DISSERTATIO ACADEMICA,

DE

FIGURA TELLURIS OPE PENDULORUM DETERMINANDA;

CUJUS

PART. VI,

CONS. AMPL. FAC. PHIL. AB.

PRÆSIDE

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ABOÆ, TYPIS FRENCKELLIANIS.

Qui allata methodo ſic eruitur valor penduli polaris $P = \frac{544150}{1232,521} = 441,4933$, vero quidem proximus judicari poteſt. Sunt tamen rationes, quæ illum dubium reddant, ſuadeantque, ut alia via certior determinari poſſe videatur. Quorundam enim locorum valor penduli pluries quam reliquorum in hacce occurrit comparatione, unde efficitur, ut ſi hic ipſe aliquo ſcateat errore obſervationis, qui a reliquis obſervationibus non corrigitur, vis ejus in valore determinando juſto major ſit. E re igitur erit, ut alia quoque ratione, quadratorum ſcilicet minimorum methodo, hæc quæſtio examinetur, quo ſic, ea ex omni parte conſiderata, veriſimillimus eruatur valor.

Facta nimirum, ut ſupra, longitudine penduli æquatorialis $= E$, differentia longitudinis penduli polaris & æquatorialis ſeu $P - E = x$, atque latitudine loci $= \lambda$, ita ut e præcedentibus habeatur ejusdem loci longitudo penduli $p = E + x. Sin \lambda^2$; ſupponamus

 ab

ab hoc valore illum, qui experimentis est determinatus, quantitate parva $= t$ aberrare, quo sic habeatur $p - E - Sin\, \lambda^2 . x = t$. Hujusmodi igitur æquatio cuique loco observationis competit sua sequens:

Spitsbergen -	:	$441,380 - E - 0,9685 . x = t_1$
Kola - - -	:	$441,348 - E - 0,8701 . x = t_2$
Mulgrave - -	:	$441,122 - E - 0,8536 . x = t_3$
Ponoi - - -	:	$441,210 - E - 0,8483 . x = t_4$
Pello - - -	:	$441,163 - E - 0,8448 . x = t_5$
Archangelop. -	:	$441,132 - E - 0,8155 . x = t_6$
Petropolis - -	:	$441,005 - E - 0,7491 . x = t_7$
Upsalia - - -	:	$440,901 - E - 0,7479 . x = t_8$
Revalia - - -	:	$440,934 - E - 0,7415 . x = t_9$
Dorpatum - -	:	$440,917 - E - 0,7252 . x = t_{10}$
Pernavia - -	:	$440,920 - E - 0,7252 . x = t_{11}$
Arensburgum -	:	$440,885 - E - 0,7231 . x = t_{12}$
Gryphisvaldia -	:	$440,830 - E - 0,6558 . x = t_{13}$
Lugdunum -	:	$440,710 - E - 0,6236 . x = t_{14}$
Londinum - -	:	$440,638 - E - 0,6127 . x = t_{15}$
Schweidnitz -	:	$440,635 - E - 0,6013 . x = t_{16}$
Nootka - - -	:	$440,479 - E - 0,5797 . x = t_{17}$
Parisii - - -	:	$440,559 - E - 0,5668 . x = t_{18}$
Vienna - - -	:	$440,550 - E - 0,5559 . x = t_{19}$
Tolosa - - -	:	$440,339 - E - 0,4755 . x = t_{20}$
Roma - - -	:	$440,310 - E - 0,4460 . x = t_{21}$
Formentera -	:	$440,155 - E - 0,3903 . x = t_{22}$
Monterey - -	:	$440,123 - E - 0,3555 . x = t_{23}$
Gades - - -	:	$439,999 - E - 0,3544 . x = t_{24}$
Melita - - -	:	440, 2[illegible]0 $- E - 0,3438 . x = t_{25}$
Megalaki - -	:	$440,051 - E - 0,2924 . x = t_{26}$
Macao - - -	:	$439,594 - E - 0,1552 . x = t_{27}$

Gua-

Guarico - -	:	439,512	— E —	0,1145.	$x = t_{28}$
Parva Goava -	:	439,470	— E —	0,1002.	$x = t_{29}$
Jamaica - -	:	439,444	— E —	0,0955.	$x = t_{30}$
Acapulco - -	:	439,412	— E —	0,0839.	$x = t_{31}$
Manilla - - -	:	439,338	— E —	0,0636.	$x = t_{32}$
Madagascar -	:	439,412	— E —	0,0585.	$x = t_{33}$
Umatog - -	:	439,226	— E —	0,0529.	$x = t_{34}$
Pondichery -	:	439,282	— E —	0,0427.	$x = t_{35}$
Porto bello -	:	439,300	— E —	0,0275.	$x = t_{36}$
Sambuangan -	:	439,268	— E —	0,0145.	$x = t_{37}$
Para - - -	:	439,249	— E —	0,0007.	$x = t_{38}$
Æquator - -	:	439,210	— E —	0,0000.	$x = t_{39}$
Lima - - -	:	439,274	— E —	0,0438.	$x = t_{40}$
Vavao - - -	:	439,482	— E —	0,1023.	$x = t_{41}$
Portus Ludovici	:	439,561	— E —	0,1188.	$x = t_{42}$
Rio Janeiro -	:	439,950	— E —	0,1514.	$x = t_{43}$
Portus Jackson	:	439,989	— E —	0,3103.	$x = t_{44}$
Promont. b. sp.	:	439,976	— E —	0,3114.	$x = t_{45}$
Monte Video -	:	440,033	— E —	0,3276.	$x = t_{46}$
Conception -	:	440,011	— E —	0,3572.	$x = t_{47}$
St. Helena -	:	440,518	— E —	0,4913.	$x = t_{48}$
Puerto Egmont	:	440,611	— E —	0,6099.	$x = t_{49}$

Hinc vero, sumtis quadratis omnium valorum t, positaque secundum methodum laudatam, pro sola quantitate E ut variabili primum considerata, summa omnium t^2 minima, scilicet $d(t_1^2 + t_2^2 + t_3^2 + \&c.) = o$, quod hoc casu, quo quantitas E eodem ubique affecta est coëfficiente, eo redit, ut summa omnium æquationum arithmetice media quæratur, eruitur æquatio:

$$440{,}1825 - E - 0{,}4102 . x = o,$$

qua e ſuperioribus ſubtracta ſequentes producuntur:

$$1,1975 - 0,5583.\ x = t_1$$
$$1,1655 - 0,4599.\ x = t_2$$
$$0,9395 - 0,4434.\ x = t_3$$
$$1,0275 - 0,4381.\ x = t_4$$
$$0,9805 - 0,4346.\ x = t_5$$
$$0,9495 - 0,4053.\ x = t_6$$
$$0,8225 - 0,3389.\ x = t_7$$
$$0,7185 - 0,3377.\ x = t_8$$
$$0,7515 - 0,3313.\ x = t_9$$
$$0,7345 - 0,3150.\ x = t_{10}$$
$$0,7375 - 0,3150.\ x = t_{11}$$
$$0,7025 - 0,3129.\ x = t_{12}$$
$$0,6475 - 0,2456.\ x = t_{13}$$
$$0,5275 - 0,2134.\ x = t_{14}$$
$$0,4555 - 0,2025.\ x = t_{15}$$
$$0,4525 - 0,1911.\ x = t_{16}$$
$$0,2965 - 0,1695.\ x = t_{17}$$
$$0,3765 - 0,1566.\ x = t_{18}$$
$$0,3675 - 0,1457.\ x = t_{19}$$
$$0,1565 - 0,0653.\ x = t_{20}$$
$$0,12\text{[illegible]}5 - 0,0358.\ x = t_{21}$$
$$-0,0275 + 0,0199.\ x = t_{22}$$
$$-0,0595 + 0,0447.\ x = t_{23}$$
$$-0,1835 + 0,0558.\ x = t_{24}$$
$$+0,0375 + 0,0664.\ x = t_{25}$$
$$-0,1315 + 0,1178.\ x = t_{26}$$
$$-0,5085 + 0,2550.\ x = t_{27}$$
$$-0,6705 + 0,2957.\ x = t_{28}$$
$$-0,7125 + 0,3100.\ x = t_{29}$$
$$-0,7385 + 0,3147.\ x = t_{30}$$
$$-0,7705 + 0,3263.\ x = t_{31}$$

$$- 0,8445 + 0,3466 . x = t_{32}$$
$$- 0,7705 + 0,3517 . x = t_{33}$$
$$- 0,9565 + 0,3573 . x = t_{34}$$
$$- 0,9005 + 0,3675 . x = t_{35}$$
$$- 0,8825 + 0,3827 . x = t_{36}$$
$$- 0,9145 + 0,3957 . x = t_{37}$$
$$- 0,9335 + 0,4095 . x = t_{38}$$
$$- 0,9725 + 0,4102 . x = t_{39}$$
$$- 0,9085 + 0,3664 . x = t_{40}$$
$$- 0,7005 + 0,3079 . x = t_{41}$$
$$- 0,6215 + 0,2914 . x = t_{42}$$
$$- 0,2325 + 0,2588 . x = t_{43}$$
$$- 0,1935 + 0,0999 . x = t_{44}$$
$$- 0,2065 + 0,0988 . x = t_{45}$$
$$- 0,1495 + 0,0826 . x = t_{46}$$
$$- 0,1715 + 0,0530 . x = t_{47}$$
$$0,3355 - 0,0811 . x = t_{48}$$
$$0,4285 - 0,1997 . x = t_{49}$$

Hinc item erit:

$$0,31169 . x^2 - 2 . 0,66856 . x - \&c. = (t_1)^2$$
$$0,21150 . x^2 - 2 . 0,53601 . x - \&c. = (t_2)^2$$
$$0,19661 . x^2 - 2 . 0,41658 . x - \&c. = (t_3)^2$$
$$0,19193 . x^2 - 2 . 0,45015 . x - \&c. = (t_4)^2$$
$$0,18887 . x^2 - 2 . 0,42612 . x - \&c. = (t_5)^2$$
$$0,16427 . x^2 - 2 . 0,38484 . x - \&c. = (t_6)^2$$
$$0,11485 . x^2 - 2 . 0,27874 . x - \&c. = (t_7)^2$$
$$0,11404 . x^2 - 2 . 0,24264 . x - \&c. = (t_8)^2$$
$$0,10976 . x^2 - 2 . 0,24897 . x - \&c. = (t_9)^2$$
$$0,09922 . x^2 - 2 . 0,23137 . x - \&c. = (t_{10})^2$$
$$0,09922 . x^2 - 2 . 0,23231 . x - \&c. = (t_{11})^2$$
$$0,09791 . x^2 - 2 . 0,21981 . x - \&c. = (t_{12})^2$$
$$0,06032 . x^2 - 2 . 0,15902 . x - \&c. = (t_{13})^2$$

$0,04554.\ x^2 - 2.\ 0,11257.\ x - \&c. = (t_{14})^2$
$0,04101.\ x^2 - 2.\ 0,09224.\ x - \&c. = (t_{15})^2$
$0,03652.\ x^2 - 2.\ 0,08647.\ x - \&c. = (t_{16})^2$
$0,02873.\ x^2 - 2.\ 0,05026.\ x - \&c. = (t_{17})^2$
$0,02452.\ x^2 - 2.\ 0,05896.\ x - \&c. = (t_{18})^2$
$0,02123.\ x^2 - 2.\ 0,05355.\ x - \&c. = (t_{19})^2$
$0,00426.\ x^2 - 2.\ 0,01219.\ x - \&c. = (t_{20})^2$
$0,00128.\ x^2 - 2.\ 0,00456.\ x - \&c. = (t_{21})^2$
$0,00040.\ x^2 - 2.\ 0,00055.\ x - \&c. = (t_{22})^2$
$0,00200.\ x^2 - 2.\ 0,00266.\ x - \&c. = (t_{23})^2$
$0,00311.\ x^2 - 2.\ 0,01024.\ x - \&c. = (t_{24})^2$
$0,00441.\ x^2 - 2.\ 0,00249.\ x - \&c. = (t_{25})^2$
$0,01388.\ x^2 - 2.\ 0,01549.\ x - \&c. = (t_{26})^2$
$0,06503.\ x^2 - 2.\ 0,15007.\ x - \&c. = (t_{27})^2$
$0,08744.\ x^2 - 2.\ 0,19827.\ x - \&c. = (t_{28})^2$
$0,09608.\ x^2 - 2.\ 0,22087.\ x - \&c. = (t_{29})^2$
$0,09904.\ x^2 - 2.\ 0,23241.\ x - \&c. = (t_{30})^2$
$0,10647.\ x^2 - 2.\ 0,25141.\ x - \&c. = (t_{31})^2$
$0,12013.\ x^2 - 2.\ 0,29270\ x - \&c. = (t_{32})^2$
$0,12369.\ x^2 - 2.\ 0,27099.\ x - \&c. = (t_{33})^2$
$0,12766.\ x^2 - 2.\ 0,34175.\ x - \&c. = (t_{34})^2$
$0,13506.\ x^2 - 2.\ 0,33094.\ x - \&c. = (t_{35})^2$
$0,14646.\ x^2 - 2.\ 0,33773.\ x - \&c. = (t_{36})^2$
$0,15658.\ x^2 - 2.\ 0,36186.\ x - \&c. = (t_{37})^2$
$0,16769.\ x^2 - 2.\ 0,38227.\ x - \&c. = (t_{38})^2$
$0,16827.\ x^2 - 2.\ 0,39892.\ x - \&c. = (t_{39})^2$
$0,13425.\ x^2 - 2.\ 0,33288.\ x - \&c. = (t_{40})^2$
$0,09480.\ x^2 - 2.\ 0,21568.\ x - \&c. = (t_{41})^2$
$0,08491.\ x^2 - 2.\ 0,18111.\ x - \&c. = t_{42}{}^2$
$0,06698.\ x^2 - 2.\ 0,06017.\ x - \&c. = (t_{43})^2$
$0,00100.\ x^2 - 2.\ 0,01933.\ x - \&c. = (t_{44})^2$
$0,00976.\ x^2 - 2.\ 0,02040.\ x - \&c. = (t_{45})^2$
$0,00682.\ x^2 - 2.\ 0,01235.\ x - \&c. = (t_{46})^2$

$0,00281. x^2 - 2.0,00909. x -$ &c. $= (t_{47})^2$
$0,00658. x^2 - 2.0,02721. x -$ &c. $= (t_{48})^2$
$0,03988. x^2 - 2.0,08557. x -$ &c. $= (t_{49})^2$

Collectis vero his omnibus æquationibus, habebitur $(t_1)^2 + (t_2)^2 + (t_3)^2 +$ &c. $= 4,23447. x^2 - 2. 9,72635. x -$ &c., unde differentiando, pro casu minimi, orietur $d((t_1)^2 + (t_2)^2 + (t_3)^2 +$ &c.$) = 0 = 4,23447. xdx - 9,72635. dx$, atque $x = 2,29695$, quo valore in æquatione supra inventa $440,1825 - E - 0,4102. x = 0$ substituto, proveniet $E = 439,2393$, atque valor longitudinis penduli simplicis generalis $p = 439,239 + 2,297 \; Sin \; l^2$.

Facta igitur hic $l = 90°$, invenitur longitudo penduli polaris $P = 441,539$, quam tamen ne ipsam quidem pro verissima adhuc esse censendam ostendemus.

Si nempe valor p ex hac formula pro quovis loco, ubi determinata habetur per experientiam longitudo penduli, eruitur; differentia ab observata longitudine adeo interdum magna, aut e vitio quodam observationis omnem exspectationem superante, aut ex inæqualitatibus terræ insignioribus, aut denique ex utraque causa simul agente proficiscens, provenit, ut in hac comparatione, qua generalis quædam intenditur determinatio, qualem omnes observationes in eundem finem amice & conjunctim conspirantes præbent, illæ observationes, quæ a reliquis nimis aber-

rare

rare videntur, plane omittantur. Accedit, quod omnes allatæ obſervationes ſimul ſumtæ talem telluri esſe oſtendant ellipticitatem, quæ cum eadem ex aliis rationibus deducta non ſatis bene convenit. Harum enim rerum ſcrutatoribus patet, non minus menſurandi operationes exactisſimas, quibus longitudines partium meridiani cujusdam ſunt determinatæ, quam plures ex Aſtronomia deſumtæ rationes (*) ſuadere, ut asſignetur terræ, utpote non homogeneæ, ſed majoris verſus centrum denſitatis, ellipticitas quam proxime $= \frac{1}{305}$.

Eſt vero ſupra (P. IV, pag. 10) oſtenſum haberi $n = \frac{1}{p} \sqrt{(P^2 + (P^2 - p^2)\, Tg\, l^2)}$, unde poſita $l = 0$, quo caſu abit p in E, invenitur $n = \frac{P}{E}$, ſeu $n - 1 = \frac{P - E}{E} = \frac{x}{E}$.

E principiis vero mechanicis, quibus æquilibrium in ſuperficie telluris verſus centrum denſioris & circa axem ſuum data celeritate rotantis ſtabilitur, demonſtrarunt *Clairaut* (**) & *La Place* (***), ob vim centrifugam,

(*) Vide: *Expoſit. des operations faites en Lapponie pour la determination d'un arc du meridien; par J. Svanberg. Stockh. 1805; Disc. prelim. p. XXVIII.*

(**) *Theorie de la Figure de la Terre, p. 250.*

(***) *Mechanik des Himmels, Th. 2, S. 121, 180.*

fugam, quæ vim gravitatis ſub æquatore terræ parte $\frac{1}{289}$ minuit, veram haberi telluris ellipticitatem $= \frac{5}{2} \cdot \frac{1}{289} - \frac{x}{E} = 0{,}00865 - \frac{x}{E}$. Si igitur applicatio inſtituitur valoris nuper determinati, prodibit ellipticitas terræ ex allatis omnibus penduli longitudinibus $= 0{,}00865 - \frac{2{,}29695}{439{,}2393} = 0{,}003421 = \frac{1}{292{,}3}$.

Quoniam vero ab hoc differt valor $\frac{1}{305}$, videtur, inter allatas obſervationes aliquas eſſe, quæ hac admiſſa ellipticitate a reliquis nimis aberrant. Et quidem, comparatione valoris p cum obſervationibus inſtituta, animadvertuntur pro locis Kola, Mulgrave, Melita, Megaſaki, Umatog, Rio Janeiro & St. Helena differentiæ partem $\frac{1}{10}$ lineæ pariſinæ ſuperantes. Horum igitur locorum valoribus penduli miſſis, calculoque cum reliquis de novo inſtituto, oriuntur ſimili ac antea methodo $x = 2{,}32941$, $E = 439{,}20943$, atque ellipticitas $= 0{,}00335 = \frac{1}{298{,}5}$.

Hac ratione plures inſtituimus comparationes, aliis aliisve omiſſis obſervationibus, quarum fides minor viſa eſt, & præbuit nobis hic calculus valores ellipticitatis $\frac{1}{312{,}6}$, $\frac{1}{309{,}8}$, $\frac{1}{303{,}7}$, $\frac{1}{301{,}4}$, qui omnes a-

 perte

perte ostendunt, verum valorem ellipticitatis terræ ex observationibus penduli derivatum, utpote intra allatos hos limites medium, valori aliunde invento non modo non repugnare, sed potius optime ita convenire, ut, si ex diversissimis similiter sitis locis haberentur observationes penduli æque certæ, nullum esse videatur dubium, quin hæ etiam ellipticitatem indicent $= \frac{1}{305}$ uti maxime probabilem,

Hæc vero, quam sic convenientia omnino admiranda confirmant phænomena diversissimi generis, pro fundamento est ponenda in valore generali longitudinis penduli determinando. Cumque nullibi locorum eadem certitudine ac Lutetiæ Parisiorum datum habeatur pendulum, ex eo cum data hac ellipticitate generalis pro universa terra deducendus est valor longitudinis penduli, qui sic invenitur,

$$p = 439,2221 + 2,3596.\ Sin\, l^2,$$

unde apparet esse longitudinem penduli æquatorialis $E = 439,2221$, seu proxime talem, qualem eam *Bouguer* determinavit, atque penduli polaris maxime probabilem $P = 441,5817$.