DISSERTATIO,

DE INVENIENDA FIGURA SPATII ILLUMINATI, QUANDO LUMEN SOLARE PER FORAMEN DATUM AD SUPERFICIEM PLANAM TRANSMITTITUR;

Quam

Conf. Amplisf. Facult. Philof. Reg. Acad. Aboënf.

PRÆSIDE

MAG. *GUST. GABR. HÅLLSTRÖM,*

PHYSICES PROFESS. REG. ET ORDIN. ATQUE REG. SOCIET. OECON. FENN. MEMBRO,

PRO GRADU PHILOSOPHICO

publicæ cenfuræ fubmittit

FRIDERIC. EPHRAIM BERGROTH,

Stip. Reg. Borea-Fenno.

In Audit. Anatomico die XXIX Maji MDCCCII,

Horis a. m. folitis.

ABOÆ, typis Frenckellianis.

19.

KYRKOHERDEN UTI IKALIS,

VÅLÅREVÖRDIGE OCH HÖGLÄRDE

Herr HENRIC JOHAN BERGROTH,

ſamt

Fru HELENA ULRICA BERGROTH,

född *PROCHMAN*,

MINE HULDASTE FÖRÄLDRAR!

Njuta Föräldrar ingen ſtörre glädje i verlden, än den att finna ſina Barn tackſamma; och emottaga de med godhet åfven de ringaſte bevis deraf, frågande mera hvad hjertat vill, än hvad förmågan mågtar åſtadkomma: ſå kan ockſå jag med tillförſigt hoppas, at detta offer ſkall äga inför Eder, Mine Huldaſte Föräldrar, något värde. Emottagen det, ſåſom ett bevis af den tackſamhet, ſom ej blott ſkall lifva mig, ſå länge jag ännu uppmuntras af Eder ömhet, vägledes af Edra råd, och ſtyrckes af Edra efterdömen, utan ſom ännu ſkall vara lika helig för mitt hjerta i de ſtunder, dem Förſynen fördröje! då jag tröſtlös och öfvergifven går att välſigna Eder aſka.

MINE HULDASTE FÖRÄLDRARS

ödmjuk-lydigſte Son

FREDRIC EPHRAIM BERGROTH.

Quando radii luminis e ſole profecti per foramen cujuscunque figuræ, in plano aliquo confectum, projiciuntur in planum aliud, ſpatium ab illis illuminatum formam habere foramini ſimilem raro invenimus, diſſimilitudoque illa pendet ab inclinatione mutua amborum planorum atque radiorum illuminantium, nec uon a diſtantia foraminis a ſpatio illuminato. Si v. c. forma foraminis eſt rectilinea quadrata, ſpatium illuminatum circulo non diſſimile eſſe poteſt. Hujus phænomeni explicationem, quam usque a temporibus ARISTOTELIS tentaſſe dicuntur optici, primus veram dedit Mathematicus Meſſanenſis FRANCISCUS MAUROLYCUS (*). Recte ſcilicet obſervavit, quum ſol non ut punctum lucens ſed ut corpus diametro aliquo præditum nobis appareat, quodcunque punctum peripheriæ foraminis conſiderandum eſſe ut verticem duorum lucis conorum oppoſitorum, quorum unus baſin habet in ſole, alter vero in peripheria ſpatii illuminati aut circulum aut ellipſin

A pin-

(*) Vide ejus librum: *Photiſmi de lumine & umbra, ad Perſpectivam radiorum & incidentiam facientes, Venetiis* 1575.

pingit. Tales circuli vel ellipses faciunt, ut spatium illuminatum figuram adipiscatur curvilineam, etiamsi foramen sit rectilineum.

Cum in arte gnomonica hujus problematis sit usus, calculo determinare constituimus figuram & dimensiones spatii illuminati, datis figura foraminis, inclinatione planorum & radiorum illuminantium, atque distantia foraminis a spatio illuminato, æquæ lectoris censuræ subjicientes conatus.

Ad puncta extra Ellipses nominatas per foramen nullum transmittitur lumen; linea itaque omnes Ellipses tangens figuram determinat spatii illuminati. Prius vero quam hujus lineæ innotescit æquatio, in illam rem inquirere debemus, qualis esset figura spatii illuminati, si ut punctum consideraretur sol.

Sit in hoc casu curva *MAR* (Fig. 1.) pars peripheriæ foraminis, per cujus puncta *A* & *M* radii luminis e sole profecti paralleli *AB* & *MN* perveniunt ad planum *NBS*, cui dato angulo inclinationis $= m$ occurrunt in punctis *B* & *N*, atque curvam *NBS* formant. Recta linea *CD* sit intersectio planorum *MAR* & *NBS*, cui normalis ex *A* ducatur recta *AC*. A puncto *A* ulterius demittatur perpendicularis *AE* ad planum *NBS*, cui occurrat in *E*, & jungantur puncta *B*, *E* & *C* rectis *BE*, *EC* & *BC*, quo facto erit ang. $ABE = m$, & planorum *MAR* & *NBS* mutua inclinatio $= ACE = n$. A puncto *M* ducatur recta *MP* perpendicularis in *AC*, & ex *P* demittatur *PQ* parallela lineæ *AB*, ut occurrat plano *NBS* in puncto *Q*, quod in recta *BC* situm erit. Junctis itaque punctis *Q* & *N* linea *NQ*, ob parallelismum laterum oppositorum erit *MPQN* parallelogrammum, adeoque $PQ =$

MN,

MN, & $QN = PM$. Factis vero $AB = a$, & Sinu toto $= 1$, erunt $AE = a \; Sin \; m$, $BE = a \; Cof \; m$, $AC = \frac{AE}{Sin \; n} = \frac{a \; Sin \; m}{Sin \; n}$, & $CE = \frac{AE}{Tg \; n} = \frac{a \; Sin \; m}{Tg \; n}$. Si quoque a B ducitur recta BF perpendicularis in planum MAR, cui occurrit in F, & a F recta FG perpendicularis in CD, ductis FA & GB, inclinatio radiorum AB & MN in planum MAR erit $FAB = l$. Est vero in triangulo CEB latus $BC = \surd(CE^2 - 2CE.\,EB.\; Cof \; CEB + EB^2) = \frac{a \; Cof \; m}{Tg \; n} \surd(Tg \; m^2 - 2 \; Tg \; m \; Tg \; n \; Cof \; CEB + Tg \; n^2)$, & $Tg \; ECB = \frac{EB.\; Sin \; CEB}{EC - EB.\, Cof \, CEB} = \frac{Tg \; n \; Sin \, CEB}{Tg \; m - Tg \, n \; Cof \, CEB} = Cotg \; GCB$, unde invenitur $Sin \; GCB = \frac{Tg \; m - Tgn \; Cof \; CEB}{\surd(Tg \; m^2 - 2 \; Tg \; m \; Tg \; n \; Cof \; CEB + Tgn^2)}$. Ulterius est $BG = BC.\; Sin \; GCB = a \; Cof \; m\left(\frac{Tg \; m}{Tg \; n} - Cof \; CEB\right)$, $BF = BG.\; Sin \; BGF = a \; Sin \; n \; Cof \; m \left(\frac{Tg \; m}{Tg \; n} - Cof \; CEB\right)$, adeoque $Sin \; l = \frac{BF}{AB} = Sin \; n \; Cof \; m \left(\frac{Tg \; m}{Tg \; n} - Cof \; CEB\right)$. Hinc vero invenitur $Cof \, CEB = \frac{Tg \; m}{Tg \; n} - \frac{Sin \; l}{Sin \; n \; Cof \; m}$, & ejus

 sub-

substitutione facta $BC = \frac{a\, Cos\, m}{Tg\, n} . \sqrt{(Tg\, n^2 + \frac{2\, Tg\, m\, Sin\, l}{Cos\, m\, Cos\, n}}$ $- Tg\, m^2)$, atque $Sin\, GCB =$

$$\frac{Sin\, l}{Cos\, m\, Cos\, n \sqrt{(Tg\, n^2 + \frac{2\, Tg\, m\, Sin\, l}{Cos\, m\, Cos\, n} - Tg\, m^2)}}$$, seu facta $k = \sqrt{(Tg\, n^2 + \frac{2\, Tg\, m\, Sin\, l}{Cos\, m\, Cos\, n} - Tg\, m^2)}$, $BC =$ $\frac{a\, k\, Cos\, m}{Tg\, n}$, & $Sin\, GCB = \frac{Sin\, l}{k\, Cos\, m\, Cos\, n}$.

Sumantur jam ab origine *A* curvæ *MAR* abſciſſæ in recta *AC*: fiat ſcilicet $AP = x$, & ordinata orthogonalis $PM = y = QN$. Erit itaque $BC : AC :: k\, Cos\, n : Tg\, m :: BQ : x$, unde eruitur $BQ = \frac{k\, Cos\, n}{Tg\, m} . x$.

Sit ulterius *BE* curvæ *NBS* linea abſciſſarum, & ducatur a *N* recta *NT* normalis in *BE*, quæ lineæ *BC* in *U* occurrat. Fiat $BT = r$ & $TN = s$. Cum antea inventus ſit $Cos\, CEB = \frac{Sin\, m\, Cos\, n - Sin\, l}{Cos\, m\, Sin\, n}$, eſt quoque $Sin\, CEB = \sqrt{\left(1 - \frac{(Sin\, m\, Cos\, n - Sin\, l)^2}{Cos\, m^2\, Sin\, n^2}\right)} = b$, & $Sin\, CBE = \frac{b . CE}{CB} = \frac{b\, Tg\, m}{k} = Cos\, TUB$. Hinc vero

in-

invenitur $Sin\ TUB = \frac{1}{k}\sqrt{k^2 - b^2\ Tg\ m^2}$, & $NU =$

$\frac{NQ\ .\ Sin\ NQU}{Sin\ NUQ} = \frac{NQ\ .\ Sin\ GCB}{Sin\ TUB} =$

$\frac{Sin\ l\ .\ y}{Cos\ m\ Cos\ n\ \sqrt{(k^2 - b^2\ Tg\ m^2)}}$, nec non $QU =$

$\frac{NQ\ .\ Sin\ QNU}{Sin\ NUQ} = \frac{NQ\ .\ Sin\ (TUB + GCB)}{Sin\ TUB} =$

$\frac{gky}{\sqrt{(k^2 - b^2\ Tg\ m^2)}}$ si facta est $g = \frac{1}{k^2\ Cos\ m\ Cos\ n}\ (b Sin\ l Tg\ m$
$+ \sqrt{(k^2 - b^2\ Tg\ m^2)}\ (k^2\ Cos\ m^2\ Cos\ n^2 - Sin\ l^2)\ )$.

Hinc autem inveniuntur $BU = BQ - QU = \frac{Cos\ n.\ kx}{Tg\ m} -$

$\frac{gky}{\sqrt{(k^2 - b^2\ Tg\ m^2)}}$, $TU = BU\ .\ Cos\ TUB = Cos\ n\ bx -$

$\frac{Tg\ m\ .\ gby}{\sqrt{(k^2 - b^2\ Tg\ m^2)}}$, adeoque $NU + UT = s =$

$\frac{(Sin\ l - gb\ Sin\ m\ Cos\ n)\ y}{Cos\ m\ Cos\ n\ \sqrt{(k^2 - b^2\ Tg\ m^2)}} + Cos\ n\ .bx$, & $BT =$

$BU\ .\ Sin\ TUB = r = \frac{Cos\ n\ \sqrt{(k^2 - b^2\ Tg\ m^2)}}{Tg\ m}\ .\ x - gy.$

Ambabus hisce æquationibus comparatis habetur

$x = \frac{Sin\, m}{Sin\, l}\left(gs + \frac{(Sin\, l - gb\, Sin\, m\, Cos\, n)r}{Cos\, m\, Cos\, n\, \sqrt{(k^2 - b^2\, Tg\, m^2)}}\right)$, atque $y =$

$$\frac{Sin\, m}{Sin\, l}\left(\frac{Cos\, n\, \sqrt{(k^2 - b^2\, Tg\, m^2)}}{Tg\, m} . x - gy\right).$$

Si jam hi valores coordinatarum x & y in data curvæ MAR æquatione substituuntur, oritur æquatio coordinatas r & s continens, & determinans curvam NBS, quæ spatii illuminati peripheriam constitueret, si sol nullo appareret diametro, vel si e centro tantum solis radii lucis advenirent. Re ipsa autem axes omnes lucis conorum in curva NBS plano $NBSG$ occurrunt, angulo inclinationis existente $= m$.

Nisi sit $m = 90°$, quando circulus oritur, lumen in plano $NBSG$ circa quodvis curvæ punctum N ellipsin pingit, cujus axis major lineæ BE est parallelus. Omnium autem ellipsium axis major a curva NBS in partes proportionales secatur. Magnitudo axis hujus pendet a distantia inter M & N, quæ variabilis est, nisi sit angulus $n = o$. Ut itaque determinentur dimensiones talium ellipsium, sit ABC (Fig 2) conus lucis, cujus intersectio elliptica cum plano NmS (Fig. 1) sit BKS (Fig. 2). Inclinatio autem axis coni AD & axis majoris BC ellipsis est ang. $ADB = m$. Ductis itaque per punctum D rectis EF in plano $ABDC$ & DK in plano ellipsis axi AD normalibus, erit $ED = DK$, adeoque, facta $AD = A$, & dimidio solis apparente diametro = ang. $BAD = \mu$, ED $= DK = A\, Tg\, \mu$, $BD = \frac{A\, Sin\, \mu}{Sin\,(m + \mu)}$, & $DC =$

A Sin

$\frac{A\, Sin\, \mu}{Sin\,(m - \mu)}$: Inde autem invenitur axis $BC =$

$$\frac{A\, Sin\, \mu}{Sin\,(m + \mu)} + \frac{A\, Sin\, \mu}{Sin\,(m - \mu)} = \frac{A\, Sin\, m\, Sin\, 2\, \mu}{Sin\,(m + \mu)\, Sin\,(m - \mu)}.$$

Secto axe BC in G in duas æquales partes, ductaque recta GH lineæ DK parallela, habetur ex natura ellipsis $BD \,.\, DC : DK^2 : BG^2 : GH^2 :: \frac{A^2\, Sin\, \mu^2}{Sin\,(m + \mu)\, Sin\,(m - \mu)}$ $:: A^2\, Tg\, \mu^2 :: \frac{A^2\, Sin\, m^2\, Sin\, 2\, \mu^2}{4\, Sin\,(m + \mu)^2\, Sin\,(m - \mu)^2} : GH^2$, unde post debitam reductionem invenitur semiaxis minor $GH = \frac{A\, Sin\, m\, Sin\, \mu}{\sqrt{Sin\,(m + \mu)\, Sin\,(m - \mu)}}$. Numerata itaque ex origine B in linea BC abscissa $= p$, factaque ordinata orthogonali $= q$, erit æquatio ellipsis quæsita: $q^2 = 2\, A\, Sin\, m\, Tg\, \mu.\, p - \frac{Sin\,(m + \mu)\, Sin\,(m - \mu)\,.\, p^2}{Cos\, \mu^2}$. Est autem (in Fig. 1) $PQ = MN = \frac{AB\,.\, CP}{AC} = a - \frac{Sin\,.\, n\, x}{Sin\, m}$ $= a - \frac{Sin\, n}{Sin\, l}\left(g\, s + \frac{(Sin\, l - g\, b\, Sin\, m\, Cos\, n)\, r}{Cos\, m\, Cos\, n \sqrt{(k^2 - b^2\, Tg\, m^2)}}\right)$; quo valore pro A substituto in æquatione ellipsis, factisque $\alpha = 2\, a\, Sin\, m\, Tg\, \mu$; $\beta = \frac{2\, g\, Sin\, m\, Sin\, n\, Tg\, \mu}{Sin\, l}$,

$$\gamma = \frac{2\, Tg\, m\, Tg\, n\, Tg\, \mu\, (Sin\, l - gb\, Sin\, m\, Cos\, n)}{Sin\, l\, \surd(k^2 - b^2\, Tg\, m^2)},$$ &

$\delta = \frac{Sin\, (m + \mu)\, Sin\, (m - \mu)}{Cos\, \mu^2}$, oritur æquatio $q^2 = (\alpha - \beta s - \gamma r) p - \delta p^2$, quæ determinat ellipsin circa punctum N in plano NBS pictam. Est autem ejus axis majoris pars $BD = \frac{Cos\, \mu}{2\, Sin\, m\, Sin\, (m - \mu)} (\alpha - \beta s - \gamma r)$,

& $DC = \frac{Cos\, \mu}{2\, Sin\, m\, Sin\, (m - \mu)} (\alpha - \beta s - \gamma r)$.

His præmissis jam inveniri potest æquatio, cujus ope innotescit figura spatii illuminati. Denotante scilicet in Fig. 3 NBS, BT & TN easdem lineas quam in Fig. 1, & sit QGH ellipsis circa punctum N pictum, cujus axis major GH parallelus est lineæ abscissarum BE in curva NBS. Sit autem linea DCQ pars peripheriæ spatii illuminati, quæ ellipsin in Q tangit. Producta ulterius EB, donec occurrat lineæ DQ in C, a Q perpendicularis in CE ducatur QP, quæ rectæ GH perpendiculariter occurrit in L. Factis itaque $BT = r$, $TN = = s$; $GL = p$, $QL = q$; $CP = t$, $PQ = v$; erit æquatio pro ellipsi QGH: I.) $q^2 = (\alpha - \beta s - \gamma r)\, p - \delta p^2$, recta autem $GN = \zeta(\alpha - \beta s - \gamma r)$, facta $\zeta = \frac{Cos\, \mu}{2\, Sin\, m\, Sin\, m - \mu}$; Præterea est $PQ = TN + LQ$, seu II.) $v = s + q$. Ducta quoque QR normali in Q ad lineam DCQ, erit quoque ejus pars

QU

QU normalis ellipsis in *Q*, adeoque *QP* : *PR* : : *QL* : *LU*, seu $dt : dv :: dp : dq$, unde habetur æquatio III.) $\frac{dt}{dv} = \frac{dp}{dq}$. Si tandem in valore lineæ *GN* ponuntur $s = o$ & $r = o$, invenitur $CB = \alpha\zeta = \frac{a \, Sin \, \mu}{Sin \, (m - \mu)}$, adeoque ob $LN = CB + BT - CP$, IV.) $t - r = \zeta(\beta s + \gamma r) + p$.

Cum itaque, ut in antecedentibus est ostensum, ex data æquatione inter *x* & *y* cognoscatur æquatio inter *r* & *s*, ope harum æquationum exterminari possunt quantitates *r*, *s*, *p* & *q*, ut restet æquatio differentialis solas coordinatas *t* & *v* cum suis fluxionibus continens, cujus integrale præbet æquationem quæsitam lineæ *DCQ*.

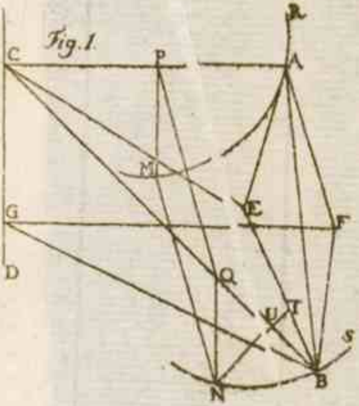

Fig. 1.

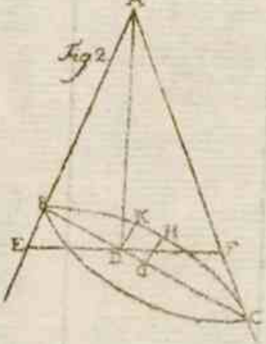

Fig 2.

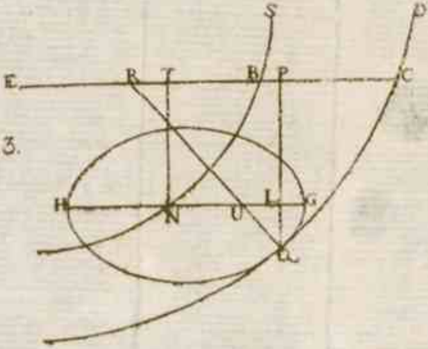

Fig. 3.