2 and 5 with the zero inefficiency scores are recognized as the benchmark suppliers. The shadow prices of $d_{21}, d_{210}, d_{211}, d_{51}, d_{510},$ and $d_{511}$ are the benchmark weights of efficient suppliers 2 and 5 for inefficient suppliers.

Table 6. Inefficiency scores of all suppliers with the optimal weights of inefficient suppliers 1, 10, and 11

<table>
<thead>
<tr>
<th>Supplier</th>
<th>$d_{k1}$</th>
<th>$d_{k2}$</th>
<th>$d_{k3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.261</td>
<td>0.216</td>
<td>0.234</td>
</tr>
<tr>
<td>2</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>3</td>
<td>0.236</td>
<td>0.165</td>
<td>0.215</td>
</tr>
<tr>
<td>4</td>
<td>0.263</td>
<td>0.208</td>
<td>0.236</td>
</tr>
<tr>
<td>5</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>6</td>
<td>0.217</td>
<td>0.140</td>
<td>0.200</td>
</tr>
<tr>
<td>7</td>
<td>0.442</td>
<td>0.372</td>
<td>0.395</td>
</tr>
<tr>
<td>8</td>
<td>0.075</td>
<td>0.047</td>
<td>0.070</td>
</tr>
<tr>
<td>9</td>
<td>0.047</td>
<td>0.029</td>
<td>0.044</td>
</tr>
<tr>
<td>10</td>
<td>0.442</td>
<td>0.349</td>
<td>0.403</td>
</tr>
<tr>
<td>11</td>
<td>0.302</td>
<td>0.253</td>
<td>0.275</td>
</tr>
<tr>
<td>12</td>
<td>0.146</td>
<td>0.100</td>
<td>0.137</td>
</tr>
</tbody>
</table>
In order to interpret the behavior of the dual-role factor, consider for instance, suppliers 1, 2, and 6. For supplier 1, R&D is behaving like an input, and lower value of such factor would increase the efficiency of the supplier. For supplier 2, R&D is behaving at equilibrium level. And for supplier 6, R&D is behaving like an output, and higher level of such factor would improve the efficiency of the supplier. Cook et al. (2006) found an interesting relationship between the sign of $\gamma - \beta$ and the return to scale of DMU$_o$. Based on their results, suppliers with $\gamma - \beta < 0$ are experiencing decreasing returns to scale. Therefore, these suppliers are not operating at the most productive scale size (MPSS) and a reduction in size would help them to reach to a better performance. Suppliers with $\gamma - \beta > 0$, are experiencing increasing returns to scale, and again they are not performing at the MPSS and suppliers with $\gamma - \beta = 0$ are experiencing constant returns to scale and are at the MPSS.

As it can be seen in Table 5, most of the suppliers have the efficiency scores of 1. Therefore, the problem now is to select a supplier from those efficient suppliers. To derive a complete ranking of suppliers, the formula (12) is used. Note that to find the $Bl_o$, the optimal weights of the criteria derived by Model (11) is used. Table 8 depicts the optimal weights of the criteria for all the suppliers.

<table>
<thead>
<tr>
<th>Supplier</th>
<th>$v_1$</th>
<th>$v_2$</th>
<th>$\beta_1$</th>
<th>$u_1$</th>
<th>$u_2$</th>
<th>$\gamma_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0019453</td>
<td>0.0333333</td>
<td>0.0166667</td>
<td>0.0027366</td>
<td>0.0013171</td>
<td>0.0148091</td>
</tr>
<tr>
<td>2</td>
<td>0.0016835</td>
<td>0.0666667</td>
<td>0.0104167</td>
<td>0.0025641</td>
<td>0.0017182</td>
<td>0.0104167</td>
</tr>
<tr>
<td>3</td>
<td>0.0014556</td>
<td>0.0185185</td>
<td>0.0626088</td>
<td>0.0046957</td>
<td>0.0015152</td>
<td>0.0222222</td>
</tr>
<tr>
<td>4</td>
<td>0.0019724</td>
<td>0.0392602</td>
<td>0.1582132</td>
<td>0.0033333</td>
<td>0.0107495</td>
<td>0.0333333</td>
</tr>
<tr>
<td>5</td>
<td>0.0015723</td>
<td>0.0370370</td>
<td>0.0250616</td>
<td>0.0019268</td>
<td>0.0019656</td>
<td>0.0208333</td>
</tr>
<tr>
<td>6</td>
<td>0.0067239</td>
<td>0.0222222</td>
<td>0.0119048</td>
<td>0.0067809</td>
<td>0.0015152</td>
<td>0.0180424</td>
</tr>
<tr>
<td>7</td>
<td>0.0015949</td>
<td>0.1879485</td>
<td>0.4715918</td>
<td>0.0055556</td>
<td>0.040469</td>
<td>0.0277778</td>
</tr>
<tr>
<td>8</td>
<td>0.0071684</td>
<td>0.0416667</td>
<td>0.0092593</td>
<td>0.0022989</td>
<td>0.0017094</td>
<td>0.0404217</td>
</tr>
<tr>
<td>9</td>
<td>0.0117965</td>
<td>0.0416667</td>
<td>0.0111111</td>
<td>0.0096535</td>
<td>0.0052454</td>
<td>0.0195947</td>
</tr>
<tr>
<td>10</td>
<td>0.0021391</td>
<td>0.0238095</td>
<td>0.0119048</td>
<td>0.0024129</td>
<td>0.0012699</td>
<td>0.0113590</td>
</tr>
<tr>
<td>11</td>
<td>0.0018823</td>
<td>0.0303030</td>
<td>0.0175439</td>
<td>0.0024170</td>
<td>0.0013891</td>
<td>0.0156850</td>
</tr>
<tr>
<td>12</td>
<td>0.0124794</td>
<td>0.0238095</td>
<td>0.0148630</td>
<td>0.0147922</td>
<td>0.0015949</td>
<td>0.0133333</td>
</tr>
</tbody>
</table>

In Table 8, the BI scores and ranking of suppliers have been displayed. As Table 6 shows, supplier 7 received the lowest BI value and is the first candidate for selection. The calculation details of Model (11) and formula (12) for supplier 1 have been presented in the Appendix.
Table 8. The BI scores and ranking of suppliers

<table>
<thead>
<tr>
<th>Supplier</th>
<th>$BI_k$</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-77.188</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>-149.431</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>-134.701</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>-391.580</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>-92.658</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>-26.675</td>
<td>11</td>
</tr>
<tr>
<td>7</td>
<td>-1511.408</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>-17.092</td>
<td>12</td>
</tr>
<tr>
<td>9</td>
<td>-61.894</td>
<td>8</td>
</tr>
<tr>
<td>10</td>
<td>-52.632</td>
<td>9</td>
</tr>
<tr>
<td>11</td>
<td>-70.709</td>
<td>7</td>
</tr>
<tr>
<td>12</td>
<td>-37.907</td>
<td>10</td>
</tr>
</tbody>
</table>

4. Concluding remarks

Selection and assessment of suppliers is an important activity within the enterprise in today’s competition commercial environment. Erroneous selection of supplier may lead to problems of operation and finance. For many manufacturers, purchase of raw materials and common parts from suppliers is the most important expenses within the company (Che and Wang, 2008).

In this paper, we conducted a comprehensive survey regarding papers which have contributed to the supplier evaluation and selection literature. To help researchers know strengths and weaknesses of different methods of selecting suppliers, a comparison among the methods was made. Considering all the aspects provided in the comparison, we concluded that DEA outweighs other methods and is an appropriate method in supplier selection problem. However, when DEA is used to such an important decision, decision makers may face with dual-role factors. There are some models, like the model proposed by Cook et al. (2006), which can help DEA users to determine the role of these types of factors. Nevertheless, these models are not useful when all the factors are to be considered as discretionary. Also, the models cannot fully measure the inefficiency of DMUs, since they are all based on the radial models. In this paper, we demonstrated how a SBM model is able to solve these problems. A SBM model in the presence of dual-role factor was developed.

Further researches can be done based on the results of this paper. Some of them are as below:
• To prevent DMUs from being efficient, due to the unrealistic weights, cross-efficiency method introduced by Sexton et al. (1986) can be used. The main idea of the cross-efficiency is to use DEA in a peer evaluation instead of a self evaluation mode. That is, the relative efficiency of DMU<sub>o</sub> should be calculated by the optimal weights of the inputs and outputs of DMU<sub>k</sub>.

• Another potential improvement is to develop a new version of the proposed model for considering imprecise data. In real world, due to the presence of uncertainty, there might be some factors that they cannot be measured precisely. Generally speaking, uncertain information or imprecise data can be expressed in interval or fuzzy number (Wang et al., 2005). To deal with all aspects of imprecise data in DEA, Cooper et al. (1999) proposed imprecise data envelopment Analysis (IDEA). In addition, Wang et al. (2005) developed a new pair of interval DEA models for dealing with imprecise data such as interval data, ordinal preference information, fuzzy data and their mixture.

Acknowledgements

The authors wish to thank two anonymous reviewers for valuable suggestions and comments.

References


Appendix

Model (11) for supplier 1:

\[
\text{max } \theta, \\
\theta + 197 v_1 + 10 v_2 + 20 \beta_1 - 90 u_1 - 187 u_2 - 20 \gamma_1 = 1, \\
90 u_1 + 187 u_2 + 20 \gamma_1 - 197 v_1 - 10 v_2 - 20 \beta_1 \leq 0, \\
130 u_1 + 194 u_2 + 32 \gamma_1 - 198 v_1 - 5 v_2 - 32 \beta_1 \leq 0, \\
200 u_1 + 220 u_2 + 15 \gamma_1 - 229 v_1 - 18 v_2 - 15 \beta_1 \leq 0, \\
100 u_1 + 160 u_2 + 10 \gamma_1 - 169 v_1 - 12 v_2 - 10 \beta_1 \leq 0, \\
173 u_1 + 204 u_2 + 16 \gamma_1 - 212 v_1 - 9 v_2 - 16 \beta_1 \leq 0, \\
170 u_1 + 192 u_2 + 28 \gamma_1 - 197 v_1 - 15 v_2 - 28 \beta_1 \leq 0, \\
60 u_1 + 194 u_2 + 12 \gamma_1 - 209 v_1 - 13 v_2 - 12 \beta_1 \leq 0, \\
145 u_1 + 195 u_2 + 36 \gamma_1 - 203 v_1 - 8 v_2 - 36 \beta_1 \leq 0, \\
150 u_1 + 200 u_2 + 30 \gamma_1 - 205 v_1 - 8 v_2 - 30 \beta_1 \leq 0, \\
90 u_1 + 171 u_2 + 28 \gamma_1 - 203 v_1 - 14 v_2 - 28 \beta_1 \leq 0, \\
100 u_1 + 174 u_2 + 19 \gamma_1 - 207 v_1 - 11 v_2 - 19 \beta_1 \leq 0, \\
200 u_1 + 209 u_2 + 25 \gamma_1 - 234 v_1 - 14 v_2 - 25 \beta_1 \leq 0, \\
v_1 \geq \frac{1}{3} \left( \frac{1}{197} \right), \\
v_2 \geq \frac{1}{3} \left( \frac{1}{10} \right), \\
\beta_1 \geq \frac{1}{3} \left( \frac{1}{20} \right), \\
u_1 \geq \frac{\theta}{3} \left( \frac{1}{90} \right), \\
u_2 \geq \frac{\theta}{3} \left( \frac{1}{187} \right),
\]
\[ \gamma_1 \geq \frac{\theta}{3} \left( \frac{1}{20} \right). \]

Formula (12) for supplier 1:

\[ BI_1 = 1608 \times (0.0027366) + 137 \times (0.0013171) + 2463 (0.0148091) - 271 (0.0019453) - 2300 (0.0333333) - 2463 (0.0166667) = -77.188. \]

© 2013 Inderscience Enterprises Ltd. retains the copyright of the paper.
Using DEA cross-efficiency evaluation for suppliers ranking in the presence of non-discretionary inputs

Abdollah Noorizadeh
Department of Industrial Management,
Lappeenranta University of Technology,
P.O. Box 20, 53851 Lappeenranta, Finland
E-mail: ab.noorizadeh@gmail.com

Mahdi Mahdiloo
Institute for Integrated and Intelligent Systems,
Griffith University,
QLD 4111, Australia
E-mail: mahdi.mahdiloo@griffithuni.edu.au

Reza Farzipoor Saen*
Department of Industrial Management,
Faculty of Management and Accounting, Karaj Branch,
Islamic Azad University,
P.O. Box 31485-313, Karaj, Iran
Fax: 0098 (263) 4418156
E-mail: farzipour@yahoo.com
*Corresponding author

Abstract: Supply chain includes all activities associated with the flow and transformation of goods from the raw material stage through to the end user. Supplier selection is one of the most important parts of supply chain management (SCM). For selecting suppliers, data envelopment analysis (DEA), as a multiple criteria decision making tool, has been applied for several times. However, sometimes in supplier selection problem, there may exist some criteria that are beyond the control of a management. These criteria are called non-discretionary or exogenously fixed factors.

Since in traditional treatment of non-discretionary inputs in DEA, free reign is given when deciding for each decision making unit (DMU) which outputs and inputs to emphasise, many different avenues are present by which a DMU can appear efficient. Therefore, it is common to have many DMUs that are relatively efficient. In addition, since each DMU has its own set of weights, all of its weight might be put on a single output and input. As a result, the objective of this paper is to propose a cross-efficiency model which is able to consider non-discretionary inputs. A numerical example demonstrates the application of the proposed model in supplier selection context.

Keywords: data envelopment analysis; DEA; cross-efficiency; supplier ranking; non-discretionary inputs.


Reza Farzipoor Saen is an Associate Professor at the Department of Industrial Management, Islamic Azad University, Karaj Branch in Iran. In 2002, he obtained his PhD in Industrial Management from Islamic Azad University, Science and Research Branch in Iran. He has published over 86 refereed papers in many prestigious journals, such as *Expert Systems with Applications*, *Int. J. of Production Economics*, *Annals of Operations Research*, *Journal of the Operational Research Society*, *European Journal of Operational Research*, *Journal of Industrial and Management Optimization*, *Applied Mathematics and Computation*, *Applied Mathematical Modelling*, *Int. J. Advanced Manufacturing Technology*, *Int. J. Applied Management and Technology*, *Asia Pacific Management Review*, etc.

1 Introduction

The purpose of supplier selection is to determine the optimal supplier who can offer the best products or services for the customer and become a part of the organisation’s supply chain (Ebrahim et al., 2009). As a supplier becomes a part of the established supply chain, it will have a lasting effect on the efficiency and effectiveness of the entire supply chain (Chen et al., 2006). In the past few decades, there have been major changes in the supplier selection practices. The competition has risen and the market has become globally operating. In such a scenario, it has become highly difficult for industries to produce low cost and high quality products successfully without proper suppliers (Weber et al., 1991). Effective supplier evaluation and selection strategies can directly impact
Using DEA cross-efficiency evaluation for suppliers ranking

supply chain performance, resulting in organisational productivity and profitability. Ghodsypour and O’Brien (1998) declare that in manufacturing industries the raw materials and component parts can equal up to 70% of the product cost. In such circumstances, the purchasing department can play a key role in cost reduction by selecting good suppliers.

This paper proceeds as follows. In Section 2, literature review is presented. Section 3, introduces the model which selects the suppliers. Numerical example and concluding remarks are discussed in Sections 4 and 5, respectively.

2 Literature review

Various studies on the supplier selection are briefly summarised in the following subsections.

2.1 Analytic hierarchy process

Xia and Wu (2007) presented an integrated approach of analytic hierarchy process (AHP) improved by rough sets theory and multi-objective mixed integer programming. This model determines the best set of suppliers and their corresponding order quantities. Ghodsypour and O’Brien (1998) proposed an integration of AHP and linear programming to consider both tangible and intangible factors in choosing the best suppliers and placing the optimum order quantities such that the total value of purchasing (TVP) becomes the maximum. Chan et al. (2007) developed an AHP-based multi-criterion decision making approach of supplier selection.

2.2 Analytic network process

Gencer and Gürpinar (2007) suggested an analytic network process (ANP)-based model for an electronic company for supplier evaluation and selection with respect to various evaluating criteria. Sarkis and Talluri (2002) believe that supplier evaluation factors would influence each other, and the internal interdependency need to be considered in the evaluation process. The authors applied ANP to evaluate and select the best supplier with respect to organisational factors and strategic performance metrics, which consist of seven evaluating criteria.

2.3 Fuzzy set theory

Lin (2009) suggested an integrated fuzzy analytic network process multi-objective linear programming (FANP-MOLP) approach for identifying top suppliers by considering the effects of interdependence among the selection criteria, as well as to achieve optimal allocation of orders among the selected suppliers. Vinodh et al. (2011) used fuzzy ANP approach for the supplier selection process in an Indian electronics switches manufacturing company. Kuo et al. (2010) proposed integration of particle swarm optimisation (PSO)-based fuzzy neural network (FNN) and artificial neural network for supplier selection. This study is intended to develop an intelligent supplier decision
A support system which is able to consider both the quantitative and qualitative factors. It is composed of:

1. the collection of quantitative data such as profit and productivity
2. a PSO-based FNN to derive the rules for qualitative data
3. a decision integration model for integrating both the quantitative data and fuzzy knowledge decision to achieve the optimal decision.

Amin et al. (2011) proposed a decisional model for supplier selection which consists of two phases. In the first phase, quantified strengths, weaknesses, opportunities and threats (SWOT) analysis are applied for evaluating suppliers. The linguistic variables and triangular fuzzy numbers are used to quantify variables. In the second phase, a fuzzy linear programming model is applied to determine the order quantity. Sarkar and Mohapatra (2006) used the performance and the capability as two major measures in the supplier evaluation and selection problem. The authors used the fuzzy set approach to account for the imprecision involved in numerous subjective characteristics of suppliers. A hypothetical case was adopted to illustrate how the two best suppliers were selected with respect to four performance-based and ten capability-based factors.

2.4 Mathematical programming

Jayaraman et al. (1999) proposed a mixed integer linear programming model to solve the supplier selection and order quantity allocation problem. Mendoza and Ventura (2010) proposed a mixed integer non-linear programming model to determine an optimal inventory policy that coordinates the transfer of items between different stages of a serial supply chain, while properly allocating orders to selected suppliers. Wadhwa and Ravindran (2007) modelled the supplier selection problem as a multi-objective programming (MOP) problem, in which there are three objective functions (i.e., minimisation of price, lead time, and rejects). Three solution approaches, including weighted objective method, goal programming (GP) method, and compromise programming, were used to compare the solutions.

2.5 Data envelopment analysis

Weber (1996) applied data envelopment analysis (DEA) in supplier evaluation for an individual product and demonstrated the advantages of applying DEA to such a system. In this study, the criteria for selecting suppliers were significant reductions in costs, late deliveries and rejected materials. Weber et al. (2000) also presented an approach for evaluating the number of suppliers to employ in a procurement situation using MOP and DEA. Wu (2009) used DEA, decision trees (DTs) and neural networks to assess supplier performance. The model consists of two modules: Module 1 applies DEA and classifies suppliers into efficient and inefficient clusters based on the resulting efficiency scores. Module 2 utilises firm performance-related data to train DT, neural networks model and apply the trained DT model to new suppliers. Wu et al. (2007) presented a so-called augmented imprecise DEA for supplier selection. The proposed model is able to handle imprecise data and allow for increased discriminatory power (i.e., to discriminate efficient suppliers from poor performing suppliers). Mahdiloo et al. (2012) proposed an
algorithm for ranking suppliers in the presence of volume discount offers in terms of multiple criteria in the context of cross-efficiency evaluation.

In this paper, DEA as a non-parametric and multiple criteria decision making tool is used to suppliers’ evaluation. DEA was first introduced by Charnes, Cooper, and Rhodes (CCR) in 1978 and it is a linear-programming-based methodology that uses multiple inputs and multiple outputs to calculate efficiency scores. Since then, DEA is used in different settings like efficiency measurement of container-terminals (Bichou, 2011), airports (Chow et al., 2010), and seaports (Kamble et al., 2010).

The efficiency score for each decision making unit (DMU) is defined as a weighted sum of outputs divided by a weighted sum of inputs, where all efficiencies are restricted to a range from 0 to 1. To avoid the potential difficulty in assigning these weights among various DMUs, a DEA model computes weights that give the highest possible relative efficiency score to a DMU while keeping the efficiency scores of all DMUs less than or equal to one under the same set of weights (Liu et al., 2000). DEA is a robust, standardised and transparent methodology. Furthermore, it possesses additional features which made it suitable to be used as a tool for evaluating the suppliers. The advantages of DEA are as below (Wong and Wong, 2008):

- DEA is an effective tool for evaluating the relative value of DMUs in the presence of multiple performance measures.
- DEA is able to address the complexity arising from the lack of a common scale of measurement.
- In DEA, one need not assume a priori the existence of a particular production function for weighting and aggregating inputs or outputs.
- The objectivity stemming from DEA weighting variables during the optimisation procedure frees the analysis from subjective estimates and randomness.

With the features and inherent characteristics of DEA discussed above, DEA is justified to be used as an evaluation tool for suppliers.

However, sometimes in suppliers’ evaluation problem, there may exist some criteria that should be considered as non-discretionary factors. Discretionary models for evaluating the efficiency of suppliers assume that all criteria are discretionary, that is, controlled by the management of each supplier and varied at its discretion. Thus, failure of a supplier to produce maximal output levels with minimal input consumption results in a decreased efficiency score. In any realistic situation, however, there may exist exogenously fixed or non-discretionary criteria that are beyond the control of a management (Farzipoor Saen, 2009b). In an analysis of a network of fast food restaurants, Banker and Morey (1986) illustrated the impact of exogenously determined inputs that are not controllable. In their study, each of the 60 restaurants in the fast food chain consumes six inputs to produce three outputs. The three outputs (all controllable) correspond to breakfast, lunch, and dinner sales. Only two of the six inputs, expenditures for supplies and expenditures for labour, are discretionary. The other four inputs (age of store, advertising level, urban/rural location, and presence/absence of drive-in capability) are beyond the control of the individual restaurant manager. Their analysis clearly demonstrated the value of accounting for the non-discretionary character of these inputs explicitly in the DEA models they employed; the result is identification of a considerably enhanced opportunity for targeted savings in the controllable inputs and targeted
increases in the outputs. Liu et al. (2000) for selecting the best suppliers considered distance and supply variety as non-discretionary input and non-discretionary output, respectively. Farzipoor Saen (2009b) proposed a new DEA model for selecting the best suppliers in the presence of non-discretionary factors and imprecise data. As well, Farzipoor Saen (2009a) proposed a model for ranking suppliers in the presence of weight restrictions, non-discretionary factors, and cardinal and ordinal data. Recently, Noorizadeh et al. (2011) developed a model to consider dual-role factors, non-discretionary inputs, and weight restrictions to select suppliers.

The approaches presented in the works of Banker and Morey (1986), Liu et al. (2000), Farzipoor Saen (2009a, 2009b) and Noorizadeh et al. (2011) had considerable contribution for considering non-discretionary factors in the DEA context. However, their treatment of non-discretionary factors in DEA models suffers from some limitations. Since, in traditional DEA, free reign is given when deciding for each DMU which outputs and inputs to emphasise, many different avenues are present by which a DMU can appear efficient. Therefore, it is common to have many DMUs that are relatively efficient. In addition, since each DMU has its own set of weights, all of its weight might be put on a single output and input. While this is permissible, it may not be realistic. To overcome these problems, we propose to incorporate non-discretionary inputs in the cross-efficiency method introduced by Sexton et al. (1986) and developed by Doyle and Green (1994). The main idea of cross-efficiency is to use DEA in a peer evaluation instead of a self-evaluation mode. Anderson et al. (2002) argue that, cross evaluation proponents often two main advantages to its use. First, it usually creates a unique ordering among the DMUs. With cross evaluation, since each DMU is rated not only by its own weighting scheme but the schemes of the others also, this amalgamation of weighting schemes makes it far more difficult to have ties and, in effect, creates a unique ordering in practice. Second, cross evaluation appears to eliminate unrealistic weighting schemes that might be used by the DMUs. Under a cross evaluation, once the DMU has a chosen weighting scheme which has been applied to all DMUs, the efficiency value given to each DMU is set aside forming a cross-efficiency matrix. Once the matrix is filled, each DMU has not only its own self-evaluation but also the peer evaluations it has received via the other DMUs in the sample. The average across self and peer evaluations represent a DMU’s cross-efficiency value. A DMU which has a high cross-efficiency value has, therefore, passed a more rigorous test since it cannot only make itself look good but is considered efficient by the majority of its peers.

The above discussions make it more reasonable to model the cross-efficiency formulation of considering non-discretionary factors in DEA models. However, by modelling the cross-efficiency formulation of non-discretionary inputs, a new problem can be occurred. Consider following function, which is the objective function of the model proposed by Banker and Morey (1986).

\[
\text{max } \sum_{r=1}^{k} u_r y_{ro} - \sum_{i \in I_{ND}} v_{i_{ND}x_{io}}
\]

Presence of \( \sum_{i \in I_{ND}} v_{i_{ND}x_{id}} \) with the negative sign in the objective function causes the cross-efficiency score of some DMUs to be negative. To avoid this problem and to
Using DEA cross-efficiency evaluation for suppliers ranking

consider non-discretionary inputs as well, we impose a kind of restriction to the model proposed by Banker and Morey (1986).

The objective of this paper is to propose a cross-efficiency model which considers non-discretionary inputs. As well, by imposing a kind of restriction into the model, the positive cross-efficiency score of all DMUs is guaranteed.

To the best of knowledge of authors, there is not any reference that uses cross-efficiency model and non-discretionary inputs simultaneously. Some of the contributions of this paper are as below:

- The proposed model evaluates suppliers in a multi criteria context.
- Supplier selection is a straightforward process carried out by the proposed model.
- The proposed model considers non-discretionary inputs for supplier selection.
- To achieve the peer appraisal of suppliers instead of their self-appraisal, the cross-efficiency model which considers non-discretionary inputs is developed.

3 Proposed model

Charnes et al. (1978) proposed CCR model (Model 2) as a mathematical programming procedure for evaluating the relative efficiency of multiple DMUs that involve multiple inputs and multiple outputs. The used nomenclatures in this paper are summarised in Table 1.

Table 1  The nomenclatures

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DMU_o</td>
<td>The decision making unit under investigation</td>
</tr>
<tr>
<td>k = 1, …, K</td>
<td>Collection of DMUs (suppliers)</td>
</tr>
<tr>
<td>r = 1, …, R</td>
<td>The set of outputs</td>
</tr>
<tr>
<td>i = 1, …, I</td>
<td>The set of inputs</td>
</tr>
<tr>
<td>ID</td>
<td>The set of discretionary inputs</td>
</tr>
<tr>
<td>IND</td>
<td>The set of non-discretionary inputs</td>
</tr>
<tr>
<td>x_{io}</td>
<td>The i^{th} input of the DMU_o</td>
</tr>
<tr>
<td>y_{ro}</td>
<td>The r^{th} output of DMU_o</td>
</tr>
<tr>
<td>v_{iD}</td>
<td>The weight for i^{th} discretionary input</td>
</tr>
<tr>
<td>v_{iND}</td>
<td>The weight for i^{th} non-discretionary input</td>
</tr>
<tr>
<td>u_r</td>
<td>The weight for r^{th} output</td>
</tr>
<tr>
<td>x_{ik}</td>
<td>The i^{th} input of DMU_k</td>
</tr>
<tr>
<td>y_{rk}</td>
<td>The r^{th} output of DMU_k</td>
</tr>
<tr>
<td>E_{ok}</td>
<td>Shows the relative efficiency of DMU_k with optimal weights for inputs and outputs of DMU_o</td>
</tr>
<tr>
<td>E_{oo}</td>
<td>Is the efficiency score of DMU_o by its optimal weights</td>
</tr>
</tbody>
</table>

The input-oriented CCR model evaluates the suppliers under investigation (DMU_o) (o = 1, …, K) by solving Model (2).
\[
\text{max } h_A = \sum_{r=1}^{R} u_r y_{ro} \\
\text{s.t.} \\
\sum_{i=1}^{I} v_i x_{io} = 1 \\
\sum_{r=1}^{R} u_r y_{rk} - \sum_{i=1}^{I} v_i x_{ik} \leq 0, \quad k = 1, ..., K, \\
v_i \geq 0, \quad i = 1, ..., I, \\
u_r \geq 0, \quad r = 1, ..., R. 
\]

(2)

3.1 CCR-non-discretionary inputs

Suppose that the input variables to be partitioned into subsets of discretionary (D) and non-discretionary (N) variables. Thus, \( i \{1, ..., I\} = I_D \cup I_N, I_D \cap I_N = \emptyset \).

Banker and Morey (1986) developed Model (3) to consider non-discretionary inputs. This model is called CCR-non-discretionary inputs.

\[
\text{max } E_{oo} = \sum_{r=1}^{R} u_r y_{ro} - \sum_{i \in I_{ID}} v_{ND} x_{io} \\
\text{s.t.} \\
\sum_{i \in I_{ID}} v_{ID} x_{io} = 1, \\
\sum_{r=1}^{R} u_r y_{rk} - \left( \sum_{i \in I_{ID}} v_{ID} x_{ik} + \sum_{i \in I_{ID}} v_{ND} x_{ik} \right) \leq 0, \quad k = 1, ..., K, \\
v_{ND} \geq 0, \quad i \in I_N, \\
v_{ID} \geq 0, \quad i \in I_{ID}, \\
u_r \geq 0, \quad r = 1, ..., R. 
\]

(3)

They proved that, the way to model such inputs is to move them to the output side, but with the opposite sign. This idea often arises in situations where there are criteria that are beyond the control of the management but influence the efficiency of DMUs. Thus, in evaluating process, these factors are generally expected to remain at their current level.

3.2 Cross-efficiency formulation of CCR-non-discretionary inputs

At this juncture to create a unique ordering among the efficient DMUs and to eliminate unrealistic weighting schemes in Model (3), we develop the cross-efficiency form of this model. For each \( DMU_o \) \((o = 1, ..., K)\), in Model (3), we can obtain a set of optimal weights (multipliers) \((u_{o}^*, v_{ND}^*, u_{ID}^*)\). Using these set of weights, the cross-efficiency for any \( DMU_k \) \((k = 1, ..., K)\), is then calculated as:

\[
E_{ok} = \frac{\sum_{r=1}^{R} u_{ro}^* y_{rk} - \sum_{i \in I_{ID}} v_{ND}^* x_{ik}}{\sum_{i \in I_{ID}} v_{ID}^* x_{ik}} 
\]

(4)

where \( E_{ok} \) shows the relative efficiency of \( DMU_k \) with optimal weights for inputs and outputs of \( DMU_o \). One can compute the average of the efficiencies in each column to get
Using DEA cross-efficiency evaluation for suppliers ranking

a measure of how the DMUs associated with the column are rated by the rest of the DMUs. Good operating practices more likely to be exhibited by relatively efficient DMUs offering high average efficiencies in their associated columns in the cross-efficiency matrix. Since Model (3) will be run \( n \) times for \( n \) DMUs, respectively, each DMU will get \( n \) efficiency scores, which construct a \( n \times n \) matrix, called cross-efficiency matrix.

For DMU\(_k\) \((k = 1, \ldots, K)\), the average of all \( E_{ok}\) \((o = 1, \ldots, K)\), can be used as an efficiency measure for DMU\(_k\), and will be referred to as the cross-efficiency score for DMU\(_k\). The average is calculated as below.

\[
\bar{E}_k = \frac{1}{n} \sum_{o=1}^{K} E_{ok}
\]

The non-uniqueness of the DEA optimal weights possibly reduces the usefulness of the cross-efficiency. To overcome this problem, Doyle and Green (1994) suggested the use of aggressive and benevolent cross evaluation. A cross evaluation is aggressive/benevolent in the sense that it selects a set of weights which not only maximise the efficiency of a particular DMU under evaluation, but also minimise/maximise the efficiencies of all other DMUs in some sense. We develop the aggressive formulation of Model (3) and represent it as Model (6). Note that the benevolent formulation has the same set of constraints except that the objective function is maximised.

\[
\text{min } h_B = u_r \sum_{i \neq k} y_{rk} - v_{ND} \sum_{i \neq o} x_{ik}
\]

s.t.

\[
\begin{align*}
& \sum_{r=1}^{R} u_r y_{rk} - \left( \sum_{i \in I_D} v_{iD} x_{ik} + \sum_{i \in I_{ND}} v_{iND} x_{ik} \right) \leq 0, \quad k \neq o, \\
& \left( \sum_{r=1}^{R} u_r y_{ro} - \sum_{i \in I_{ND}} v_{iND} x_{ro} \right) - E_{oo} \left( \sum_{i \in I_D} v_{iD} x_{ro} \right) = 0, \quad k = 1, \ldots, K, \\
& v_{ND} \geq 0, \quad i \in I_{ND}, \\
& v_{ID} \geq 0, \quad i \in I_D, \\
& u_r \geq 0, \quad r = 1, \ldots, R.
\end{align*}
\]

where \( E_{oo} \) is the efficiency of DMU\(_o\) obtained from Model (3).

### 3.3 Modified CCR-non-discretionary inputs

The \( E_{ok} \) for some DMUs can be negative. The negative \( E_{ok} \) score is due to \( \sum_{r=1}^{R} u_r y_{rk} - \sum_{i \in I_{ND}} v_{iND} x_{rk} < 0 \) for some DMU\(_k\), i.e., some DMU\(_k\) will have negative efficiency ratios when they use a set of optimal weights obtained when DMU\(_o\) is under evaluation. Normally, we want every output-input efficiency ratio to be positive regardless of the chosen weights. To avoid negative efficiency ratios problem, Wu et al. (2009) proposed adding \( \sum_{r=1}^{R} u_r y_{rj} - w_d \geq 0 \) into variable return to scale (VRS) model when calculating the cross-efficiency scores. Therefore, we suggest adding
\[ \sum_{r=1}^{R} u_{r} y_{rk} - \sum_{i \in I_{ND}} v_{ND} x_{ik} \geq 0 \]

into the Model (3) to prevent negative efficiency ratios.

This idea leads to the Model (7) which is called the modified CCR-non-discretionary inputs model.

\[
\max \quad h_{C} = \sum_{r=1}^{R} u_{r} y_{ro} - \sum_{i \in I_{ND}} v_{ND} X_{io}
\]

s.t.

\[
\sum_{i \in I_{D}} v_{D} X_{io} = 1,
\]

\[
\sum_{r=1}^{R} u_{r} y_{rk} - \left( \sum_{i \in I_{D}} v_{D} x_{ik} + \sum_{i \in I_{ND}} v_{ND} x_{ik} \right) \leq 0, \quad k = 1, \ldots, K, \tag{7}
\]

\[
\sum_{r=1}^{R} u_{r} y_{rk} - \sum_{i \in I_{ND}} v_{ND} x_{ik} \geq 0,
\]

\[
v_{ND} \geq 0, \quad i \in I_{ND},
\]

\[
v_{D} \geq 0, \quad i \in I_{D},
\]

\[
u_{r} \geq 0, \quad r = 1, \ldots, R.
\]

### 3.4 Modified cross-efficiency formulation of CCR-non-discretionary inputs

Where suppliers use the best possible set of weights for their inputs and outputs, traditional DEA can suffer from unrealistic weighting scheme of some suppliers. These suppliers try to disregard those criteria on which they have weakly performed by giving them a zero weight. It follows that these suppliers might be identified as efficient, while this efficiency score of 1 is only due to the type of weights that they have used. In addition, when the number of inputs and outputs increases, the discrimination power of traditional DEA models may decrease. Consequently, these cannot distinguish among those suppliers on the efficiency frontier. To overcome these kinds of problems, a modified cross-efficiency evaluation model which can consider non-discretionary inputs is developed. The proposed model is as below.

\[
\min \quad h_{D} = u_{r} \sum_{k \neq o} y_{rk} - v_{ND} \sum_{k \neq o} x_{ik}
\]

s.t.

\[
v_{D} \sum_{k \neq o} x_{ik} = 1,
\]

\[
\sum_{r=1}^{R} u_{r} y_{rk} - \left( \sum_{i \in I_{D}} v_{D} x_{ik} + \sum_{i \in I_{ND}} v_{ND} x_{ik} \right) \leq 0, \quad k \neq o,
\]

\[
\sum_{r=1}^{R} u_{r} y_{rk} - \sum_{i \in I_{ND}} v_{ND} x_{ik} \geq 0, \quad k = 1, \ldots, K, \tag{8}
\]

\[
\left( \sum_{r=1}^{R} u_{r} y_{ro} - \sum_{i \in I_{ND}} v_{ND} X_{io} \right) - E_{oo} \left( \sum_{i \in I_{D}} v_{D} x_{io} \right) = 0, \quad k = 1, \ldots, K,
\]

\[
v_{ND} \geq 0, \quad i \in I_{ND},
\]

\[
v_{D} \geq 0, \quad i \in I_{D},
\]

\[
u_{r} \geq 0, \quad r = 1, \ldots, R.
\]
4 Numerical example

In order to demonstrate the application of the proposed model in supplier selection context, the dataset is partially taken from Farzipoor Saen (2009a). The inputs for selecting suppliers include total cost of shipments (TC), and distance (D). The outputs utilised in the study are number of shipments to arrive on time (NOT), number of bills received from the supplier without errors (NB), and number of parts (supply variety, SV) that a supplier supplies. Distance (D) is considered as a non-discretionary input. We divide non-discretionary factors into two parts of temporary and permanent. The temporary refers to those factors that DMU can control them after spending time and energy. Suppliers distance can be considered as an example of this type. On the other hand, Permanent non-discretionary factors refer to those factors that DMU cannot control them at all. For example, in the efficiency evaluation of farming lands, amount of rains as an input, is out of the control permanently.

Table 2 displays the dataset for 18 suppliers.

<table>
<thead>
<tr>
<th>Supplier (DMU)</th>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TC (1,000$)</td>
<td>D(km)</td>
</tr>
<tr>
<td>1</td>
<td>253</td>
<td>249</td>
</tr>
<tr>
<td>2</td>
<td>268</td>
<td>643</td>
</tr>
<tr>
<td>3</td>
<td>259</td>
<td>714</td>
</tr>
<tr>
<td>4</td>
<td>180</td>
<td>1,809</td>
</tr>
<tr>
<td>5</td>
<td>257</td>
<td>238</td>
</tr>
<tr>
<td>6</td>
<td>248</td>
<td>241</td>
</tr>
<tr>
<td>7</td>
<td>272</td>
<td>1,404</td>
</tr>
<tr>
<td>8</td>
<td>330</td>
<td>984</td>
</tr>
<tr>
<td>9</td>
<td>327</td>
<td>641</td>
</tr>
<tr>
<td>10</td>
<td>330</td>
<td>588</td>
</tr>
<tr>
<td>11</td>
<td>321</td>
<td>241</td>
</tr>
<tr>
<td>12</td>
<td>329</td>
<td>567</td>
</tr>
<tr>
<td>13</td>
<td>281</td>
<td>567</td>
</tr>
<tr>
<td>14</td>
<td>309</td>
<td>967</td>
</tr>
<tr>
<td>15</td>
<td>291</td>
<td>635</td>
</tr>
<tr>
<td>16</td>
<td>334</td>
<td>795</td>
</tr>
<tr>
<td>17</td>
<td>249</td>
<td>689</td>
</tr>
<tr>
<td>18</td>
<td>216</td>
<td>913</td>
</tr>
</tbody>
</table>
Table 3  Results of evaluation using Model (3)

<table>
<thead>
<tr>
<th>Supplier (DMU)</th>
<th>Efficiency score</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.926</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>0.877</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>0.821</td>
<td>12</td>
</tr>
<tr>
<td>8</td>
<td>0.746</td>
<td>15</td>
</tr>
<tr>
<td>9</td>
<td>0.737</td>
<td>16</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>0.678</td>
<td>17</td>
</tr>
<tr>
<td>12</td>
<td>0.821</td>
<td>11</td>
</tr>
<tr>
<td>13</td>
<td>0.816</td>
<td>13</td>
</tr>
<tr>
<td>14</td>
<td>0.778</td>
<td>14</td>
</tr>
<tr>
<td>15</td>
<td>0.968</td>
<td>8</td>
</tr>
<tr>
<td>16</td>
<td>0.587</td>
<td>18</td>
</tr>
<tr>
<td>17</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>18</td>
<td>0.993</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 4  Efficiency scores where ‘D’ is treated as a discretionary input

<table>
<thead>
<tr>
<th>Supplier (DMU)</th>
<th>Efficiency score</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.929</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>0.879</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>0.827</td>
<td>13</td>
</tr>
<tr>
<td>8</td>
<td>0.746</td>
<td>16</td>
</tr>
<tr>
<td>9</td>
<td>0.741</td>
<td>17</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>0.842</td>
<td>11</td>
</tr>
<tr>
<td>12</td>
<td>0.841</td>
<td>12</td>
</tr>
<tr>
<td>13</td>
<td>0.816</td>
<td>14</td>
</tr>
<tr>
<td>14</td>
<td>0.778</td>
<td>15</td>
</tr>
<tr>
<td>15</td>
<td>0.968</td>
<td>8</td>
</tr>
<tr>
<td>16</td>
<td>0.600</td>
<td>18</td>
</tr>
<tr>
<td>17</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>18</td>
<td>0.993</td>
<td>7</td>
</tr>
</tbody>
</table>
Table 3 illustrates the efficiency scores of suppliers, using Model (3), and their ranking results. In this model, each supplier seeks to maximise its efficiency score by choosing a set of optimal weights for all inputs and outputs. In this evaluation, the best suppliers are suppliers 3, 4, 5, 6, 10, and 17 which their efficiency scores equal to unity.

Now, we analyse the effects of considering ‘D’ as a non-discretionary input on the results. Therefore, we re-solve the problem by considering ‘D’ as a discretionary factor. The results are shown in Table 4. By comparing the results, it can be seen that the ranking of some suppliers by two strategies (considering distance as a non-discretionary and discretionary factor) is different. A remarkable difference is found in supplier 11, whose efficiency score increased by 0.164. As a result, supplier 11 promotes its rank from 17 to 11. This is due to the fact that when we consider distance as a discretionary input, weight of this factor increases from 0.000124 to 0.004149. The rationale behind this is that supplier 11 is the second best supplier regarding this factor.

Table 5 Results of evaluation via Model (7) and Model (8)

<table>
<thead>
<tr>
<th>Supplier (DMU)</th>
<th>Efficiency score obtained by Model (7)</th>
<th>Rank</th>
<th>Modified cross-efficiency obtained by Model (8)</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.926</td>
<td>9</td>
<td>0.622</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>0.877</td>
<td>10</td>
<td>0.704</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>0.854</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>0.618</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td>0.940</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1</td>
<td>0.963</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>0.821</td>
<td>11</td>
<td>0.440</td>
<td>17</td>
</tr>
<tr>
<td>8</td>
<td>0.746</td>
<td>15</td>
<td>0.623</td>
<td>12</td>
</tr>
<tr>
<td>9</td>
<td>0.737</td>
<td>16</td>
<td>0.608</td>
<td>15</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>1</td>
<td>0.661</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>0.678</td>
<td>17</td>
<td>0.518</td>
<td>16</td>
</tr>
<tr>
<td>12</td>
<td>0.818</td>
<td>12</td>
<td>0.681</td>
<td>9</td>
</tr>
<tr>
<td>13</td>
<td>0.816</td>
<td>13</td>
<td>0.702</td>
<td>8</td>
</tr>
<tr>
<td>14</td>
<td>0.778</td>
<td>14</td>
<td>0.654</td>
<td>11</td>
</tr>
<tr>
<td>15</td>
<td>0.968</td>
<td>8</td>
<td>0.832</td>
<td>5</td>
</tr>
<tr>
<td>16</td>
<td>0.587</td>
<td>18</td>
<td>0.391</td>
<td>18</td>
</tr>
<tr>
<td>17</td>
<td>1</td>
<td>1</td>
<td>0.841</td>
<td>4</td>
</tr>
<tr>
<td>18</td>
<td>0.993</td>
<td>6</td>
<td>0.805</td>
<td>6</td>
</tr>
</tbody>
</table>

The problem now becomes selecting a supplier from efficient suppliers (when distance is considered as a non-discretionary input). As is seen, Model (3) cannot give a complete ranking and there are ties among six efficient suppliers. Therefore, Model (6) is used to derive the suppliers’ cross-efficiency score and their complete ranking. However, by using Model (6) which is the aggressive formulation of Model (3), the cross-efficiency scores of some DMUs are negative. For example, the cross-efficiency scores of suppliers 1, 2, 3, 4, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, and 18 have negative efficiency ratios when they use a set of optimal weights obtained when DMU_{10} is under evaluation. Therefore, to guarantee the non-negativity of all suppliers cross-efficiency, Models (7) and (8) are
used. The new cross-efficiency matrix is shown in Table 6 in the Appendix. Table 5 shows the suppliers final efficiency scores obtained by Model (7) and their cross-efficiency scores. Also, suppliers’ final ranking derived by Model (7) and cross-efficiency approach is presented in this table. Cross-efficiency ranking is used as the final ranking of suppliers.

As the last column of Table 5 presents, supplier #6 is the most efficient supplier and is the first candidate for selection.

5 Concluding remarks

Supply chain focuses on the improvement of customer service, profitability and business performance. Strategic partnership with better suppliers needs to be formed to improve quality, flexibility as well as to reduce lead time. The problem of supplier selection is a multi-criteria decision making (MCDM) problem in the presence of many criteria. A decision maker needs to make use one of the MCDM methods (Ayag and Ozdemir, 2009). To this end, we used DEA as a multiple criteria decision making tool to evaluate suppliers. To rank suppliers and to consider non-discretionary inputs as well, cross-efficiency formulation of the model proposed by Banker and Morey (1986) is developed.

The problem considered in this study is at the initial stage of investigation and further researches can be done based on the results of this paper. Some of them are as below.

- Similar research can be repeated in the presence of imprecise data and fuzzy data.
- One of the assumptions of all the classical models of DEA is based on complete homogeneity of DMUs (suppliers), where as this assumption in many real applications cannot be generalised. In other words, some inputs and/or outputs are not common for all the DMUs occasionally. Therefore, there is a need for a model that deals with these conditions.
- Similar research can be repeated for suppliers ranking in the presence of dual-role factors. Dual-role factors refer to those factors which could serve as either inputs or outputs.

Acknowledgements

The authors wish to thank the anonymous reviewer for the valuable suggestions and comments which improved the quality of this paper.

References


Notes

1 In the fractional programming of Model (3), \( \sum_{r=1}^{R} m_r Y_{r0} = \sum_{i \in I_D} V_{iD} X_{i0} / \sum_{i \in I_D} V_{iD} X_{i0} \) can be regarded as the aggregated output to input ratio.
Using DEA cross-efficiency evaluation for suppliers ranking

### Table 6: Matrix of cross-efficiency

<table>
<thead>
<tr>
<th>Supplier (DMU)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.926</td>
<td>0.850</td>
<td>1.000</td>
<td>0.762</td>
<td>1.000</td>
<td>0.973</td>
<td>0.727</td>
<td>0.653</td>
<td>0.721</td>
<td>0.606</td>
<td>0.678</td>
<td>0.761</td>
<td>0.687</td>
<td>0.717</td>
<td>0.757</td>
<td>0.562</td>
<td>0.819</td>
<td>0.842</td>
</tr>
<tr>
<td>2</td>
<td>0.878</td>
<td>0.877</td>
<td>1.000</td>
<td>1.000</td>
<td>0.989</td>
<td>0.816</td>
<td>0.731</td>
<td>0.737</td>
<td>0.710</td>
<td>0.661</td>
<td>0.758</td>
<td>0.730</td>
<td>0.771</td>
<td>0.829</td>
<td>0.586</td>
<td>0.917</td>
<td>0.919</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.479</td>
<td>0.585*</td>
<td>1.000</td>
<td>0.193</td>
<td>0.968</td>
<td>0.984</td>
<td>0.000</td>
<td>0.477</td>
<td>0.573</td>
<td>0.300</td>
<td>0.427</td>
<td>0.816</td>
<td>0.755</td>
<td>0.636</td>
<td>0.829</td>
<td>0.233</td>
<td>0.617</td>
<td>0.856</td>
</tr>
<tr>
<td>4</td>
<td>0.832</td>
<td>0.814</td>
<td>0.956</td>
<td>1.000</td>
<td>0.893</td>
<td>0.871</td>
<td>0.802</td>
<td>0.665</td>
<td>0.688</td>
<td>0.583</td>
<td>0.610</td>
<td>0.715</td>
<td>0.661</td>
<td>0.725</td>
<td>0.727</td>
<td>0.566</td>
<td>0.800</td>
<td>0.870</td>
</tr>
<tr>
<td>5</td>
<td>0.623</td>
<td>0.616</td>
<td>0.950</td>
<td>0.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.046</td>
<td>0.447</td>
<td>0.580</td>
<td>0.353</td>
<td>0.510</td>
<td>0.777</td>
<td>0.696</td>
<td>0.577</td>
<td>0.766</td>
<td>0.266</td>
<td>0.613</td>
<td>0.733</td>
</tr>
<tr>
<td>6</td>
<td>0.289</td>
<td>0.489</td>
<td>0.412</td>
<td>0.000</td>
<td>0.895</td>
<td>1.000</td>
<td>0.000</td>
<td>0.546</td>
<td>0.396</td>
<td>1.000</td>
<td>0.369</td>
<td>0.406</td>
<td>0.622</td>
<td>0.410</td>
<td>0.897</td>
<td>0.121</td>
<td>0.961</td>
<td>0.474</td>
</tr>
<tr>
<td>7</td>
<td>0.879</td>
<td>0.852</td>
<td>1.000</td>
<td>1.000</td>
<td>0.945</td>
<td>0.921</td>
<td>0.821</td>
<td>0.689</td>
<td>0.720</td>
<td>0.609</td>
<td>0.645</td>
<td>0.750</td>
<td>0.691</td>
<td>0.751</td>
<td>0.760</td>
<td>0.587</td>
<td>0.834</td>
<td>0.899</td>
</tr>
<tr>
<td>8</td>
<td>0.824</td>
<td>0.860</td>
<td>0.950</td>
<td>1.000</td>
<td>0.996</td>
<td>1.000</td>
<td>0.790</td>
<td>0.746</td>
<td>0.717</td>
<td>0.781</td>
<td>0.637</td>
<td>0.727</td>
<td>0.735</td>
<td>0.760</td>
<td>0.860</td>
<td>0.561</td>
<td>0.961</td>
<td>0.905</td>
</tr>
<tr>
<td>9</td>
<td>0.878</td>
<td>0.877</td>
<td>1.000</td>
<td>1.000</td>
<td>0.989</td>
<td>0.816</td>
<td>0.731</td>
<td>0.737</td>
<td>0.710</td>
<td>0.661</td>
<td>0.758</td>
<td>0.730</td>
<td>0.771</td>
<td>0.829</td>
<td>0.586</td>
<td>0.917</td>
<td>0.919</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.045</td>
<td>0.294</td>
<td>0.060</td>
<td>0.060</td>
<td>0.582</td>
<td>0.704</td>
<td>0.000</td>
<td>0.443</td>
<td>0.202</td>
<td>1.000</td>
<td>0.192</td>
<td>0.126</td>
<td>0.415</td>
<td>0.230</td>
<td>0.702</td>
<td>0.027</td>
<td>0.845</td>
<td>0.243</td>
</tr>
<tr>
<td>11</td>
<td>0.926</td>
<td>0.850</td>
<td>1.000</td>
<td>0.762</td>
<td>1.000</td>
<td>0.973</td>
<td>0.727</td>
<td>0.653</td>
<td>0.721</td>
<td>0.606</td>
<td>0.678</td>
<td>0.761</td>
<td>0.687</td>
<td>0.717</td>
<td>0.757</td>
<td>0.562</td>
<td>0.819</td>
<td>0.842</td>
</tr>
<tr>
<td>12</td>
<td>0.480</td>
<td>0.591</td>
<td>1.000</td>
<td>0.194</td>
<td>0.981</td>
<td>1.000</td>
<td>0.000</td>
<td>0.487</td>
<td>0.577</td>
<td>0.325</td>
<td>0.431</td>
<td>0.818</td>
<td>0.764</td>
<td>0.641</td>
<td>0.845</td>
<td>0.234</td>
<td>0.637</td>
<td>0.861</td>
</tr>
<tr>
<td>13</td>
<td>0.463</td>
<td>0.673</td>
<td>1.000</td>
<td>0.729</td>
<td>0.963</td>
<td>1.000</td>
<td>0.286</td>
<td>0.641</td>
<td>0.623</td>
<td>0.524</td>
<td>0.432</td>
<td>0.801</td>
<td>0.816</td>
<td>0.746</td>
<td>0.937</td>
<td>0.332</td>
<td>0.817</td>
<td>0.993</td>
</tr>
<tr>
<td>14</td>
<td>0.805</td>
<td>0.852</td>
<td>1.000</td>
<td>1.000</td>
<td>0.999</td>
<td>1.000</td>
<td>0.749</td>
<td>0.734</td>
<td>0.723</td>
<td>0.709</td>
<td>0.623</td>
<td>0.765</td>
<td>0.754</td>
<td>0.778</td>
<td>0.865</td>
<td>0.551</td>
<td>0.926</td>
<td>0.946</td>
</tr>
<tr>
<td>15</td>
<td>0.280</td>
<td>0.560</td>
<td>0.581</td>
<td>0.458</td>
<td>0.900</td>
<td>1.000</td>
<td>0.168</td>
<td>0.643</td>
<td>0.472</td>
<td>0.943</td>
<td>0.359</td>
<td>0.515</td>
<td>0.715</td>
<td>0.558</td>
<td>0.968</td>
<td>0.202</td>
<td>1.000</td>
<td>0.702</td>
</tr>
<tr>
<td>16</td>
<td>0.879</td>
<td>0.852</td>
<td>1.000</td>
<td>1.000</td>
<td>0.945</td>
<td>0.921</td>
<td>0.821</td>
<td>0.689</td>
<td>0.720</td>
<td>0.609</td>
<td>0.645</td>
<td>0.750</td>
<td>0.691</td>
<td>0.751</td>
<td>0.760</td>
<td>0.587</td>
<td>0.834</td>
<td>0.899</td>
</tr>
<tr>
<td>17</td>
<td>0.236</td>
<td>0.508</td>
<td>0.471</td>
<td>0.243</td>
<td>0.888</td>
<td>1.000</td>
<td>0.060</td>
<td>0.602</td>
<td>0.419</td>
<td>1.000</td>
<td>0.340</td>
<td>0.445</td>
<td>0.674</td>
<td>0.481</td>
<td>0.952</td>
<td>0.141</td>
<td>1.000</td>
<td>0.590</td>
</tr>
<tr>
<td>18</td>
<td>0.463</td>
<td>0.673</td>
<td>1.000</td>
<td>0.729</td>
<td>0.963</td>
<td>1.000</td>
<td>0.286</td>
<td>0.641</td>
<td>0.623</td>
<td>0.524</td>
<td>0.432</td>
<td>0.801</td>
<td>0.816</td>
<td>0.746</td>
<td>0.937</td>
<td>0.332</td>
<td>0.817</td>
<td>0.993</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td>0.622</td>
<td>0.704</td>
<td>0.854</td>
<td>0.618</td>
<td>0.940</td>
<td>0.963</td>
<td>0.440</td>
<td>0.623</td>
<td>0.608</td>
<td>0.661</td>
<td>0.518</td>
<td>0.681</td>
<td>0.702</td>
<td>0.654</td>
<td>0.832</td>
<td>0.391</td>
<td>0.841</td>
<td>0.805</td>
</tr>
<tr>
<td><strong>Rank</strong></td>
<td>13</td>
<td>7</td>
<td>3</td>
<td>14</td>
<td>2</td>
<td>1</td>
<td>17</td>
<td>12</td>
<td>15</td>
<td>10</td>
<td>16</td>
<td>9</td>
<td>8</td>
<td>11</td>
<td>5</td>
<td>18</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

Notes: *0.585 represents the cross-efficiency score of supplier #2 in terms of optimal weights of supplier #3. **Italic numbers in the leading diagonal are the efficiency scores obtained by Model (3).
Article IV


© 2012 Inderscience Enterprises Ltd. retains the copyright of the paper.
A data envelopment analysis model for selecting suppliers in the presence of both dual-role factors and non-discretionary inputs

Abdollah Noorizadeh
Department of Industrial Management,
Lappeenranta University of Technology,
P.O. Box 20, 53851 Lappeenranta, Finland
E-mail: ab.noorizadeh@gmail.com

Mahdi Mahdiloo
Institute for Integrated and Intelligent Systems,
Griffith University,
QLD 4111, Australia
E-mail: mahdi.mahdiloo@griffithuni.edu.au

Reza Farzipoor Saen*
Department of Industrial Management,
Faculty of Management and Accounting,
Islamic Azad University – Karaj Branch,
P.O. Box 31485-313, Karaj, Iran
Fax: 0098 (261) 4418156
E-mail: farzipour@yahoo.com
*Corresponding author

Abstract: Supplier selection is the strategy adopted by the manufacturer, to evaluate and select suppliers, which can fulfil the requirements of the manufacturer. To this end, data envelopment analysis (DEA), as a multiple criteria decision-making tool, has been applied for several times. However, conventional DEA models cannot simultaneously consider dual-role and non-discretionary factors. The objective of this paper is to propose a DEA model for ranking suppliers in the presence of both dual-role factors and non-discretionary inputs. A numerical example demonstrates the application of the proposed model.

Keywords: data envelopment analysis; DEA; supplier selection; dual-role factors; non-discretionary inputs.


1 Introduction

Most manufacturing enterprises are organised as networks of manufacturing and distribution sites that procure raw materials, transform them into intermediate and finished products, and distribute the finished products to customers (Lee and Billington, 1992). The short-term objective of supply chain management (SCM) is primarily to increase productivity and reduce the entire inventory and the total cycle time, while the long-term objective is to increase customer satisfaction, market share, and profits for all organisations in the supply chain (Tan et al., 1998). Shin et al. (2000) argue that several important factors have caused the current shift to single sourcing or a reduced supplier base. First, multiple sourcing prevents suppliers from achieving the economies of scale based on order volume and learning curve effect. Second, a multiple supplier system can be more expensive than a reduced supplier base. For instance, managing a large number of suppliers for a particular item directly increases costs, including the labour and order processing costs to managing multiple source inventories. Meanwhile multiple sourcing lowers overall quality levels because of the increased variation in incoming quality among suppliers. Third, a reduced supplier base helps eliminate mistrust between buyers
and suppliers due to lack of communication. Fourth, worldwide competition forces firms to find the best suppliers in the world.

The objective of this paper is to propose a new data envelopment analysis (DEA) model for ranking suppliers in the presence of both non-discretionary inputs and dual-role factors.

DEA is proposed by Charnes et al. (1978) and provides a non-parametric methodology for evaluating the efficiency of each of a set of comparable decision-making units (DMUs), relative to one another. DEA is a non-parametric mathematical programming technique that determines an efficient frontier of the most efficient DMUs and calculates the efficiency of each DMU relative to this efficient frontier based on multiple observed inputs and outputs. An efficiency score of a DMU is generally defined as the weighted sum of outputs divided by the weighted sum of inputs, while weights need to be assigned. To avoid the potential difficulty in assigning these weights among various DMUs, a DEA model computes weights that give the highest possible relative efficiency score to a DMU while keeping the efficiency scores of all DMUs less than or equal to one under the same set of weights (Liu et al., 2000).

In DEA, DMU is evaluated against the performance of the remaining DMUs in the sample via a ratio of the sum of weighted outputs to the sum of weighted inputs. Two restrictions are applied. The first restriction is that the weights must be non-negative. The second restriction is that the weighting scheme used will be applied to all other DMUs in the sample and none of them may have a ratio greater than one. Therefore, an inefficient DMU is one for which a weighting scheme cannot be found that evaluates it better than all other DMUs. An attempt is made to find the weighting scheme for each DMU that casts it in the most favourable light possible and the resulting ratio is designated the DMU’s efficiency value (Anderson et al., 2002). In DEA formulations, the assessed DMUs can freely choose the weights or values to be assigned to each input and output in a way that maximises its efficiency, subject to this system of weights being feasible for all other DMUs. This freedom of choice shows the DMU in the best possible light, and is equivalent to assuming that no input or output is more important than any other. The free imputation of input-output values can be seen as an advantage, especially as far as the identification of inefficiency is concerned. If a DMU (supplier) is free to choose its own value system and some other suppliers uses this same value system to show that the first supplier is not efficient, then a stronger statement is being made. The primary problem associated with arbitrary weights (which is mostly used in MCDM methods) is that they are subjective, and it is often a difficult task for the decision-maker (DM) to accurately assign numbers to preferences. It is a daunting task for the DM to assess weighting information as the number of performance criteria increased. DEA does not demand exact weights from the DM. Since classical techniques always require intuitive judgements that have biases, DEA helps DMs to select the suppliers without relying on intuitive judgements (Farzipoor Saen, 2010b).

In applying DEA, there is a strong argument for permitting certain factors to simultaneously play the role of both inputs and outputs. Such factors such as suppliers research and development (R&D) cost clearly constitute an output measure, but at the same time it is an important component of the supplier, hence, it is an input. From the perspective of DM who intends to select the best supplier, such measures may play the role of proxy for ‘suppliers’ innovation’. R&D results in the technology that brings new products and services to the market place or strengthens better processes. Innovation
results in high quality jobs, successful businesses, better goods and services and more efficient processes. That is why R&D can reasonably be classified as output. On the other hand, from the perspective of supplier, it can be considered as input that imposes special expenses to the supplier.

On the other hand, discretionary models for evaluating the efficiency of suppliers assume that all criteria are discretionary, i.e., controlled by the management of each supplier and varied at its discretion. Thus, failure of a supplier to produce maximal output levels with minimal input consumption results in a decreased efficiency score. In any realistic situation, however, there may exist exogenously fixed or non-discretionary criteria that are beyond the control of a management. For example, consider suppliers distance from the factory which is an input. It will not be acceptable from the supplier’s perspective to decrease the distance in order to improve its performance.

Clearly, there may exist a situation that these two factors (i.e., the dual-role factors and non-discretionary inputs) should be considered simultaneously and a technique that can deal with these two factors in a single model is needed to better model such situation.

Another issue which has been discussed frequently in the suppliers ranking literature has been the lack of discrimination in DEA applications, in particular when the number of inputs and outputs is too high relative to the number of DMUs. The basic DEA models classify the DMUs into two groups, efficient and inefficient. Often DMs are interested in a complete ranking in order to refine the evaluation of the units. To this end, we use ‘virtual best’ DMU concept to derive the complete ranking of suppliers.

This paper proceeds as follows. In Section 2, literature review is presented. Section 3 introduces the method which ranks the suppliers in the presence of both dual-role factors and non-discretionary inputs. Numerical example and concluding remarks are discussed in Sections 4 and 5, respectively.

2 Literature review

2.1 Supplier selection

Some mathematical programming approaches have been used for supplier selection in the past. Nydick and Hill (1992), Barbarosoglu and Yazgac (1997), and Narasimhan (1983) used analytic hierarchy process (AHP) to support supplier selection decisions. Akarte et al. (2001) developed a web-based AHP system to evaluate the casting suppliers with respect to 18 criteria. In this system, suppliers should first register, and then input their casting specifications. To evaluate the suppliers, buyers determine the relative importance weightings for the criteria based on the casting specifications, and then assign the performance rating for each criterion using a pairwise comparison. Chan (2003) developed an interactive selection model with AHP to facilitate DMs in selecting suppliers. Kahraman et al. (2003) suggested fuzzy AHP for selecting the best supplier providing the most satisfaction for the determined criteria. Ghodsypour and O’Brien (1998) used AHP and linear programming to select suppliers.

Sarkis and Talluri (2002) believe that supplier evaluation factors would influence each other, and the internal interdependency need to be considered in the evaluation process. The authors applied analytic network process (ANP) to evaluate and select the best supplier with respect to organisational factors and strategic performance metrics, which consist of seven evaluating criteria.
Lee (2008) proposed a mean-variance approach to determine the optimal number of suppliers in the presence of supplier failure risks. The mean value approach assumes that the firm has a linear utility function with respect to the supply disruptions. Hu and Xie (2010) considered the value of practicing early order commitment (EOC) in a supply chain with demand uncertainty and lost sales. They also examined the impact of forecasting errors and inventory policies used by the retailers on the performance of the supply chain. Xiao et al. (2010) developed a two-period game model of a supply chain consisting of one manufacturer and one retailer to investigate the pricing and effort investment decisions when customer satisfaction is considered. Dharmapala (2008) used the DEA model with intrinsic assurance regions (IAR) and focused on how to make cost savings in a supply chain by projecting inefficient supply units on to the efficient frontier.

Choy and Lee (2002) proposed a generic model using the case-based reasoning (CBR) technique for supplier selection. Various evaluating criteria were grouped into three categories: technical capability, quality system, and organisational profile. The model was implemented in a consumer products manufacturing company, which had stored the performance of past suppliers and their attributes in a database system. Choy et al. (2005) applied the CBR-based model to aid DMs in the supplier selection problem.

Lin and Chen (2004) presented a fuzzy decision-making framework for selecting the most favourable strategic supply chain alliance under limited evaluation resources. Holt (1998) and Li et al. (1997) applied fuzzy sets theory in supplier selection. Sarkar and Mohapatra (2006) suggested that performance and capability are two major measures in the supplier evaluation and selection problem. The authors used the fuzzy set approach to account for the imprecision involved in numerous subjective characteristics of suppliers. A hypothetical case was adopted to illustrate how the two best suppliers were selected with respect to four performance-based and ten capability-based factors. Talluri and Baker (2002) developed a binary integer linear programming model to evaluate alternative supplier bids based on ideal targets for bid attributes set by the buyer, and to select an optimal set of bids by matching demand and capacity constraints. Based on four variations of model, effective negotiation strategies were proposed for unselected bids.

Karpak et al. (2001) constructed a goal programming (GP) model to evaluate and select the best suppliers. Three goals were considered in the model, including cost, quality, and delivery reliability. Wadhwa and Ravindran (2007) modelled the supplier selection problem as a multi-objective programming (MOP) problem, in which there are three objective functions, such as minimisation of price, lead time, and rejects. Three solution approaches, including weighted objective method, GP method, and compromise programming were used to compare the solutions. Vokurka et al. (1996) proposed to incorporate expert system technology into a decision-support framework. Their expert system integrates the judgement and expertise of purchasing professionals with the formal approaches of earlier works. Ndubisi et al. (2005) used a multiple regression model for supplier selection and found that the selection of supplier based on technology is important for the manufacturer whose focus is on product and launch flexibility. Rezaei and Davoodi (2008) considered the problem of supply chain with multiple suppliers and multiple products. Their supplier evaluation includes four major assumptions:

a suppliers have limited capacity
b received items from suppliers are not of perfect quality
c the demand over a finite planning horizon is known
A. Noorizadeh et al.

the buyer has a maximum storage space in each period.

Weber (1996) applied DEA in supplier evaluation for an individual product and demonstrated the advantages of applying DEA to such a system. In this study, the criteria for selecting suppliers were significant reductions in costs, late deliveries and rejected materials. Weber et al. (2000) also presented an approach for evaluating the number of suppliers to employ in a procurement situation using MOP and DEA. Talluri et al. (2006) developed a chance-constrained DEA model for selecting suppliers. Talluri and Narasimhan (2003) developed a max-min DEA model for supplier selection problem. Mohammady Garfamy (2006) presented the methodology of applying DEA to compare overall supplier performances based on total cost of ownership (TCO) concept and demonstrated this application through a study for a hypothetical firm.

2.2 Dual-role factors

In applying DEA, there is a strong argument for permitting certain factors to simultaneously play the role of both inputs and outputs. Beasley (1990, 1995), in a study of the efficiency of university departments, treated research funding on both the input and output sides. However, as Cook et al. (2006) addressed, the model proposed by Beasley (1990, 1995) has two limitations. The first limitation is that in the absence of constraints (e.g., assurance region or cone-ratio) on the multipliers, each DMU may be 100% efficient. The second limitation is that the dual-role factor is considered differently on the input than on the output side. Cook et al. (2006) developed a new model that has not the above mentioned limitations. Recently, Farzipoor Saen (2010a) proposed a model which can consider multiple dual-role factors for selecting third-party reverse logistics (3PL) providers. In his study, the ratings for service-quality experience and service-quality credence on selecting third-party reverse logistics providers are used as dual-role factors. As well, Farzipoor Saen (2010b) proposed a method for selecting suppliers in the presence of a dual-role factor and weight restrictions. In this study, the R&D cost is considered as both an input and an output.

Recently, Mahdiloo et al. (2011) addressed the problem of a factor in supplier selection analysis which may be classified either an input or an output. They demonstrated the validity of their proposed approach via comparing the results with conventional models. Farzipoor Saen (2010b) and Mahdiloo et al. (2011) used R&D cost of suppliers as a dual-role factor. However, they did not consider non-discretionary inputs.

2.3 Non-discretionary inputs

Discretionary models for evaluating the efficiency of suppliers assume that all criteria are discretionary, that is, controlled by the management of each supplier and varied at its discretion. Thus, failure of a supplier to produce maximal output levels with minimal input consumption results in a decreased efficiency score. In any realistic situation, however, there may exist exogenously fixed or non-discretionary criteria that are beyond the control of a management. In an analysis of a network of fast food restaurants, Banker and Morey (1986) illustrated the impact of exogenously determined inputs that are not controllable. In their study, each of the 60 restaurants in the fast food chain consumes six inputs to produce three outputs. The three outputs (all controllable) correspond to
breakfast, lunch, and dinner sales. Only two of the six inputs, expenditures for supplies and expenditures for labour, are discretionary. The other four inputs (age of store, advertising level, urban/rural location, and presence/absence of drive-in capability) are beyond the control of the individual restaurant manager. Their analysis clearly demonstrates the value of accounting for the non-discretionary character of these inputs explicitly in the DEA models they employ; the result is identification of a considerably enhanced opportunity for targeted savings in the controllable inputs and targeted increases in the outputs. In the case of supplier selection, distance and supply variety are generally considered as non-discretionary criteria. To select suppliers, Liu et al. (2000) considered supply variety as a non-discretionary output. As well, Farzipoor Saen (2009a) used distance of suppliers from the factory as a non-discretionary input. However, they did not consider dual-role factors in their paper. Recently, Noorizadeh et al. (in press) developed a model to consider dual-role factors, non-discretionary inputs and weight restrictions. Nevertheless, their proposed model can not rank all the suppliers.

2.4 Augmented DEA

While DEA is an appropriate model for supplier evaluation, if the number of inputs and outputs being used increases, the discrimination power of DEA models may decrease. Therefore, in the context of supplier evaluation and selection, DEA may not derive a complete ranking of efficient suppliers. To overcome this problem, Appalla (2003) proposed an augmented DEA, which enhances the capability of discriminating efficient suppliers further by introducing a ‘virtual best’ supplier. Wu et al. (2007) used augmented DEA for supplier ranking which can operate under conditions of imprecise data. As well, Wu and Blackhurst (2009) developed an augmented DEA model which can derive a complete ranking of suppliers. The idea of augmented DEA is based on the introduction of a new virtual DMU called the ‘virtual best’ DMU, which is created by selecting the best values of each criterion from the existing DMU base. This method changes the efficient frontier of the model and thus increases the discriminatory power of the basic DEA model. The efficiency of each DMU is obtained with respect to the efficient frontier of the ‘virtual best’ DMU, which can then be used to rank the DMUs (Wu et al., 2007).

However, all of the above mentioned references which use the concept of virtual best DMU to rank suppliers do not consider dual-role factors and non-discretionary inputs in their research. A technique that can deal with both dual-role factors and non-discretionary inputs in an augmented DEA model is needed to better model such situation.

To the best of knowledge of authors, there is not any reference that discusses suppliers ranking in the presence of both dual-role factors and non-discretionary inputs. The approach presented in this paper has some distinctive contributions.

- Supplier selection is a straightforward process carried out by the proposed model.
- The increasing number of decision-making criteria, complicates the supplier selection process. This paper presents a robust model to solve the multiple-criteria problem.
- The proposed model can be easily computerised, enabling it to serve as a decision-making tool to assist DMs.
The proposed model does not demand exact weights from the DM. Since classical techniques always require intuitive judgements that have biases, this paper helps DMs to select the suppliers without relying on intuitive judgements.

The proposed model considers dual-role factors for supplier selection.

The proposed model considers non-discretionary inputs for supplier selection.

The proposed model incorporates both dual-role factors and non-discretionary inputs into a single model.

The proposed model can derive a complete ranking of suppliers.

3 Proposed model

Consider a situation where members \( k \) of a set of \( K \) DMUs are to be evaluated in terms of \( R \) outputs \( Y_k = (y_{rk})^{R}_{r=1} \) and \( I \) inputs \( X_k = (x_{ik})^{I}_{i=1} \). In addition, assume that a particular factor is held by each DMU in the amount \( w_k \), and serves as both an input and output factor. The used nomenclatures in this paper are summarised in Table 1.

Table 1 The nomenclatures

<table>
<thead>
<tr>
<th>DMU ( o )</th>
<th>The decision-making unit under investigation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k = 1, \ldots, K )</td>
<td>Collection of DMUs (suppliers)</td>
</tr>
<tr>
<td>( r = 1, \ldots, R )</td>
<td>The set of outputs</td>
</tr>
<tr>
<td>( i = 1, \ldots, I )</td>
<td>The set of inputs</td>
</tr>
<tr>
<td>( I_D )</td>
<td>Set of discretionary inputs</td>
</tr>
<tr>
<td>( I_{ND} )</td>
<td>Set of non-discretionary inputs</td>
</tr>
<tr>
<td>( f = 1, \ldots, F )</td>
<td>The set of dual-role factors</td>
</tr>
<tr>
<td>( x_{io} )</td>
<td>The ( i )th input of the DMU ( o )</td>
</tr>
<tr>
<td>( y_{ro} )</td>
<td>The ( r )th output of DMU ( o )</td>
</tr>
<tr>
<td>( w_o )</td>
<td>Level of dual-role factor of DMU ( o )</td>
</tr>
<tr>
<td>( v_{iD} )</td>
<td>The weight for ( i )th discretionary input</td>
</tr>
<tr>
<td>( v_{iND} )</td>
<td>The weight for ( i )th non-discretionary input</td>
</tr>
<tr>
<td>( u_r )</td>
<td>The weight for ( r )th output</td>
</tr>
<tr>
<td>( x_{ik} )</td>
<td>The ( i )th input of DMU ( k )</td>
</tr>
<tr>
<td>( y_{rk} )</td>
<td>The ( r )th output of DMU ( k )</td>
</tr>
<tr>
<td>( w_{fk} )</td>
<td>The ( f )th dual-role factor of DMU ( k )</td>
</tr>
<tr>
<td>( \gamma_f )</td>
<td>The weight for dual-role factor when it is treated on the output side</td>
</tr>
<tr>
<td>( \beta_f )</td>
<td>The weight for dual-role factor when it is treated on the input side</td>
</tr>
<tr>
<td>( x_{iD, v} )</td>
<td>The ( i )th discretionary input of ‘virtual best’ DMU</td>
</tr>
<tr>
<td>( x_{iND, v} )</td>
<td>The ( i )th non-discretionary input of ‘virtual best’ DMU</td>
</tr>
<tr>
<td>( y_{rv} )</td>
<td>The ( r )th output of ‘virtual best’ DMU</td>
</tr>
<tr>
<td>( w_{pv} )</td>
<td>The ( p )th dual-role factor of ‘virtual best’ DMU</td>
</tr>
</tbody>
</table>
Model (1) is proposed by Cook et al. (2006) for considering a single dual-role factor in DEA.

$$\text{Max} \frac{\sum_{r=1}^{R} u_r \cdot y_{ro} + (\gamma - \beta) \cdot w_o}{\left( \sum_{i=1}^{I} v_i \cdot x_{io} \right)}$$

s.t. $$\left( \sum_{r=1}^{R} u_r \cdot y_{rk} + (\gamma - \beta) \cdot w_k \right) \leq 1, \quad k = 1, ..., K,$$

$$u_r, v_i, \gamma, \beta \geq 0$$

Using a standard technique (see, e.g., Charnes et al., 1978) to transform the above fractional model (1) into a linear model, there will be the following linear programming model.

$$\text{Max} \sum_{r=1}^{R} u_r \cdot y_{ro} + (\gamma - \beta) \cdot w_o$$

s.t. $$\sum_{i=1}^{I} v_i \cdot x_{io} = 1,$$

$$\sum_{r=1}^{R} u_r \cdot y_{rk} + (\gamma - \beta) \cdot w_k - \sum_{i=1}^{I} v_i \cdot x_{ik} \leq 0, \quad k = 1, ..., K,$$

$$u_r, v_i, \gamma, \beta \geq 0.$$

To consider multiple dual-role factors in DEA models, Farzipoor Saen (2010a) proposed model (3). Assume that some factors are held by each DMU in the amount \( w_k \) \((f = 1, ..., F)\), and serve as both an input and output factors. The proposed model for considering multiple dual-role factors is as follows:

$$\text{Max} \sum_{r=1}^{R} u_r \cdot y_{ro} + \sum_{f=1}^{F} (\gamma_f - \beta_f) \cdot w_{fo}$$

s.t. $$\sum_{i=1}^{I} v_i \cdot x_{io} = 1,$$

$$\sum_{r=1}^{R} u_r \cdot y_{rk} + \sum_{f=1}^{F} (\gamma_f - \beta_f) \cdot w_{fk} - \sum_{i=1}^{I} v_i \cdot x_{ik} \leq 0, \quad k = 1, ..., K,$$

$$u_r, v_i, \gamma_f, \beta_f \geq 0.$$

Now, to demonstrate how to incorporate dual-role factors and non-discretionary inputs simultaneously into a single model, model (4) is proposed.
The objective function of the model (4) seeks to maximise the efficiency score of the DMU by choosing a set of weights for all discretionary and non-discretionary inputs, outputs and dual-role factors. The first constraint set of model (4) ensures that, under the set of chosen weights, the efficiency scores of all DMUs are less than or equal to 1. Other constraint sets of model (4) guarantee the non-negativity of all weights. Since we want to maximise the ratio, the way to achieve that goal is decreasing the denominator; therefore, the model suggests that inputs should be decreased. Outcome of model (4) is an efficiency score equal to one for efficient DMUs and less than one for inefficient DMUs. Model (4) can be converted into a linear programming problem as follows:

\[
\text{Max} \; \frac{\sum_{r=1}^{R} u_r \cdot y_{ro} + \sum_{f=1}^{F} (y_f - \beta_f) \cdot w_{fo} - \sum_{i=1}^{I} v_{iND} \cdot x_{io}}{\left( \sum_{i=1}^{I} v_i \cdot x_{io} \right)}
\]

s.t.

\[
\frac{\sum_{r=1}^{R} u_r \cdot y_{rk} + \sum_{f=1}^{F} (y_f - \beta_f) \cdot w_{fk} - \sum_{i=1}^{I} v_{iND} \cdot x_{ik}}{\left( \sum_{i=1}^{I} v_i \cdot x_{ik} \right)} \leq 1, \quad k = 1, \ldots, K, \tag{4}
\]

\[v_{iND}, \beta_f \geq 0,\]

\[v_{ID}, u_r, y_f \geq \varepsilon,\]

\[\varepsilon > 0, \text{ (non-Archimedean)}.
\]

Now, one of three possibilities exists in regard to the sign of \(\hat{\gamma} - \hat{\beta}\), where \(\hat{\gamma}, \hat{\beta}\) are the optimal values from model (3); \(\hat{\gamma} - \hat{\beta} > 0, = 0, \text{ or } < 0 \) (Cook et al., 2006).

Case 1 If \(\hat{\gamma} - \hat{\beta} < 0\), then the dual-role factor is ‘behaving like input’. Hence, less of this factor is better, and would lead to an increase in efficiency.

Case 2 If \(\hat{\gamma} - \hat{\beta} > 0\), then the dual-role factor is ‘behaving like output’. Hence, more of this factor is better, and would lead to an increase in efficiency.
Case 3  If $\hat{\gamma} - \hat{\beta} = 0$, then dual-role factor is at equilibrium level.

However, efficiency scores calculated by model (5) can not give a complete ranking of suppliers. To derive a complete ranking, a ‘virtual best’ DMU is incorporated into model (5). Therefore, model (6) is an augmented DEA model which considers both dual-role factors and non-discretionary inputs.

$$\begin{align*}
\text{Max} & \sum_{r=1}^{R} u_r \cdot y_{ro} + \sum_{f=1}^{F} (\gamma_f - \beta f) \cdot w_{fo} - \sum_{i=1}^{I} v_{ND} \cdot x_{io} \\
\text{s.t.} & \sum_{i=1}^{I} v_{ID} \cdot x_{io} = 1, \\
& \sum_{r=1}^{R} u_r \cdot y_{rk} + \sum_{f=1}^{F} (\gamma_f - \beta f) \cdot w_{f} - \left( \sum_{i=1}^{I} v_{ID} \cdot x_{ik} + \sum_{i=1}^{I} v_{ND} \cdot x_{ik} \right) \leq 0, \quad k = 1, ..., K, \\
& \sum_{r=1}^{R} u_r \cdot y_{rv} + \sum_{f=1}^{F} (\gamma_f - \beta f) \cdot w_{f} - \left( \sum_{i=1}^{I} v_{ID} \cdot x_{iv} + \sum_{i=1}^{I} v_{ND} \cdot x_{iv} \right) \leq 0 \\
& v_{ND}, \beta f \geq 0, \\
& v_{ID}, u_r, \gamma_f \geq \varepsilon, \\
& \varepsilon > 0, \text{ (non-Archimedean)}. 
\end{align*}$$

Note that, model (6) is applied only for selecting a supplier from efficient suppliers and resulting of this model is less than 1 for efficient DMUs. The amount of discretionary and non-discretionary inputs, outputs and dual-role factors associated with the ‘virtual best’ supplier is created in the following form.

$$\begin{align*}
y_r &= \max \left\{ y_{rk} \right\}, \quad r = 1, ..., R, \quad k \in \text{efficient DMUs} \\
x_{iv} &\in I_D \quad \min \left( x_{ik} \right), \quad i \in I_D, \quad k \in \text{efficient DMUs} \\
x_{iv} &\in I_{ND} \quad \min \left( x_{ik} \right), \quad i \in I_{ND}, \quad k \in \text{efficient DMUs} \\
w_f &\in F \quad \max \left( w_{f} \right), \quad f = 1, ..., F, \quad k \in \text{efficient DMUs, when dual-role factor is treated on the output side} \\
w_f &\in F \quad \min \left( w_{f} \right), \quad f = 1, ..., F, \quad k \in \text{efficient DMUs, when dual-role factor is treated on the input side}. 
\end{align*}$$

4  Numerical example

In order to demonstrate the application of the proposed approach in supplier selection context, the dataset for this study is partially taken from Farzipoor Saen (2010b). The inputs for selecting suppliers include total cost of shipments (TC), number of shipments per month (NS), and R&D cost. The outputs utilised in the study are number of
shipments to arrive on time (NOT), number of bills received from the supplier without errors (NB), product quality (PQ), and R&D. R&D plays the role of both input and output. Distance (D) is considered as a non-discretionary input. Table 2 shows the dataset for 18 suppliers.

Table 2  

| Supplier no. | TC (1,000$) | NS | D (km) | R&D (1,000$) | NOT | NB | PQ
|--------------|-------------|----|--------|---------------|-----|----|----
| 1            | 253         | 197| 249    | 20            | 187 | 90 | 1  |
| 2            | 268         | 198| 643    | 32            | 194 | 130| 5  |
| 3            | 259         | 229| 714    | 15            | 220 | 200| 3  |
| 4            | 180         | 169| 1809   | 10            | 160 | 100| 4  |
| 5            | 257         | 212| 238    | 16            | 204 | 173| 1  |
| 6            | 248         | 197| 241    | 28            | 192 | 170| 2  |
| 7            | 272         | 209| 1404   | 12            | 194 | 60 | 5  |
| 8            | 330         | 203| 984    | 36            | 195 | 145| 3  |
| 9            | 327         | 208| 641    | 30            | 200 | 150| 2  |
| 10           | 330         | 203| 588    | 28            | 171 | 90 | 3  |
| 11           | 321         | 207| 241    | 19            | 174 | 100| 1  |
| 12           | 329         | 234| 567    | 25            | 209 | 200| 2  |
| 13           | 281         | 173| 567    | 18            | 165 | 163| 1  |
| 14           | 309         | 203| 967    | 27            | 199 | 170| 4  |
| 15           | 291         | 193| 635    | 22            | 188 | 185| 2  |
| 16           | 334         | 177| 795    | 31            | 168 | 85 | 3  |
| 17           | 249         | 185| 689    | 50            | 177 | 130| 5  |
| 18           | 216         | 176| 913    | 15            | 167 | 160| 4  |

Notes: ¹This variable is a qualitative criterion. Assume that for this qualitative variable each supplier is rated on a 5-point Likert scale, where the particular point on the scale is chosen through a consensus on the part of executives within the organisation. 5-point scales are common for evaluating in terms of qualitative data, and are often accompanied by interpretations such as: 1 = very bad, 2 = bad, 3 = medium, 4 = good, 5 = very good, which are easily understood by DM.

Table 3 reports the results of efficiency score obtained by model (5). Also, the behaviour of dual-role factor for 18 suppliers is depicted in this table. Model (5) identified suppliers 2, 3, 4, 5, 6, 7, 14, 15, 17, and 18 to be efficient with a relative efficiency score of 1. The remaining 8 suppliers with relative efficiency score of less than 1 are considered to be inefficient. Note that, each DEA model seeks to determine which of the n DMUs define an envelopment surface that represents best practice, referred to as the empirical production function or the efficient frontier. DMUs that lie on the surface are deemed efficient in DEA, while those DMUs that do not, are termed inefficient. DEA provides a comprehensive analysis of relative efficiencies for multiple input-multiple output situations by evaluating each DMU and measuring its performance relative to an envelopment surface composed of other DMUs (Farzipoor Saen, 2009b). In order to interpret the behaviour of dual-role factor, consider, for instance, suppliers 1 and 2. For
A data envelopment analysis model for selecting suppliers

supplier 1, with a negative $\hat{y}_1 - \hat{\beta}_1$, R&D is behaving like an input, and lower value of such factor would increase the efficiency of the supplier. For supplier 2, with a positive $\hat{y}_1 - \hat{\beta}_1$, R&D is behaving like an output, and higher level of such factor would improve the efficiency of the supplier.

Table 3  Efficiency scores and output/input behaviour using model (5)

<table>
<thead>
<tr>
<th>Supplier no.</th>
<th>Efficiency scores in the presence of both dual-role factor and non-discretionary input</th>
<th>$\hat{y}_1$</th>
<th>$\hat{\beta}_1$</th>
<th>$\hat{y}_1 - \hat{\beta}_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9728</td>
<td>0.0001</td>
<td>0.001072786</td>
<td>–0.000972786</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.0000981117</td>
<td>0</td>
<td>0.000981117</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0.0001</td>
<td>0.000758839</td>
<td>–0.000658839</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0.0001</td>
<td>0.09193314</td>
<td>–0.09183314</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0.0001</td>
<td>0.001004062</td>
<td>–0.000904062</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>0.000882344</td>
<td>0</td>
<td>0.000882344</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>0.0001</td>
<td>0.006941042</td>
<td>–0.006841042</td>
</tr>
<tr>
<td>8</td>
<td>0.9807</td>
<td>0.000923908</td>
<td>0</td>
<td>0.000923908</td>
</tr>
<tr>
<td>9</td>
<td>0.9776</td>
<td>0.000196352</td>
<td>0</td>
<td>0.000196352</td>
</tr>
<tr>
<td>10</td>
<td>0.8524</td>
<td>0.000164275</td>
<td>0</td>
<td>0.000164275</td>
</tr>
<tr>
<td>11</td>
<td>0.8574</td>
<td>0.0001</td>
<td>0.0010197</td>
<td>–0.0009197</td>
</tr>
<tr>
<td>12</td>
<td>0.9291</td>
<td>0.0001</td>
<td>0.003157961</td>
<td>–0.003057961</td>
</tr>
<tr>
<td>13</td>
<td>0.9977</td>
<td>0.0001</td>
<td>0.01230339</td>
<td>–0.01220339</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td>0.000234812</td>
<td>0</td>
<td>0.000234812</td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>0.0007871516</td>
<td>0</td>
<td>0.0007871516</td>
</tr>
<tr>
<td>16</td>
<td>0.9603</td>
<td>0.001106689</td>
<td>0</td>
<td>0.001106689</td>
</tr>
<tr>
<td>17</td>
<td>1</td>
<td>0.004458744</td>
<td>0</td>
<td>0.004458744</td>
</tr>
<tr>
<td>18</td>
<td>1</td>
<td>0.006624602</td>
<td>0</td>
<td>0.006624602</td>
</tr>
</tbody>
</table>

Now, we analyse the effects of considering ‘D’ as a non-discretionary input on the results. Therefore, we re-solve the problem by considering ‘D’ as a discretionary factor. The results are shown in Table 4. In this time, 9 out of 18 suppliers are efficient. By comparing Tables 3 and 4, it can be seen that the ranking of some suppliers by two strategies (considering distance as a non-discretionary or discretionary factor) are different.

The problem now becomes selecting a supplier from those ten efficient suppliers (when distance is considered as a non-discretionary input). Therefore, we use model (6) to derive the suppliers’ score and their complete ranking. The scores derived by using model (6) and final ranking of suppliers have been displayed in Table 5. As Table 5 shows, supplier 17 receives the highest score in the presence of virtual best DMU, and is the first candidate for selection. If they are able to use the minimum inputs to produce the maximum outputs, they are DEA efficient; otherwise, they are inefficient. Therefore, DM can choose one or more of these efficient suppliers. Samples of models (5) and (6) for supplier 2 have been presented in Appendix. $\varepsilon$ has been set to be 0.0001.
Table 4  Efficiency scores when ‘D’ is treated as a discretionary input

<table>
<thead>
<tr>
<th>Supplier no.</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9711</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>0.9236</td>
</tr>
<tr>
<td>9</td>
<td>0.9416</td>
</tr>
<tr>
<td>10</td>
<td>0.8410</td>
</tr>
<tr>
<td>11</td>
<td>0.8606</td>
</tr>
<tr>
<td>12</td>
<td>0.9375</td>
</tr>
<tr>
<td>13</td>
<td>0.9973</td>
</tr>
<tr>
<td>14</td>
<td>0.9817</td>
</tr>
<tr>
<td>15</td>
<td>1</td>
</tr>
<tr>
<td>16</td>
<td>0.9236</td>
</tr>
<tr>
<td>17</td>
<td>1</td>
</tr>
<tr>
<td>18</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5  Efficiency scores and ranking of efficient suppliers in the presence of virtual best DMU

<table>
<thead>
<tr>
<th>Rank</th>
<th>Supplier no.</th>
<th>Efficiency scores obtained by model (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17</td>
<td>0.8975</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.8373</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>0.7988</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0.7894</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>0.7842</td>
</tr>
<tr>
<td>6</td>
<td>18</td>
<td>0.7626</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>0.7436</td>
</tr>
<tr>
<td>8</td>
<td>14</td>
<td>0.7431</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>0.7332</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>0.7322</td>
</tr>
</tbody>
</table>

5  Concluding remarks

Today, manufacturing companies are facing intense global competition and consequently an incredible pressure to reduce the cost and development time of a new product. It is well known that a substantial proportion of the cost of a typical engineering product is accounted for in raw material, components and other supplies; on average,
manufacturers’ purchases of goods and services amounts to 55% of revenue (Akarte et al., 2001). Purchasing is thus one of the most crucial and vital activities of business, as it has a significant impact on finance, operations and competitiveness of the organisation (Stainer et al., 1996).

This paper has provided a model for selecting suppliers in the presence of both dual-role factors and non-discretionary inputs.

The problem considered in this study is at the initial stage of investigation and further researches can be done based on the results of this paper. Some of them are as below:

- Similar research can be repeated for supplier selection in the presence of fuzzy data.
- Preferences of DM can be incorporated into the proposed algorithm by restricting the feasible region of the inputs and outputs’ weights.
- Similar research can be repeated in the presence of stochastic data.

Acknowledgements

The authors wish to thank the three anonymous reviewers for their valuable suggestions and comments.

References


A data envelopment analysis model for selecting suppliers


Appendix

Model (5) for supplier #2:

\[
\begin{align*}
\text{Max} & = 194* u_1 + 130* u_2 + 1* u_3 + 32* y_1 - 32* \beta_1 - 643* v_{1ND}, \\
\text{s.t.} & \quad 268* v_{1D} + 198* v_{2D} = 1, \\
& \quad 187* u_1 + 90* u_2 + 1* u_3 + 20* y_1 - (253* v_{1D} + 197* v_{2D} + 20* \beta_1 + 249* v_{1ND}) \leq 0, \\
& \quad 194* u_1 + 130* u_2 + 5* u_3 + 32* y_1 - (268* v_{1D} + 198* v_{2D} + 32* \beta_1 + 643* v_{1ND}) \leq 0, \\
& \quad 220* u_1 + 200* u_2 + 3* u_3 + 15* y_1 - (259* v_{1D} + 229* v_{2D} + 15* \beta_1 + 714* v_{1ND}) \leq 0, \\
& \quad 160* u_1 + 100* u_2 + 4* u_3 + 10* y_1 - (180* v_{1D} + 169* v_{2D} + 10* \beta_1 + 1,809* v_{1ND}) \leq 0, \\
& \quad 204* u_1 + 173* u_2 + 1* u_3 + 16* y_1 - (257* v_{1D} + 212* v_{2D} + 16* \beta_1 + 238* v_{1ND}) \leq 0, \\
& \quad 192* u_1 + 170* u_2 + 2* u_3 + 28* y_1 - (248* v_{1D} + 197* v_{2D} + 28* \beta_1 + 241* v_{1ND}) \leq 0, \\
& \quad 194* u_1 + 60* u_2 + 5* u_3 + 12* y_1 - (272* v_{1D} + 209* v_{2D} + 12* \beta_1 + 1,404* v_{1ND}) \leq 0, \\
& \quad 195* u_1 + 145* u_2 + 3* u_3 + 36* y_1 - (330* v_{1D} + 203* v_{2D} + 36* \beta_1 + 984* v_{1ND}) \leq 0, \\
& \quad 200* u_1 + 150* u_2 + 2* u_3 + 30* y_1 - (327* v_{1D} + 208* v_{2D} + 30* \beta_1 + 641* v_{1ND}) \leq 0, \\
& \quad 171* u_1 + 90* u_2 + 3* u_3 + 28* y_1 - (330* v_{1D} + 203* v_{2D} + 28* \beta_1 + 588* v_{1ND}) \leq 0, \\
& \quad 174* u_1 + 100* u_2 + 1* u_3 + 19* y_1 - (231* v_{1D} + 207* v_{2D} + 19* \beta_1 + 241* v_{1ND}) \leq 0, \\
& \quad 209* u_1 + 200* u_2 + 2* u_3 + 25* y_1 - (329* v_{1D} + 324* v_{2D} + 25* \beta_1 + 567* v_{1ND}) \leq 0, \\
& \quad 165* u_1 + 163* u_2 + 1* u_3 + 18* y_1 - (281* v_{1D} + 173* v_{2D} + 18* \beta_1 + 567* v_{1ND}) \leq 0, \\
& \quad 199* u_1 + 170* u_2 + 4* u_3 + 27* y_1 - (309* v_{1D} + 203* v_{2D} + 27* \beta_1 + 967* v_{1ND}) \leq 0, \\
& \quad 188* u_1 + 185* u_2 + 2* u_3 + 22* y_1 - (291* v_{1D} + 193* v_{2D} + 22* \beta_1 + 635* v_{1ND}) \leq 0, \\
& \quad 168* u_1 + 85* u_2 + 3* u_3 + 31* y_1 - (334* v_{1D} + 177* v_{2D} + 31* \beta_1 + 795* v_{1ND}) \leq 0, \\
& \quad 177* u_1 + 130* u_2 + 5* u_3 + 50* y_1 - (249* v_{1D} + 185* v_{2D} + 50* \beta_1 + 689* v_{1ND}) \leq 0, \\
& \quad 167* u_1 + 160* u_2 + 4* u_3 + 15* y_1 - (216* v_{1D} + 176* v_{2D} + 15* \beta_1 + 913* v_{1ND}) \leq 0,
\end{align*}
\]

\[
v_{1D} \geq 0.0001, \\
v_{2D} \geq 0.0001, \\
u_1 \geq 0.0001, \\
u_2 \geq 0.0001, \\
u_3 \geq 0.0001, \\
y_1 \geq 0.0001, \\
\beta_1 \geq 0, \\
v_{1ND} \geq 0.
\]
Model (6) for supplier #2:

\[
\begin{align*}
\text{Max} & \quad 194u_1 + 130u_2 + 1u_3 + 32\gamma_1 - 32\beta_1 - 643v_{1ND}, \\
\text{s.t.} & \quad 268v_{1D} + 198v_{2D} = 1, \\
187u_1 + 90u_2 + 1u_3 + 20\gamma_1 - (253v_{1D} + 197v_{2D} + 20\beta_1 + 249v_{1ND}) & \leq 0, \\
194u_1 + 130u_2 + 5u_3 + 32\gamma_1 - (268v_{1D} + 198v_{2D} + 32\beta_1 + 643v_{1ND}) & \leq 0, \\
220u_1 + 200u_2 + 3u_3 + 15\gamma_1 - (259v_{1D} + 229v_{2D} + 15\beta_1 + 714v_{1ND}) & \leq 0, \\
160u_1 + 100u_2 + 4u_3 + 10\gamma_1 - (180v_{1D} + 169v_{2D} + 10\beta_1 + 1,809v_{1ND}) & \leq 0, \\
204u_1 + 173u_2 + 1u_3 + 16\gamma_1 - (257v_{1D} + 212v_{2D} + 16\beta_1 + 238v_{1ND}) & \leq 0, \\
192u_1 + 170u_2 + 2u_3 + 28\gamma_1 - (248v_{1D} + 197v_{2D} + 28\beta_1 + 241v_{1ND}) & \leq 0, \\
194u_1 + 60u_2 + 5u_3 + 12\gamma_1 - (272v_{1D} + 209v_{2D} + 12\beta_1 + 1,404v_{1ND}) & \leq 0, \\
195u_1 + 145u_2 + 3u_3 + 36\gamma_1 - (330v_{1D} + 203v_{2D} + 36\beta_1 + 984v_{1ND}) & \leq 0, \\
200u_1 + 150u_2 + 2u_3 + 30\gamma_1 - (327v_{1D} + 208v_{2D} + 30\beta_1 + 641v_{1ND}) & \leq 0, \\
171u_1 + 90u_2 + 3u_3 + 28\gamma_1 - (330v_{1D} + 203v_{2D} + 28\beta_1 + 588v_{1ND}) & \leq 0, \\
174u_1 + 100u_2 + 1u_3 + 19\gamma_1 - (231v_{1D} + 207v_{2D} + 19\beta_1 + 241v_{1ND}) & \leq 0, \\
209u_1 + 200u_2 + 2u_3 + 25\gamma_1 - (329v_{1D} + 324v_{2D} + 25\beta_1 + 567v_{1ND}) & \leq 0, \\
165u_1 + 163u_2 + 1u_3 + 18\gamma_1 - (281v_{1D} + 173v_{2D} + 18\beta_1 + 567v_{1ND}) & \leq 0, \\
199u_1 + 170u_2 + 4u_3 + 27\gamma_1 - (309v_{1D} + 203v_{2D} + 27\beta_1 + 967v_{1ND}) & \leq 0, \\
188u_1 + 185u_2 + 2u_3 + 22\gamma_1 - (291v_{1D} + 193v_{2D} + 22\beta_1 + 635v_{1ND}) & \leq 0, \\
168u_1 + 85u_2 + 3u_3 + 31\gamma_1 - (334v_{1D} + 177v_{2D} + 31\beta_1 + 795v_{1ND}) & \leq 0, \\
177u_1 + 130u_2 + 5u_3 + 50\gamma_1 - (249v_{1D} + 185v_{2D} + 50\beta_1 + 689v_{1ND}) & \leq 0, \\
167u_1 + 160u_2 + 4u_3 + 15\gamma_1 - (216v_{1D} + 176v_{2D} + 15\beta_1 + 913v_{1ND}) & \leq 0, \\
220u_1 + 200u_2 + 5u_3 + 50\gamma_1 - (180v_{1D} + 169v_{2D} + 50\beta_1 + 238v_{1ND}) & \leq 0,
\end{align*}
\]
\[
\begin{align*}
v_{1D} & \geq 0.0001, \\
v_{2D} & \geq 0.0001, \\
u_1 & \geq 0.0001, \\
u_2 & \geq 0.0001, \\
u_3 & \geq 0.0001, \\
\gamma_1 & \geq 0.0001, \\
\beta_1 & \geq 0, \\
v_{1ND} & \geq 0.
\end{align*}
\]