DISSERTATIO

DE

# *FIGURA FLUIDI IMMOTI, AB EXTERNA VI SOLLICITATI.*

CUJUS PARTEM PRIOREM,

*Consensu Ampl. Facult. Philos. Aboëns.*

Publico subjiciunt Examini

Fratres


*JOHANNES FRID. AHLSTEDT,*

Mathes. puræ Docens,

&

*CAROLUS GUSTAVUS AHLSTEDT,*

Stip. Reg. Satacundenses.


*In Aud. Majori die I Jun. MDCCCIII.*

Horis a. m. consvetis.

---


ABOÆ, typis Frenckellianis.


2.

Fluidum immotum, figuram, quam centri ſui attractionis vi relictum ſibi vindicat ſphæricam, accedente vi aliena in diſtantia ſollicitante, conſervare non poſſe, niſi æqualiter & ſub directionibus parallelis quæcunque ejus premeretur particula, plurimi affirmarunt æſtus marini Inveſtigatores. Hinc vim inæqualem pro diverſis particulis & ad unum punctum directam, novam fluido inducere figuram, eo judicarunt certius, quo clarius perſpexerunt, duas has res inſimul in forma fluidi alienanda verſari. Hoc judicium quamvis de quacunque Lege attractionis pronuntiari non posſit *), æſtum tamen marium ex his causſis, pro attractione, quadratis diſtantiarum reciproce proportionali, in eodem producendo effectu conſpirantibus, oriri, indubiis confirmarunt demonſtrationibus. Figuræ autem hinc ortæ inveſtigandæ rationem cum tam motus & inertia fluidi, quam vis externæ continua loci mutatio, difficillimam reddant, problema hoc ad methodum hydroſtaticam primo ſolvendum ſibi

 pro-

*) Infra enim patebit, figuram ſphæricam immutatam fluido competere, ſi quæcunque ejus particula in ratione diſtantiarum directa ſollicitatur.

proposuerunt, sicque inquisiverunt in figuram Telluris, aqua, omni inertia carente, cinctæ, immotæ, a vi Solis & Lunæ quiescentium sollicitatæ. Ast, quoniam sub his quoque conditionibus, æquatio differentialis, ad quam ex genuinis principiis pervenire licet, adeo fit composita, ut parum successus in integrali completo inveniendo sperari possit, ope approximationis hanc rem assequendam esse judicarunt. His invigilantes, integrale completum, non modo Eorum vestigia premendo, pro voto inveniri posse, verum etiam methodo concinniori sponte se offerre, comperuimus. Persuasi igitur, problematis hujus plene soluti usum in aliis quoque casibus, ubi approximatio usitata minime admitti potest, esse frequentem, disquisitionem nostram Tuæ, L. C. mitiori censuræ submittere constituimus.

Quo consensus methodi nostræ cum illa, quam exposuit Illust. Eulerus, in *Inquisitione physica in Causam fluxus ac refluxus maris*, (præmio a Celebri Academia Scientiarum Parisiensi condecorata), appareat, æquationem differentialem primo hac via quærendam esse censuimus.

Sint $S$ & $T$ centra duorum corporum, quorum massæ vocentur $m$, $m'$ respective, distantia centrorum $ST = a$, $M$ particula quæcunque fluidi a $T$ & $S$ attracti, ejusque distantia $TM = r$ & $SM = u$. Sit porro $AMCBD$ sectio fluidi in æquilibrio constituti, per $M, T, S$ transiens, & demisso a puncto $M$ perpendiculo in $TS$, sit abscissa $TP = x$, ordinatim applicata $PM = y$, earumque fluxiones $Pp = mn = dx$, $Mn = -dy$, unde $SP = a - x$, $r^2 = x^2 + y^2$, $u^2 = (a-x)^2 + y^2$, $Mm = \sqrt{dx^2 + dy^2}$ $(= ds.)$ Agant jam vires centrorum $T, S$ in particulam $M$ & in se mutuo, in composita ratione massarum $m', m$ & potentiæ $\lambda$ distantiarum,

rum, atque resolvatur vis $mu^{\lambda}$, qua $S$ particulam $M$ in directione $MS$ attrahit, in vires laterales secundum $TS$ & $MT$, unde $u : a : : mu^{\lambda} : mau^{\lambda-1}$ = vi in directione $TS$ sive lineæ parallelæ $MH$, & $u : r : : mu^{\lambda} : mru^{\lambda-1}$ = vi in directione $MT$. Ut vero centrum $T$, ab $S$ in directione $TS$ vi $ma^{\lambda}$ sollicitatum, ad quietem perducatur, applicetur hæc vis particulæ $M$ in directione opposita $ME$, quare residuum tantum virium, hoc est: $mau^{\lambda-1} - ma^{\lambda}$ guttulam $M$ in directione $MH$ urget. Capiatur $MQ$ huic residuo proportionalis, & ducta $QV$ ipsi $TM$ productæ perpendiculari, erit ob similitudinem triangulorum $TMP$, $QMV$; $MQ : MV : : r : x : : mau^{\lambda-1} - ma^{\lambda} : mau^{\lambda-1} r^{-1} x - ma^{\lambda} r^{-1} x$ = vi in directione $MV$, quæ vi antea inventæ $mru^{\lambda-1}$ est opposita, & $r : y : : mu^{\lambda-1} a - ma^{\lambda} : mar^{-1} u^{\lambda-1} y - ma^{\lambda} r^{-1} y$ = vi horizontali in directione $MG$ ipsi $VQ$ parallelæ. Gravitat vero guttula $M$ ad $T$ vi = $m' r^{\lambda}$, ita ut conjunctim ab omnibus his viribus in directione $MT$ urgeatur vi

$$m' r^{\lambda} + mru^{\lambda-1} - mar^{-1} u^{\lambda-1} x + ma^{\lambda} r^{-1} x.$$

 Sit

Sit *MN* media directio virium in directionibus *MT* & *MG* agentium, erit tangens anguli *TMN* (posito radio = 1) = vi secundum *MG*: vim secundum *MT* =

$$\frac{mar^{-1}u^{\lambda-1}y - ma^{\lambda}r^{-1}y}{m'r^{\lambda} + mru^{\lambda-1} - mar^{-1}u^{\lambda-1}x + ma^{\lambda}r^{-1}x}.$$

Jam, quoniam ob æquilibrium fluidi eadem media directio curvæ *AMB* in puncto *M* normalis esse debet, erit *Tangens* anguli *NMP* = *Mmn* = $-\frac{dy}{dx}$. Est autem *Tangens* anguli *TMP* = $\frac{x}{y}$, unde anguli *TMP* — *NMP* =

$$TMN \text{ Tangens} = \frac{\frac{x}{y} + \frac{dy}{dx}}{1 - \frac{xdy}{ydx}} = \frac{xdx + ydy}{ydx - xdy},$$ quo valore cum superiori comparato, sequens emergit æquatio fluxionalis: $$\frac{xdx + ydy}{ydx - xdy} =$$

$$\frac{mar^{-1}u^{\lambda-1}y - ma^{\lambda}r^{-1}y}{m'r^{\lambda} + mru^{\lambda-1} - mar^{-1}u^{\lambda-1}x + ma^{\lambda}r^{-1}x}.$$

Quamquam hæc æquatio, per coordinatas *x* & *y* expressa, adeo est implicita, ut omnes in integrando vires spernere videatur, attamen, insertis pro variabilibus *x* & *y* valoriloribus per $r$ & $u$ definitis, integrale completum obtineri posse comperuimus. Ad hoc commodissime præstandum, ducatur æquatio inventa in productum denominatorum, quo pacto erit:

$$m' r^{\lambda} xdx + m' r^{\lambda} ydy + mru^{\lambda - 1} xdx + mru^{\lambda - 1} ydy$$

$$- mar^{-1} u^{\lambda - 1} x^2 dx - mar^{-1} u^{\lambda - 1} xydy +$$

$$ma^{\lambda} r^{-1} x^2 dx + ma^{\lambda} r^{-1} xydy = mar^{-1} u^{\lambda - 1} y^2 dx -$$

$$mar^{-1} u^{\lambda - 1} xydy - ma^{\lambda} r^{-1} y^2 dx + ma^{\lambda} r^{-1} xydy,$$

five $m' r^{\lambda} (xdx + ydy) + mru^{\lambda - 1} (xdx + ydy) +$

$$ma^{\lambda} r^{-1} (x^2 + y^2) dx = mar^{-1} u^{\lambda - 1} (x^2 + y^2) dx.$$

($A$.) Cum vero erat $x^2 + y^2 = r^2$ & $a^2 - 2ax + x^2 + y^2 = u^2$, erit, fubducendo hanc æquationem ab illa, $2ax - a^2 = r^2 - u^2$, & fumtis fluxionibus $2adx = 2rdr - 2udu$, unde $dx = \frac{rdr - udu}{a}$. Differentiata æquatione $x^2 + y^2 = r^2$, oritur, $xdx + ydy = rdr$. Quibus pro functionibus $x$ & $y$ infertis valoribus, æquatio ($A$) abibit in hanc:

$$m' r^{\lambda + 1} dr + mr^2 u^{\lambda - 1} dr + ma^{\lambda - 1} r^2 dr =$$

$ma^{\lambda - 1} rudu + mr^2 u^{\lambda - 1} dr - mru^{\lambda} du$, five deletis delendis:

$m' r^{\lambda} dr + ma^{\lambda - 1} rdr = ma^{\lambda - 1} udu \quad mu^{\lambda} du$, quæ æquatio facillime integrari poteſt.

Nimiæ hujus prolixitatis evitandæ ſtudioſi, in aliam incidimus methodum, quæ directe ad eandem ducit æquationem.

Ductis hunc ob finem perpendiculis $TR$ & $SO$ in normalem $MN$ utrinque productam, iisdemque ſervatis denominationibus, erit $PN = -\frac{ydy}{dx}$, $TN = \frac{xdx + ydy}{dx}$ $= \frac{rdr}{dx}$ & $SN = \frac{(a - x)\, dx - ydy}{dx} = -\frac{udu}{dx}$ atque ob ſimilitudinem triangulorum $Mmn$, $TNR$, $SNO$, $Mm\,(= ds)$ : $mn\,(= dx)$ : : $TN\left(= \frac{rdr}{dx}\right)$ : $TR = \frac{rdr}{ds}$ : : $SN\left(= -\frac{du}{dx}\right)$ : $SO = -\frac{udu}{ds}$.

Reſoluta vi $ma^{\lambda}$, qua $T$ ab $S$ attrahitur, in vires laterales ſecundum $TM$ & $MS$, oritur $illa = ma^{\lambda - 1} r$ & $hæc = ma^{\lambda - 1} u$. Effectus vero hinc oriundus cum viribus oppoſitis annihilari debeat, quo quieſcat corpus $T$, attrahetur guttula $M$ in directione $MT$ vi $m' r^{\lambda} + ma^{\lambda - 1} r$ & in directione $MS$ vi $= mu^{\lambda} - ma^{\lambda - 1} u$. Reſolutis iterum

iterum hisce in vires æquivalentes ſecundum directiones *RT*, *MR* & *OS*, *MO*, vires in directionibus parallelis *RT* & *OS*, oppoſitis & normali *MN* perpendicularibus, aquam in *M* ita afficiant neceſſe eſt, ut nullus inde cieatur motus, quocirca hæ vires æquales eſſe debent. Hinc cum ſit

$$r : \frac{rdr}{ds} :: m'r^{\lambda} + ma^{\lambda-1}r : \frac{m'r^{\lambda}dr + ma^{\lambda-1}rdr}{ds}$$

$=$ vi in directione *RT*, & $u : -\frac{udu}{ds}$

$$:: mu^{\lambda} - ma^{\lambda-1}u : \frac{ma^{\lambda-1}udu - mu^{\lambda}du}{ds}$$ $=$ vi in directione *OS*, erit

$$m'r^{\lambda}dr + ma^{\lambda-1}rdr = ma^{\lambda-1}udu - mu^{\lambda}du, \quad (B)$$

(viribus inventis æqualibus conſtitutis & in $ds$ ductis), plane ut ſupra allata methodo invenimus. Sequens ergo integrando oritur æquatio ſectionis *AMCBD* naturam definiens:

$$\frac{m'r^{\lambda+1}}{\lambda+1} + \tfrac{1}{2}ma^{\lambda-1}r^{2} = \tfrac{1}{2}ma^{\lambda-1}u^{2} - \frac{mu^{\lambda+1}}{\lambda+1} +$$

*conſt.* (*D*), quæ in omnibus aliis caſibus eſt algebraica, excepto illo, in quo attractio diſtantiis reciproce ponatur proportionalis, ſive $\lambda = -1$; hoc enim valore in æquatione (*B*) adhibito, oritur integrando:

$m'$ Log. hyp. $r + \frac{mr^2}{2a^2} = \frac{mu^2}{2a^2} - m$ Log. hyp. $u +$ *const.*

Sectiones vero Axi *AB* perpendiculares omnes erunt circuli.

Posita in æquatione (*B*) $dr = o$, erit $ma^{\lambda - 1} udu - mu^{\lambda} du = o$, sive $u = a$, unde patet, radium corporis fluidi esse aut maximum aut minimum quando fuerit $MS = TS$. *) Cum autem pro generali valore litteræ λ nulla alia proprietas perspici possit, diversos ipsi λ tribuemus valores, ut sic curvæ hujus natura explorari possit.

Si attractio ponatur distantiis directe proportionalis, erit $\lambda = 1$, quo inserto valore, æquatio (*D*) hanc induit formam:

$\frac{1}{2} m' r^2 + \frac{1}{2} mr^2 = \frac{1}{2} mu^2 - \frac{1}{2} mu^2 + c = c$, sive $r^2 = \frac{2c}{m' + m}$, quæ expressio cum circuli centro *T* præditi naturam definiat, patet, fluidum hac in hypothesi sub pristina sua sphærica forma quiescere; quare generaliter dici non potest, attractionem quamcunque inæqualem & ad unum

*) Differt in hoc approximatio Euleriana, secundum quam ima aqua puncto ab *A* ad 90° distanti responderet. Differentia vero hæc sensibilis non est nisi pro refluxu a Luna effecto, qui angulo 28′ 31″ propius ad *A* accedit, ac approximatione reperitur.

unum punctum directam, figuram fluidi immutare, æstusque ciere. Medium enim tenet hic casus illorum, in quibus fuerit aut $\lambda > 1$, aut $\lambda < 1$. In *illo*, aquæ in $C$ & $D$, quæ puncta distant ab $S$ intervallo $= a$, maxime elevantur, in $A$ vero & $B$ maxime deprimuntur, ita tamen, ut altitudo aquarum in $A$ semper sit major altitudine in $B$. Contra autem in *hoc*, erit refluxus maris in $C$ & $D$ maximus, in $A$ vero & $B$ maximus affluxus.

Posita attractione constanti, sive $\lambda = 0$, hæc oritur æquatio: $r^2 + \frac{2am'}{m} r = u^2 - 2au + C$. Sit radius fluidi $CT = TD = 1$, unde *Constans* $C = 1 + \frac{2am'}{m} + a^2$. Quo altitudo aquarum in $A$ & $B$ eruatur, substituatur valor $a \mp r$ loco $u$, quo pacto erit:

$r^2 + \frac{2am'}{m} r = r^2 - a^2 + 1 + \frac{2am'}{m} + a^2$, & $r = 1 + \frac{m}{2am'}$. Æstus ergo hac in hypothesi æqualis est in $A$ & $B$.

Evolvamus vero casum pro $\lambda = -2$, sive pro attractione quadratis distantiarum reciproce proportionali, quam legem Naturæ propriam esse abunde constat. Hoc introducto valore & constanti ita determinato, ut pro $u = a$, fiat $r = TC = TD = 1$, æquatio ($D$) transibit in: $\frac{mr^2}{2a^3} - \frac{m'}{r} = \frac{mu^2}{2a^3} + \frac{m}{u} - m' + \frac{m}{2a^3} - \frac{3m}{2a}$, ($E$), quæ ad

B hanc

hanc reducitur; $r^3 - (u^2 + \frac{2a^3}{u} - \frac{2a^3 m'}{m} - 3a^2 + 1) r - \frac{2a^3 m'}{m} = 0$, (C) unde pro quovis valore ipsi $u$ tributo, $r$ investigari potest ad Regulam CARDANI notissimam. Hæc autem æquatio, per coordinatas $x$ & $y$ expressa & ad rationalitatem perducta, involvit potentiam *duodecimam*, cunctasque inferiores ipsius $x$, & *octavam* ordinatæ $y$, cujus ergo solutionem hac via ne quidem tentabimus.

Quo altitudo fluidi in $A$ & B innotescat, ponatur $u = a \mp r$, quo facto æquatio ($E$) abit in

$$\frac{mr^2}{2a^3} - \frac{m'}{r} = \frac{m}{2a} \mp \frac{mr}{a^2} + \frac{mr^2}{2a^3} + \frac{m}{a \mp r} - m' + \frac{m}{2a^3} - \frac{3\,m}{2a}$$

sive $- \frac{m'}{r} = \mp \frac{mr}{a^2} + \frac{m}{a \mp r} - m' + \frac{m}{2a^3} - \frac{m}{a}$, quæ ducta

in $\frac{(a \mp r)\, a^2 r}{m}$ & rite disposita evadit

$$r^3 \pm (\frac{a^2 m'}{m} - \frac{1}{2a})\, r^2 - (\frac{a^3 m'}{m} \pm \frac{a^2 m'}{m} - \tfrac{1}{2})\, r + \frac{a^3 m'}{m} = o \; (F)$$

ubi signa superiora valent pro $r = TA$, inferiora autem pro $r = TB$.

Ad effectum Solis in Mari elevando computandum, erit ex elementis astronomicis, posita *massa Telluris* $m' = 1$, *massa Solis* $m = 329809$, distantia centrorum $T$ & $S$ =

$a = 23702$

$a = 23702$ semidiametrorum Terræ hinc:

$a^2 = 561784804$, $a^3 = 13315423424408$,

$\frac{a^2 m'}{m} = 1703,36408$, $\frac{a^3 m'}{m} = 40373135,43417$, $\frac{1}{2a} = 0,00002$,

$\frac{a^2 m'}{m} - \frac{1}{2a} = 1703,36406$, $\frac{a^3 m'}{m} + \frac{a^2 m'}{m} - \frac{1}{2} = 40374838,29825$

$\frac{a^3 m'}{m} - \frac{a^2 m'}{m} - \frac{1}{2} = 40371431,57009$, unde

$r^3 + 1703,36406 r^2 - 40374838,29825 r + 40373135,43417 = o$, $(G)$

&

$r^3 - 1703,36406 r^2 - 40371431,57009 r + 40373135,43417 = o$, $(H)$

Ex æquatione $(G)$ obtinetur

$$r = TA = 1,0000000371545$$

& ex æqu. $(H)$ $r = TB = 1,0000000371524$

Hinc, posito radio Telluris medio $= 21482225$ ped. Svec. invenitur, altitudo maris in $A$ supra libellam in $C$, sive

$$TA - TC = 0,79816 \ \&$$
$$TB - TC = 0,79812 \text{ ped. Svec.}$$

Reliquæ radices æquationis $(G)$ sunt 5558,69 & — 7263,06, atque æquationis $(H)$ — 5559.56 & 7261,92 *semidiam. terræ* in quibus omnibus distantiis curva axin $TS$ secabit. Pluribus autem ex æquatione $(C)$ erutis valoribus, perspeximus, *tres* diversas oriri *curvas*, quarum *duæ* sunt *ovales* & finitam comprehendunt aream, *tertia* vero eſt *aſymptotica*, cujus aſymptoton axin *TS* verticaliter ſecat in diſtantia $= 25405,36406$ a *T* verſus *B*, quam ex æquatione (*E*), per coordinatas *x* & *y* expreſſa, invenimus. Eſt nim. $\frac{m(x^2+y^2)}{2a^3} - \frac{m'}{\sqrt{x^2+y^2}} = \frac{m(a^2-2ax+x^2+y^2)}{2a^3}$

$+ \frac{m}{\sqrt{(a-x)^2+y^2}} - m' + \frac{m}{2a^3} - \frac{3m}{2a}$, unde, poſita $y = \infty$,

ſequens naſcitur æquatio: $\frac{m}{2a} - \frac{mx}{a^2} - m' + \frac{m}{2a^3} - \frac{3m}{2a} = 0$,

ſive $x = \frac{1}{2a} - \frac{a^2 m'}{m} - a = - 25405,36406.$

Quo æſtus a Luna in *A* & *B* excitatus innoteſcat, ponatur $a = 60,25$ = *mediæ diſtantiæ Lunæ*, $m$ = *maſſæ Lunæ* = $0,017035775$, manentibus ceteris ut ante; hinc
$a^2 = 3630,06250$; $a^3 = 218711,26562$;

$\frac{1}{2a} = 0,00829$;

$\frac{a^2 m'}{m} = 213084,66875$; $\frac{a^3 m'}{m} = 12838351,29218$;

quibus in æquatione (*F*) inſertis valoribus, ſequentes oriuntur æquationes:

$r^3 + 213084,66045 r^2 - 1305435,46094 r + 12838351,29219 = 0$

$r^3 - 213084,66045 r^2 - 12625266,12344 r + 12838351,29219 = 0$

A prio-

A priori æquatione invenitur

$$r = TA = 1{,}000000118152$$

& a poſteriori $r = TB = 1{,}000000115565$, quare erit

$$TA - TC = 2{,}538169 \quad \&$$

$$TB - TC = 2{,}482608 \quad \text{ped. ſvec.}$$

Reliqui valores ipſius $r$ in æquatione prima ſunt 60,26 & — 213145,86 & in ſecunda 60,23 & — 213143,89 *ſemidiam. terræ*. Aſymptoton autem axin $TS$ ſecabit in diſtantia 213144,91046 *ſem. diam. terræ*, a $T$ verſus $B$.

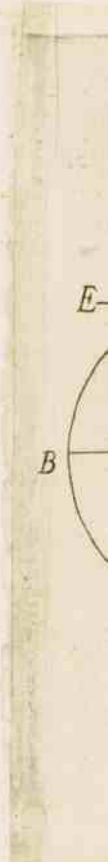
Quod ſi corporis $S$ figura, a $T$ alienata, inveſtiganda foret, æquatio ($E$) transibit in

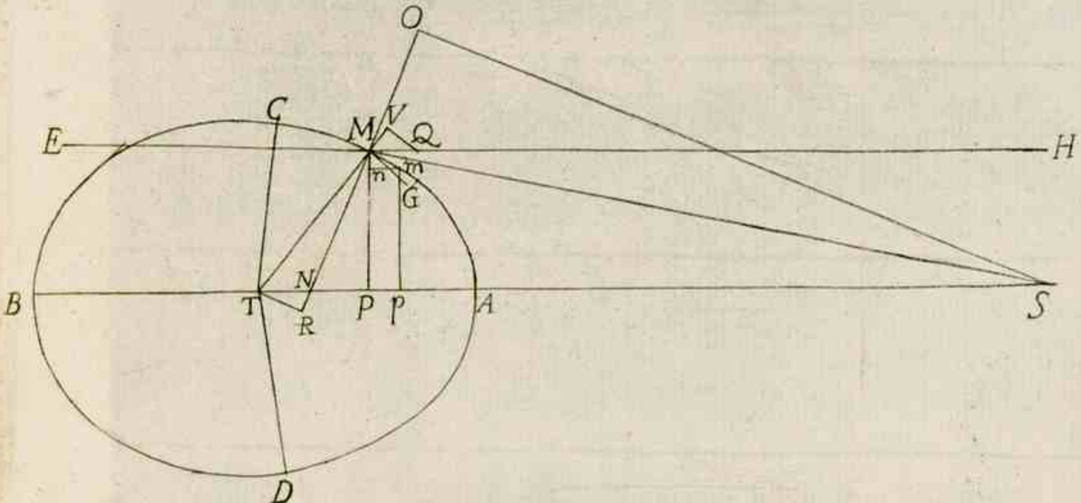
$$\frac{m'}{r} + \frac{m'r^2}{2a^3} = \frac{m'u^2}{2a^3} - \frac{m}{u} + \mathit{conſt}.$$

Ceterum, ſi aut maſſa corporis in diſtantia ſollicitantis evaneſcat, aut diſtantia $a$ ponatur infinite magna, ſectio $AMBDA$ erit in hac hypothſi circulus, æquatione $r = \frac{c}{m'}$ definitus. Generaliter autem ſi centra $T$ & $S$ coëant, hoc eſt, $a = o$ & $u = r$, curva $AMBDA$ abit in circulum; eſt enim $\frac{m' + m}{\lambda + 1} r^{\lambda + 1} = \pm C$, ſive

$$r = \left(\frac{\pm(\lambda + 1)C}{m' + m}\right)^{\frac{1}{\lambda + 1}}$$

, unde patet, fluidum quod ad unicum gravitat centrum, ſub ſphærica ſemper acquieſcere forma.




O
C
M
V
Q
E
H
m
n
G
N
B
T
R
P
p
A
S
D

Errata.

Pag. 4. lin. 7 *Tangens* anguli $NMP = Mmn$ — leg. *anguli* $NMP = Mmn$ *Tangens*

Pag. 6. lin. 10 — $\frac{du}{dx}$ — leg. — $\frac{udu}{dx}$.

Pag. 8. lin. 10. iu- — leg. in-