DISSERTATIO PHYSICO-MATHEMATICA,

# *PHÆNOMENA LUMINIS, VIRIBUS ATTRACTIVIS & REPULSIVIS CORPORUM SUBJACERE & EX HIS DERIVARI POSSE,*

STATUENS;

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CUJUS PARTEM QUARTAM,

CONSENTIENTE AMPLISS. ORDINE PHILOSOPH.
IN IMPERIALI ACADEMIA ABOËNSI,

PRÆSIDE

*Mag. JOH. FREDR. AHLSTEDT,*
*Mathem. Professore, Publ. & Ordin.*

PRO GRADU,

PUBLICE VENTILANDAM SISTIT

*OTTO WILHELMUS ROSENLEW,*
*Nobilis, Stipend. Publ. Borealis,*

In Auditorio Jurid. die 19 Junii 1819.
h. a. m. solitis.

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ABOÆ, Typis FRENCKELLIANIS.

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$$dy\left(\frac{C \pm \tfrac{1}{2}(a-r)y^2 \mp \tfrac{1}{3}.\left(\frac{A^3}{a^2}-\frac{R^3}{r^2}\right)\frac{1}{y} \pm \frac{1.2}{2.4.3}.\left(\frac{A^6}{a^5}-\frac{R^6}{r^5}\right)\frac{1}{y^4} \mp \ldots}{D-C \mp \tfrac{1}{2}(a-r)y^2 \pm \tfrac{1}{3}\left(\frac{A^3}{a^2}-\frac{R^3}{r^2}\right)\frac{1}{y} \pm \frac{1.2}{2.4.3^2}.\left(\frac{A^6}{a^5}-\frac{R^6}{r^5}\right)\frac{1}{y^4} \pm \ldots}\right)^{\frac{1}{2}},$$

quæ nullam alia conditione admittere videtur evolutionem in seriem, ubi crescente $y$ hæc decresceret, nisi fuerit $a = r$. In hoc autem casu, positis:

$$\frac{C}{D-C} = \alpha;\ \tfrac{1}{2}\left(\frac{a-r}{D-C}\right) = o;\ \frac{1}{3(D-C)}.\left(\frac{A^3}{a^2}-\frac{R^3}{r^2}\right) = \beta;$$

$$\frac{1.2}{2.4.3^2\,(D-C)}.\left(\frac{A^6}{a^5}-\frac{R^6}{r^5}\right) = \gamma;$$

$$\frac{1.2.5}{2.3.7.3^3\,(D-C)}.\left(\frac{A^9}{a^8}-\frac{R^9}{r^8}\right) = \delta;$$

$$\frac{1.2.5.8}{2.3.4.10.3^4\,(D-C)}.\left(\frac{A^{12}}{a^{11}}-\frac{R^{12}}{r^{11}}\right) = \varepsilon;$$

$$\frac{1.2.5.8.11}{2.3.4.5.13.3^5\,(D-C)}.\left(\frac{A^{15}}{a^{14}}-\frac{R^{15}}{r^{14}}\right) = \zeta;\ \ldots$$

formula nostra differentialis abit in:

$$dx = dy\left(\frac{\alpha \mp \frac{\beta}{y} \pm \frac{\gamma}{y^4} \mp \frac{\delta}{y^7} \pm \frac{\varepsilon}{y^{10}} \mp \frac{\zeta}{y^{13}} \pm \ldots}{1 \pm \frac{\beta}{y} \mp \frac{\gamma}{y^4} \pm \frac{\delta}{y^7} \mp \frac{\varepsilon}{y^{10}} \pm \frac{\zeta}{y^{13}} \mp \ldots}\right)^{\frac{1}{2}}.$$

Ad hanc æquationem integrandam, ponatur

$$z = \pm \frac{\beta}{y} \mp \frac{\gamma}{y^4} \pm \frac{\delta}{y^7} \mp \frac{\varepsilon}{y^{10}} \pm \frac{\zeta}{y^{13}} \mp \ldots, \text{ unde}$$

$$z^2 = \frac{\beta^2}{y^2} - \frac{2\beta\gamma}{y^5} + \frac{2\beta\delta+\gamma^2}{y^8} \ldots;$$

$$z^3 = \pm \frac{\beta^3}{y^3} \mp \frac{3\beta^2\gamma}{y^6} \pm \frac{3\beta^2\delta}{y^9} \mp \ldots;$$

$$z^4 = \frac{\beta^4}{y^4} - \frac{4\beta^3\gamma}{y^7} + \ldots;$$

$$z^5 = \pm \frac{\beta^5}{y^5} \mp \frac{5\beta^4\gamma}{y^8} \pm \ldots;$$

$$z^6 = \frac{\beta^6}{y^6} - \frac{6\beta^5}{y^9} \ldots \text{; quo facto, oritur:}$$

$$\left(\frac{\alpha \mp \frac{\beta}{y} \pm \frac{\gamma}{y^4} \mp \frac{\delta}{y^7} \pm \frac{\varepsilon}{y^{10}} \mp \frac{\zeta}{y^{13}} \pm \ldots}{1 \pm \frac{\beta}{y} \mp \frac{\gamma}{y^4} \pm \frac{\delta}{y^7} \mp \frac{\varepsilon}{y^{10}} \pm \frac{\zeta}{y^{13}} \mp \ldots}\right)^{\frac{1}{2}} = \left(\frac{a-z}{1+z}\right)^{\frac{1}{2}}.$$

Fiat $\left(\frac{\alpha-z}{1+z}\right)^{\frac{1}{2}} = A_{,} + B_{,}z + C_{,}z^2 + D_{,}z^3 + E_{,}z^4 + F_{,}z^5 + G_{,}z^6 + H_{,}z^7 + \ldots$; (ŧ)

erit, sumtis utrinque Logarithmis:

$\frac{1}{2}$ Log

$\frac{1}{2}$ Log $(\alpha - z)$ — $\frac{1}{2}$ Log $(1 + z) =$
Log $(A_{,} + B_{,}z + C_{,}z^2 + D_{,}z^3 + E_{,}z^4 + F_{,}z^5 + G_{,}z^6 + \ldots,)$
quibus differentiatis, simulque in binarium ductis, emergit:

$$- dz \left(\frac{1}{\alpha - z} + \frac{1}{1 + z}\right) = - dz.\ \frac{\alpha + 1}{\alpha + (\alpha - 1)\, z - z^2} =$$

$$dz \left(\frac{2 B_{,} + 4 C_{,}z + 6 D_{,}z^2 + 8 E_{,}z^3 + 10 F_{,}z^4 + 12 G_{,}z^5 + \ldots}{A_{,} + B_{,}z + C_{,}z^2 + D_{,}z^3 + E_{,}z^4 + F_{,}z^5 + G_{,}z^6 + \ldots}\right).$$

Dividatur hæc æquatio per $dz$, & reducatur ad eundem denominatorem, erunt, terminis numeratoris ad unam partem æquationis transpositis, coëfficientibusque cujusvis potentiæ ipsius $z$ cyphræ æquatis:

$$\left.\begin{array}{l}
A_{,}\,(\alpha + 1) + 2\,B_{,}\alpha = 0, \\
B_{,}\,(\alpha + 1) + 4\,C_{,}\alpha + 2\,B_{,}\,(\alpha - 1) = 0, \\
C_{,}\,(\alpha + 1) + 6\,D_{,}\alpha + 4\,C_{,}\,(\alpha - 1) - 2\,B_{,} = 0, \\
D_{,}\,(\alpha + 1) + 8\,E_{,}\alpha + 6\,D_{,}\,(\alpha - 1) - 4\,C_{,} = 0, \\
E_{,}\,(\alpha + 1) + 10\,F_{,}\alpha + 8\,E_{,}\,(\alpha - 1) + 6\,D_{,} = 0, \\
F_{,}\,(\alpha + 1) + 12\,G_{,}\alpha + 10\,F_{,}\,(\alpha - 1) - 8\,E_{,} = 0, \\
G_{,}\,(\alpha + 1) + 14\,H_{,}\alpha + 12\,G_{,}\,(\alpha - 1) - 10\,F_{,} = 0, \&c
\end{array}\right\} (\varrho)$$

Valor qvantitatis $A_{,}$ oritur ex equat. ($\delta$), facto $z = 0$, scilicet

$A_{,}$

$A_{,} = \sqrt{\alpha}$ , & reliquarum ex (2) inveniuntur:

$$B_{,} = -\frac{(\alpha + 1)\sqrt{\alpha}}{2\,\alpha},$$

$$C_{,} = \frac{(3\alpha - 1)\,(\alpha + 1)\,\sqrt{\alpha}}{2\,.\,4\,.\,\alpha^{2}},$$

$$D_{,} = -\frac{3\,(5\,\alpha^{2} - 2\,\alpha + 1)\,(\alpha + 1)\,\sqrt{\alpha}}{2\,.\,4\,.\,6\,.\,\alpha^{3}},$$

$$E_{,} = \frac{3\,(35\,\alpha^{3} - 15\,\alpha^{2} + 9\,\alpha - 5)\,(\alpha + 1)\,\sqrt{\alpha}}{2\,.\,4\,.\,6\,.\,8\,.\,\alpha^{4}},$$

$$F_{,} = -\frac{15(63\,\alpha^{4} - 28\,\alpha^{3} + 18\,\alpha^{2} - 12\,\alpha + 7)\,(\alpha + 1)\,\sqrt{\alpha}}{2\,.\,4\,.\,6\,.\,8\,.\,10\,\alpha^{5}},$$

$$G_{,} = \frac{45\,(231\,\alpha^{5} - 105\,\alpha^{4} + 70\,\alpha^{3} - 50\,\alpha^{2} + 35\alpha - 21)(\alpha + 1)\,\sqrt{\alpha}}{2\,.\,4\,.\,6\,.\,8\,.\,10\,.\,12\,.\,\alpha^{6}},$$

&:cra.

Hisce prælibatis, habebitur:

$$dx = dy\left(\frac{\alpha \mp \frac{\beta}{y} \pm \frac{\gamma}{y^{4}} \mp \frac{\delta}{y^{7}} \pm \frac{\varepsilon}{y^{10}} \mp \ldots}{1 \pm \frac{\beta}{y} \mp \frac{\gamma}{y^{4}} \pm \frac{\delta}{y^{7}} \mp \frac{\varepsilon}{y^{10}} \pm \ldots}\right)^{\frac{1}{2}} = (A_{,} + B_{,}z$$

$$+ C_{,}z^{2} + D_{,}z^{3} + E_{,}z^{4} + F_{,}z^{5} + G_{,}z^{6} + \ldots)\,dy\,;$$

& insertis valoribus ipsius $z$, $z^{2}$, . . .

$dy =$

$$dx = dy \Big(A_{,} \pm B_{,} \Big(\frac{\beta}{y} - \frac{\gamma}{y^4} + \frac{\delta}{y^7} - \frac{\varepsilon}{y^{10}} + \ldots\Big)$$

$$+ C_{,} \Big(\frac{\beta^2}{y^2} - \frac{2\beta\gamma}{y^5} + \frac{2\beta\delta + \gamma^2}{y^8} - \ldots\Big)$$

$$\pm D_{,} \Big(\frac{\beta^3}{y^3} - \frac{3\beta^2\gamma}{y^6} + \frac{3\beta\delta}{y^9} - \ldots\Big)$$

$$+ E_{,} \Big(\frac{\beta^4}{y^4} - \frac{4\beta^3\gamma}{y^7} + \ldots\Big) \pm F_{,} \Big(\frac{\beta^5}{y^5} - \frac{5\beta^4\gamma}{y^8} + \ldots\Big)$$

$$+ G_{,} \Big(\frac{\beta^6}{y^6} - \frac{6\beta^5}{y^9} + \ldots\Big) \pm \ldots\Big);$$

quibus collectis, emergit:

$$dx = dy \Big(A_{,} \pm \frac{B_{,}\beta}{y} + \frac{C_{,}\beta^2}{y^2} \pm \frac{D_{,}\beta^3}{y^3} + \frac{E_{,}\beta^4 \mp B_{,}\gamma}{y^4}$$

$$\pm \frac{F_{,}\beta^5 \mp 2C_{,}\beta\gamma}{y^5} + \frac{G_{,}\beta^6 \mp 3D_{,}\beta^2\gamma}{y^6} \pm \ldots\Big),$$ cujus

demum Integrale æquationem Lineæ curvæ a particula defcriptæ exhibet:

$$x = A_{,}y \pm B_{,}\beta \, Log \, y - \frac{C_{,}\beta^2}{y} \mp \frac{D_{,}\beta^3}{2y^2} - \frac{E_{,}\beta^4 \mp B_{,}\gamma}{3y^3} \mp$$

$$\frac{F_{,}\beta^5 \mp 2C_{,}\beta\gamma}{4y^4} - \frac{G_{,}\beta^6 \mp 3D_{,}\beta^2\gamma}{5y^5} \mp \ldots + Conft. \ldots (9).$$

In

In altero vero cafu, ubi $y$ decrefcit, **Formula** $(A)$ abit in:

$$dx = dy\left(\frac{C \mp (R-A)y \pm \frac{1}{3.4}\left(\frac{a^3}{A^2}-\frac{r^3}{R^2}\right)y^4 \mp \frac{1.2}{2.7.3}\left(\frac{a^6}{A^5}-\frac{r^6}{R^5}\right)y^7 \pm \ldots}{D-C \pm (R-A)y \mp \frac{1}{3.4}\left(\frac{a^3}{A^2}-\frac{r^3}{R^2}\right)y^4 \pm \frac{1.2}{2.7.3^2}\left(\frac{a^6}{A^5}-\frac{r^6}{R^5}\right)y^7 \mp \ldots}\right)^{\frac{1}{2}},$$

quæ, factis: $\frac{C}{D-C} = \alpha_{\prime}$; $\frac{R-A}{D-C} = \beta_{\prime}$;

$$\frac{1}{3 \cdot 4 \cdot (D-C)}\left(\frac{a^3}{A^2}-\frac{r^3}{R^2}\right) = \gamma_{\prime};$$

$$\frac{1 \cdot 2}{2 \cdot 7 \cdot 3^2 \cdot (D-C)}\left(\frac{a^6}{A^5}-\frac{r^6}{R^5}\right) = \delta_{\prime};$$

$$\frac{1 \cdot 2 \cdot 5}{2 \cdot 3 \cdot 10 \cdot 3^3\,(D-C)}\left(\frac{a^9}{A^8}-\frac{r^9}{R^8}\right) = \varepsilon_{\prime}; \ldots$$

hanc induit formam:

$$dx = dy\left(\frac{\alpha_{\prime} \mp \beta_{\prime} y \pm \gamma_{\prime} y^4 \mp \delta_{\prime} y^7 \pm \varepsilon_{\prime} y^{10} \mp \ldots}{1 \pm \beta_{\prime} y \mp \gamma_{\prime} y^4 \pm \delta_{\prime} y^7 \mp \varepsilon_{\prime} y^{10} \pm \ldots}\right)^{\frac{1}{2}}.$$

Sit $z_{\prime} = \pm \beta_{\prime} y \mp \gamma_{\prime} y^4 \pm \delta_{\prime} y^7 \mp \varepsilon_{\prime} y^{10} \pm \ldots$, erit

$$dx = dy\left(\frac{\alpha_{\prime} - z_{\prime}}{1 + z_{\prime}}\right)^{\frac{1}{2}} = (A_{\prime\prime} + B_{\prime\prime} z_{\prime} + C_{\prime\prime} z_{\prime}^2 + D_{\prime\prime} z_{\prime}^3$$

$$E_{\prime\prime} z_{\prime}^4 + F_{\prime\prime} z_{\prime}^5 + G_{\prime\prime} z_{\prime}^6 + \ldots)dy.$$

Coëfficientibus $A_{\prime\prime}$, $B_{\prime\prime}$, $C_{\prime\prime}$, ... eadem methodo ac fupra inventis, exoritur:

$dx =$

$$dx = dy\,(A_{,,} + B_{,,}\,(\pm \beta_{,} y \mp \gamma_{,} y^4 \pm \delta_{,} y^7 \mp ..)$$
$$+ C_{,,}(\beta_{,}^2 y^2 - 2\beta_{,}\gamma_{,} y^5 + (2\beta_{,}\delta_{,} + \gamma_{,}^2) y^8 - ..)$$
$$+ D_{,,}\,(\pm \beta_{,}^3 y^3 \mp 3\beta_{,}^2\gamma_{,} y^6 \pm ..)$$
$$+ E_{,,}(\beta_{,}^4 y^4 - 4\beta_{,}^3\gamma_{,} y^7 + ..)$$
$$+ F_{,,}\,(\pm \beta_{,}^5 y^5 \mp 5\beta_{,}^4\gamma_{,} y^8 \pm ..)$$
$$+ G_{,,}\beta_{,}^6 y^6 \pm H_{,,}\beta_{,}^7 y^7 + I_{,,}\beta_{,}^8 y^8 + \ldots$$

& membris rite ordinatis:

$$dx = dy\,(A_{,,} \pm B_{,,}\beta_{,} y + C_{,,}\beta_{,}^2 y^2 \pm D_{,,}\beta_{,}^3 y^3$$
$$+ (E_{,,}\beta_{,}^4 \mp B_{,,}\gamma_{,})\, y^4 \pm (F_{,,}\beta_{,}^5 \mp 2C_{,,}\beta_{,}\gamma_{,})\, y^5$$
$$+ (G_{,,}\beta_{,}^6 \mp 3D_{,,}\beta_{,}^2\gamma_{,}) y^6 \pm (H_{,,}\beta_{,}^7 \mp 4E_{,,}\beta_{,}^3\gamma_{,} \pm B_{,,}\delta_{,}) y^7$$
$$+ (I_{,,}\beta_{,}^8 \mp 5F_{,,}\beta_{,}^4\gamma_{,} + C_{,,}(2\beta_{,}\delta_{,} + \gamma_{,}^2))\, y^8 \pm ..),$$

cujus Integrale eſt:

$$x = Conſt. + A_{,,} y \pm \frac{B_{,,}\beta_{,} y^2}{2} + \frac{C_{,,}\beta_{,}^2 y^3}{3}$$
$$\pm \frac{D_{,,}\beta_{,}^3 y^4}{4} + (E_{,,}\beta_{,}^4 \mp B_{,,}\gamma_{,}) \cdot \frac{y^5}{5}$$
$$\pm (F_{,,}\beta_{,}^5 \mp 2C_{,,}\beta_{,}\gamma_{,}) \cdot \frac{y^6}{6} + (G_{,,}\beta_{,}^6 \mp 3D_{,,}\beta_{,}^2\gamma_{,}) \cdot \frac{y^7}{7}$$
$$\pm (H_{,,}\beta_{,}^7 \mp 4E_{,,}\beta_{,}^3\gamma_{,} \pm B_{,,}\delta_{,}) \cdot \frac{y^8}{8}$$
$$+ (I_{,,}\beta_{,}^8 \mp 5F_{,,}\beta_{,}^4\gamma_{,} + C_{,,}(2\beta_{,}\delta_{,} + \gamma_{,}^2))\, \frac{y^9}{9} \pm \ldots (\odot).$$

Evol-

Evolvamus æquationem ($B$) pag. 6. sive

$$dx = dy\left(\frac{D_{\prime}}{C - D_{\prime} \mp \int P dy}\right)^{\frac{1}{2}} = dy\sqrt{D_{\prime}}\,(C - D_{\prime} \mp \int P dy)^{-\frac{1}{2}};$$

quæ, inſerto valore ($\mathfrak{A}$), evadit:

$$dx = dy\sqrt{D_{\prime}}\Big(C - D_{\prime} \mp \tfrac{1}{2}(a-r)y^2 \pm \tfrac{1}{3}\cdot\left(\frac{A^3}{a^2} - \frac{R^3}{r^2}\right)\frac{1}{y}$$

$$\mp \frac{1.2}{2.4.3^2}\cdot\left(\frac{A^6}{a^5} - \frac{R^6}{r^5}\right)\frac{1}{y^4} \pm \frac{1.2.5}{2.3.7.3^3}\cdot\left(\frac{A^9}{a^8} - \frac{R^9}{r^8}\right)\frac{1}{y^7}$$

$$\mp \frac{1.2.5.8}{2.3.4.10.3^4}\cdot\left(\frac{A^{12}}{a^{11}} - \frac{R^{12}}{r^{11}}\right)\frac{1}{y^{10}} \pm \ldots\Big)^{-\frac{1}{2}}$$

$$= \frac{dy}{y}\sqrt{D_{\prime}}\Big(\mp \tfrac{1}{2}(a-r) + \frac{C - D_{\prime}}{y^2} \pm \tfrac{1}{3}\left(\frac{A^3}{a^2} - \frac{R^3}{r^2}\right)\frac{1}{y^3}$$

$$\mp \frac{1.2}{2.4.3^2}\cdot\left(\frac{A^6}{a^5} - \frac{R^6}{r^5}\right)\frac{1}{y^6}$$

$$\pm \frac{1.2.5}{2.3.7.3^3}\cdot\left(\frac{A^9}{a^8} - \frac{R^9}{r^8}\right)\frac{1}{y^9} \mp \ldots\Big)^{-\frac{1}{2}},$$

duos complectens caſus, ſcilicet, 1:o exiſtente $a < r$, & 2:o, $a > r$. In priori habetur, poſitis:

$$-\tfrac{1}{2}(a-r) = \alpha_{\prime\prime},\ C - D_{\prime} = \beta_{\prime\prime},\ \tfrac{1}{3}\left(\frac{A^3}{a^2} - \frac{R^3}{r^2}\right) = \gamma_{\prime\prime},$$

$$\frac{1.2}{2.4.3^2}\cdot\left(\frac{A^6}{a^5} - \frac{R^6}{r^5}\right) = \delta_{\prime\prime},\ \ldots$$

$dx =$

Rv. Mf. 6187