SPECIMINIS ACADEMICI,

DE INVENIENDA LEGE EXPANSIONIS VAPORUM AQUÆ IN DIVERSIS CALORIS TEMPERATURIS,

CONTINUATIO;

QVAM

Conf. Ampl. Fac. Philos. Reg. Acad. Aboënf.

PRÆSIDE

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Publico Subjicit Examini

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ABOÆ, Typis FRENCKELLIANIS.

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SPECTATISSIMÆ

IN SACRAM REG. MAJESTATEM

FIDEI VIRO,

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DIRECTORI SUPREMO REI GEODETICÆ EIQUE
PER FINLANDIAM PRÆFECTO,

Ut gratum Benefactori maximo testificaretur Animum, has pagellas dicare voluit

JOHANNES SANDRY.

Præter generalem illam æqvationem, cujus ope determinari poteſt expanſio vaporum aqvæ in diverſis caloris gradibus, & qvæ in disſertatione de hac re nuper edita proponitur, plures aliæ eidem ſini inſervientes excogitari posſunt. Inter illas unam qvoqve exponentialem afferre liceat, qvæ ad omnia D:ni DALTON experimenta & omnes ejus concluſiones & determinationes applicanda, eo præ illa gaudet commodo, qvod paulo accuratius cum iis conveniat, & interdum breviorem reqvirat calculum.

Exiſtente nempe e in pollicibus anglicis altitudine illa mercurii in barometro, qvæ vim vaporum expanſivam exprimit, & c gradu caloris in thermometro centigrado tali, qvod 100° oſtendit in aqva, qvæ ebullit exiſtente altitudine barometri = 30 poll. angl., asſumamus esſe

$$\text{Log.}\ \frac{e}{30} = Aa^c + Bb^c + C',$$

cujus valoris qvantitates conſtantes & adhuc ignotæ A, a, B, b & C' ſecundum experimenta D:ni DALTON ſunt determinandæ. Id autem ſeqventi modo fecimus.

 Pro

Pro $c = 0;\ 40;\ 80;\ 120;\ 160;$

fit Log. $\frac{e}{30} = e_0;\ e_1;\ e_2;\ e_3;\ e_4;$

qvibus valoribus fubftitutis oriuntur qvinqve feqventes æqvationes:

$$e_0 = A + B + C',$$
$$e_1 = Aa^{40} + Bb^{40} + C',$$
$$e_2 = Aa^{80} + Bb^{80} + C',$$
$$e_3 = Aa^{120} + Bb^{120} + C',$$
$$e_4 = Aa^{160} + Bb^{160} + C'$$

unde exterminando qvantitatem C' obtinentur

$$e_1 - e_0 = A(a^{40} - 1) + B(b^{40} - 1),$$
$$e_2 - e_1 = Aa^{40}(a^{40} - 1) + Bb^{40}(b^{40} - 1),$$
$$e_3 - e_2 = Aa^{80}(a^{40} - 1) + Bb^{80}(b^{40} - 1),$$
$$e_4 - e_3 = Aa^{120}(a^{40} - 1) + Bb^{120}(b^{40} - 1),$$

atqve hinc exterminata qvantitate A

$$e_2 - e_1 - a^{40}(e_1 - e_0) = B(b^{40} - a^{40})(b^{40} - 1),$$
$$e_3 - e_2 - a^{40}(e_2 - e_1) = Bb^{40}(b^{40} - a^{40})(b^{40} - 1),$$
$$e_4 - e_3 - a^{40}(e_3 - e_2) = Bb^{80}(b^{40} - a^{40})(b^{40} - 1).$$

Exterminata ulterius qvantitate $B(b^{40} - a^{40})(b^{40} - 1)$, inveniuntur

$$b^{40}(e_2 - e_1 - a^{40}(e_1 - e_0)) = e_3 - e_2 - a^{40}(e_2 - e_1),$$
$$b^{40}(e_3 - e_2 - a^{40}(e_2 - e_1)) = e_4 - e_3 - a^{40}(e_3 - e_2),$$

atqve hinc

$$\frac{e_3 - e_2 - a^{40}(e_2 - e_1)}{e_2 - e_1 - a^{40}(e_1 - e_0)} = \frac{e_4 - e_3 - a^{40}(e_3 - e_2)}{e_3 - e_2 - a^{40}(e_2 - e_1)},$$

qvæ

qvæ æqvatio rite correcta præbet hanc;

$$a^{80} + \frac{(e_4 - e_3)(e_1 - e_0) - (e_3 - e_2)(e_2 - e_1)}{(e_2 - e_1)^2 - (e_3 - e_2)(e_1 - e_0)} a^{40}$$

$$= \frac{(e_4 - e_3)(e_2 - e_1) - (e_3 - e_2)^2}{(e_2 - e_1)^2 - (e_3 - e_2)(e_1 - e_0)}.$$ Cum autem ex experimentis Daltonianis ſint

$e_0 = -2,1760913$ & $e_1 - e_0 = 1,0232525$
$e_1 = -1,1528388$ $e_2 - e_1 = 0,8193567$
$e_2 = -0.3334821$ $e_3 - e_2 = 0,6077939$
$e_3 = 0,2743118$ $e_4 - e_3 = 0,3789007$
$e_4 = 0,6532125$

invenitur per debitam ſubſtitutionem horum valorum $a^{80} - 2,2317\, a^{40} = -1,193$, unde tandem eruitur
$a^{40} = 1,11585 + 0,2283024 = 1,3441524$,
$b^{40} = 1,11585 - 0,2283024 = 0,8875476$,
nec non $a = 1,0074215$, Log. $a = 0,003211212$,
& $b = 0,9970220$, Log. $b = 0,99870479 - 1$.
His cognitis invenitur ex præcedentibus

$$B = \frac{e_2 - e_1 - a^{40}(e_1 - e_0)}{(b^{40} - a^{40})(b^{40} - 1)} = -10,82938,$$ &

Log. $B = 1,0346038$, nec non

$$A = \frac{e_1 - e_0 - B(b^{40} - 1)}{a^{40} - 1} = -0,5652655,$$

Log. $A = 0,7522525 - 1$, & $C' = e_0 - A - B = 9,2185542$.

Facta igitur $C = \text{Log.}\, 30 + C' = 10,6956755$, erit

$$\text{Log.}\, e = Aa^c + Bb^c + C.$$

 Con-

Convenientia hujus æqvationis cernitur ex seqventibus, ubi $V(e)$ significat variationem vis expansivæ e pro variatione caloris unius gradus in temperatura juxta qvam positum est hocce signum.

c	e experimentis inventa	e calculo inventa.	Differentia.	
− 40	0,013	0,012	− 0,001	Error $< 1,4 V(e)$.
− 20		0,052		
0	0,200	0,200	0,000	
+ 20	0,676	0,689	+ 0,013	Error $< \frac{1}{3} V(e)$.
40	2,110	2,110	0,000	
60	5,740	5,751	+ 0,011	Error $< \frac{1}{26} V(e)$.
80	13,920	13,920	0,000	
100	30,000	29,842	− 0,158	Error $< \frac{1}{6} V(e)$.
120	56,420	56,424	+ 0,004	Error $< \frac{1}{200} V(e)$.
140	93,230	93,558	+ 0,328	Error $< \frac{1}{6} V(e)$.
160	135,000	135,046	+ 0,046	Error $< \frac{1}{44} V(e)$.

Æqvales, qvi hic calculo committuntur errores, in ipsis qvoqve experimentis non evitantur, si vel $\frac{1}{100}$ pollicis anglici in mensura qvantitatis e, vel $\frac{1}{5}$ gradus in observatione thermometri negligitur, unde intelligitur, æqvatione allata legem expansionis vaporum aqvæ satis accurate exprimi.

Si autem r significat gradus thermometri Reaumuriani, manentibus A, B & C invariatis, & positis $a = 1,0092855$; Log. $a = 0,004014015$;

$b =$

$b = 0,9962789$; Log. $b = 0,99838099 - 1$;
erit Log. $e = Aa^r + Bb^r + C$.

Ex inventis generalibus æqvationibus inveniri qvidem posset æqvatio, cujus ope pro data vi expansiva vaporum determinaretur gradus caloris, cui respondet. Nimis autem implicita hæc evaderet & prolixum in applicatione reqvireret calculum, qvare melius putavimus esse, ut ex partiali illa erueremus valorem ipsius c, qvo uti possumus existente $e > 30$, sicut formulæ D:ni SOLDNER adhibendæ sunt pro $e < 30$. Facta igitur barometri altitudine $= E$, in pollicibus anglicis expressa, qvando in aqva ebulliente determinatus est centesimus gradus thermometri centigradi, invenitur

$$c = 206,6 - \sqrt{\left(11371,37 - 14091,51 . \text{Log.} \frac{e}{E}\right)},$$

seu pro scala Reaumuriana

$$r = 165,3 - \sqrt{\left(7277,68 - 9018,56 . \text{Log.} \frac{e}{E}\right)},$$

qva uti oportet pro tertia æqvatione D:ni SOLDNER, qvando $e > 30$.

Allatæ autem æqvationes alias præbent, qvæ pro $e > 30$ substitui possunt loco formularum reliqvarum D:ni SOLDNER, & qvas iisdem cum Soldnerianis, qvibus conveniunt, indicibus designatas in seqventibus proponemus.

Existente B altitudine barometri, circa qvam thermometri punctum ebullitionis aqvæ determinatum est, & b altitudine barometri circa qvam aqva

ca-

calefit, invenitur calor r hujus aqvæ, qvando ebullit, ope hujus æqvationis:

$$\text{IV.)}\ r = 80 + 52{,}86\ \mathrm{Log.}\ \frac{b}{B} + 16{,}38\ \left(\mathrm{Log.}\ \frac{b}{B}\right)^2 + 10{,}15\ \left(\mathrm{Log.}\ \frac{b}{B}\right)^3 + \&c.$$

Qvando b parum differt a B, negligi possunt altiores dignitates qvantitatis Log. $\frac{b}{B}$, adeoqve etiam termini hujus æqvationis reliqvi præter primum & secundum. Si igitur B est normalis qvædam altitudo barometri, ad qvam reducendæ sunt observationes thermometricæ, & circa qvam determinari deberet punctum aqvæ ebullientis in thermometro, hæc autem determinatio revera fit circa altitudinem barometri b, qvæ major est qvam B; in loco ita determinato scribi debet non 80 pro r seu 100 pro c, sed seqventes valores:

$$\text{V.)}\ r = 80 + 53\ \mathrm{Log.}\ \frac{b}{B}\ ;\ c = 100 + 66\ \mathrm{Log.}\ \frac{b}{B}.$$

Si itaque volumus, ut gradus thermometrorum determinati sint circa altitudinem barometri B, thermometrum autem aliqvod, cujus gradus inventi sunt circa altitudinem b, ostendit calorem t, gradus x qvod hoc ostendere debet, est

$$x = t + \tfrac{2}{3}\ t\ \mathrm{Log.}\ \frac{b}{B}$$

Existente R gradu, in thermometro Reaumuri-

ano

ano, liqvidi alicujus, qvando hoc circa altitudinem barometri E ebullit; erit vis expansiva e vaporum hujus liqvidi in temperatura r, qvæ major est ipsa R, hoc modo inventa:

VI.) $\text{Log. } e = \text{Log. } E + 0{,}0189186\,(r - R) - 0{,}00011088\,(r - R)^2$.

Si igitur liqvidum aliqvod sub pressione aëris B ebullit in temperatura R, sub pressione aëris majore b ebullit in temperatura r, qvæ est

VII.) $r = R + 85{,}3 - \sqrt{(7277{,}68 - 9018{,}56 \text{ Log. } \frac{b}{B})}$.

Similiter pro gradibus caloris C & c, qvorum $c > C$, in thermometro Celsii, invenitur in hoc casu

VIII.) $\text{Log. } e = \text{Log. } E + 0{,}0151349\,(c - C) - 0{,}00007096\,(c - C)^2$, & IX.) $c = C + 106{,}6 - \sqrt{(11371{,}37 - 14091{,}51 \text{ Log. } \frac{b}{B})}$.

Cum calor liqvidi cujusvis major illo, in qvo ebullit, evadere neqveat; formulæ Soldnerianæ X, XI, XII, XIII & XIV correctione nostra non egent. Sit B altitudo Barometri, circa qvam calor ebullitionis aqvæ in thermometro Reaumuriano adhibito est determinatus, & sub pressione aëris atmosphærici $= b$ qværatur vis expansiva e vaporum aqvæ in temperatura r, qvæ major est qvam

$R = 80 + 53 \text{ Log. } \frac{b}{B}$ si $b > B$, sed major qvam

$R = 80 + 50 \text{ Log. } \frac{b}{B}$, si est $b < B$.

Erit

Erit XV.) Log. $c = $ Log. $b + 0{,}0189186\,(r - R)$ $- 0{,}00011088\,(r - R)^2$.

Formulæ Soldnerianæ XVI, XVII, & XVIII evaporationem aqvæ respiciunt in aëre atmosphærico, cujus calor minor semper est calore aqvæ ebullientis, qvare pro calore aliqvo hoc majore non egent correctione. Æqvatio autem ejus ultima hæc:

$$\text{Log.}\,\frac{\beta}{e} = \text{Log.}\,\frac{V}{v} = \frac{(200 + u)\,u}{10280},$$

ubi β est altitudo barometri, circa qvam gradus thermometri Reaumuriani sunt determinati & liqvidum aliqvod ebullit, V autem est qvantitas liqvidi hujus dato tempore sub ebullitione evaporati, v qvantitas ejusdem liqvidi eodem tempore evaporati in alia temperatura, qvæ u gradibus minor est calore ebullitionis, & e vis expansiva vaporum, in minore hac temperatura surgentium. Observat D:nus Soldner, negativam in hac æqvatione sumendam esse u, si temperatura illa, in qva vaporum elasticitas est e, u gradibus major est calore ebullitionis liqvidi. Ex iis autem, qvæ supra exposuimus, patet, coëfficientes qvantitatis u, (qvando hæc est differentia inter calorem ebullitionis & aliqvem alium majorem in experimentis observatum), a D:no Soldner inventas, pro majore hoc calore secundum experimenta Daltoniana non valere, sed ex æqvatione nostra VI vel XV sumendas esse, ut sit Log. $\frac{\beta}{e} = - 0{,}0189186\,u + 0{,}00011088\,u^2$.