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ABSTRACT : By using the two-period model, this study analyzes the effects of forest taxation on the short-term timber supply of nonindustrial private forest owners. The study consists of three separate articles and a note that have been published in forest economic journals. In each the assumption is made that either timber price or the real interest rate is uncertain; moreover, the capital market is either perfect or imperfect. The articles examine the effects of these factors on harvesting and the optimal design of forest taxes in terms of incentives and social welfare. The results of the analyses and the two-period model are compared to the Faustmann rotation framework in an extensive survey of forest economic literature. The survey concludes that the models are suited to different, yet complementary ends: the two-period model provides a tool for analyzing the market behavior of private forestry, and the Faustmann rotation model supplies the means for the management of public forests.

Keywords: forest taxation, timber supply, uncertainty

ABSTRAKTI: Tutkimuksessa analysoidaan metsäverotuksen vaikutusta yksityismetsänomistajien lyhyen aikavälin puun tarjontaan kahden periodin mallissa. Tutkimus koostuu kolmesta täydestä ja yhdestä note-tyyppisestä artikkelista, jotka on julkaistu kansainvälisissä metsäekonomisissa aikakauslehdissä. Kussakin artikkelissa oletetaan, että joko puun hinta tai reaalikorko on epävarma, kun pääomamarkkinat ovat vaihtoehtoisesti täydelliset tai epätäydelliset. Artikkeleissa tutkitaan näiden tekijöiden vaikutusta hakkuisiin ja määritetään yhteiskunnallisesti optimaalinen metsäverojärjestelmä, kun julkisen vallan tavoitteena on joko kannustaa puun tarjontaa tai maksimoida yhteiskunnallista hyvinvointia. Artikkeleiden tuloksia ja kahden periodin mallia verrataan Faustmannin malliin laajassa katsausartikkelissa. Katsauksen johtopäätöksenä on, että mallit soveltuvat erilaisiin, toisiaan täydentäviin tarkoituksiin: kahden periodin malli on hyvä väline yksityismetsänomistajien markkinakäyttäytymisen tutkimiseen ja Faustmannin malli puolestaan julkisten metsien käytön suunnitteluun.

Asiasanat: metsäverotus, puun tarjonta, epävarmuus

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PREFACE

This thesis consists of the following four papers and an introductory essay which follows this preface and acknowledgements:

1. Forest Taxation and the Timing of Private Nonindustrial Forest Harvests Under Interest Rate Uncertainty. *Canadian Journal of Forest Research*, 20, 1923-1929, 1990.
2. A Mean-Variance Approach to Short-Term Timber Selling and Forest Taxation Under Multiple Sources of Uncertainty. *Canadian Journal of Forest Research*, 23, 573-581, 1993.
3. The Effect of Nontimber Taxes on the Harvest Timing - The Case of Private Nonindustrial Forest Owners: A Note. *Forest Science*, 37, 356-363, 1991.
4. The Analytics of Timber Supply and Forest Taxes Under Endogenous Credit Rationing - Separability After All. *Journal of Forest Economics*, 1996 forthcoming.

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Helsinki, May 1996

Markku Ollikainen

TIMBER SUPPLY AND FOREST TAXATION: A REVIEW OF THE LITERATURE

1. INTRODUCTION

The classical *rotation problem* in forest economics is defined by the following question: at which age should one cut a tree or an even-aged forest? This question has puzzled tax officials, foresters and economists since the 18th century. In his article "Economics of forestry in an evolving society" (1976) Paul A. Samuelson gives a comprehensive analytical treatment of the rotation problem. He delineates the previous *intellectual development* of the issue and carefully formulates the assumptions behind the rotation analysis. Discussing at length the weaknesses of many biological and economic solutions to the problem, Samuelson shows that the correct solution to the rotation problem in a stationary world with perfect markets is obtained by maximizing the soil expectation formula originated by a German forester, Martin Faustmann (1849), and solved independently by young Bertil Ohlin (1921).

Samuelson's article represents an important turning point in the development of forest economics. The article was followed by active forest economic research within the newly-founded Faustmann model in two broad lines. First, the basic Faustmann framework was enlarged in scope, i.e. to contain new issues such as the multiple-use of forests (Hartman 1976) and silvicultural activity as an input of the forest growth function (Hyde 1980, Jackson 1980 and Chang 1983). Second, various policy issues were studied within the basic or enlarged Faustmann model. The most notable contributions are the studies of the effects of forest taxation (e.g. Klemperer 1976, Chang 1982), the implementation of multiple-use principles in practice (e.g. Calish, Fight and Teeguarden 1978, Bowes and Krutilla 1985), and the effects of improved biotechnology (Johansson and Löfgren 1985).

During the 1980s a new research strand, outside the Faustmann model, was delineated in forest economics. Instead of analyzing an infinite series of rotations, some analysts condensed the problem of harvest timing into a two-period Fisherian consumption-saving-cutting model. The first formulation of the two-period model by Johansson and Löfgren (1985), followed an elaborated version of Koskela (1989a) and (1989b). The basic version of the two-period model was quite rapidly extended to include the multiple-use/recreation aspects (Max and Lehman 1988 and Ovaskainen 1992) and silvicultural effort (Ovaskainen 1992). The effects of forest taxes and the design of optimal forest taxation have also been extensively analyzed within the two-period models (Koskela 1989a, 1989b, Ollikainen 1990, 1991, 1993, Amacher and Brazee 1995, and Koskela and Ollikainen 1995a and 1995b). Thus this new framework has been used to explain the same phenomena as has the Faustmann model.

The emergence of two alternative models raises a host of questions. What is the new insight obtained by introducing the two-period harvesting model? Do the results derived within this model differ from those of the Faustmann model? Are there differences in the forest tax policy recommendations? This essay offers an analysis and synthesis of the current state of the art in forest economics in terms of these two model types. The aim is to define the major theoretical developments within both models and to discuss how they are related to each other. The discussion of both models is organized on the basis of Samuelson's research agenda of relaxing the heroic assumptions to include the effects of uncertainty and market imperfections.¹

To anticipate the results, this essay demonstrates that there is a need for further theoretical work to clarify the relationship between these two models. The Faustmann model assumes that the owner lives infinitely, or that the owner, in fact, equals society. The former assumption needs more profound justification to show how the market behavior of forest owners with a finite life span generates an infinite time horizon. The latter is consistent with infinite series of rotations and gives the framework the status of a planning and management model. By omitting the planting of a new tree generation, the two-period model focuses only on the short-term liquidation of timber inventory. To determine the relationship between these two theoretical frameworks, one should examine under which assumptions the market behavior described in the two-period model might generate the Faustmann rule. Altruism in the form of giving timber bequests to the next generation is one attractive way of introducing such an infinite time horizon to the behavior of private (nonindustrial) forest owners. This emphasizes the need to further develop the two-period model into an overlapping generations model. The first step in this direction was in an inspiring paper by Löfgren (1991).

This paper will advocate the view that, at their present stage, the Faustmann and the two-period models are not substitutes but complements. Despite the difference in the time horizon, both models share considerable similarities. Most importantly, they are revenue maximization models with two assets; trees and money. In both models monetary capital earns a constant return given by the real interest rate, while timber capital earns first growing and then declining interest given by the forest growth function. The Faustmann model with its stress on long-term timber supply is an excellent model for studying socially optimal forestry and for managing the public forests. The two-period model with its stress on short-term timber supply, is superior in analyzing the market behavior of private nonindustrial forest owners. Thus the research problem at hand determines the choice of the model type.

The essay begins in section 2 with a short preview of Samuelson's analysis which offers a synthesis of the older literature and a starting point for subsequent development. This is followed

¹ There are two related reviews of the forest economic literature by Newman (1984) and Montgomery and Adams (1995). They focus basically on the work done within the Faustmann model, but they do not relate the two-period and Faustmann models to each other.

by a discussion on how the Faustmann model is applied in order to relax the heroic assumptions. The basic features of the two-period model are discussed in section 3, and its results with regard to the relaxation of the heroic assumptions are dealt with. Section 4 gives a synthesis of the analysis of forest taxes and forest tax policy in both models. The conclusion suggests some outlines for future research.

2. THE FAUSTMANN ROTATION MODEL

2.1 Samuelson's synthesis of the rotation analysis

Samuelson's analysis aims at developing a proper formulation of the rotation problem. At the beginning of his article, Samuelson makes assumptions that allow one to *focus only on the problem of the optimal timing of harvest without any other decision variables*. By combining and interpreting Samuelson's various remarks, one can extract a definition of the general rotation problem (i.e. general enough to be the common standpoint of all economic and biological approaches to the rotation problem). The definition is given through the following four general assumptions.

A definition of the rotation problem: timber production

- G1. The property rights over a given land area are well-defined.
- G2. There are no external effects involved in the forestry.
- G3. The forest owner grows forests in order to sell timber in the market.
- G4. There is a constant point-input/ point-output technology of growing trees.

Given the assumptions G1 - G4, the goal of the forest owner is identical to the problem of finding an optimal cutting time t^ .*

The role of the four assumptions is quite obvious. Assumption G1 guarantees that the owner has command over the piece of land and ownership of its trees. By this assumption one rules out the problem of open access common property resources, which is well-known and extensively analyzed in the economics of fisheries. Assumption G2 excludes those nontimber products that are public goods -- e.g. multiple use, recreation by the general public, flood control, etc. -- and could, therefore, provide a reason for government intervention. G3 rules out the possibility of the owner having any recreational or other motives that might affect the cutting decision. Together G2 and G3 allow one to focus solely on timber production and given G1, these three offer a narrow definition of the rotation problem. By excluding the choice of silvicultural management intensity, assumption G4 allows one to focus only on one decision variable, namely, timing, as time is the only variable production factor.

To analyze the rotation problem in the capital theoretical framework requires that some *additional* assumptions hold. Samuelson labels them as heroic. The economic analysis of the rotation problem could, of course, be solved at the most general level by maximizing the forest owner's utility from timber selling revenue. But, as is well-known, under certain circumstances, the utility maximization problem becomes identical to maximizing the net present value of an asset. This occurs when the Fisherian result of the separability of preferences from the production decision holds (Hirschleifer 1970). This has been nicely demonstrated for the Faustmann model in two important papers by Mitra and Wan (1985, 1986). They analyze optimal harvesting when the social planner has a concave utility function. Mitra and Wan demonstrate that under positive and zero interest rates the Faustmann rule is optimal for the linear utility function. It is also optimal for the strictly concave utility function under a positive interest rate, while under zero interest rate this function leads to the maximum sustained yield rule. Thus the role of Samuelson's heroic assumption is to point out the circumstances in which separability holds and the use of the capital theoretical analysis of the rotation problem is valid. When discussing the role of capital markets, he states very clearly the implication of separability: "Each person's longevity and degree of patience to spend becomes immaterial in a competitive market place with borrowing, lending and capitalizing interest rate that encapsulates all which is relevant about society's effective time preferences" (Samuelson 1976, 474).

To reduce the utility maximization problem to a simpler form of the capital theoretical approach, Samuelson employs the following heroic assumptions.²

- A1. The forest owner has perfect foresight concerning all future timber and input prices and future interest rates.
- A2. Capital markets are perfect so that the forest owners can borrow or lend infinite amounts of money at the prevailing interest rate.
- A3. Forest land markets are perfect.
- A4. The growth function of a tree (and of an even-aged forest) is known for certain.

Given the assumptions G1-G4 and A1-A4, the forest owner can find the optimal cutting time t^ by maximizing the net present value of harvest income.*

The role of these assumptions is clear. Perfect foresight of all relevant economic variables

² Notice that assumptions A1-A2 are given a formulation which differs from that given by Samuelson. He formulates them as follows. A1': Future lumber prices at which all outputs can be freely sold, and future wages of all inputs are known for certain. A2': The future interest rate at which enterprise can both borrow and lend in indefinite amounts is known for certain. One can see, however, that A2' includes in fact two assumptions - that of perfect foresight and of perfect capital markets. Therefore I prefer to collect together the perfect foresight assumptions at market level and state the perfect capital market assumption explicitly and separately.

restricts the analysis to the special case of certainty, thus excluding uncertainty in the sense of both economic risk and of unexpected events. By A1-A2 and A4 the forest owner knows exactly the present value of the infinite chains of rotations. Samuelson acknowledges the strong restrictions imposed by assumptions A1-A2: "... in other than a first approximation, the assumptions ... need to be complicated" (Samuelson (1976), 470). Assumptions A2 and A3 specify more explicitly what the competitive environment in the forestry context means. They help to exclude all kinds of imperfections in the capital and land markets. Perfect capital markets make it possible for the owner to either borrow or lend money according to his consumption plans so that he never has to cut his forest at a less than optimal point of time. The assumption of perfect land markets guarantees that there is competition among numerous forest owners, and land is, thus, allocated efficiently between forestry and other possible uses.

In addition to the heroic assumptions, Samuelson also makes many other assumptions. For the purpose of the partial analysis of the determination of timber supply only, it suffices here to state one of them.

A5. Future timber price and the real interest rate are constant.

Adding assumption A5 to the heroic assumptions A1-A4 implies that the optimal solution of the rotation problem will be a steady-state solution, which means that each cycle will be of equal length.

Samuelson points out that, given these assumptions, the correct solution to the rotation problem defined by G1-G4 and A1-A5 is given by maximizing the *Faustmann or soil expectation value formula* over an infinite series of rotations

$$[1] \quad \underset{\{t\}}{\text{Max}} \quad V = [pf(t)e^{-rt} - c](1 - e^{-rt})^{-1},$$

where p is net timber price (stumpage price), r is the real discount rate, c is the (fixed) planting costs and $f(t)$ is the growth function of a tree, where t is simultaneously the age of the tree and time. Moreover, by assumption, the growth function is concave so that for $f(t) = 0$ if $t \leq a > 0$ and for $t > a$ holds that $f'(t) > 0$ and $f''(t) < 0$.³ In what follows I will call the model [1] with $c \geq 0$ the *basic version of the Faustmann model* to separate it from its later modifications which endogenize either silvicultural management intensity or the valuation of nontimber benefits of the standing forest.⁴

³ Alternatively, some analysts assume logistic growth, i.e., there are points of time, t_n and t_m such that if $t \leq t_m$ then $f'(t) \geq 0$ and if $t \leq t_n$ $f''(t) > 0$ but for $t > t_n$ it holds that $f''(t) < 0$.

⁴ The *multiple-use version of the Faustmann model* is given by the following expression

The Faustmann model includes two assets, timber capital and monetary capital and the owner has to decide how to allocate his money between these two. Choosing t so as to maximize [1] leads to *the Faustmann rule* for the optimal rotation period [2a].

$$[2a] \quad pf'(t) = rpf(t) + rV$$

According to this rule, the cutting time is optimal when the value growth of the trees is equal to the opportunity cost of cutting consisting of the interest on the timber stock and the forest soil which shows the opportunity cost of growing a new tree generation.

For zero planting costs, the Faustmann rule reduces to

$$[2b] \quad \frac{f'(t)}{f(t)} = r \left[1 - \frac{1}{1 - e^{-r}} \right].$$

Besides establishing the correctness of the Faustmann model, Samuelson demonstrated that the weakness of the other economic solutions -- the Jevons-Wicksell model of one rotation period and Boulding's model of internal rate of return -- lies in the fact that they ignore the land rent component rV . In the Jevons-Wicksell model the forest owner maximizes his net present value of harvest income over one rotation period [3].⁵

$$[3] \quad \underset{(t)}{\text{Max}} \hat{V} = pf(t)e^{-rt} - c$$

Choosing t optimally produces the harvesting rule [4] according to which the forest should be cut when the relative growth is equal to the interest rate. Interestingly and contrary to the Faustmann rule, the price of timber and the planting costs have no effect whatsoever on the optimal solution.

$V = \left[pf(t)e^{-rt} - c + \int_0^t a(t)dn \right] (1 - e^{-rt})^{-1}$, where a denotes the valuation of nontimber services of the forest stand (with $a'(t) > 0$ and $a''(t) < 0$). Respectively, *the extended Faustmann model* is given by

$$V = [pf(t,s)e^{-rt} - cs](1 - e^{-rt})^{-1}, \text{ where } s \text{ denotes the silvicultural effort.}$$

⁵ The other economic and biological models and harvesting rules dealt with in Samuelson's article are not important for the purposes of the present paper. It suffices just to note that maximizing either the internal rate return, the net revenue over one rotation period, the forest rent (which Samuelson misleadingly calls net maximum sustained yield model, see Newman 1984, 6-7) or the maximum sustained yield give misleading rules for the optimal rotation period.

$$[4] \quad \frac{f'(t)}{f(t)} = r$$

Timber supply in the Faustmann model

Samuelson did not discuss what kind of a timber supply function can be derived from the Faustmann solution. This was done by others, in particular by Clark (1976) in the basic version of the model, and by Jackson (1980), Hyde (1980) and Chang (1983) in the model that also includes silvicultural activity. They demonstrated that the optimal rotation period depends positively on timber price and the real interest rate and negatively on the planting cost,

$$[5a] \quad t^* = t^*(p, r, c)$$

+ + -

Clark translated these findings into the terms of timber supply. An increase in the timber price increases the short-term timber cutting as some existing stands become overmature. The long-term effect on timber supply, however, is opposite to *this short-term adjustment*: the supply will fall as the price increases (Clark, 1976, 263). This yields a backward-bending supply curve of timber, which, according to Clark, can be regarded as a counterpart of the supply function of the Schäfer model in the economics of fisheries. Jackson (1980) and Hyde (1980) accepted this view but stressed that endogenizing silvicultural management in the model yields reasonable comparative statics of timber supply. Especially, a higher timber price boosts long term timber supply.

The problem of an appropriate long-term timber supply function was dealt with later by Williams and Nautiyal (1990). They criticized Clark for having omitted the fact that a higher cutting frequency may compensate for the lower volume. Thus, Clark did not after all exclusively determine the linkage between the rotation period length and timber supply. Williams and Nautiyal hold the annual long-term supply function to be the appropriate timber supply concept. By applying Hotelling's lemma, this function can be derived from the annual forestry rent (profit) defined by the land expectation value [1]. Williams and Nautiyal demonstrate that the long-term timber supply, defined as the annual long term timber supply X^s is positively sloped in terms of price, establishing that [5b] holds for both the basic and the extended version of the Faustmann model.

$$[5b] \quad X^s = X^s(p, r, c)$$

+ + -

In the Faustmann model the increase of the timber price is assumed to be permanent. This opens a question of the short-term timber supply function which would show what happens to the

quantity of supplied timber *when, ceteris paribus, the timber price increases in a particular period* (Johansson and Löfgren 1985, 111). Hyde also regards this issue important, arguing that short-term fluctuation may be even more important than long-term supply. What is more, short-term harvesting may affect, and even deny, the feasibility of the long-term results (Hyde, 1980, 80). An answer to these questions has not yet been given but it is closely tied to the problem of showing how the Faustmann model can be generated from the behavior of private forest owners.

These considerations are closely tied to the question of who actually owns the forests, the government or the private sector. The stress on long-term supply is natural if the government is the owner, and the model basically tends to the management of public forests. If the forests, however, belong to private owners, the situation is different. Due to their finite life span (which is sometimes even shorter than that of the trees), it becomes extremely important to examine the timber price variability. Unfortunately, there are no analyses that state how the short-term market behavior of the private sector may lead to the Faustmann rule.

2.2. Relaxing the heroic assumptions in the Faustmann model

Samuelson maintained that assumptions A1-A4 are very restricting and must be completed by second approximations that deal with uncertainties and market imperfections. His intention was to improve the model's suitability for a positive analysis of forest owner's harvesting behavior. Samuelson was quite confident that the second approximations can be made within the Faustmann framework. Analyzing the subsequent development of forest economics reveals that he is right to a certain extent, but, contrary to his optimism, there still remain many areas that are not covered by this model.

The Faustmann model under price uncertainty

An interesting and important, but neglected, paper by Bhattacharyya and Snyder (1988) offers the first application of the Faustmann model under timber price uncertainty, relaxing thus assumption A1.⁶ Bhattacharyya and Snyder assume that future timber price is uncertain.⁷ The risk-averse forest owner knows its mean and distribution and maximizes his utility over the uncertain present value of the infinite chains of rotations. Under these assumptions with zero planting costs, the forest owner chooses the optimal cutting time t so as to maximize the expected land site value

⁶ The simplifying assumption A5 concerning the constancy of timber prices, costs and the interest rate can easily be relaxed in the Faustmann framework. The effect of increasing prices and costs on the cutting decision was first analyzed by McConnell, Dabekow and Hardie (1983), Hardie, Dabekow and McConnell (1984), and Bare and Waggener (1980). They argue that, under steadily increasing prices and costs, one cannot obtain a simple steady-state optimum, because the subsequent cycles are not of equal length. Increasing prices and costs imply changes in the allocation of land between forestry, agriculture, and other land uses.

⁷ There is an earlier analysis of the effects of uncertainty by Norström (1975) in the Jevons-Wicksell framework with risk-neutral agent.

$$[6] \quad \underset{(t)}{\text{Max}} E[u(V)] = E\left[u(\tilde{p}f(t)e^{-r}(1-e^{-r})^{-1})\right],$$

where the tilde above the timber price denotes its stochasticity.

Choosing t optimally so as to maximize the expected utility of the present value from harvesting yields the following first-order condition

$$[7] \quad \frac{dE[u(V)]}{dt} = E\left[u'(V)(\tilde{p}f'(t) - \frac{1}{\lambda}\tilde{p}f(t))\right] = 0,$$

$$\text{where } \lambda = \frac{(1-e^{-r})}{r}.$$

By making various assumptions concerning the owner's risk-bearing behavior, one can derive results from [7]. First, if the forest owner is risk-neutral, it means that the utility function in [7] would be linear in terms of \tilde{p} . He then simply substitutes the uncertain variables for their expected values in the soil expectation value formula [2] and ends up with the Faustmann rule [2b]. Second, if the owner is risk-averse, then for the Faustmann rule it holds that⁸

$$[8] \quad \frac{f'(t)}{f(t)} < r \left[1 - \frac{1}{1-e^{-r}} \right].$$

According to [8], the rotation age of the forest is longer under timber price uncertainty than it is under certainty. This implies that the long-term annual timber supply will be smaller than under certainty. Due to uncertainty, timber is now a risky asset, and more capital is shifted to money, which gives a certain return.

Bhattacharyya and Snyder's analysis offers a good starting point for applying the Faustmann model to various forms of uncertainty. There are currently no other applications of the model, for example, in the analysis of forest taxation under timber price uncertainty. There are also no analyses of the effects of the uncertain interest rate on long-term timber supply.

Finally, it is useful to note that Bhattacharyya and Snyder implicitly assume that the forest owner regards his consumption as a perfect substitute at every point of time. Consequently, price uncertainty does not affect the consumption-saving decision of the risk-averse forest owner. This feature implies that their model suits well the cases in which the forest owner is a risk-averse

⁸ For the proof, see Bhattacharyya and Snyder (1988), 308-309.

enterprise or the government. It does not, however, apply to the analysis of the harvesting behavior of nonindustrial private forest owners, who can be expected to care for their consumption in each and every period, which implies that uncertainty also affects their consumption-saving decision. This, in turn, means that the separability of the forest owner's preferences from the cutting decision no longer holds at any level. Under nonseparability, the owner's preferences affect the cutting decision not only via risk-aversion but also via distorting the consumption-saving decision. The simultaneous existence of these features cannot be reflected in the capital theoretical rotation analysis, and one needs a more general description of the forest owner's target function.

The Faustmann model under stochastic growth due to exogenous stand destruction

The forest owners always face the possibility that their stands may be destroyed by fire, strong wind, or other natural causes. These elements were first incorporated into the Faustmann model by Reed (1984). He analyzed the cases of total and partial destruction of the stand and an extension where the probability of fire was assumed to depend on the age of the forest stand. Clarke and Reed (1989 and 1990) extended the analysis by including also a timber price variability into the model and by allowing the natural catastrophe to take the form of age-dependent and size-dependent stochastic growth. These papers are of great importance in the intellectual developments within the Faustmann paradigm.

Reed (1984) demonstrates that under risk-neutrality, when fire follows a Poisson process and causes a total destruction of the stand, the optimal policy is to follow a Certainty-Equivalence strategy by adding a risk premium (given by the probability of fire) into the real interest rate in the Faustmann rule [2] to yield

$$[9] \quad pf'(t) = (r + \lambda)pf(t) + (r + \lambda)V,$$

where λ is the probability of fire.

Interestingly, under risk-neutrality and age-dependent stochastic growth, the Certainty-Equivalence approach is optimal (Clarke and Reed 1989). The reason for the relatively minor modification of the Faustmann rule is that in these cases the stochasticity enters linearly into the objective function of the owner. Under size-dependent stochastic growth, the stochasticity affects the objective function nonlinearly via the forest growth function and results in a more complicated harvesting rule.

Imperfect capital markets and the Faustmann model

If the capital market is not perfect this may cause "liquidity" for the forest owner, as Samuelson pointed out. Capital market imperfections may occur in many forms such as a loan ceiling that the owner would like to, but cannot be allowed to, exceed. Alternatively, the saving and

borrowing rates may differ which implies that the present value of the infinite series of rotations is not uniquely determined as the owner may act over time in the role of a borrower as well as of a lender. As generally shown by Hirschleifer (1970), the production decision is not separable from the consumption decision under these circumstances.

Murphy, Fortson and Bethune (1977) take nonseparability as their starting point when studying the effects of imperfect capital markets on the harvesting decision in terms of forest rotation. They develop a dynamic programming model for scheduling timber harvest and for determining the optimum size of the land base for an even-aged stand through maximizing the liquidation value of forest land. The model can be solved only numerically, which shows how hard it is to relax assumption A2 at the theoretical level within the Faustmann model.

Summing up

The previous discussion suggests that there are many opportunities for increasing the applicability of the Faustmann model to cope with the effects of uncertainty. There are three key questions, however, that still remain unanswered within its framework: developing a meaningful short-term supply function; offering a more detailed analysis of the effects of timber price uncertainty and introducing interest rate uncertainty; and, finally, developing an analysis of the effects of capital market imperfection on the length of the rotation period and long-term timber supply. At its present stage, the Faustmann model gives but little insight into the market behavior of nonindustrial private forest owners. It is, nevertheless, an excellent planning model for determining the socially optimal forestry and for managing public forests. It needs a complementary approach which analyzes the market behavior of private nonindustrial forest owners. The two-period model offers such an alternative.

3. THE TWO-PERIOD CUTTING MODEL

3.1 General features of the two-period model

Historically, the first published application of the Fisherian two-period model to forestry was by Johansson and Löfgren (1985). They formulated the model as a problem of choosing between the amount harvested now and in the future so as to maximize the utility from the net present value of harvest income. This formulation shares the same weakness as the analysis of Bhattacharyya and Snyder: it implicitly assumes that the forest owner regards consumption during the first and the second period as perfect substitutes. The two-period model was completed to cover both consumption-saving and harvesting decisions by Koskela (1989a).

The structure of the *basic version* of the two-period consumption-saving-cutting model can be given as follows. The preferences of the representative owner define an additive and separable

utility function over periods now and the future.⁹ Assuming that future timber price is uncertain, the owner maximizes expected utility [10]. The owner has originally a given amount of timber Q and no other sources of income. The growth function $g(Q-x)$ is assumed to be concave in $(Q-x)$, i.e., $g'(Q-x) > 0$ and $g''(Q-x) < 0$. The total growth increases with a decreasing rate in $(Q-x)$. The amount of future harvesting z is uniquely determined by the current harvesting and the forest growth rate according to equation [11]. At the beginning of the first period the owner receives timber sale revenue p_1x and allocates it between current consumption c_1 and saving s according to $c_1 = p_1x - s$. At the beginning of the second period he either gets stochastic harvest revenue \tilde{p}_2z and the return of saving, $R=(1+r)$, where r is the real interest rate. This is expressed as the forest owner's intertemporal budget constraint [12].

$$[10] \quad U = u(c_1) + \beta E[u(c_2)]$$

$$[11] \quad z = (Q-x) + g(Q-x)$$

$$[12] \quad \tilde{c}_2 = \tilde{p}_2z + R(p_1x - c_1)$$

As the intertemporal budget constraint [12] indicates, the owner considers his income stream from harvesting. Here too -- just as in the Faustmann model -- the owner has two assets, timber stock and money, which can bring return. Thus the basic ingredients of both models are the same when the consumption decision is set aside. The growth rate of the forest stand in [11] is, however, defined differently, namely, in terms of biomass instead of time. Because of this feature, Montgomery and Adams (1995) interpret the two-period model to describe the harvesting of an uneven-aged forest. Under selective harvesting, the density of the forest changes and yields a concave growth function in terms of biomass in a similar manner as in the standard fishery models, where the focus only includes biomass and not the age-class distribution. While interesting and accurate as such, one can alternatively think that the two-period model describes the harvesting of even-aged stands in a given forest plot. Assume that the original volume of timber Q is given by n identical trees which grow according to the growth function $f(t)$ defined in section 2. Thus the existing timber stock at the beginning of period 1 is $Q = nf(t)$. Suppose now that the owner decides to harvest m trees at the beginning of the first period, so that current harvesting is given by $x = mf(t)$ and the rest $(n-m)$ of the trees are left for future harvesting. When m trees have been taken away the growth rate of the remaining trees will increase, because of the concavity of the growth function.

The forest owner's problem is to choose the amount of current (and future) consumption and the amount of harvesting now (and in the future) so as to maximize the expected utility function [8].

⁹ Alternatively, one could apply a more general utility function, $u = u(c_1, c_2)$, from which the separable utility function can be generated by assuming that the cross-derivative $\frac{\partial^2 u(c_1, c_2)}{\partial c_1 \partial c_2} = 0$, which, of course, simplifies the results (see Sandmo, 1969).

One obtains the following optimum condition for harvesting

$$[13] \quad \beta E[u'(\tilde{c}_2)e] = 0,$$

where $e = Rp_1 - (1 + g')\tilde{p}_2$.

Assuming that future timber price is certain allows one to write equation [13] without the expectation operator yielding $\beta u'(c_2)e = 0$. As the marginal utility is always positive, the expression can be zero only if $e = 0$, yielding thus at the interior solution the following harvesting rule

$$[14a] \quad Rp_1 - (1 + g')p_2 = 0$$

and to facilitate the interpretation, equivalently

$$[14b] \quad p_1 = \frac{(1 + g')}{(1 + r)} p_2$$

According to [14b] the owner chooses to increase current harvesting until the marginal revenue p_1 is equal to the opportunity cost of harvesting, which is given by the present value of the last unit cut if it were left to grow, $\frac{(1 + g')}{(1 + r)} p_2$.¹⁰

Suppose further that timber price is constant over both periods and known for certain. Then one can immediately see that

$$[15] \quad g' = r$$

indicates that the relative growth must be equal to the interest rate. A similar result was obtained as equation [4] in the Jevons-Wicksell model, where the relative growth was defined by the ratio $\frac{f'(t)}{f(t)}$.

Equations [14] and [15] show that the forest owner ignores the land rent component in the

¹⁰ Each and every type of the two-period model has so far simply assumed that an interior solution exists. One may, however, also end up with corner solutions. If $Rp_1 - (1 + g')p_2 < 0$ then $x = 0$, i.e., at the prevailing stock, the value of marginal growth from every unit harvested is higher than is the marginal revenue that the owner could obtain. If $Rp_1 - (1 + g')p_2 > 0$, then $x = Q$, indicating that the owner cuts all his timber during the first period. However, if one assumes that the Koopmans-Inada conditions hold, it is never optimal to completely cut the forest during the first period.

harvesting decision. This is the basic difference of the two-period model relative to the Faustmann model.

Short-term timber supply function

Two-period models produce a conventional short-term supply function which tells how much the current timber supply will increase, *ceteris paribus*, if the exogenous variables increase. The properties of the short-term supply function are, in qualitative terms, similar to those of the long-term supply function of the Faustmann model (and to the short-term adjustment effects). The comparative statics effects of timber prices and interest rate on current timber supply are given by

$$[16] \quad x^s = x^s(p_1, p_2, r).$$

The comparative statics of future timber supply is defined as $z_\theta = -(1+g')x_\theta$, where the subindex θ refers to any exogenous variable. Due to the growth function [11] and the restriction of the analysis to two periods only, the second period's timber supply is uniquely defined by the current timber supply. This does not correspond to the long-term timber supply function of the Faustmann model. To overcome this weakness, Ovaskainen (1992) extended the number of periods to three so that there will be timber left after the second period. In a stationary world, the first period's harvesting adjusts the forest stock to its steady-state level. Then the second period's harvesting gives the stationary long-term timber supply, which is equal to forest growth at the optimal growing stock (Ovaskainen 1992, 19-28). Even though this shows some robustness in the harvesting rule of the two-period model, one still needs to expand the number of planning periods to see how the increased dynamics affects the results of the model. A question of utmost interest is under which circumstances one could derive the Faustmann rule from the two-period model.

3.2. Relaxing the heroic assumption in the two-period model

Harvesting under timber price uncertainty

Under timber price uncertainty, equation [13] is the product of two uncertain variables, \tilde{c}_2 and \tilde{p}_2 . Because of risk-aversion the harvesting decision no longer is separable from the owner's preferences. Moreover, the owner will harvest more in period one and less in period two relative to certainty as indicated by the following harvesting rule under uncertainty.

$$[17] \quad Rp_1 - (1+g')\bar{p}_2 < 0.^{11}$$

¹¹ The result is justified as follows. Utilizing the rule $E(ab) = E(a)E(b) + \text{cov}(a,b)$ allows one to write the optimal condition [13] as follows, $E[u'(c_2)]E[e] - \beta \text{cov}(u'(c_2), e) = 0$. Due to risk-aversion, the covariance term is negative, implying that $E[e] < 0$, because $E[u'(c_2)] > 0$ for [13] to hold.

The comparative static results under uncertainty depend on assumptions concerning risk-bearing behavior. One can assume alternatively that the Arrow-Pratt measure of absolute risk-aversion, $A(c_2) = -\frac{u''(c_2)}{u'(c_2)}$ defined in terms of future consumption may be either constant or decreasing ($A'(c_2) \leq 0$). Under constant absolute risk-aversion, equation [16] holds for the effect of current timber price, expected future timber price and the real interest rate. Under decreasing absolute risk-aversion the overall effects of price changes are ambiguous, but one can show that the income effect is positive and the substitution effect is negative. In addition, the effect of a change in the pure risk on timber supply is a priori ambiguous.

Harvesting under interest rate uncertainty

Both the Faustmann and the two-period models place great weight on the real interest rate as the determinant of optimal harvesting. Therefore, it is also interesting to ask how the stochasticity of the real interest rate affects harvesting behavior. Ollikainen (1990) and (1993) focus both on interest rate uncertainty and on double uncertainty caused simultaneously by interest rate and future timber price. Under sole interest rate uncertainty, the forest owner's position in the capital markets and his risk-bearing behavior turn out to be important for the cutting decision as can be seen from the following harvesting rule

$$[18] \quad \bar{R}p_1 - (1 + g')p_2 \geq (-)0, \text{ as } s \geq (-)0,^{12}$$

where positive s denotes savers and negative s borrowers.

According to harvesting rule [18], risk-averse savers will decrease and borrowers increase their cutting relative to certainty (given by [14a-b]). Savers decrease their harvesting, because, due to risk-aversion, they prefer to hold more of their capital in the form of timber which gives them a certain return on capital instead of the uncertain return of savings. Risk-averse borrowers increase their harvesting in order to avoid the risks associated with borrowing at an uncertain interest rate. The effects of double uncertainty depend on whether or not the interest rate and timber price are independent random variables. In both cases, however, both the owners' position in the capital market and their risk-bearing behavior play an important role in the determination of harvesting.

Harvesting under imperfect capital market

Capital market imperfection has been analyzed extensively by Koskela (1989b), Kuuluvainen (1990) and by Ollikainen (1996). The results of these studies depend heavily on the assumptions

¹² To justify the harvesting rule [18], note that the optimal condition [13] under uncertain interest rate can be expressed as $E[u'(c_2)]E[e] - \beta \text{cov}(u'(c_2), e) = 0$ where the sign of the covariance term is $\text{sgn}(u'(\bar{c}_2), \tilde{r}) = \text{sgn}(u''(\bar{c}_2)s)$, i.e. positive or negative depending on the sign of s .

concerning the imperfectness of capital markets. Assuming an exogenously given credit rationing in the form of a loan ceiling, both Koskela and Kuuluvainen separately showed that the owners cannot necessarily harvest their forest stands at the optimal point of time, because they have to finance their consumption. In fact, they will harvest more than under perfect capital markets, if the interest rate is assumed to be the same in both cases.¹³

While credit rationing in Koskela's model was just an exogenous assumption, Ollikainen (1996) endogenized it in a model with identical forest owners as borrowers, risk-neutral banks as lenders and an uncertain future timber price, which causes a default risk for the banks and is the source of credit rationing. The analysis shows that the cutting decision is separable from the forest owner's preferences under endogenous credit rationing even though the owner cuts more than under perfect capital market conditions. This finding undermines the nonseparability result of exogenous credit rationing models and shows a situation where the application of the Faustmann model may be accurate.¹⁴ One must note, however, that under asymmetric information and various types of borrowers the result may not be robust.

Summing up

The two-period cutting model reduces the infinite time horizon to two periods only. This has the advantage of producing a meaningful concept for short-term timber supply, which is well suited to the analysis of the harvesting behavior of private nonindustrial forest owners. This property is, however, also the weakness of the model. Because the model ignores growing future forest stands, it becomes inapplicable to the long-term analysis. The problem can be alleviated by adding more periods, which also increases the scope of the model to include such new issues as the effect of the owner's planning horizon on timber supply. Another necessary line of developing the long-term analysis is to assume altruism in the form of timber bequests, which basically leads to an examination under an infinite time horizon and makes it possible to contrast the model with the Faustmann framework. So far this option remains virtually unexplored. Therefore, at the present stage, the two models can be regarded as complements of each other.

4. FOREST TAXATION AND TIMBER SUPPLY

Following well-established lines (see Klemperer, 1976, 135; Montgomery and Adams, 1995) one can distinguish three broad categories of forest taxes. The first class consists of capital income taxes, based on the yield potentiality of a given site and thus called *site productivity*

¹³ It is more plausible to think that the interest rate in the imperfect capital market differs from that in the perfect capital market. Unless the interest rate is rationed by the government, one can expect it to be higher in the imperfect capital market, which thus tends to even further increase current harvesting.

¹⁴ Thus, given the results of exogenous and endogenous credit rationing assumption, A5 may not be as restrictive as Samuelson thought.

taxes. They are annual taxes grounded on the estimated average yield of the forest by site quality irrespective of the actual harvests or existing timber stock. The second tax type is *yield taxes*, levied on the value of timber harvested at the time of harvest. The third type are *ad valorem property taxes* which are levied on the market value of the forest property including the value of the standing timber and/or land. All of these types have been studied extensively.

The literature on forest taxation is huge, even though the basic results date back to some central articles. The studies deal with many specific topics, which can broadly be classified to cover three issues. First, many analyses study *the ceteris paribus effects* of various forest tax forms on the rotation period or on timber supply. The most notable references are Johansson and Löfgren (1985) for the basic Faustmann model, and Chang (1982 and 1983) for the version including silvicultural effort; Koskela (1989a and 1989b), Ollikainen (1990, 1991 and 1993), and Ovaskainen (1992) for the two-period model. Second, there are some recent attempts to discuss *the incidence of forest taxation* between forest owners and forest industry by endogenizing the determination of equilibrium timber prices and quantities at the market level. The first attempt was by Stier and Chang (1983) in the Faustmann framework. It was followed by studies by Johansson and Löfgren (1985), Aronsson (1990), and Koskela and Ollikainen (1995b). Third, a number of studies have aimed at defining *the optimal forest taxes* in terms of inter- and intrasectoral efficiency. One can subsume under this heading many traditionally important forest tax debates, for example, Fairchild's argument concerning *the deferred bias* (1909; see Klemperer 1977, Stier and Chang 1983, and Boyd 1986) for the subsequent discussion); Klemperer's concept of *the site burden* of forest taxes (Klemperer 1976, 1977, 1982); and the discussion of the *Austrian sector* problem (Kovenock and Rothschild 1983, Kovenock 1986, Ovaskainen 1992).

4.1 The ceteris paribus effects of forest taxes

The two-period models have focused on two forms of forest taxes, namely the yield tax on harvesting and the (unmodified) site productivity tax, which is a lump-sum tax. The two-period model is not amenable to the analysis of ad valorem property taxes when they are defined in the typical manner, because the value of land cannot be determined within its framework.¹⁵ This clearly is one of its major weaknesses. Therefore the following discussion will be restricted only to yield and site productivity taxes.

¹⁵ One alternative is, of course, to add one period to the analysis and levy an ad valorem property tax on the value of standing timber, as is done in Ovaskainen (1992). He finds that this tax increases short-term timber supply, which is in line with the findings of Kovenock (1986), and Gamponia and Mendelsohn (1987). Notice also that one could incorporate a productivity tax on the forest growth into the two-period model, but so far nobody has analyzed how the productivity tax (capital gains tax) affects short-term timber supply under certainty and uncertainty.

It is useful to first clarify the terminology employed in the interpretation of the results. Following the terminology of taxation literature, I will refer by the term *neutral* to the case in which the tax has no effect on timber supply whatsoever; in technical terms, both substitution and wealth effects are absent. A tax is *non-neutral* if it causes a wealth effect but no substitution effect. The term *distortionary* refers to the case in which the tax distorts the prevailing market prices, causing a substitution effect.

The effects of site productivity and yield taxes under certainty

It is useful to start by supposing that assumptions A1-A4 hold which imply perfect foresight and perfect capital market. Then the effects of the site productivity tax and the yield tax on the long-term timber supply X^s of the Faustmann model and the short-term timber supply of the two-period model x^s are given by

$$\text{[18a]} \quad X_T^s = x_T^s = 0$$

$$\text{[18b]} \quad X_\tau^s \leq 0 \text{ as } c \geq 0$$

$$\text{[18c]} \quad x_\tau^s = 0$$

The site productivity tax is neutral in terms of both short-term and long-term supply. That the yield tax distorts timber prices and causes a positive substitution effect, which increases the rotation period and decreases the long-term timber supply is a well-known result. Mason Gaffney (1975) was the first to realize the dependence of this effect on the planting costs: the higher the costs, the more the long-term supply decreases. If the planting costs are zero the yield tax is neutral. In the two-period model the yield tax is always neutral, as the planting of new tree generations is ignored. Thus the difference between the models in the qualitative effects of the yield tax is caused by planting costs. If harvesting costs are included into the two-period model, the yield tax becomes distortionary (Koskela and Ollikainen 1995b).

The effects of taxes under uncertainty

The two-period models have produced results of the effects of forest taxes under uncertainty and imperfect capital markets. Even though the effects under price uncertainty on timber supply could be examined in the Bhattacharyya and Snyder application of the Faustmann model, they have so far remained unanalyzed. Therefore, the following discussion is based on the results of the two-period models. Under non-increasing absolute risk-aversion, $A' \leq 0$, the effects of taxes can be expressed as

$$\text{[19a]} \quad x_T^s \geq 0 \text{ as } A' \leq 0$$

$$\text{[19b]} \quad x_\tau^s = yx_T + x_\eta,$$

where $y = (Rp_1x + \bar{p}_2z) > 0$ and x_η describes the 'adjusted risk-effect' (see Koskela 1989a).

Under timber price uncertainty the site productivity tax is either neutral or non-neutral depending on the risk-bearing behavior of the owner. A higher site productivity tax makes the forest owner worse off and under decreasing absolute risk-aversion, he is less willing to bear the risk of future timber price, which makes him increase current harvesting. Under constant absolute risk-aversion, a change in the level of future consumption does not affect risk-bearing, and the owner does not adjust the timber supply.

The yield tax is always distortionary. Even though a higher yield tax does not distort the marginal revenue and the opportunity cost of harvesting at the interior solution, it decreases the post-tax risk of harvest revenues. Under decreasing absolute risk-aversion, higher yield tax boosts short-term timber supply via a positive wealth effect, while the effect of risk is a priori ambiguous. This ability of the yield tax to decrease the future timber price risk will have important consequences for determining the optimal design of forest taxes under price uncertainty. Under constant absolute risk-aversion, the yield tax is distortionary and decreases current timber supply.

The two-period model can be also used to define the effects of site productivity and yield taxes on short-term timber supply under interest rate uncertainty. Ollikainen (1990) and (1993) have focused both on sole interest rate uncertainty and on double uncertainty caused by the interest rate and timber price. The effects of forest taxes under sole interest rate uncertainty are

$$[20a] \quad x_T^s = 0 \text{ as } A' = 0, \text{ otherwise ambiguous}$$

$$[20b] \quad x_T^s = \gamma x_T + x_T^c$$

Under decreasing absolute risk-aversion, the site productivity tax is non-neutral and a priori ambiguous. The interest rate variability affects two things: the return on the saving (the cost of borrowing) and thus the consumption-saving decision, and the marginal revenue of harvesting. They tend in opposite directions, causing an indeterminate outcome. The yield tax is distortionary and ambiguous with a negative substitution effect. Under constant absolute risk-aversion the site productivity tax is neutral, while the yield tax is distortionary decreasing timber supply. Finally, under double uncertainty, constant absolute risk-aversion and zero covariance between the interest rate and timber price, the effects of the site productivity tax on short-term timber supply will remain qualitatively the same as above, while the yield tax is distortionary.

The effects of taxes under credit rationing

Relaxing the assumption of perfect capital markets towards exogenous credit rationing, while maintaining the assumption of timber price uncertainty, Koskela demonstrated that the site productivity tax causes a liquidity effect, being thus non-neutral, but ambiguous. Because of the loan ceiling, the owner can adjust consumption under the higher tax only by changing timber supply. Current harvesting will increase or decrease depending on whether the higher site

productivity tax affects more the current or the future marginal utility of consumption, which is the same as saying on which period he is more severely constrained. The yield tax is distortionary just as under timber price uncertainty.¹⁶

Under endogenous credit rationing and uncertain timber price, the site productivity tax is, however, always neutral as demonstrated by Ollikainen (1996). The yield tax may be either neutral or distortionary depending on who has the right to first collect the money from the defaulted owner. If the banks have the right, the yield tax is neutral; but if the government has it, the yield tax decreases timber supply. The reason for neutrality is that under equilibrium credit rationing risk-neutral banks function as a risk-shifting device, eliminating thus the distortions of taxes and promoting social optimum (see Ollikainen 1996, 7-9).

4.2 The incidence of forest taxes

The applicability of the *ceteris paribus* effects of forest taxes is, however, of limited scope. They implicitly assume that forest owners bear the whole tax, which need not be the case. In fact, the theory of tax incidence implies that forest owners bear taxes fully only if timber supply is inelastic to timber price, or if the demand for timber is infinitely elastic. Both cases are actually highly unrealistic. Therefore the effects of taxes have to be analyzed in the market level to determine their incidence.

Stier and Chang (1983) were the first to explicitly consider forest tax incidence. They employ the Faustmann framework to analyze the land use implications of the *ad valorem* tax. They did not, however, explicitly address the determination of market equilibrium, but made some exogenous assumptions about the incidence and thus about the equilibrium stumpage price. Johansson and Löfgren (1985) suggest a static framework for the analysis of market equilibrium, but they do not analyze tax incidence. The first explicit analysis of tax incidence comes from Aronsson (1990). He develops a market model in which timber supply is derived within the two-period model and which determines the tax incidence in order to test it empirically by using Swedish data.

Koskela and Ollikainen (1995b) offer a systematic and purely theoretical attempt to analyze both the incidence of forest taxes and the optimal forest taxation under uncertain endogenous timber prices and rational expectations.¹⁷ After some simplifying assumptions (the forest owner is risk-

¹⁶ Notice that under exogenous credit rationing nontimber taxes also affect the harvesting decision as is established in Ollikainen (1991).

¹⁷ This paper distinguishes -- for the first time in the forest economic literature -- between idiosyncratic and aggregative risk. Under idiosyncratic or individual risk, the risk is independent across forest owners implying that some do well and others badly. Idiosyncratic risk may be caused, for instance, by the volatility of regional timber prices. Aggregate risk is common to all forest owners, implying that all

averse), they establish the following result. *In terms of tax incidence, the site productivity tax is like a pure profit tax and fully borne by forest owners, while the burden of the yield tax is generally shared by both sides of the market.*

The role of the site productivity tax can be explained as follows. The tax is neutral as $x_T^s = 0$. Thus timber supply does not react to it and the owners bear it fully. On the other hand, yield tax is distortionary, $x_T^c < 0$ and thus affects timber supply. It turns out that forest owners bear a higher share of the yield tax when the price elasticity of the demand for timber is high, or when the price elasticity of timber supply is low. Interestingly, the price elasticity of timber supply depends on the risk-aversion of the forest owners. Because of this feature, the short-term timber supply is a decreasing function of the yield tax and, therefore, this tax can be used to affect the market equilibrium and the volatility of the equilibrium timber price.

4.3. Optimal forest taxation

The discussion of *optimal* forest taxes has traditionally been carried out only in terms of neutrality. The reason for this is obvious: the less the government intervenes with timber supply, the better. The worry concerning the distortionary effects of forest taxes in relation to other sectors has been expressed in terms of the *deferred yield bias* and the *site burden of taxes*. According to the deferred yield bias, any given property tax implies a higher burden on forestry with a long-term production period than on properties that provide an annual cash flow of income, making them unattractive to use (Fairchild 1909). Klemperer's term, "site burden of taxes," refers to the relative tax-induced reduction in the value of land (e.g., the value of the forest land relative to that of agricultural land) (Klemperer 1974, 1976, 1982). Klemperer stresses the requirement of a neutral tax system which guarantees that the highest and the best use of land before taxation will remain so also after taxes (1982, 294).

The same inter- and intrasectoral considerations have been dealt with at a more general level as the *Austrian sector problem*. This term refers to an economy containing a capital asset (forest, wine) which increases in value as it ages. By formulating a model of an economy with an Austrian and an ordinary sector, Kovenock and Rothschild (1983) reformulated and developed further the problem of optimal forest taxation, which had already been tackled in the articles by Fairchild and Klemperer, as well as in those by Bentick (1980) and Chisholm (1975). They assumed that society has to use a capital gains taxation and studied its effects on two types of Austrian assets in terms of the intra- and intersectoral efficiency of investments. A capital gains tax drives resources from the Austrian sector, leading to a situation in which the social return on the Austrian asset is less than is the rate of return in the ordinary sector. Kovenock (1986) continued the analysis by examining the effect of land value and income taxation in an Austrian

either gain or lose. Aggregate risk may be caused by the volatility of general timber price level. The results reported in this survey deal only with the idiosyncratic risk.

Economy. Kovenock demonstrated that both inter- and intrasectoral efficiency is obtained if a properly chosen property tax is levied on the Austrian sector. By assuming a pre-existing distortion caused by capital income tax, Ovaskainen (1992) pushed the analysis of the Austrian sector further and showed that an ad valorem property tax on the standing timber can be used as a means of restoring the neutrality of taxation in forestry.

An implicit assumption in most of these analyses is that society is in a first-best situation in the sense that the government is free to choose any tax form without a budget constraint (tax revenue requirement). Then the task of the government is simply to choose taxes so that the intra- and intersectoral return on investments will be the same. These results are interesting and relevant as such. They do, however, neglect the fact that the government is not usually free to choose taxes without taking into account the need to finance public spending. Then the design of optimal taxation has to be solved as a second-best solution, which does not necessarily lead to a neutral tax system. The optimal choice of the land site and yield taxes are obtained by maximizing the social welfare function subject to the government tax revenue requirement, as given in its simplest form in equation [21]. The social welfare function in this partial model consists of the forest owner's indirect utility function. The expected tax revenue is a sum of site productivity and yield taxes, and the risk-neutral government uses the expected value of uncertain future timber price.

$$\begin{aligned}
 & \underset{\{T, \tau\}}{\text{Max}} \quad SW = EU^*(T, \tau, \dots) \\
 \text{[21]} \quad & \text{s.t.} \quad \bar{G} = (1 + R^{-1})T + (p_1x + R^{-1}\bar{p}_2z)\tau
 \end{aligned}$$

The optimal design of taxes resulting from [21] can be characterized as follows (see Amacher and Brazee (1995a) and, Koskela and Ollikainen (1995b)). *It is optimal to use both the site productivity tax and the yield tax at the same time. The former is used to raise the tax revenue, while the latter promotes social insurance by decreasing the post-tax variability of the uncertain future timber price.* Under perfect capital market, a full insurance with 100% yield tax rate is optimal (and to distribute the tax revenue as lump-sum subsidies to the forest owners). However, as demonstrated in Ollikainen (1996), under endogenous credit rationing, the optimal yield tax rate is less than 100%, indicating that it is not optimal to eliminate uncertainty altogether due to the risk-shifting device provided by the banks.

If there are externalities involved within forestry, they have to be taken into account in the social welfare function. The most common source of externalities are the public goods services that forests provide. When the forest owner harvests his forests, he does not take into account the

utility that non-owners derive from forests, causing them thus an externality.¹⁸ The social welfare function now reads

$$[22] \quad SW = EU^*(T, \tau, \dots) + (n-1)(v(k_1) + \beta v(k_2)),$$

where k_i , with $i = 1, 2$ is the volume of standing timber in the beginning of periods one and two, and n is the number of agents in the economy and $v(k_i)$ denotes the utility from the public goods services.

Under these circumstances the problem of socially optimal taxation becomes more intriguing, as the number of targets of forest taxes increases. The choice of the yield tax rate is affected by the considerations of social insurance and tax revenue requirement just as in the previous case. In addition, the yield tax can be used as a Pigouvian tax to internalize the externality. Drawing on the analysis of Koskela and Ollikainen (1995a), and on that of Amacher and Brazee (1995b), the optimal tax design under externalities can be characterized as follows. *The government uses the site productivity tax to collect the required tax revenue. In addition, it introduces the yield tax, $0 < \tau^* < 1$. The optimal yield tax rate depends on three things: i) the social value of forest stands, ii) timber price risk, and iii) the properties of compensated timber supply.*

This result qualifies in an interesting fashion the previous one. Under certainty and in the presence of externalities, it is optimal to introduce the yield tax as a pure Pigouvian tax with the tax rate less than 100%, while in the absence of externalities it is not optimal to use the yield tax at all and to collect all the taxes by the site productivity tax.

5. DISCUSSION

This survey has shown that both the Faustmann rotation model and the two-period model are important and useful frameworks in forest economics. They both analyze optimal harvesting when the forest owner has two assets, money and trees. The main difference between the models concerns the time horizon and thus the concept of timber supply. The Faustmann model produces a long-term timber function, while the two-period model offers that of the short-term.

Despite this difference, the qualitative effects of timber price(s) and the interest rate on short-term and long-term timber supply turned out to be similar. Under certainty and perfect markets, also the ceteris paribus effects of site productivity are similar. This also holds true for the yield tax, if the planting costs are assumed to be zero. The two-period model has the weakness of not

¹⁸ Notice that the owner may have recreational motives. There is, however, no guarantee that his preferences are identical to those of the whole society as Englin and Klan (1990) have pointed out.

being amenable to the analysis of conventional ad valorem property taxes, because the land value is not properly defined. On the other hand, this model is more amenable to the analysis of tax incidence and the design of optimal taxes under government tax revenue requirement.

There are more applications of the two-period model to the conditions of uncertainty and market imperfections. The Faustmann rotation model works extremely well as the framework for determining socially optimal forestry and managing public forests. The two-period model is very convenient in analyzing the market behavior of private nonindustrial forest owners. The Faustmann model is widely applied in the United States and Canada, where the public ownership of forests is prevalent. The two-period model was developed in Sweden and Finland, where the market behavior of nonindustrial forest owners crucially affects the roundwood market.

This survey reveals that there still remains a great deal to be done within both model types. As for the Faustmann model, one can push further the analysis of various determinants of timber supply under uncertainty. The models by Bhattacharyya and Snyder, and by Clarke and Reed offer promising starting points for this enterprise. The same holds also true for the two-period model. One important direction for research would be to generate a more general t -period model, which both narrows the gap between the two-period model and the Faustmann model and allows the explicit study of the effects that the planning horizon has on the harvesting behavior of nonindustrial private forest owners (see Kuuluvainen 1989). Another promising direction is to use two-period models in the overlapping generations framework started by Löfgren (1991; see also Hultkrantz, 1993), to allow for an endogenous determination of prices and equilibrium in the forestry sector. In his paper Löfgren, for example, demonstrates for the first time in forest economics that timber supply depends on the time structure of timber prices and the interest rate. Such issues as the structure of the roundwood market and the implications of international competition on harvesting, and the multiple use of forests still remain virtually unexplored.

Finally, the need to clarify the relationship between the Faustmann and the two-period frameworks has become more than evident. The theoretical challenge is basically to show how the market behavior of the private sector generates the Faustmann rule. One promising possibility is to combine market behavior and timber bequests to produce an overlapping generation framework with an infinite time horizon.

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II SUMMARIES OF THE ARTICLES

THE EFFECTS OF NONTIMBER TAXES ON THE HARVEST TIMING - THE CASE OF PRIVATE NONINDUSTRIAL FOREST OWNERS: A NOTE

Under timber price uncertainty and risk-aversion, the harvesting decision is no longer separable from the forest owner's preferences, as shown by Koskela (1989a). This has important consequences on harvesting behavior. Koskela used a two-period model and demonstrated that under timber price uncertainty, forest owners increase their current harvesting and that the land site tax (as a lump-sum tax) is distortionary relative to the case of certainty. Given the nonseparability of preferences, one can also expect other taxes than forest taxes to affect the harvesting decision. This feature, however, has not been studied at length in forest economics.

This paper presents an attempt to enlarge the analysis of taxation and tax policy to include certain nontimber taxes in a two-period framework. The aim is to examine how nontimber taxes affect harvesting under perfect and imperfect capital markets when future timber price is uncertain. The model includes three tax forms. One example of the forest taxes is the land site tax, which is a lump-sum tax levied independently on harvesting and silvicultural activities. The second tax is the labor income tax which is levied on the exogenous labor income, and the third one is the capital income tax, levied on the return of savings and interpreted as an interest rate tax deductibility for borrowers. The paper analyzes the properties of the harvesting rule and develops the *ceteris paribus* effects of forest and nontimber taxes under perfect and imperfect capital markets. These are used to study the incentive effects of choosing the tax base in terms of timber supply when the government has an exogenous tax revenue requirement.

The harvesting rule in the presence of nontimber taxes is qualitatively similar to that derived in Koskela (1989a) with the proviso that the capital income tax is distortionary and decreases the marginal return of harvesting under both perfect and imperfect capital markets. As for the *ceteris paribus* effects of taxes, the labor income and the land site taxes cause only a wealth effect, while a change in the capital income tax can be decomposed into wealth and substitution effects. Under perfect capital markets, both the land site and the labor income tax boost current harvesting, while a higher capital income tax has a priori ambiguous (decreasing) effect on the current harvesting for lenders (borrowers). The latter result comes from the fact that the wealth effect is positive for lenders, but negative for borrowers. Under credit rationing, the wealth effects of the land site and the labor income taxes are ambiguous, while the capital income tax (as an interest payment tax deductibility) boosts current harvesting.

The analysis of the incentive effects of nontimber taxes under perfect capital markets reveals that shifting the tax base from the land site tax towards the labor income tax has no effect whatsoever

on current harvesting, while a shift towards capital income tax decreases current harvesting. Under credit rationing, the tax switch between the land site and the labor income tax has an ambiguous effect on harvesting. Shifting the tax base towards capital income taxation from land site or labor income taxation, however, increases current harvesting.

FOREST TAXATION AND THE TIMING OF PRIVATE NONINDUSTRIAL HARVESTS UNDER INTEREST RATE UNCERTAINTY

In the two-period model the decision whether or not to harvest under certainty is made by comparing the marginal revenue to the opportunity cost of harvesting according to condition $Rp_1 = (1 + f')p_2$, where $R = (1 + r)$ is the real interest rate, p_i , $i=1,2$ timber price in period i , and f' is the growth rate of the forest stand. Uncertainty may affect the harvesting decision either via economic variables, timber price and interest rate, or biological variables in the form of forest growth. The effect of stochasticity in the forest growth has been analyzed, for example, by Brock et al. (1979), and the timber price uncertainty has been introduced by Koskela (1989a), but there are no studies that deal with the effects of the stochastic interest rate on harvesting behavior. As one can, however, notice from the harvesting rule, interest rate uncertainty may have important implications for harvesting behavior.

This paper analyzes how interest rate uncertainty affects the harvesting behavior of risk-averse nonindustrial private forest owners under perfect and imperfect capital market conditions. Under perfect capital markets, the stochasticity of the real interest rate does not only emerge in the harvesting rule, but it also distorts the consumption-saving decision of the forest owners relative to the case of certainty. The same mechanisms are operative also under credit rationing, but one has to trace how they operate under credit rationing, which is modeled as a binding loan ceiling. The paper examines separately three issues under perfect and imperfect capital markets. The properties of the harvesting rule under interest rate uncertainty are examined and the ceteris paribus effects of forest taxes are developed. These are used to study the incentive effects of the choice of the forest tax base in terms of timber supply when the government has an exogenous tax revenue requirement.

The most important and new result of the paper concerns the properties of the harvesting rule under perfect capital markets. It is shown that interest rate uncertainty affects asymmetrically forest owners depending on whether they are lenders or borrowers. Risk-averse lenders decrease and borrowers increase their current harvesting relative to certainty. Lenders prefer to hold more of their capital in the form of timber with a certain growth rate instead of money, which gives uncertain return on saving. Borrowers wish to reduce the risk associated with borrowing at an uncertain interest rate and increase their current harvesting. This feature for borrowers remains, and is even strengthened, under credit rationing.

The *ceteris paribus* effects of forest taxes under perfect capital markets depend on the further assumptions of risk-bearing behavior. Under constant absolute risk-aversion, the current (future) land site tax increases (decreases) current harvesting. For the effects of the yield tax a Slutsky decomposition is developed. The substitution effects of the current (future) yield are negative (positive). Thus, given the opposite signs of the wealth effects, the overall effect is a priori ambiguous. Under credit rationing, the liquidity effects of the land site tax are opposite to those of perfect capital markets. As for the incentive effects of the forest tax base, one obtains that a shift from current (future) land site taxes towards current (future) yield taxation reduces current harvesting under both perfect and imperfect capital markets.

A MEAN-VARIANCE APPROACH TO SHORT-TERM TIMBER SELLING AND FOREST TAXATION UNDER MULTIPLE SOURCES OF UNCERTAINTY

The basic idea of the paper is to investigate how the double uncertainty caused by stochastic timber price and interest rate affects the harvesting behavior of nonindustrial private forest owners in the two-period consumption-saving-harvesting model. The reference-points for the study are Koskela's (1989a) analyses of timber price uncertainty and Ollikainen's (1990) examination of interest rate uncertainty. These studies reveal that timber price uncertainty and interest rate uncertainty affect timber supply differently. Under timber price uncertainty, the risk-averse forest owner increases current harvesting as a response to the risk, but under interest rate uncertainty, current harvesting may increase or decrease depending on the owner's position (lender, borrower) in the capital markets. It also turns out that comparative static effects of forest taxes on harvesting differ.

The focus of the study is, therefore, on the effects the double uncertainty has on current harvesting at the margin. The paper also develops the *ceteris paribus* effects of forest taxation. To sharpen the analytics, the model is formulated as a mean-variance analysis which reduces the risk to two quantities: mean as the expected value of uncertain variables; and variances and covariance as the pure risk. By utilizing these assumptions, the paper defines harvesting rules under double uncertainty and the *ceteris paribus* effects of forest taxes. These results are used to define the incentive effects of forest taxes in terms of current harvesting, when the government chooses a tax base which maximizes timber supply subject to an exogenously given tax revenue requirement. Finally, the paper discusses the possible sign of covariance by utilizing the Finnish data.

The basic findings in terms of the harvesting rule reveal that the asymmetry of lenders and borrowers emerges also under double uncertainty. If future timber price and interest rate are

independent random variables, there exists an asymmetry between lenders and borrowers in the same fashion as it exists in the single interest rate uncertainty case. Borrowers cut more and lenders cut less as a response to double uncertainty relative to single price uncertainty. If timber price and the interest rate are correlated, the sign of the covariance is crucial for the interpretation of the optimality conditions. If covariance is negative, borrowers still tend to increase their cutting, but the behavior of lenders depends on the relative magnitudes of risks associated with the interest rate and timber price, which is indicated by variances and covariance.

Also, the comparative static results depend on the sign of covariance. The current (future) land site tax boosts (decreases) current harvesting if the covariance is negative or zero, which is suggested by empirical evidence. For the unit tax one can develop Slutsky equation, with negative (positive) substitution effects of current (future) taxes on harvesting. While the current yield tax has the same qualitative effect as the current unit tax, the future yield tax has the additional effect of reducing the risk via variance and covariance terms. Finally, it turns out that the incentive effects of forest tax policy are similar to the single interest rate uncertainty and differ from the case of timber price uncertainty only for the switch between the land site tax and the yield tax. As for the incentive effects of forest taxes, the paper finds that Finnish data can be used to evaluate the size of various risks and the sign of covariance.

ANALYTICS OF TIMBER SUPPLY AND FOREST TAXES UNDER ENDOGENOUS CREDIT RATIONING - SEPARABILITY AFTER ALL

The famous Faustmann model, assumes, among other things, that capital markets are perfect. Under perfect capital market, the forest owner can borrow or lend money at a constant interest rate to adjust his consumption. This ensures that he harvests timber only at the optimal point of time given by the Faustmann Rule. This means that the owner's preferences are separable from the harvesting decision. The assumption of perfect capital market was relaxed by Koskela (1989b) in a two-period model. He assumed that forest owners face a loan ceiling which they would like to exceed but cannot. Under imperfect capital market nonseparability occurs and the liquidity effect affects current harvesting. The weakness of the analysis is that credit rationing is exogenous to the model.

This paper endogenizes credit rationing by introducing the behavior of the banking sector to the two-period cutting model, which has now the following features. The owner asks for a bank loan in the first period and pays it back during the second period. The future timber price, however, is uncertain so that the owner may default. By assumption, the owner has only a limited liability. While retaining the standard assumptions of risk-averse forest owners, the banking sector is assumed to be risk-neutral and competitive. Moreover, the probability of default is known by

both parties so that the information is imperfect but symmetric. The paper derives the harvesting rule as a contract between the owner and the bank. It also develops the *ceteris paribus* effects of forest taxes, analyzes the incentive effects of choosing the forest tax base, and the design of optimal forest taxation when the government faces an exogenous tax revenue requirement.

It turns out that the harvesting decision is separable from the owner's preference in this kind of model contrary to the findings of the exogenous credit rationing models, because risk-neutral banks shift the risk from risk-averse owners to themselves. According to the resulting harvesting rule, the optimal current harvesting equalizes the marginal revenue with the expected opportunity cost of harvesting. What is more, the order in which the debtors can receive their money in the case of default also affects the harvesting rule. If the banks have the right to take back their money before other debtors do, current harvesting remains the same as in the risk-neutral case. If the government has the right to collect its taxes first, this right affects the default risk of the banks and increases current harvesting.

The *ceteris paribus* effects of forest taxes differ from those of exogenous credit rationing. First the land site tax is neutral and the yield tax causes a positive (no) substitution effect depending on the order in which the debtors are allowed to collect their money in the case of default. The analysis of incentive effects reveals that a switch from yield taxation towards land site taxation decreases current supply. As for the optimal design of forest taxes, the paper shows that, given the optimal land site tax, it is desirable to introduce the yield tax as an insurance device. The optimal yield tax rate is greater than zero, but less than 100%.

1

Forest taxation and the timing of private nonindustrial forest harvests under interest rate uncertainty

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The effects of forest taxation (lump sum, unit, and yield taxes) on timber supply are analyzed in a two-period model with an uncertain real interest rate under perfect or credit-rationed capital market conditions. The results differ considerably from the implications in price uncertainty models. First, an uncertain interest rate introduces asymmetry between lenders and borrowers, unknown in those models. Lenders tend to decrease and borrowers to increase forest cutting as a response to interest rate uncertainty. Second, the qualitative effects of timber taxes on supply also differ between the models. The reason for this lies in the change of the source of uncertainty. Interest rate uncertainty implies that the wealth effects of lump sum taxes are not zero even under constant absolute risk aversion, as they are zero under price uncertainty. Third, these differences imply differences in the relative effectiveness of taxes.

OLLIKAINEN, M. 1990. Forest taxation and the timing of private nonindustrial forest harvests under interest rate uncertainty. *Can. J. For. Res.* **20** : 1823-1829.

Les effets de divers modes de taxation (montant forfaitaire, taxe par unité produite et taxe au rendement) sur l'offre de matière ligneuse des propriétaires forestiers non industriels sont étudiés à l'aide d'un modèle où le taux d'intérêt réel représente la variable associée à l'incertitude. Ce modèle comporte une analyse répartie sur deux périodes et s'inscrit dans un marché financier où le crédit est limité. Les résultats obtenus se démarquent de ceux particuliers aux modèles utilisant le prix pour caractériser l'incertitude. Premièrement, une asymétrie entre les prêteurs et les emprunteurs est remarquable quand le taux d'intérêt spécifie l'incertitude. Dans la position d'emprunteur, un propriétaire intensifie la récolte de matière ligneuse en réaction à l'incertitude. À l'inverse, un propriétaire-prêteur diminue sa pression de récolte devant des taux d'intérêt de plus en plus incertains. Deuxièmement, l'emploi du taux d'intérêt préférentiellement au prix induit des différences qualitatives quant à l'impact de la taxation sur l'offre de matière ligneuse. Des taux d'intérêt incertains impliquent des effets d'enrichissement associés aux taxes établies sur une base forfaitaire qui ne sont pas nuls. Cette observation se maintient même dans le cas où, à l'instar des modèles utilisant les prix pour marquer l'incertitude, l'aversion du risque est considérée constante. Troisièmement, en conséquence des distinctions précédentes, l'efficacité relative des différents modes de taxation varie.

[Traduit par la revue]

Introduction

Timber supply under price uncertainty and market imperfections has been treated in some recent studies, following the Fisherian two-period consumption-saving-cutting model (see e.g., Johansson and Löfgren 1985; Koskela 1989a, 1989b; Ollikainen 1990). This literature has shed some new light on the effects of forest taxation. The basic effects of forest taxation under certainty are derived in Chang (1982, 1983), Klemperer (1976), and Hyde (1980), among others.

In the two-period consumption-saving-cutting model, the representative forest owner compares the value growth of his forest with the interest rate. If F denotes the growth function of the forest stand, r the interest rate, and p_1 and p_2 current and future timber prices, the formula $(1 + F')p_2/p_1 = 1 + r$ serves as the cutting rule under a perfect capital market and certainty. Thus, the owner compares the marginal revenue of cutting (right-hand side) with the opportunity cost of cutting (left-hand side). If these are equal he decides to cut, but if the left-hand side is greater he delays cutting until a future period.

Uncertainty may now involve three variables in the cutting rule: future timber price, forest growth rate, and interest rate. The effects of an uncertain future timber price on cutting are now known from the studies just mentioned. Preliminary conclusions about the implications of an uncertain forest growth rate are also outlined in Brock et al. (1979). The effects of an uncertain interest rate, however, remain unanalyzed. What happens if the interest rate is uncertain?

What kind of cutting rule emerges and what are the consequences of uncertainty on the comparative static results? These questions are addressed in this paper.

Thus, the research problem can be stated as follows: how interest rate uncertainty affects timber supply and the effectiveness of forest taxes under perfect and (alternatively) imperfect capital markets; timber supply meaning the timing of forest stand harvesting. The issue is not new in many other fields of economics or in the type of model used in this paper. Sandmo (1969), for example, used the Fisherian two-period model to analyze how an uncertain interest rate affects saving in the context of portfolio choice.

The present paper focuses on just one particular group, namely private nonindustrial forest owners. The reason for this lies in the institutional features of Scandinavia, where these owners dominate timber supply. Their behaviour has also been studied in the Faustmann rotation framework (see the works of Binkley 1981 and Knapp 1981).

The research was carried out in two phases. First, first-order conditions for optimum were analyzed and the usual comparative static results were developed. Second, attention was given to the relative effectiveness of taxes. It is assumed that government wishes to encourage timber supply and is therefore seeking a tax base that guarantees ample supply by raising one tax and compensating this by lowering another within government budget constraints. The forest taxes under study are yield tax, unit tax, and lump

sum tax. Yield tax is defined as a tax on timber revenue. Unit tax is a production tax levied on the volume of timber to be cut. Lump sum tax is a fixed amount, which is independent of the amount of cutting. For the sake of comparability between this model and price uncertainty models, it is assumed that tax rates differ between periods. Temporary changes in tax rates can thus be analyzed.

Interest rate uncertainty and timber supply

The basic framework and optimum conditions

The representative forest owner is assumed to have a 3 times differentiable, intertemporally additive utility function of consumption, C_i , $i = 1, 2$. The rate of the time preference factor is denoted by $\beta = (1 + \delta)^{-1}$.

$$[1a] \quad V = U(C_1) + \beta U(C_2), \quad U' > 0, U'' < 0$$

The assumption of forest-cutting technology is stated in eq. 2, where Q denotes the original volume of timber, x denotes cutting during the first period, F denotes the (concave) growth function of the forest stand ($F'(Q) > 0$, $F''(Q) < 0$), and z denotes cutting during the second period. Thus, cutting during the second period utilizes all the timber left over from the first period ($Q - x$) and the growth of the remaining stand $F(Q - x)$.

$$[2] \quad z = (Q - x) + F(Q - x)$$

During the first period, the forest owner receives exogenous labour income, I_1 , and timber sales income, and pays yield tax, τ_1 , unit tax, t_1 , and lump sum tax T_1 . The pre-tax timber price is denoted by p_i , $i = 1, 2$, and the after-tax timber price is defined as $p_i = p_i(1 - \tau_i) - t_i$. The owner can allocate his income either to consumption or saving, S . The owner's flow of funds equation during the first period is thus

$$[3] \quad C_1 = p_1^*x + I_1 - T_1 - S$$

Consumption during the second period consists of after-tax timber sales income, exogenous labour income, and capital income from savings (interest payments when savings are negative). The interest rate is assumed to be uncertain and is denoted by \hat{r} . As the interest rate is uncertain, consumption during the period will be uncertain as well. If $\hat{R} = 1 + \hat{r}$, future consumption can be written as

$$[4] \quad \hat{C}_2 = p_2^*z + I_2 + \hat{R}S - T_2$$

The intertemporal budget constraint of the representative forest owner can now be defined by solving S from eq. 3 and using it [4], as in eq. 5.

$$[5] \quad \hat{C}_2 = p_2^*z + I_2 - T_2 + \hat{R}(p_1^*x + I_1 - T_1 - C_1)$$

The budget constraint [5] is stochastic. This raises the question of how the owner is assumed to behave under uncertainty. Because the focus of analysis lies in private non-industrial forest owners, the expected utility hypothesis will be used. Adding the expectation factor, E , to [1a], the objective function can now be written as

$$[1b] \quad V = U(C_1) + \beta E[U(C_2)]$$

By substituting \hat{C}_2 in [1b] for intertemporal budget constraint, the maximization problem of the owner can be stated as eq. 6. His economic problem is to choose current consumption and cutting so as to maximize the expected utility from consumption:

$$[6] \quad \max. V = U(C_1) + \beta E[U(p_2^*z + I_2 - T_2 + \hat{R}(p_1^*x + I_1 - T_1 - C_1))]$$

The first-order conditions for the maximization of V can be expressed as the following equations (denoting the partial derivatives V/C_1 and V/x by V_c and V_x , respectively)

$$[7] \quad V_c = U'(C_1) - \beta E[U' \hat{R}] = 0$$

$$[8] \quad V_x = \beta E[U' e] = 0$$

where

$$e = \hat{R}p_1^* - (1 + F')p_2^*$$

The cutting rule [8] can be developed further. $V_x = 0$ is equivalent to $E[U']E[e] + \text{cov}(U', e) = 0$, where cov stands for covariance. This means that $E[e]$ is not zero, as it is under certainty. Before its sign can be determined, the sign of the covariance term must be established. The sign of $\text{cov}(U', e) = \text{sign}(U', \hat{r}) = \text{sign}(U''S)$. Following the common line of the expected utility approach, it is assumed that the owner is risk averse, which implies that $U''(C_2)$ is negative. Savings may be positive or negative depending on the owner's position in the capital market. Thus, the covariance is positive if savings are negative and vice versa. This implies that $E[e]$ must be positive for lenders and negative for borrowers on the grounds of first-order conditions. Using the definition of e for lenders (borrowers), $\hat{R}p_1^* > (<) (1 + F')p_2^*$, which is familiar from analysis of the firm under uncertainty. It can be concluded that interest rate uncertainty affects lenders and borrowers asymmetrically, and more precisely, the following.

Proposition 1—Compared with pure certainty, an uncertain interest rate induces lenders to decrease and borrowers to increase their cutting. Risk-averse lenders prefer to hold more of their capital in the form of timber, which guarantees a certain return on their capital instead of an uncertain return on savings. The reaction of risk-averse borrowers is the opposite. They increase their cutting to avoid risks that are connected with the uncertain interest costs of borrowing.

The proposition differs in outcome from future price uncertainty models, where the representative forest owner increases his current cutting irrespective of his position in the capital markets. Both models imply, however, that the preferences of the owner affect the cutting decision.

The second-order conditions [9] and [10] for the maximization problem are fulfilled because of the assumptions of risk aversion and concavity of forest growth. The determinant, D , of the Hessian matrix is defined by $D = V_{cc}V_{xx} - (V_{cx})^2$. It is positive, as required for a maximum.

$$[9] \quad V_{cc} = U''(C_1) + \beta E[U'' \hat{R}^2] < 0$$

$$[10] \quad V_{xx} = \beta E[U''(C_2)e^2] + \beta p_2^* F'' E[U'(C_2)] < 0$$

The cross derivative, $V_{cx} = -\beta E[U'' e \hat{R}]$, has an important role in the comparative static results. Its sign is not self-evident, and more assumptions are needed. The Arrow-Pratt absolute and relative risk-aversion measures give useful information about risk-bearing behaviour and will be used in the determination of sign. Absolute risk aversion is defined by $A(C_2) = -U''(C_2)/U'(C_2)$, and relative risk aversion by $R(C_2) = -U''(C_2)C_2/U'(C_2)$ (see Arrow 1974). The former is typically assumed to be decreasing in terms of consumption. There is no general agreement about

he behaviour of the latter, but constant relative risk aversion is often used as a benchmark case. It is shown in Appendix 1 that V_{cx} is always positive under decreasing or constant absolute risk aversion.

The ceteris paribus effects of interest rate and forest taxes

Given the second-order conditions, the first-order conditions implicitly define C_1 and x as functions of exogenous variables, $C_1^* = C_1^*(\cdot)$ and $x^* = x^*(\cdot)$, and they can be used to study how the optimum will change with changing exogenous parameters. In the following section, the comparative static results of current cutting are developed. The same could be done for current consumption, but it is omitted here because the primary interest is in cutting. The results are expressed using the respective exogenous variable as a subindex for X , which denotes current cutting. Thus, the effect on current cutting of a change in labour income during the first period is expressed as X_{I1} and so forth.

Income effects

It would be appropriate to begin the comparative static analysis with the effects of a change in exogenous income on current cutting. In deriving the results, usual comparative static methods are used. Thus, the first-order conditions are first, differentiated with respect to I_1 . Then, the resulting equation system is solved for X_{I1} by using Cramer's rule. This produces eqs. 11 and 12 as the results.

$$11) \quad X_{I1} = D^{-1} [U''(C_1) V_{cx}] < 0, \quad A'_c \leq 0$$

$$12) \quad X_{I2} = D^{-1} (-V_{cc} V_{xI2} + V_{cx} V_{cI2}), \quad A'_c < 0$$

$$= D^{-1} \{-E[U'' \hat{R}] V_{cx}\} > 0, \quad A'_c = 0$$

The effect on current cutting of an increase in exogenous income during the first period is negative under decreasing or constant absolute risk aversion. In contrast, X_{I2} has the opposite sign under constant absolute risk aversion. Under decreasing absolute risk aversion, X_{I2} is negative (a priori indeterminate) for borrowers (lenders) as shown in Appendix 1. Thus, under interest rate uncertainty, the income effects are not zero under constant absolute risk aversion as is suggested in price uncertainty models.

Interpretation of the results is facilitated by looking at the flow of funds eq. 3. Writing it as $I_1 + p_1 x = C_1 + S$ shows the left-hand side to be the financial source of the two uses on the right-hand side. Differentiating with respect to I_1 gives $(1 + p_1 X_{I1} = C_{I1} + S_{I1})$, where C_{I1} and S_{I1} are positive. Thus, an increase in income allows cutting to be reduced during the first period ($p_1 X_{I1} < 0$, but $1 + p_1 X_{I1}$ must be positive because of the definition of [3]). Thus, it can be concluded that exogenous income and timber selling income are substitute financial sources for consumption and saving, respectively.

In the same way, there is substitutability of timber selling income and exogenous income during the second period. The flow of funds eq. 4 provides the conclusion. Given an uncertain interest rate and its distribution, an increase in I_2 allows the forest owner to secure a given distribution of future consumption with lower timber selling income during the second period, which implies increased cutting during the first.

To keep the analysis simple, constant absolute risk aversion will be assumed from here on.

The effect of a change in the uncertain interest rate

Next, the effect of an increase in pure risk on timber supply is analyzed. To do this, the mean preserving spread in the probability distribution of r is defined. If ϵ denotes the expectation value of the interest rate and the multiplicative parameter Ω its variation around the mean, then \hat{r} can be expressed as $\hat{r} = \epsilon + \Omega r$. The mean preserving spread is now defined as a change in the multiplicative parameter Ω , which is compensated by a change in ϵ to restore the mean, so that $dE(\epsilon + \Omega r) = 0$ (for the notation and closer analysis, see Sandmo 1971, pp. 67-68). By the same procedure as described earlier, the following result is obtained:

$$[13] \quad \begin{matrix} X_{\Omega} \\ \epsilon \\ \Omega \end{matrix} \begin{matrix} \Big| \\ \\ \Big| \end{matrix} \begin{matrix} = D^{-1} (-V_{cc} V_{x\Omega} + V_{cx} V_{c\Omega}) \\ = 0 \\ = 1 \end{matrix}$$

V_{cc} is negative and V_{cx} is positive. The sign of the whole expression depends on the uncertainty terms. In Appendix 2 it is shown that $V_{x\Omega}$ is positive (negative) if savings are negative (positive), but the sign of $V_{c\Omega}$ is a priori unknown for both lenders and borrowers. The overall sign of the equation remains a priori unknown. However, a closer analysis of the sign of $V_{c\Omega}$ is possible by using specified forms of the utility function. In Appendix 2 it is shown, for example, that under the logarithmic utility function, $V_{c\Omega}$ is positive (negative) if savings are positive (negative). In all cases, however, the sign of the expression [13] is indeterminate.

The effects of forest taxation

Attention is now turned to forest taxes. A change in lump sum taxes produces only wealth effects. This differs clearly from the respective results under price uncertainty. In those models, the wealth effects are zero.

$$[14] \quad X_{T1} = -X_{I1} > 0$$

$$[15] \quad X_{T2} = -X_{I2} < 0$$

A change in unit tax, t_i , produces a substitution effect in addition to the wealth effect. This emerges because of a change in the opportunity costs of cutting: an increase in present (future) unit tax decreases the post-tax marginal return from cutting and tends to decrease (increase) current cutting. By utilizing a dual approach, the outcome for current cutting can be decomposed as Slutsky equations in the form of wealth and substitution effects. Substitution effects are defined as $X_{t1}^c = \beta E[U' \hat{R}] V_{cc} < 0$ and $X_{t2}^c = -(1 + F') X_{t1}^c > 0$.

$$[16] \quad X_{t1} = x X_{T1} + X_{t1}^c = ?$$

$$[17] \quad X_{t2} = z X_{T2} + X_{t2}^c = ?$$

Interpretation of the results is straightforward. A change in unit tax during the first (second) period causes a positive (negative) wealth effect, indicating increased (decreased) cutting during the first period. The substitution effect is negative (positive), because after-tax timber income is lower (higher) in the first period, making [16] and [17] a priori unknown.

The respective effects of a change in yield taxes on timber supply can be defined using these results, as eqs. 18 and 19 reveal. Interpretation of the results is also analogous:

$$[18] \quad X_{r1} = p_1 X_{t1} = ?$$

$$[19] \quad X_{r2} = p_2 X_{t2} = ?$$

The results of eqs. 16–19 differ from the respective effects of forest taxes under price uncertainty. If the future timber price is uncertain and constant absolute risk aversion is assumed, the wealth effects are zero and [16] is negative and [17] positive because of the substitution effects. Moreover, [18] is negative, but the sign of [19] is indeterminate because of the indeterminate uncertainty effect. The source of the difference is the existence of income effects under interest rate uncertainty.

The relative effectiveness of forest taxes

Suppose that government wishes to seek a tax base that would generate the greatest supply of timber. The tax authorities are assumed to keep government tax revenue constant and to raise one tax and to decrease another accordingly.

The expected present value tax revenue function, Γ , of government is stated in [20]. Tax revenue is uncertain, because the real interest rate is uncertain. This raises the question of government attitudes towards risk. Government is assumed to be risk neutral, which means that it is indifferent between a certain outcome and an outcome with the same expectation value. Tax rates and tax income are assumed to correlate positively, i.e., the so-called Laffer curve is assumed to be upward sloping.

$$[20] \quad \Gamma = (1 + \bar{R}^{-1})T + (p_1x + \bar{R}^{-1}p_2z)\tau + (x + \bar{R}^{-1}z)t$$

where \bar{R} is the expectation value of the interest rate.

During the first period, government has three alternative ways of changing the tax base: switching lump sum tax with unit or yield tax and switching unit and yield tax. This also applies to the second period. The effects of these changes on supply are explained in proposition 2.

Proposition 2—Under interest rate uncertainty, assuming an upward-sloping Laffer curve and government’s aim to keep expected tax revenue constant, a tax switch towards current (future) unit or yield tax from current (future) lump sum tax reduces current cutting because of the negative substitution effect, but a switch from current (future) unit tax towards yield tax has no effect on timber supply.

Proof

The start point is the compensated change between T_1 and t_1 . Differentiating [20] with respect to T_1 , t_1 , and x gives $dT_1 = -xdt_1 - mdX$. Substituting this for dT_1 in the equation $dX = x_{T1}dT_1 + x_{t1}dt_1$ produces the final result $dX/dt_1 = [1 + x_T(1 + \bar{R})m]^{-1} x'_{t1} < 0$, where the assumption of an upward-sloping Laffer curve implies that $m = \{\tau p + t - \bar{R}^{-1}[\tau p_2(1 + F') + t(1 + F')]\} > 0$. (For proof of the positivity of the numerator in a similar context, see Koskela 1989a.)

The same simple procedure applies to other cases, too. One additional result is worth proving here, namely the tax switch between τ_2 and t_2 , which differs from the price uncertainty model. Differentiating [20] with respect to τ_2 , t_2 , and x gives $dt_2 = -p_2d\tau_2 - \bar{R}z^{-1}mdx$. Substituting this for dt_2 in equation $dX = x_{\tau_2}d\tau_2 + x_{t_2}dt_2$ results in $dX/d\tau_2 = (1 + x_{\tau_2}z^{-1}m)^{-1}(x_{\tau_2} - p_2x_{t_2})$. A look at the comparative static results [17] and [19] shows that the supply effect is simply zero, when it was a priori unknown because of the risk effect under price uncertainty.

Taxation and timber supply under credit rationing and an uncertain interest rate

In the previous section it was assumed that forest owners were able to borrow or lend money without limits at a constant but uncertain interest rate. This assumption will not be relaxed. The capital market is assumed to be imperfect by the existence of binding quantitative credit rationing in the sense that forest owners would be willing to borrow more at the prevailing interest rate, but they are not allowed to. Credit rationing is exogenous to the model, i.e., the behaviour of the banking sector is ignored here. It is possible to incorporate the behaviour of banks and forest owners in the same model. In that case the nature of the model would be slightly different.¹

Under credit rationing, the forest owner is subject to binding upper borrowing limit, $B = \bar{B}$, which he would like to exceed but cannot. For him the flow of funds equation during the first period is defined by [21], according to which consumption cannot exceed the sum of net income plus the amount of the loan:

$$[21] \quad \bar{B} \geq C_1 + T_1 - p_1^*x - I_1$$

During the second period, the owner has to pay back his loan and interest on it. The size of interest payments is uncertain because of the uncertain interest rate. The flow of funds equation during the second period is $C_2 = p_2^*z + I_2 - T_2 - \bar{R}B$. By substituting B for $-S$ (S as defined earlier in this equation, the intertemporal budget constraint is obtained.

By assumption, the owner consumes his entire net income during the first period. This implies equality in equation [21]. Therefore his economic choice can be analyzed as a constrained maximization problem. The owner chooses current consumption and cutting so as to maximize the utility of consumption over the two periods in question under the constraint of [21]. The Lagrangian function is stated as eq. 22 where μ is the Lagrangian multiplier:

$$[22] \quad L = U(C_1) + \beta E[U(p_2^*z + I_2 - T_2 + \bar{R}(p_1^*x + I_1 - T_1 - C_1))] + \mu[p_1^*x + I_1 - T_1 + \bar{B} - C_1]$$

The first-order conditions of the problem are stated in eq. 23:

$$L_c = U'(C_1) - \beta E[U'R] - \mu = 0$$

$$[23] \quad L_x = \beta E[U'e] + \mu p_1^* = 0$$

$$L_\mu = p_1^*x + I_1 - T_1 + \bar{B} - C_1 = 0$$

In cutting rule L_x , e is again defined as $\bar{R}p_1^* - (1 + F')p_2^*$. Condition $L_x = 0$ implies that $\beta\{E[U'e]E[e] + \text{cov}(U'e)\} + \mu p_1^* = 0$. The covariance turned out to be positive in the previous section. For the first-order condition to hold, $E[e]$ must be negative. This can be achieved only by increasing the forest growth rate, i.e., by increasing current cutting. Thus, interest rate uncertainty increases timber supply under credit rationing. Furthermore, $E[U'e]$ is also negative (on the grounds of first-order conditions), although it was zero under perfect capital market conditions. This means that

¹M. Ollikainen. 1990. Timber supply and forest taxation under equilibrium credit rationing. Unpublished manuscript.

credit rationing, together with uncertainty, tend to increase the timber supply of borrowers compared with the perfect capital market uncertainty case.

The second-order conditions are stated in eq. 24. They hold because of risk aversion and concavity of the growth function. The determinant of the Hessian matrix, H , can be shown to be positive as required for a maximum:

$$L_{cc} = U''(C_1) + \beta E[U''\hat{R}^2] < 0$$

$$[24] \quad L_{xx} = \beta\{E[U'(C_2)e^2] - p_2^*F''E[U']\} < 0$$

$$L_{\mu\mu} = 0$$

Attention is now devoted to the comparative static results. Because optimum properties differ from those under perfect capital markets, \bar{X} is now used instead of X as the expression of current cutting. The results are developed only for current cutting.

Liquidity and uncertainty effects

Again it seems useful to start by defining the effects on supply of a change in exogenous income. This is done in eqs. 25 and 26. A rise in current (future) income decreases (increases) current cutting because of the liquidity effect:

$$[25] \quad \bar{X}_{I1} = H^{-1}\{p_1^*U''(C_1)\} < 0$$

$$[26] \quad \bar{X}_{I2} = H^{-1}\{-\beta(1 + F')p_2^*E[U'']\} > 0$$

The effect on timber supply of a change in pure uncertainty of interest rate is depicted in [27]. The sign of $E[U''(\hat{r} - \bar{r})] = \text{sign cov}(U'', \hat{r}) = \text{sign}(U''S)$. When S is negative and U'' is positive because of the assumption of constant absolute risk aversion, the sign of the expectation and of the whole expression is negative. The interpretation is as follows. An increase in interest rate uncertainty induces forest owners to reserve more timber for future contingencies, which decreases current timber selling.

$$[27] \quad \bar{X}_{\Omega} \begin{cases} = H^{-1}\{-\beta(1 + F')(p_2/p_1) \\ \quad \times SE[U''\hat{R}(r - \bar{r})]\} < 0 \\ \epsilon = 0 \\ \Omega = 1 \end{cases}$$

The effects of changes in forest taxation

The effects of changes in forest taxes on timber supply are considered next. An increase in lump sum tax causes liquidity effects, which have opposite signs to [25] and [26].

$$[28] \quad \bar{X}_{T1} = -\bar{X}_{I1} > 0$$

$$[29] \quad \bar{X}_{T2} = -\bar{X}_{I2} < 0$$

A change in unit taxes can be stated as the sum of liquidity and substitution effects when $\bar{X}_{T1}^c = -H^{-1}\{\beta E[U'\hat{R}]\} < 0$ and $\bar{X}_{T2}^c = -(1 + F')\bar{X}_{I1}^c > 0$:

$$[30] \quad \bar{X}_{T1} = x\bar{X}_{T1} + \bar{X}_{T1}^c = ?$$

$$[31] \quad \bar{X}_{T2} = z\bar{X}_{T2} + \bar{X}_{T2}^c = ?$$

As before, the effect of a change in yield tax on timber supply can be defined by using [30] and [31]: $\bar{X}_{T1} = p_1\bar{X}_{T1}$ and $\bar{X}_{T2} = p_2\bar{X}_{T2}$. In the cases of unit and yield taxes, the liquidity and substitution effects are pulling in opposite directions, producing an a priori ambiguous total effect. These results have the same sign as those under price uncertainty with one exception: the risk effect is now absent in X_{T2} as the risk is connected with the interest rate

The relative effectiveness of forest taxes

The effects of tax switches in forest taxes, when government keeps its expected tax revenue constant, are similar to those under perfect capital markets. Therefore, proposition 2 also holds true for credit rationing. (The results and the proof are available from the author upon request.)

Discussion

The policy implications of the above analyses deserve mention. Proposition 1 showed that the tendency among borrowers to increase timber supply was offset by that among lenders to decrease cutting. Compared with perfect certainty, the development of aggregate timber supply under interest rate uncertainty is thus ambiguous. A taxation scheme that reduces the interest rate uncertainty faced by lenders without changing the situation of borrowers would obviously increase timber supply. Such a scheme, however, would hardly be easy to implement. Fortunately, proposition 2 shows that the same effect can be achieved by suitable changes in forest taxation.

Lump sum taxes turned out to be a convenient way of affecting timber supply under uncertain interest rates. Moreover, asymmetry between lenders and borrowers does not matter; both types of forest owner change their supply in the same direction. In this framework, yield and unit taxes are in some sense equivalent forms, and compensated change between them does not affect timber supply.

In conclusion, under price or interest rate uncertainty, the effects of forest taxation on timber supply differ (given the assumption of constant absolute risk aversion). In the former case, the wealth effects of lump sum taxes are zero, but not in the latter case, as shown in this paper. In reality, both future timber price and real interest rates may be uncertain. To select appropriate instruments for forest policy, the forest authorities must carefully investigate which type of uncertainty dominates. Changing lump sum taxes when price uncertainty prevails would certainly not produce the desired effect on timber supply.

It may be possible, however, that only interest rate uncertainty is relevant to some forest owners and only timber price uncertainty to others. This creates an interesting task for further research, namely, to determine the effects of double uncertainty of future timber price and real interest rate on timber supply and on the functioning of forest taxation.

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Appendix 1: the sign of the cross derivative V_{cx}

In the text it was derived that $V_{cx} = -\beta E[U''e\hat{R}]$. Solving \hat{R} from the budget constraint and using it for \hat{R} , gives $V_{cx} = -\beta E[U''eC_2]S^{-1} + \beta E[U''e]wS^{-1}$, where $w = (p_2z + I_2 - T_2) > 0$.

Start with the sign of $-\beta E[U''eC_2]S^{-1}$ and assume increasing relative risk aversion. Let R_c be a measure of relative risk aversion when $e = 0$. Assume that the forest owner is a lender. If \hat{r} in e rises (declines), e becomes positive (negative) and future consumption rises (declines). Comparing the resulting measure of relative risk aversion with R_c now gives a basis for the following analysis:

$$-U''C_2/U' > (<) R_c \rightarrow -U''C_2 > (<) R_cU' \rightarrow U''C_2 < (>) -R_cU' \rightarrow U''C_2e < -R_cU'e \\ \rightarrow E[U''C_2e] < -R_cE[U'e] = 0 \text{ (see first-order conditions)}$$

So, $E[U''eC_2] < 0$ for $S > 0$.

By a similar technique it can be shown that $E[U''eC_2] > 0$ for $S < 0$. For constant relative risk aversion, just change the inequalities to equalities listed, thus $E[U''C_2e] = 0$.

The next task is to analyze the sign of $\beta E[U''e]wS^{-1}$. Assume decreasing absolute risk aversion and define A_c as its measure, when $e = 0$. Now the analysis is carried out for the borrowers. A rise (decline) in \hat{r} implies that e becomes positive (negative) and C_2 declines (rises).

$$-U''/U' > (<) A_c \rightarrow -U'' > (<) A_cU' \rightarrow U'' < (>) -A_cU' \rightarrow U''e < -A_cU'e \\ \rightarrow E[U''e] < -A_cE[U'e] = 0 \text{ (see first-order conditions)}$$

So, $E[U''e] < 0$ for $S < 0$.

Similarly it can be shown that $E[U''e]$ is positive for $S > 0$. Moreover, if absolute risk aversion is constant, $E[U''e] = 0$.

The overall sign of V_{cx} can now be concluded. Assuming decreasing absolute and constant relative risk aversion, $V_{cx} = \beta E[U''e]wS^{-1} > 0$ for all S , positive or negative. Under constant absolute and increasing relative risk aversion, $V_{cx} = -\beta E[U''eC_2]S^{-1} > 0$ for all S , positive or negative.

These results can then be used to analyze the sign of X_{I_2} .

$$[A1] \quad X_{I_2} = D^{-1}\{-\beta U''(C_1)E[U''e] + \beta^2E[U''\hat{R}]E[U''e\hat{R}] - \beta^2E[U''\hat{R}^2]E[U''e]\}$$

Under decreasing absolute and constant relative risk aversion, the result can be written in the following form using the analysis of the sign of V_{cx} .

$$X_{I_2} = D^{-1}\{-\beta U''(C_1)E[U''e] + \beta^2E[U''\hat{R}]E[U''e]wS^{-1} - \beta^2E[U''\hat{R}^2]E[U''e]\}$$

The first term in braces is positive (negative) when S is positive (negative). The second term in braces is always negative, but the third term is positive (negative) when S is positive (negative). Thus, under decreasing absolute risk aversion, the overall sign of X_{I_2} is negative for borrowers and a priori unknown for lenders.

Under constant relative risk aversion, all $E[U''e]$ terms in [A1] are zero producing ([12] in the text).

Appendix 2: the mean preserving spread with uncertain interest rate

In the text the following formulation was obtained:

$$X_{\cap} = D^{-1}\{-V_{cc}V_{\cap} + V_{cx}V_{c\cap}\}$$

The task is now to determine the sign of $V_{x\cap}$ and $V_{c\cap}$. Let us start with $V_{x\cap} = \beta\{SE[U''e(r - \bar{r})] + p_1E[U'(r - \bar{r})]\}$.

Using the definition of $e, r - \bar{r} = (e - \bar{e})/p_1$ and

$$V_{x\cap} = \beta\{p_1^{-1}SE[U''e(e - \bar{e}) + E[U'(e - \bar{e})]\}$$

$$V_{c\cap} = \beta p_1^{-1}\{p_1^{-1}S(E[U''e^2] - \bar{e}E[U''e]) - \bar{e}E[U']\}$$

All the terms in the brackets are positive if savings are negative and negative if savings are positive. Thus $V_{x\cap}$ is positive (negative) if $S < (>) 0$, as was stated in the text.

The same analysis can be carried out for $V_{c\cap}$:

$$V_{c\cap} = -\beta\{SE[U''\hat{R}(r - \bar{r})] + E[U'(r - \bar{r})]\}$$

$$V_{c\cap} = -\beta p_1^{-1}\{SE[U''\hat{R}(e - \bar{e})] + E[U'(e - \bar{e})]\}$$

$$V_{c\cap} = -\beta p_1^{-1}\{SE[U''\hat{R}e] - S\bar{e}E[U''\hat{R}] + \bar{e}E[U']\}$$

The first equation is negative for lenders and borrowers, the second is positive for both, and the third is positive (negative) for lenders (borrowers). The overall sign is now a priori unknown for both.

However, the analysis can be carried further. Recall the definition of $V_{c\cap}$ stated earlier and use of the covariance technique. $V_{c\cap} = -\beta\{S \text{cov}(U''\hat{R}, \hat{r}) + \text{cov}(U', \hat{r})\}$ to get the form $V_{c\cap} = -\beta S\{SU''\hat{R} + 2U''\}$.

Deriving the measure of relative risk aversion with respect to C_2 allows U''' to be expressed as $U''' = U''/U'(1 + R_c) + (U'/C_2)R'_c$. Substituting this for U''' in $V_{c\cap}$ then gives $V_{c\cap} = -\beta S\{U''[-(\hat{R}S)(1 + R_c)/C_2 + 2] - \hat{R}SR'_cU'\}$. Using the intertemporal budget constraint gives $C_2 = Y + \hat{R}S$, where $S = p_1x + I_1 - T_1 - C_1$ and $Y = p_2z + I_2 - T_2$. By substituting this for C_2 in $V_{c\cap}$ and manipulating it, the following formula is obtained:

$$[A2] \quad V_{c\cap} = -\beta SU''/C_2\{2Y + \hat{R}S[1 + R_c - R'_c(U'/U'')]\}$$

Some utility function specifications can be used for eq. A2:

- (A) *Logarithmic function:* $U(C_2) = \log C_2$ with $U' = 1/C_2, U'' = -1/C_2^2, R_c = 1$, and $R'_c = 0$ produces $V_{c\cap} = \beta(S/C_2)2Y$, which is positive (negative) when S is negative (positive). The overall sign of X_{\cap} still remains unknown.
- (B) *Quadratic function:* $U(C_2) = aC_2 - 1/2bC_2^2$ with $U' = a - bC_2, U'' = -b, R_c = bC_2/(a - bC_2)$, and $R'_c = ab/(a - bC_2)^2$ produces $V_{c\cap} = \beta Sb/C_2\{2Y + \hat{R}S(2a/(a - bC_2))\} > 0$ for $S > 0$, but a priori unknown for $S < 0$. The overall sign of X_{\cap} remains unknown a priori.
- (C) *Exponential function:* $U(C_2) = -k e^{-AC_2}$ with $U' = kA e^{-AC_2}, U'' = -kA^2 e^{-AC_2}, R_c = AC_2$, and $R'_c = A > 0$ produces $V_{c\cap} = -\beta S(kA^2 e^{-AC_2})/C_2\{2Y + \hat{R}S(2 + AC_2)\} > 0$ for $S > 0$, but a priori unknown for $S < 0$. The sign of X_{\cap} still remains unknown.

2

A mean-variance approach to short-term timber selling and forest taxation under multiple sources of uncertainty¹

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The effects of forest taxation on timber supply under the double uncertainty of real interest rate and future timber price are analyzed in a mean-variance framework. If future timber price and interest rate are independent random variables, there exists an asymmetry between lenders and borrowers, as there is in the single interest rate uncertainty case. Borrowers cut more and lenders cut less as a response to double uncertainty relative to single price uncertainty. If timber price and interest rate are correlated, the sign of the covariance is crucial for the interpretation of the optimality conditions and for the comparative static analysis results. If covariance is negative, borrowers still tend to increase their cutting, but the behaviour of lenders depends on the relative magnitudes of risks associated with interest rate and timber price, which is indicated by variances and covariance. Finnish data are used to evaluate the size of various risks and the sign of covariance. The evidence suggests that a statistically insignificant negative correlation exists and that the covariance is zero.

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Un cadre d'analyse variance-moyenne est utilisé pour comprendre les effets d'un régime de taxation forestière sur l'offre de bois en regard de la double incertitude engendrée par la détermination d'un taux d'intérêt réel et d'un prix futur pour le bois. À l'instar du cas où le taux d'intérêt représente la seule source d'incertitude, il existe une asymétrie entre les prêteurs et les emprunteurs quand le taux d'intérêt et le prix du bois sont des variables indépendantes et aléatoires. Comparativement à la situation où l'incertitude résulte seulement du mouvement du prix, le cas de la double incertitude amène les emprunteurs à augmenter leurs coupes et les prêteurs à réduire les quantités récoltées. Si le prix du bois et le taux d'intérêt sont corrélés, le signe de la covariance devient déterminant pour l'interprétation des conditions d'optimalité et des résultats obtenus en analyse statique comparée. Ainsi, quand la covariance est négative, les emprunteurs ont toujours tendance à accroître leur effort de récolte alors que le comportement des prêteurs dépend de l'amplitude relative des risques associés au taux d'intérêt et au prix du bois, laquelle est indiquée par les variances et la covariance. Des données finlandaises sont utilisées afin d'évaluer l'ampleur des différents risques et le signe de la covariance. L'analyse révèle qu'une corrélation statistiquement non significative existe et que la covariance est égale à zéro.

[Traduit par la rédaction]

Introduction

Timber supply and forest taxation have been studied recently using the Fisherian two-period consumption-savings-cutting model (see Johansson and Löfgren 1985; Koskela 1989a, 1989b; Ollikainen 1990, 1991). In the model, the forest owner maximizes the utility of consumption by choosing between harvesting now and harvesting tomorrow. Optimum harvesting is defined by comparing the growth rate of the forest stand with the prevailing interest rate, given after-tax timber prices. Under certainty and perfect capital market conditions, the cutting decision is separate from the consumption decision (preferences). Then, the optimum condition for harvesting and the comparative static analysis of forest taxation are qualitatively the same as the basic results under certainty in the rotation framework derived, for example, in Chang (1982, 1983) and Klemperer (1976).

Under uncertainty (or imperfect capital market conditions), separability no longer holds, preferences affect the cutting decision (Koskela 1989a, 1989b), and as a result many previously exogenous variables, even nontimber taxes, affect timber supply (Ollikainen 1991). Uncertainty may emerge in the optimality conditions through three basic channels: future timber price, interest rate, and forest growth (growth uncer-

tainty will probably be of great importance in the future because of increasing problems caused by global warming and acid rain.) The effects of timber price uncertainty on timber supply were first studied in Johansson and Löfgren (1985). In their model, the forest owner maximizes his utility from the present value of income under an implicit assumption that consumption during both periods is a perfect substitute. They found that, under price uncertainty, the risk-averse forest owner will cut more relative to certainty. Further, lump sum tax (as an approximation of the land site tax) will increase today's cutting if the owner has decreasing absolute risk aversion.

In Koskela's model, the owner maximizes the utility of consumption over both periods. In both perfect and imperfect capital markets, the results confirmed an earlier finding that, under price uncertainty, the owner cuts more now and less in the future because of risk aversion (Koskela 1989a). Further, if the capital market is imperfect in the sense that there is a ceiling on bank loans, the owner has to cut more today than if the capital market were perfect (Koskela 1989b). These studies enlarged the analysis of forest taxation to include three types of taxes: (i) lump sum tax, paid for the land independently of cutting (or silvicultural) activity; (ii) unit tax, levied on the quantity of timber sold; (iii) yield tax, levied on the timber selling revenue. Comparative static results confirmed Johansson's and Löfgren's (1985) findings (for other results see Table 1).

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Introducing interest rate uncertainty into two-period models (Ollikainen 1990) revealed that an uncertain interest rate has different implications for timber supply than price uncertainty. The owner's position in the capital market turned out to be important. Risk-averse lenders decrease and borrowers increase their current cutting. Lenders want to keep a greater share of their capital in the form of timber, the yield of which is certain. Borrowers increase their cutting to avoid risky loans. The analysis was also enlarged to cover the imperfect capital market. At the margin, the owner cuts more than under perfect capital market conditions (for tax results see Table 1).

Interest rate and price uncertainty affect timber supply differently. It is therefore interesting to ask how these two uncertain variables together affect the supply. The question is not only theoretically interesting, but also empirically relevant, because both are plausibly uncertain. What are the effects of double uncertainty on timber supply at the margin? How does forest taxation affect the cutting decision under these conditions? Further, are the results derived under a single uncertain variable robust under double uncertainty? The purpose of this paper is to answer these questions.

Analyzing many uncertain variables at the same time is a complicated task. There is now limited but growing literature on the subject (Ross 1981; Kihlström *et al.* 1981; Pratt and Zeckhauser 1987; Machina and Neilson 1987). The existing research shows the limited applicability of the theory of economic behaviour under uncertainty. In particular, Arrow-Pratt measures of risk aversion, which are well suited to the analysis of a single uncertain variable, are not strong enough for comparative static analysis under two uncertain variables (Ross 1981).

This indeterminacy problem of comparative static analysis can be avoided by simplifying some of the complexities of the expected utility approach as a first approximation. Traditionally mean-variance analysis is used, which reduces the analysis of risk to two quantities, expected yield and risk. Risk is defined as the variance and covariance of uncertain variables. Besides avoiding the indeterminacy problem, this gives the mean-variance analysis some additional advantages, which have contributed to its popularity. First of all, it is simple to use, because it incorporates a special case of the expected utility hypothesis without contradicting it. Secondly, a combination of constant absolute risk aversion and mean-variance analysis allows the study of risks to focus on substitution effects by taking away the income effects, thus concentrating on the potential changes in market allocation (see Newbery and Stiglitz 1981, pp. 85-88).²

Mean-variance analysis is used here as a device for analyzing the timber supply of private nonindustrial forest owners under an uncertain future timber price and a real interest rate. The costs of using this simplifying analysis are negligible in

²Discussion about the legitimacy of the mean-variance approach has resulted in the definition of five cases where its use is justified. (1) Uncertain variables are normally distributed. (2) The objective functions of the agents are quadratic. (3) Uncertain variables are normally distributed and the agent's objective function includes constant absolute risk aversion. (4) Risks are relatively small compared with the wealth of agents. (5) The distributions of uncertain variables can be described by compact probabilities so that all the probability distributions converge to a certain outcome. (See Newbery and Stiglitz 1981, pp. 85-91 and, especially, for the fourth and fifth point Tsing 1972 and Samuelson 1967, 1970.)

this case. It produces the same qualitative results for cutting at the margin as the more general expected utility hypothesis as is shown later in the paper.

Timber supply means the forest owner's timing of cutting between periods one and two. For the interpretation of the results, it is useful to restate the connection between two period models and the rotation problem. Two-period models in fact, address the problem of forest rotation. Under certainty the two-period model produces the same results as the one rotation-period model. It is therefore useful to think of the model in terms of a regulated forest that the forest owner will liquidate over the next two periods. The decision to cut now or tomorrow thus concerns cutting the oldest acreage down to the same age-class and letting the remaining volume grow until the next period when it will be liquidated. The model and its properties are examined in the next section. Then, the *ceteris paribus* effects of forest taxes are derived and the relative effectiveness of forest taxation are studied. The results are discussed in the light of Finnish empirical data.

Timber supply under price and interest rate uncertainty

The basic framework

The forest owner decides between cutting today (x), or tomorrow (z). Cutting during the second period is uniquely determined by the original volume of timber (Q), current cutting (x), and the concave growth function of the forest stand (F), with $F'(Q) > 0$ and $F''(Q) < 0$:

$$[1] \quad z = (Q - x) + F(Q - x)$$

Timber prices for the first and second periods are denoted by p_i , $i = 1, 2$. By assumption, forest taxation is through three channels, two of which influence effective timber prices. The owner has to pay yield and unit tax (τ_i, t_i) from his timber selling income and, in addition, lump sum tax (T_i). By assumption, tax rates differ between periods ($\tau_i, t_i, T_i, i = 1, 2$). Thus, after-tax timber prices are denoted by $p_i^* = p_i(1 - \tau_i) - t_i$.

During the first period, the forest owner receives exogenous labour income (I_1) and after-tax timber selling income. The owner allocates his income to consumption (c_1) and saving (S):

$$[2] \quad c_1 = p_1^* x + I_1 - T_1 - S$$

The timber price during the second period (p_2), and interest rate (r), are uncertain and are denoted by \hat{p}_2 and \hat{r} . Both are assumed to be normally distributed, which means that the forest owner knows their means, variances, and covariance. The expected values are denoted by a bar above the uncertain variable, variances are denoted by σ_p^2 and σ_r^2 and the covariance is denoted by σ_{pr} . If timber price and interest rate are uncorrelated, i.e., independent, then covariance is zero. If they are correlated, a high interest rate may be associated with a high timber price ($\sigma_{pr} > 0$) or with a low timber price ($\sigma_{pr} < 0$). When price and interest rate are uncertain, future consumption is uncertain as well. Because price and interest rate are normally distributed, future consumption is also normal (see Mardia *et al.* 1982, pp. 231-235).

Given the tax structure and exogenous income in the second period, the uncertain consumption during the second period can be written as

$$[3] \quad \hat{c}_2 = \hat{p}_2^* z + I_2 - T_2 + (1 + \hat{r})S$$

Living saving from [2], the owner's intertemporal budget constraint can be expressed as

$$] \quad \hat{c}_2 = \hat{p}_2^* z + I_2 - T_2 + (1 + \hat{r}) \times [p_1^* x + I_1 - T_1 - c_1]$$

The preferences of the risk-averse representative forest owner are represented by an intertemporally additive exponential utility function with constant absolute risk aversion (A) and a time preference factor of δ , so that rate factor $\beta = (1 + \delta)^{-1}$:

$$] \quad V = -\exp(-Ac_1) - \beta \exp(-A\hat{c}_2)$$

where $A = -U''/U'$ is a constant, so that the derivative of A , equals 0.

Given the normality of consumption during the second period, it is completely characterized by its expected value (mean) and variance. Therefore, under constant absolute risk aversion, the expected utility of consumption can be found from the moment-generating function for the normal distribution (see Newbery and Stiglitz 1981, pp. 85-86; Aitchison and Brown 1957).³ Thus, [5] can be written as

$$] \quad V_x = \beta A \pi \exp\left[-A\left(\bar{c}_2 - \frac{A \text{var}(c_2)}{2}\right)\right] = 0$$

here:

$$\pi = p_1^* \{(1 + \bar{r}) - A[S\sigma_r^2 + (1 - \tau_2)z\sigma_{pr}]\} - (1 + F')\{\bar{p}_2^* - A[(1 - \tau_2)^2 z\sigma_p^2 + (1 - \tau_2)S\sigma_{pr}]\} = 0$$

The second-order conditions of the model are defined in eq. 9. V_{cc} and V_{xx} are negative, and the determinant of the Hessian matrix is positive. The sign of the cross-derivative (V_{cx}) depends on interest rate variance and covariance. The sign $\{V_{cx}\} = \text{gn}\{\sigma_p^2 p_1^* - (1 + F')(1 - \tau_2)\sigma_{pr}\}$. Assume, firstly, that covariance is zero, then $\text{sign}\{V_{cx}\} = \text{sign}\{\sigma_p^2 p_1^*\} > 0$. Secondly, a high interest rate is associated with a low future timber price, the covariance term is negative and $\text{sign}\{V_{cx}\} = \text{gn}\{\sigma_p^2 p_1^* - (1 + F')(1 - \tau_2)\sigma_{pr}\} > 0$. Thirdly, if the covariance term is positive, the sign of V_{cx} is a priori ambiguous and depends on the relative magnitudes of the variance and covariance terms. It will turn out that the sign of V_{cx} is decisive to the sign of the wealth effects of forest taxes described in the next section.

$$] \quad V_{cc} = -A^2 \exp(-Ac_1) - \beta A^2 [\phi^2 + \sigma_r^2] \exp\left[-A\left(\bar{c}_2 - \frac{A \text{var}(c_2)}{2}\right)\right] < 0$$

$$V_{xx} = \beta A \{F''[\bar{p}_2^* - A((1 - \tau_2)^2 z\sigma_p^2 + (1 - \tau_2)S\sigma_{pr})] + A[(1 - \tau_2)(1 + F')(2p_1^* \sigma_{pr} - \sigma_p^2) - \sigma_r^2 p_1^*]\} \times \exp\left[-A\left(\bar{c}_2 - \frac{A \text{var}(c_2)}{2}\right)\right] < 0$$

$$V_{cx} = \beta A^2 [\sigma_r^2 p_1^* - (1 + F')(1 - \tau_2)\sigma_{pr}] \exp\left[-A\left(\bar{c}_2 - \frac{A \text{var}(c_2)}{2}\right)\right]$$

$$D = V_{cc} V_{xx} - (V_{cx})^2 > 0$$

$$[6] \quad V = -\exp(-Ac_1) - \beta \exp\left[-A\left(\bar{c}_2 - \frac{A \text{var}(c_2)}{2}\right)\right]$$

where

$$\bar{c}_2 = \bar{p}_2^* z + I_2 - T_2 + (1 + \bar{r}) \times [p_1^* x + I_1 - T_1 - c_1]$$

$$\text{var}(c_2) = (1 - \tau_2)^2 z^2 \sigma_p^2 + S^2 \sigma_r^2 + 2(1 - \tau_2)zS\sigma_{pr}$$

The forest owner maximizes his expected utility over current and future periods by choosing the amounts of current consumption (c_1), and current cutting (x). The first-order conditions are

$$[7] \quad V_{c1} = A \exp(-Ac_1) - \beta A \phi \times \exp\left[-A\left(\bar{c}_2 - \frac{A \text{var}(c_2)}{2}\right)\right] = 0$$

where

$$\phi = (1 + \bar{r}) - A[\sigma_r^2 S + (1 - \tau_2)z\sigma_{pr}] > 0$$

³An alternative way to derive the target function is to use the so-called moments of distribution method, where the utility function is generated by Taylor series expansion taken at the mean of the uncertain variable (Newbery and Stiglitz 1981, pp. 90-92; Anderson *et al.* 1971, pp. 96-99).

Interpreting the optimum conditions

Condition [7] defines the optimum allocation of consumption between the two periods. It states that the marginal utility of consumption during the first period must be equal to the utility in the second period, which is weighted by a time preference factor, interest rate, and covariance terms. The optimum condition for cutting suggests that $V_x = 0$ is equivalent to $\pi = 0$, which produces

$$[10] \quad p_1^* \{ (1 + \bar{r}) - A [S \sigma_p^2 + (1 - \tau_2) z \sigma_{pr}] \} \\ = (1 + F') \{ \bar{p}_2^* - A [(1 - \tau_2)^2 z \sigma_p^2 + (1 - \tau_2) S \sigma_{pr}] \}$$

which includes interest rate and timber price variances and covariance.

Five cases can be generated from eq. 10 that are useful for the purposes of interpretation.

Case 1: no uncertainty ($\sigma_p^2 = \sigma_r^2 = \sigma_{pr} = 0$)

Setting all $\sigma = 0$ in eq. 10 yields the benchmark case of certainty:

$$[11] \quad (1 + r) p_1^* = (1 + F') p_2^*$$

Properties of this equation have been examined by Johansson and Löfgren (1985) and Koskela (1989a). The optimum amount of harvested timber is found by equalizing the marginal revenue (on the left-hand side of the equation (LHS)) and the opportunity cost of cutting (on the right-hand side of the equation (RHS)).

Case 2: future price uncertainty ($\sigma_p^2 > 0$, $\sigma_r^2 = \sigma_{pr} = 0$)

Allowing only timber price to be uncertain in eq. 10 yields

$$[12] \quad (1 + r) p_1^* = (1 + F') \{ \bar{p}_2^* - A (1 - \tau_2)^2 z \sigma_p^2 \}$$

The opportunity cost of cutting is decreased in this case. To keep equality between marginal return and opportunity cost, the forest owner increases cutting. Thus, under future timber price uncertainty, the forest owner cuts more relative to the benchmark case [11]. The rate of yield tax, τ_2 , plays an interesting role. The higher the tax rate is, the less uncertainty there will be in future income. This tends to decrease the RHS of [12] and thus current cutting. At the extreme, $\tau_2 = 1$ and taxation guarantees full insurance for price uncertainty, thus giving the risk-neutral case. This case was analyzed in a more general manner by Koskela (1989a).

Case 3: interest rate uncertainty ($\sigma_r^2 > 0$, $\sigma_p^2 = \sigma_{pr} = 0$)

Assuming only interest rate uncertainty produces

$$[13] \quad p_1^* [(1 + \bar{r}) - A \sigma_r^2 S] = (1 + F') p_2^*$$

This is analyzed more comprehensively in Ollikainen (1990). For the first time, saving emerges in the equation and its sign is decisive for the marginal revenue of cutting. If saving is positive, the owner is a lender and the variance term decreases the marginal return of cutting. If saving is negative, however, the owner is a borrower and the variance term increases the LHS. Therefore, the LHS in [13] is smaller than the LHS in [11], making the RHS in [13] smaller than the LHS in [11] for lenders, since $F' > 0$. The opposite holds for borrowers. Thus, risk-averse lenders cut less relative to the benchmark case, whereas risk-averse borrowers cut more. This

asymmetry turns out to have a strong influence under doubt uncertainty.

The results of cases 1–3 are described graphically in Fig. where marginal revenue (MR) and opportunity cost (OC) eqs. 11–13 are plotted. MR is a horizontal line, and OC is concave function resulting from forest growth assumption. The plots under certainty, under price uncertainty, and under interest rate uncertainty are denoted by 1, 2, and 3, respectively. Under price uncertainty, MR is the same as under certainty, but the OC differs. The benchmark case, cutting under certainty, is denoted by x_1 and cutting under price uncertainty, by x_2 . Under interest rate uncertainty, MR differs from certainty and is denoted by MR(3, $S < 0$) for a borrower and MR(3, $S > 0$) for a lender; OC is the same as under certainty. The borrower's and lender's cutting are denoted by x_3 and x_3 .

Case 4: price and interest rate are random and uncorrelated ($\sigma_r^2 > 0$, $\sigma_p^2 > 0$, $\sigma_{pr} = 0$)

If independence between uncertain price and interest rate is assumed, it implies that covariance is zero, yielding

$$[14] \quad p_1^* [(1 + \bar{r}) - A \sigma_r^2 S] = (1 + F') \\ \times \{ \bar{p}_2^* - A (1 - \tau_2)^2 z \sigma_p^2 \}$$

Because of interest rate uncertainty, the owner's position in the capital market, i.e., whether he acts as a lender or borrower, has a decisive influence on the cutting decision at the margin.

There are three potential interpretations of eq. 14, based on S values. (1) $S = 0$. The LHS reduces to $p_1^* (1 + \bar{r})$ and price uncertainty prevails, because uncertainty in the capital market is not relevant. Thus, the owner cuts more today relative to the benchmark case [11]. (2) The owner is a borrower ($S < 0$). The LHS is higher than when $S = 0$, indicating a rise in the marginal revenue of current cutting. This requires that the RHS should increase. This is possible only by increasing F' , i.e., increasing current cutting. Therefore, it can be concluded that risk-averse borrowers cut even more relative to single price uncertainty. (3) The owner is a lender ($S > 0$). The LHS is lower than when $S = 0$, indicating a decline in the marginal revenue of cutting. This requires the RHS to decrease, which is possible only through cutting less. Thus, the risk-averse lender cuts less than when $S = 0$ because he is facing a higher risk in the capital market than in the roundwood market. The level of harvesting may still be lower than in the risk-neutral case, depending on the relative size of risks associated with timber price and interest rate.

The overall conclusion can be summarized as follows: Uncertain uncorrelated future timber price and real interest rate affect lenders and borrowers asymmetrically. Risk-averse borrowers increase their cutting and the cut even more than under single price uncertainty. The reaction of risk-averse lenders is less clear. They cut less relative to the single price uncertainty case, because the risk associated with the interest rate works in opposite direction to the risk associated with timber price. This is illustrated in Fig. 2. The hypothetical starting point of the analysis is $S = 0$, leading to price uncertainty and cutting x_2 . If saving does not equal zero, MR differs depending on whether the owner is a lender or a borrower, which leads to asymmetry in cutting. Cutting by borrowers and lenders is denoted by x_4 and x_4 .

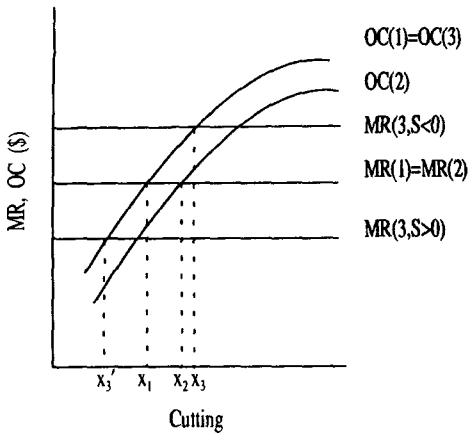


FIG. 1. Cutting (x) under price and interest rate uncertainty. Subscripts for x describe the level of cutting under conditions of (1) price and interest rate certainty; (2) price uncertainty; and (3) interest rate certainty. Vertical broken line gives the optimum cutting strategy under each condition. MR, marginal revenue; OC, opportunity costs; saving. Marginal revenue under price uncertainty, MR(2), is the same as under certainty, MR(1); opportunity costs under interest rate uncertainty, OC(3), are the same as under certainty, OC(1).

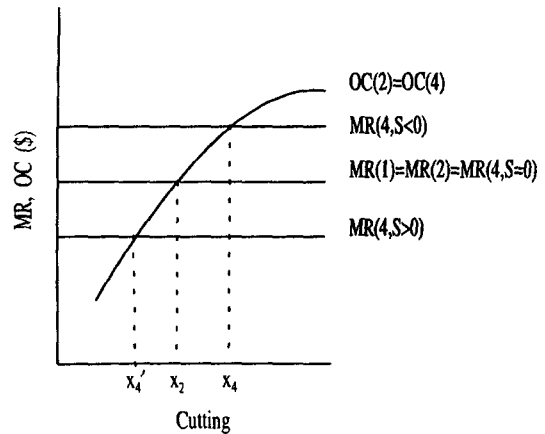


FIG. 2. Cutting (x) under independent price and interest rate uncertainty. x_4 , Cutting for a borrower; x_4' , cutting for a lender.

Case 5: price and interest rate are random and correlated ($\sigma_r^2 > 0, \sigma_p^2 > 0, \sigma_{pr} \neq 0$)

Finally, the general assumptions produce the optimum condition, derived in eq. 10. Because covariance is not determined in the model, it is assumed to be either positive or negative. Therefore there are several possible interpretations. The owner may be a borrower or a lender and covariance may be negative or positive.

Assume first that the owner is a borrower and covariance negative. Then the LHS of [10] increases relative to case 4, where covariance was assumed to be zero. At the same time, the covariance term decreases the RHS of [10] relative to case 4. This means that the owner cuts more relative to case 4 and also more than under single price uncertainty. This results from the association of a higher interest rate with a lower future timber price. The higher the interest rate, the lower the opportunity cost of cutting, which strengthens the tendency of interest rate uncertainty to increase current cutting. Thus the risk-averse borrower has double reason to increase current cutting over the levels associated with pure price uncertainty.

Assume now that covariance is positive (with $S < 0$). Then, on the LHS of [10], interest rate variance and covariance terms are of opposite signs and the level of marginal return depends on their relative magnitudes. On the RHS of [10], in turn, the timber price variance and covariance terms are working in opposite directions, leaving it ambiguous whether the RHS increases or decreases. The reaction of cutting relative to other cases can be identified only by defining variances and covariance empirically.

If the owner is a lender and covariance is negative, the interest rate variance and covariance terms on the LHS and timber price variance and covariance terms in the RHS are again working in opposite directions. If the covariance is positive, both variance and covariance terms decrease the LHS and the RHS of [10]. The net effect on cutting depends

on which one declines more. In principle, all outcomes are possible. Timber supply may increase, decrease, or remain constant depending on relative magnitudes of variances and covariance.

Case 5 for negative covariance is illustrated in Fig. 3A and 3B. (The same can easily be done for positive covariance.) In Fig. 3A, the owner is assumed to be a borrower and in Fig. 3B, a lender. Figure 3A shows how the borrower cuts even more than he does in case 4 or under single price uncertainty. The resulting cutting is denoted by x_5 . Two possibilities are drawn in Fig. 3B. First, timber price variance is assumed to be very high and interest rate variance and covariance terms about the same size. These assumptions generate typical price uncertainty outcome, where cutting increases (x_5''). Second, interest rate variance is very high and price variance is approximately the size of the covariance. Then, interest rate uncertainty dominates and the owner cuts less (x_5').

Forest taxation and timber supply

What happens to timber supply if the rate of forest taxes changes and other things remain constant? What happens when the "other things" that were held constant are allowed to vary? The tax rates change as a result of the discretionary policy of the (forest) tax authorities. They behave according to public goals and constraints which have to be taken into account. It is assumed that the tax authorities keep government tax revenue constant but by changing tax rates they seek a tax base that will lead to the highest current level of timber supply.

The ceteris paribus effects of forest taxation

According to the standard comparative static analysis, the effects of taxation are decomposed into wealth and substitution effects, often called the Slutsky decomposition (see Varian 1992; Atkinson and Stiglitz 1989; in the forest context, Koskela 1989a). The basic idea of Slutsky equations is to decompose the change in endogenous variables into changes in utility and market behaviour. The first is called the wealth effect and the second, the substitution effect, which in the context of this paper shows the change in the timing of cutting between now and the future due to a change in the marginal

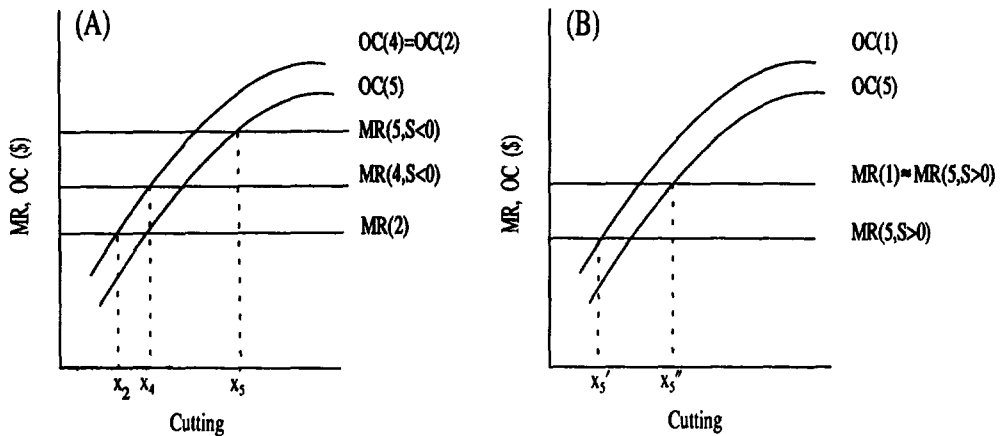


FIG. 3. Cutting (x) under two negatively correlated random variables. (A) Borrower. x_5 , Cutting for a borrower. (B) Lender. x_5' , Cutting for a lender under high interest rate variance; x_5'' , cutting for a lender under high timber price variance.

revenue and the opportunity cost of cutting. (This decomposition is obtained by calculating the "compensated substitution" terms, denoted by x_{T1}^c , x_{T2}^c , etc., according to the procedure of Koskela 1989a.)

The wealth effect of lump sum taxes

The effect of a change in the lump sum tax on timber supply during the first period is

$$[15] \quad x_{T1} = D^{-1} \{ V_{cx} A^2 \exp(-Ac_1) \} > 0 \text{ if } \sigma_{pr} \leq 0$$

$$= ? \text{ if } \sigma_{pr} > 0$$

For a change in the lump sum tax during the second period, it holds that $x_{T2} = -x_{T1}$. Thus, taxes have a symmetrical effect, but in opposite directions. It is immediately clear that the sign of x_{T1} is the same as the sign of V_{cx} . Therefore, the result will depend on the interest rate variance and covariance. If the covariance is nonpositive, the wealth effect is negative, otherwise it is a priori unknown. This means the wealth effect is not zero even though constant absolute risk aversion is assumed. The result is familiar from single interest rate uncertainty models (Ollikainen 1990), and in sharp contrast with the price uncertainty model (Koskela 1989a), where constant absolute risk aversion implied that wealth effects were zero.

The emergence of the wealth effect results from a decrease in income due to the tax change decreasing current consumption and saving, because $1 - c_{11} > 0$. First-order conditions show that decreased saving increases the marginal revenue of cutting via interest rate variance and affects the opportunity cost of cutting through the covariance term, which reflects the effect of price uncertainty. Therefore, if covariance is zero, marginal revenue goes up and the owner cuts more. If a high interest rate is associated with a low future timber price, then negative covariance strengthens the effect of interest rate uncertainty and the owner cuts more. If the covariance is positive, indicating that high interest rate is associated with high future timber price, everything depends on the relative magnitudes of interest rate variance and covariance. In principle, all possible outcomes are available; timber supply may increase or decrease.

This implies an important conclusion. Under uncertain real interest rate and future timber price, lump sum taxes are not neutral, even if the absolute risk aversion of the representative forest owner is assumed to be constant. Further, if the tax authorities wish to affect timber supply by changing the lump sum taxes, they have to know the sign of covariance. If covariance is positive, the sign of the wealth effect depends on the relative magnitudes of interest rate variance and covariance. Timber supply may even decrease as a response to increased lump sum tax during the first period! Thus, lump sum taxation is no longer a convenient, neutral instrument in the hands of government.

Unit taxes

A change in unit tax decreases post-tax timber prices and the owner's income and causes both wealth and substitution effects. A change in unit tax during the first period decreases the post-tax marginal return of cutting and tends to decrease timber supply during the first period, as $x_{T1}^c = D^{-1} \{ \beta A \exp(-Ac_1) V_{cc} \} < 0$ suggests. The substitution effect during the second period, $x_{T2}^c = -(1 + F') \phi^{-1} x_{T1}^c > 0$. The whole outcome is a priori ambiguous, because the wealth effects during both periods are of the opposite sign to the substitution effects.

$$[16] \quad x_{T1} = x x_{T1} + x_{T1}^c$$

$$[17] \quad x_{T2} = z x_{T2} + x_{T2}^c$$

Yield taxes

A change in yield tax during the first period produces almost the same result for unit tax: $x_{T1} = p_1 x_{T1}$. However, considerable differences emerge in the supply effect of a change in yield tax during the second period:

$$[18] \quad x_{T2} = [\bar{p}_2 - A((1 - \tau_2)z\sigma_p^2 + S\sigma_{pr})]x_{T2} + z\sigma_{pr}x_{T1}^c$$

The result is expressed through eq. 17 and compensated interest rate term effect. If the covariance is zero, eq. 18 reduces to $x_{T2} = [\bar{p}_2 - A(1 - \tau_2)z\sigma_p^2]x_{T2}$, which is ambiguous owing to eq. 17. If the covariance is not zero, the outcome can be decomposed into two parts. The first part is expressed

TABLE 1. Comparative static results of cutting under constant absolute risk aversion

Reference	x_{T1}	x_{t1}	$x_{\tau1}$	x_{T2}	x_{t2}	$x_{\tau2}$
Koskela 1989	0	—	—	0	+	?
Ollikainen 1990	+	?	?	+	?	?
Ollikainen 1991	+	?	?	+	?	?

NOTE: T , lump sum tax; t , unit tax; τ , yield tax.

1a unit tax and the second, via the interest rate substitution effect.⁴ This new term can be explained as a change in future yield tax that changes the risk associated with future timber price and interest rate, thus changing the marginal return of cutting. If covariance is positive (negative), it goes up (down) and tends to increase (decrease) current cutting. The overall sign of [18], however, remains ambiguous owing to [17]. The comparative static results of this and single uncertainty models presented in Table 1 indicate the sign of the change in timber supply under constant absolute risk aversion.

The relative effectiveness of taxation

It is assumed that government keeps tax revenue constant and wishes to increase timber supply by increasing one tax and decreasing another. By assumption, government is risk neutral and uncertainty has no implications on its behaviour. Thus, government uses the expected values of interest rate and future timber price. Further, it is assumed that tax revenue and tax rates are positively correlated, i.e., a higher tax rate brings higher tax revenue to the government. Another way of expressing this is to say that the so-called Laffer curve is positively sloped (for the proof and the technique see Koskela 1989a). The expected present value tax function of government is expressed as

$$19] \Gamma = T_1 + t_1x + \tau_1p_1x + (1 + \bar{r})^{-1} \times [T_2 + t_2z + \tau_2\bar{p}_2z]$$

In the following, the different forms of taxation are switched while keeping tax revenue constant. These findings can be called the relative effectiveness of forest taxation. This study leads to five different cases, which are developed, using the comparative static results of cutting, and interpreted below.

Case 1: decrease T_1 and increase t_1

Assume that government decreases lump sum tax and increases unit tax during the first period. The planned switch must be chosen so that the total tax revenue will not change. This holds when the change in tax rates and the possible change in timber supply fulfill the condition that the total derivative of the tax revenue function is zero, i.e., $d\Gamma = 0$. From this condition it follows that the decrease in T_1 must be effected as follows: $dT_1 = -(x dt_1 + m-dx)$, where $m = t_1 + \tau_1p_1 - (1 + \bar{r})^{-1}(1 + F') [t_2 + \tau_2\bar{p}_2]$. Defining the change

⁴The effect of a change in the interest rate can be analyzed as a change in the expected value (with its variance as a constant) or as a change in the variance (with its expected value as a constant). The Slutsky decomposition for the former is $x_r = Sx_{r2} + x_r^{\#} > 0$ for positive S and nonpositive σ_{pr} , otherwise a priori unknown. The substitution effect is defined as follows: $x_r^{\#} = D^{-1}\{\beta A \exp(-AC_2) [p^{\#}V_{cc} + V_{cc}]\} > 0$. The Slutsky decomposition for the latter is simply $x_{\sigma_r} = -Ax_r$.

in timber supply due to the change in the forest tax base as $dx = x_{T1}dT_1 + x_{t1}dt_1$ and substituting $dT_1 = -(xdt_1 + m dx)$ yields $dx/dt_1 = (1 + mx_{T1})^{-1}[x_{t1} - xx_{T1}]$. The denominator is positive owing to the upward-sloping Laffer curve. Thus, the nominator determines the sign. Using the comparative static analysis results in eqs. 15 and 16 yields

$$[20] \frac{dx}{dt_1} \Big|_{(T_1, t_1)} = [1 + mx_{T1}]^{-1} x_{t1}^c < 0$$

Thus, switching from lump sum tax to unit tax decreases timber supply, because unit tax decreases the marginal revenue of cutting. The same holds for the tax switch between lump sum and yield tax.

Case 2: decrease t_1 and increase τ_1

Now government shifts the tax base from unit taxation towards yield taxation. This tax switch produces

$$[21] \frac{dx}{d\tau_1} \Big|_{(t_1, \tau_1)} = 0$$

showing that timber supply does not change. It does not matter, in terms of timber supply, which one of these two taxes is used.

Case 3: decrease T_2 and increase t_2

The second-period tax switch between T_2 and t_2 that satisfies the condition of constant tax revenue produces

$$[22] \frac{dx}{dt_2} \Big|_{(T_2, t_2)} = [1 + (1 + \bar{r})mx_{T2}]^{-1} x_{t2}^c > 0$$

The outcome depends on the sign of the substitution effect of the unit tax. The result of the tax switch is to increase current timber supply, because higher unit tax decreases the opportunity cost of cutting. The outcome is the same as in the price or interest rate uncertainty models.

Case 4: decrease T_2 and increase τ_2

A tax switch between future unit and yield tax produces

$$[23] \frac{dx}{d\tau_2} \Big|_{(T_2, \tau_2)} = [1 + (1 + \bar{r})mx_{T2}]^{-1} \times [x_{\tau_2} - \bar{p}_2zx_{T2}] > 0 \text{ if } \sigma_{pr} = 0$$

$$= ? \text{ if } \sigma_{pr} \neq 0$$

The result is positive if the covariance is zero, but is otherwise ambiguous.

Case 5: decrease t_2 and increase τ_2

The switch from unit to yield taxation during the second period produces

$$[24] \frac{dx}{d\tau_2} \Big|_{(t_2, \tau_2)} = [1 + (1 + \bar{r})^{-1}mz_{t2}]^{-1} \times [x_{\tau_2} - \bar{p}_2zx_{t2}] = ?$$

which is a priori unknown, because of the opposite sign of the wealth and substitution effects of unit taxation.

The results from cases 1-5 are collected in Table 2, which also includes the respective results from the single uncertainty models of Koskela (1989a) and Ollikainen (1990). Interesting

TABLE 2. Relative effectiveness of forest taxes in terms of timber supply

Reference	Tax switch between the first and second period		
	T, t	T, τ	t, τ
Koskela 1989	-, +	-, ?	0, ?
Ollikainen 1990	-, +	-, +	0, ?
Ollikainen 1991	-, +	-, +	0, ?

TABLE 3. Nominal borrowing rate, lending rate, and timber price and consumer price index for Finland (1960-1991)

	Nominal borrowing rate	Nominal lending rate	Nominal timber price	Consumer price index
1960	6.97	4.52	21.9	138
1961	6.99	4.48	25.4	141
1962	7.03	4.50	25.2	147
1963	7.14	4.54	26.8	154
1964	7.25	5.35	31.9	170
1965	7.47	5.04	38.4	178
1966	7.53	4.87	35.7	185
1967	7.56	5.57	32.2	195
1968	7.70	5.70	27.5	212
1969	7.71	4.22	31.5	217
1970	7.76	4.53	35.6	223
1971	8.74	5.31	41.5	237
1972	8.17	4.81	39.4	254
1973	8.96	5.57	47.2	284
1974	9.83	6.30	106.3	333
1975	10.02	6.31	112.0	392
1976	10.14	6.32	91.3	449
1977	9.99	6.05	109.2	506
1978	8.51	4.44	97.1	544
1979	8.51	4.47	104.0	583
1980	10.17	6.23	120.0	651
1981	10.19	6.29	147.5	729
1982	9.58	5.79	160.0	797
1983	9.88	6.13	153.8	865
1984	10.57	6.71	163.8	925
1985	11.57	6.33	182.5	980
1986	10.65	4.76	178.5	1015
1987	10.60	4.61	168.9	1052
1988	10.79	5.53	184.5	1104
1989	12.12	5.79	207.2	1177
1990	13.77	6.26	216.8	1249
1991	13.56	6.26	221.6	1301

NOTE: Borrowing and lending rates were obtained from the Bank of Finland. The timber price was the price for coniferous sawn logs obtained from the statistical yearbook of Finland. Consumer price index was also obtained from the yearbook.

comparisons can be made. First, tax switches during the first period lead to the same qualitative outcome in every model. Second, tax switches between T_2 and t_2 yield the same results in every model. The differences come from yield tax in the second period, because of differing assumptions about future timber price. It was considered certain in Ollikainen (1990), uncertain in Koskela (1989a), and uncertain in a different way in this paper. One important conclusion can be made: the way uncertainty prevails does not change the relative effectiveness of forest taxes.

TABLE 4. Basic statistics

Variable	Real borrowing rate	Real lending rate	Real timber price
n	32	32	32
Mean	1.86	-2.02	2.5
SD	4.22	3.74	22.69
Variance	17.84	13.95	514.60
Minimum	-7.70	-11.41	-33.02
Maximum	9.39	2.30	107.97

Discussion and empirical evidence

Mean-variance analysis as the selected framework for analyzing the effects of two sources of uncertainty on cutting implies some limitations on the results, but these mainly concern the effect that a change in the risk of an uncertain interest rate or future timber price has on timber supply. Nevertheless, a number of new interesting findings emerged and many old ones were established as special cases in the model.

In most cases, the sign of covariance or the relative sizes of variance were decisive for the results. An unambiguous conclusion emerged for independent or negatively correlated ($\sigma_{pr} < 0$) variables. Most importantly, there were some features of asymmetry between the harvesting behaviour of lenders and borrowers, as in single interest rate uncertainty models. Borrowers will cut more than under price uncertainty and lenders will cut less; how much less depends on the size of the covariance and variances. For positively correlated price and interest rate variables, everything depends on the size of risks associated with timber price and interest rate.

Empirical evidence from Finnish data (1960-1991) are presented to illustrate the effects of the price and interest rate variances and covariance on timber supply (Table 3). These nominal variables are transformed into real variables as follows. By subtracting the annual inflation rate (e_t) from nominal interest rate (i_t) the real interest rate (r_t) is obtained. The annual inflation rate is calculated from the consumer price index. The most normal way of obtaining real timber prices is to deflate them, for example, by the consumer price index. The model analyzed in this paper, however, requires that the size of interest rate and price variances must be comparable. This is made possible by defining the real relative change in timber price (\hat{p}_t) as $\hat{p}_t = (p_t - p_{t-1})/p_{t-1} - e_t$.

The size of variance describes the degree of uncertainty associated with timber and financial capital. Using transformed data, Table 4 shows that timber price risks are larger than interest rate risks. This is expected because of the cyclical nature of world demand for paper and other wood products.

The sign and size of the covariance in a small, open, wood product exporting economy like Finland or Sweden is predicted to be negative for the following reasons. Timber price is highly dependent on the world price of wood products. A high timber price means indirectly that the volume of export is high and the whole economy is healthy. Raising exporting and timber prices tends to lower the real interest rate, because the nominal interest rate was controlled up to the 1990s in both countries. Therefore, the covariance between timber price and interest rate is probably negative.

Borrowing and lending rates correlate negatively with timber price and positively with each other (Table 5). The correlation coefficient between price and interest rate is negative,

TABLE 5. Correlation matrix

	Real borrowing rate	Real lending rate	Real timber price
Real borrowing rate	1.000	0.940	-0.213
Real lending rate		1.000	-0.218
Real timber price			1.000

Note: Significant correlation at $\alpha = 0.05$ for $n = 30$ is 0.288.

TABLE 6. Covariance matrix

	Real borrowing rate	Real lending rate	Real timber price
Real borrowing rate	17.840	14.820	-20.376
Real lending rate		13.947	-18.502
Real timber price			514.600

but is not significant at the 5% level in a one-sided test. The covariances between interest rate and timber price are negative and slightly larger than the interest rate variances (Table 6).

What further light does this shed on the first-order conditions of double uncertainty? First, the case of two independent random variables seems to be the most relevant, at least in Finland. Regardless of the size of variance the borrower cuts more than the owner under single price uncertainty. The behaviour of the lender is dependent on the size of variance in eq. 14. Empirical judgement of eq. 14 is complicated by the savings and yield tax rates, but some conclusions can be drawn. For countries (like Finland and Sweden) with low savings rates and low or zero yield tax rates, the opportunity cost decreases more than the marginal return and the owner cuts more relative to the risk-neutral case, but clearly less than under single price uncertainty.

Second, the possibility of negatively correlated random variables in eq. 10 cannot be ruled out. In this case, the borrower increases his cutting but the reactions of the lender are dependent on the relative sizes of interest rate variance and covariance. The former turned out to be slightly smaller, but covariance is multiplied with the yield tax rate and depends on its size. Therefore, firm conclusions can be drawn only in a country that uses some other forest tax than yield tax. For this scenario, under double uncertainty, risk-averse borrowers harvest more timber than under single price or interest rate uncertainty, whereas risk-averse lenders cut less than under price uncertainty.

The third important and empirically open question is the wealth effect of the lump sum tax. If covariance is zero or negative, a rise in lump sum tax increases current harvesting. Therefore, lump sum taxes are not neutral taxes, although constant absolute risk aversion is assumed. The same result that was generated in the single interest rate uncertainty model is different from the single price uncertainty model where the wealth effect was zero.

Finally, a provisional statement concerning the tax base can be made. If forest tax policy aims at diminishing the risks associated with timber price and interest rate, yield tax has

the advantage of reducing price uncertainty, which is the more important of the two. This would lead to a lower level of harvesting. If this is considered as an undesirable goal, the land site (lump sum) tax is preferable.

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3

The Effect of Nontimber Taxes on the Harvest Timing—The Case of Private Nonindustrial Forest Owners: A Note

Markku Ollikainen

ABSTRACT. The paper focuses on how lump sum, exogenous labor income and capital income taxes influence timber supply in a two-period model, when future timber price is uncertain. First, the usual comparative static results are derived. Second, the relative effects of taxes on the timber supply are studied, on the assumption that government keeps the tax revenue constant. It will be established that the entire taxation system, i.e., even nontimber taxes, affect timber supply. Moreover, the supply effects of government tax policy depend crucially on capital market conditions. Under perfect capital markets a switch from lump sum tax towards labor income tax has no effect on supply, and on the other hand, a tax switch toward capital income tax (increased interest payment deductability) decreases timber supply. Under credit rationing the sign of the former switch depends on the relative magnitudes of labor income during the first and second period, and the latter increases timber supply. FOR. SCI. 37(1):356-363.

ADDITIONAL KEY WORDS. Timber supply, taxation system, capital market, risk-bearing.

THE TRADITIONAL FISHERIAN TWO-PERIOD MODEL has recently proved useful in the analysis of timber supply (Johansson and Löfgren 1985, Koskela 1989a, 1989b, Max and Lehman 1988). By adding the growth function of trees to the intertemporal consumption-saving setting, the model can be used to analyze how various parameters affect the timing of the cuttings and timber supply in the short run (or one rotation period), which differs from the supply concept of rotation models.

The two-period model omits the classical rotation aspect and sustainable yield requirements. The justification of this model lies in its simplicity. Under imperfect capital market the net present value of forest rotations is not well defined, because it depends on the time pattern of forest owner's consumption. This would require reformulation of the rotation model (more closely, see the presentation of Tönö Puu's work in Johansson and Löfgren 1985, 11-16). This interdependence of consumption and cutting decision can be analyzed quite easily in the two-period consumption-cutting model.

Using the two-period model, Koskela studied how forest taxation (lump sum, yield, and unit tax) affects the cutting decision. He analyzed timber price uncertainty under perfect and imperfect capital markets. In both cases the Fisherian separation theorem does not hold, and the preferences of producers affect the cutting decision. This clearly changes the traditional forest taxation results based on the optimum rotation framework (see Chang 1982, 1983, or Klemperer 1976).

This note presents an attempt to enlarge the analysis of taxation and taxation policy. It is evident that under nonseparation, all exogenous variables affect timber supply. Therefore the taxation issue can and must be enlarged to include the whole taxation system. The focus of attention is now: how do nontimber taxes affect timber supply? It is my intention to analyze the supply effects of lump sum, exogenous labor income and capital income taxes. The analysis concentrates on the nonindustrial forest owners, who are the dominant suppliers of timber in Scandinavia.

The lump sum tax is defined as a fixed amount of tax, which is independent of the amount of timber sold or other sources of income. The labor income tax is levied on the exogenous labor income of the forest owner. The capital income tax is assumed to be a tax levied on savings returns. In the case of negative savings it is interpreted as interest payment deductability. Taxes have wealth, liquidity, and substitution effects depending on the state of the capital markets and assumptions about the risk bearing behaviour.

Once the first-order conditions and the usual comparative static results have been derived, taxation policy will be studied more closely from the point of view of the relative effectiveness of taxes. The question will be asked: given the tax forms and the behavior of forest owners, what tax structure will guarantee the largest supply of timber given government's goals? In order to define the relative effectiveness of taxes, government is supposed to raise one tax and compensate it with a decline in another, thus keeping its tax revenue constant.

TIMBER SUPPLY AND TAXATION UNDER PERFECT CAPITAL MARKETS CONDITIONS AND PRICE UNCERTAINTY

THE MODEL

The theoretical framework of the two-period model is described in detail in the two above-mentioned studies and will be dealt with only briefly here. Those readers who are interested in more thorough coverage are referred to Koskela 1989a and 1989b. Actually, the assumptions made are the standard ones (see, e.g., Sandmo 1969 or Atkinson and Stiglitz 1980). The basic point is just to add the growth function of trees to the model.

The forest owner is assumed to have a three times differentiable, intertemporally additive utility function with the consumption of both periods as arguments.

$$U = U(C_1) + (1 + \delta)^{-1}U(C_2), \quad (2.1)$$

where C_i denotes consumption and $(1 + \delta)$ is the rate of time preference, and it is assumed that $U' > 0$, $U'' < 0$.

Let Q denote the volume of forest in a given forestland area. The growth function of the forest is assumed to be increasing at a decreasing rate in Q ; i.e., $F'(Q) > 0$, $F''(Q) < 0$. Given the amount of cutting during the first period x , and the initial density of forest Q , the cutting during the second period is uniquely determined.

$$z = (Q - x) + F(Q - x) \quad (2.2)$$

Equation (2.2) shows immediately that $\partial z/\partial x = -(1 + F') < 0$, thus the more timber cut today, the less available tomorrow. Note that all the timber is assumed to be cut during the second period.

Timber prices are denoted by p_1 and \hat{p}_2 . The circumflex above p_2 denotes price uncertainty during the second period. Following the standard method of analyzing decisions under uncertainty, it is assumed that the forest owner knows the expectation value and the probability distribution of the uncertain price. (For the basic rationality postulates of consumer behavior under uncertainty, see Arrow 1974, 90-109.) The owner gets an exogenous labor income I_i , $i = 1, 2$. The tax rate on labor income is t , so the after-tax labor income is $(1 - t)I_i$. Now we can define the owner's flow of funds [Equation (2.3)], where savings are denoted by S and lump sum tax by T .

$$C_1 = p_1x + (1 - t)I_1 - T - S \quad (2.3)$$

During the second period, the forest owner sells his/her timber at an uncertain price, receives an after-tax exogenous labor income and interest income on the savings. The capital income tax, denoted by t_r , is levied on the interest income. If the pretax interest rate is denoted by $(1 + r) = \hat{R}$, then the aftertax interest rate is $(1 + r(1 - t_r)) = R$. Therefore, consumption during the second period is as follows

$$C_2 = \hat{p}_2z + (1 - t)I_2 - T + RS. \quad (2.4)$$

The owner's intertemporal budget constraint can be expressed as Equation (2.5), where savings are deduced from Equation (2.3) and substituted for S in Equation (2.4).

$$C_2 = \hat{p}_2z + (1 - t)I_2 - T + R[p_1x + (1 - t)I_1 - T - C_1] \quad (2.5)$$

TAXATION AND TIMBER SUPPLY UNDER UNCERTAINTY AND PERFECT CAPITAL MARKET CONDITIONS

The price of timber during the second period being uncertain, consumption is also uncertain. Therefore the owner maximizes his expected utility over the two periods: $\max U = U(C_1) + (1 + \delta)^{-1}E[U(C_2)]$. The forest owner is assumed to be risk averse, $U''(C_2) < 0$. The decision variables are cutting and consumption. Substituting the right hand side of

the intertemporal budget constraint for C_2 in this expression, a formula for maximization is obtained.

$$\text{MAX}_{(C_1, x)} V = U(C_1) + (1 + \delta)^{-1} E[U(\hat{p}_2 x + (1 - t)I_2 - T + R(p_1 x + (1 - t)I_1 - T - C_1))] \quad (2.6)$$

The first-order conditions for optimum are now

$$V_c = U'(C_1) - \beta E[U'(C_2)] = 0 \quad (2.7)$$

$$V_x = (1 + \delta)^{-1} E[U'(C_2)e] = 0, \quad (2.8)$$

where

$$\beta = R(1 + \delta)^{-1} \text{ and } e = -(1 + F')\hat{p}_2 + Rp_1.$$

As it was pointed out in Johansson and Löfgren 1985 and Koskela 1989a, two important implications can be drawn from the first-order condition [Equation (2.8)]. First, price uncertainty implies that cutting today will increase compared with the certainty case. Secondly, the cutting decision is no longer separable from the forest owner's preferences, and the consumption decision affects the cutting decision. Therefore the two decisions have to be analyzed simultaneously.

The second-order conditions are

$$V_{cc} = U''(C_1) + \beta RE[U''(C_2)] < 0 \quad (2.9)$$

$$V_{xx} = (1 + \delta)^{-1} \{E[U''(C_2)e^2]\} + F''E[U'(C_2)\hat{p}_2] < 0. \quad (2.10)$$

The determinant D of the Hessian matrix is positive as required for a maximum. Assuming the forest owners to have decreasing absolute and constant relative risk aversion the cross derivative, $V_{cx} = -\beta E[U''(C_2)e] \geq 0$ (for the proof see Koskela 1988a).

Given the second-order conditions, first-order conditions define implicitly C_1^* and X^* as a function of exogenous variables, $C_1^* = C_1^*(T, t, t_p, \dots)$ and $X^* = X^*(T, t, t_p, \dots)$. The supply effects of changes in the taxes can be studied by the comparative statics analysis.

A change in the lump sum and labor income tax causes only a wealth effect, as Equations (2.11) and (2.12) show. An increase in the lump sum (labor income) tax will decrease a forest owner's future consumption. With lowered consumption possibilities, decreasing absolute risk aversion implies that he or she becomes less willing to take future risks and therefore increase their present cutting. Under constant absolute risk aversion, a change in consumption possibilities will not change the level of risk and there will be no supply effects

$$X_T = D^{-1} \{(1 + \delta)^{-1} (1 + R) U''(C_1) E[U''(C_2)e]\} \geq 0 \quad (2.11)$$

$$X_t = (1 + R)^{-1} (RI_1 + I_2) X_T \geq 0 \quad (2.12)$$

The effect of a change in the capital income tax on supply can be expressed as a sum of wealth and substitution effects by using the dual approach.

$$X_{t^*} = (1 + R)^{-1} S X_T + X_{t^*}^c = ? \text{ as } S > 0 < 0 \text{ as } S < 0 \quad (2.13)$$

The wealth effect is positive (negative) to the lender (borrower) under decreasing absolute risk aversion. A rise in the capital income tax reduces the lender's consumption, and decreasing absolute risk aversion implies increased cutting. A rise of the interest payment deductibility increases the borrower's consumption, and the decreasing absolute risk aversion implies less cutting. The substitution effect describes a change in the profitability of forestry due to a change in the effective interest rate. It is negative irrespective of the owner's position in the capital market. Under constant absolute risk aversion, the wealth effect is zero and the substitution effect determines the reaction of supply.

TIMBER SUPPLY INCENTIVES AND TAXATION POLICY

Throughout this section it will be assumed that the government wishes to find a tax base that will lead to the maximum current supply of timber. For this purpose the government

is assumed to simultaneously alter two taxes by increasing one and compensating it by decreasing the other, keeping the tax revenue constant.

In order to simplify the analysis, tax rates and government tax revenue are assumed to correlate positively, which means that the so-called Laffer-curve is upward-sloping (see Fullerton 1982). Government collects its income from three sources: exogenous labor income, capital income, and lump sum taxes according to Equation (2.14), where $\hat{R} = (1 + r)$, i.e., the pretax discount rate. The tax revenue is certain because uncertainty concerns only future timber price.

$$\Gamma = (I_1 + \hat{R}^{-1}I_2)t + \hat{R}^{-1}t_rS + (1 + \hat{R}^{-1})T, \quad (2.14)$$

where

$$S = p_1x + (1 - t)I_1 - T - C_1.$$

The results of tax switches are stated directly as proposition 1, which is followed by a detailed interpretation.

Proposition 1

When the capital market is perfect, the future timber price is uncertain and the target of taxation policy is to keep government tax revenue constant, then irrespective of risk-bearing behavior (i) an increase in the exogenous labor income tax, which is compensated by an equivalent decrease in the lump sum tax, has no effect on timber supply, and (ii) an increase in the capital tax (interest payment deductibility), which is compensated by an equivalent change in the lump sum (labor income) tax, decreases timber supply.

The supply effect of a compensated change between lump sum and exogenous labor income taxes will be worked out in details. Other cases will be similarly derived, and only the final results will be stated and interpreted.

Differentiating Equation (2.14) with respect to t , T , x and C_1 with Γ constant and solving it for dT gives

$$dT = -\alpha_1 dt - \alpha_2 dx + \alpha_3 dC_1, \quad (2.15)$$

where

$$\alpha_1 = [RI_1 + I_2]/[1 + R], \quad \alpha_2 = p_1\alpha_3, \quad \alpha_3 = [t_r]/[1 + R]$$

and $R = (1 + r(1 - t_r))$ is the aftertax discount rate.

The effect of tax change on consumption and cutting can be expressed as in Equations (2.16) and (2.17).

$$dx = x_t dt + x_T dT \quad (2.16)$$

$$dC_1 = c_t dt + c_T dT \quad (2.17)$$

Substitute Equation (2.15) for dT and solve it for dx .

$$(1 + \alpha_2 x_T) dx = (x_t - \alpha_1 x_T) dt + \alpha_3 x_T dC_1 \quad (2.18)$$

Use then Equation (2.15) in (2.17) and solve it for dC_1 .

$$dC_1 = (1 - \alpha_3 c_T)^{-1} [(c_t - \alpha_1 c_T) dt - \alpha_2 c_T dx] \quad (2.19)$$

Substitute Equation (2.19) for dC_1 in (2.18) and solve it for x in Equation (2.20), where $\Phi = (1 + \alpha_2 x_T - \alpha_3 c_T)$.

$$dX^*/dt = \Phi^{-1} [x_t - \alpha_3 c_T x_t - \alpha_1 x_T + \alpha_3 x_T c_t] \quad (2.20)$$

According to the assumption of positively sloping Laffer-curve, the denominator Φ is always positive, so the analysis can be concentrated on the sign of the numerator. (The positivity of Φ is shown in Appendix 2 for a compensated change between T and t_r . Other cases operate similarly.) Equation (2.20) turns out to be zero when the comparative static results derived earlier and the respective results of consumption presented in Appendix 1 are substituted in the equation and calculated for. This means that the compensated tax

change has no effect on the supply of timber. Both taxes are independent of forestry, not changing the profitability of forestry and no change in cutting behaviour will take place.

$$\left. \begin{aligned} dX^*/dt &= 0 \\ \Gamma &= \Gamma^0 \\ T &- \text{compensation} \end{aligned} \right\} \quad (2.21)$$

The result of a compensated change between the capital tax and the lump sum tax is stated in Equation (2.22), where α_i 's and Φ are defined as before and $m = 1/(1 - t)$. The equation is characterized by the compensated supply and consumption terms X_{tr}^c and C_{tr}^c , which are weighted by income effects. The compensated change between taxes changes the effective interest rate, which affects the conditions of cutting and consumption and is the cause for the emergence of the substitution terms. The interpretation of X_{tr}^c (and it can be applied to C_{tr}^c) is as before in Equation (2.9). The substitution terms reinforce each other, and timber supply will decline.

$$\left. \begin{aligned} dX^*/dt_r &= \Phi^{-1}[(1 + mt_r C_{11})X_{tr}^c - mt_r X_{12} C_{tr}^c] < 0 \\ \Gamma &= \Gamma^0 \\ T &- \text{compensation} \end{aligned} \right\} \quad (2.22)$$

The supply effects of a compensated change between labor income and capital income taxes are the same as (2.30), only the denominator, Φ differs and is defined as $\Phi = (1 + \alpha_2 x_t + \alpha_3 c_t) > 0$, $\alpha_2 = p_1 \alpha_3$ and $\alpha_3 = t_r / (RI_1 + I_2)$. So, the change will decrease timber supply.

Proposition 1 has now been established.

TAXATION AND TIMBER SUPPLY UNDER PRICE UNCERTAINTY AND CREDIT RATIONING

In the previous section, forest owners were assumed to be able to borrow or lend money without limit at a constant interest rate. This assumption will now be relaxed. It is assumed that capital market is imperfect. It is assumed that a binding quantitative credit rationing exists in the sense that forest owners would be willing to borrow more at the prevailing interest rate, but they are not allowed to. Credit rationing is exogenous to the model, i.e., the behavior of the bank sector is omitted here.

Under credit rationing the forest owner is subject to a binding upper limit of borrowing, $B = \bar{B}$, which he/she would like to, but cannot, exceed. The flow of funds during the first period is defined by Equation (3.1) according to which consumption cannot be greater than the sum of timber and labor income and loan minus taxes.

$$C_1 \leq p_1 x + (1 - t)I_1 + \bar{B} - T \quad (3.1)$$

Substituting B for $-S$ (S as defined earlier) in the flow of funds, Equation (3.1), the intertemporal budget constraint of the forest owner can be expressed as follows:

$$C_2 = \hat{p}_2 z + (1 - t)I_2 - T + R(p_1 x + (1 - t)I_1 - T - C_1) \quad (3.2)$$

Given the utility function of the owner, the constrained maximization problem produces the following first-order conditions, where μ is the Lagrangian multiplier.

$$\begin{cases} L_c = U'(C_1) - \beta E[U'(C_2)] - \mu = 0 \\ L_x = (1 + \delta)^{-1} E[U'(C_2)e] + \mu p_1 = 0 \\ L_\mu = p_1 x + (1 - t)I_1 - T + \bar{B} - C_1 = 0 \end{cases} \quad (3.3)$$

Condition $L_x = 0$ implies that $(1 + \delta)^{-1} E[U'(C_2)e]$ is now negative. This means that credit rationing increases timber supply compared with the previous perfect capital market uncertainty case where $V_x = 0$ implied that $(1 + \delta)^{-1} E[U'(C_2)e]$ was zero. The nonseparability of preferences from the production decision is also obvious.

Second-order conditions are given in Equation (3.4), and the determinant H , of the Hessian matrix, is positive as maximum requires.

$$\begin{cases} L_{cc} = U''(C_1) + \beta RE[U'(C_2)] < 0 \\ L_{xx} = (1 + \delta)^{-1} \{E[U'(C_2)e^2] - F''E[U'(C_2)\hat{p}_2]\} < 0 \\ L_{\mu\mu} = 0 \end{cases} \quad (3.4)$$

The comparative static results of tax changes are conveniently described by using the income effects of Equation (3.5). It shows that the effects of a change in the labor income are not zero even under constant absolute risk aversion. This means that liquidity effects substitute for the wealth effects, which were important under perfect capital markets. The results are marked with a tilde because their properties differ from the perfect capital market case.

$$\begin{cases} \tilde{X}_{I1} = H^{-1}\{\hat{p}_1 U''(C_1)\} < 0 \\ \tilde{X}_{I2} = H^{-1}\{-(1 + \delta)^{-1}(1 + F')(1 - t)E[U''(C_2)\hat{p}_2]\} > 0 \end{cases} \quad (3.5)$$

The supply effects of changes in lump sum and labor income taxes induce liquidity effects (3.6) and (3.7). A rise in the lump sum (labor income) tax requires increased cuttings during the first period in order to cover increased expenditure. On the other hand, increased tax payments during the second period require more timber to be sold then, and this tends to reduce cutting during the first period. The final outcome depends on the relative magnitude of these two opposite liquidity effects

$$\tilde{X}_T = -(1 - t)^{-1}(X_{I1} + X_{I2}) = ? \quad (3.6)$$

$$\tilde{X}_t = -(1 - t)^{-1}(I_1 X_{I1} + I_2 X_{I2}) = ? \quad (3.7)$$

Under credit rationing capital tax is interpreted as the (unlimited) deductibility of interest payments. The use the indirect utility approach shows that the substitution effect X_{tr}^c is zero, only liquidity effect remains operative.

$$\tilde{X}_{tr} = -(1 - t)^{-1} r S X_{I2} > 0 \quad (3.8)$$

The positive liquidity effect might be interpreted as follows: A rise in interest payment deductibility decreases the effective interest rate and also lowers interest payments during the second period. Therefore the forest owner may diminish the volume of timber reserved for the second period and increase the cutting during the first period. Note too, that a rise in the interest rate decreases the current cutting, which is a counterintuitive but reasonable result—the higher the interest rate, the more the forest owner needs money during the second period.

Finally, the relative effectiveness of taxes in terms of timber supply under credit rationing is summarized in proposition 2. The derivation of these results is exactly the same as earlier, therefore the proof is omitted here.

Proposition 2

When there exists (exogenous) credit rationing, the future timber price is uncertain, and the target of tax policy is to keep government tax revenue constant, then irrespective of the risk-bearing behavior: (i) an increase in the labor income tax, which is compensated by a decrease in the lump sum tax, leads to increased (decreased) timber supply if the forest owner's income is greater (smaller) in the first period than in the second period, and (ii) an increase in the interest payment deductibility, which is compensated by an equal rise in the lump sum or income tax increases, the supply of timber.

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APPENDIX 1: COMPARATIVE STATICS OF TAXES WITH RESPECT TO FIRST PERIOD'S CONSUMPTION

PERFECT CAPITAL MARKETS

$$C_T = D^{-1}\{-\beta(1+\delta)^{-1}(1+R)\{E[U(C_2)]E[U(C_2)e] - (E[U(C_2)])^2\} + F''E[U'(C_2)\hat{p}_2]E[U''(C_2)]\} = ?$$

$$C_t = (1+R)^{-1}(RI_1 + I_2)C_T = ?$$

$$C_{tr} = (1+R)^{-1}rSC_T + C_{tr}^c = ?$$

where

$$C_{tr}^c = -D^{-1}\{(1+\delta)^{-2}rE[U'(C_2)]\{F''E[U'(C_2)\hat{p}_2] + (1+F')E[U''(C_2)e\hat{p}_2]\} > 0$$

CREDIT RATIONING

$$C_T = -(1-t)^{-1}[C_{T1} + C_{T2}] < 0$$

$$C_t = -(1-t)^{-1}[I_1C_{T1} + I_2C_{T2}] < 0$$

$$C_{tr} = -(1-t)^{-1}rSC_{T2} > 0,$$

where

$$C_{T1} = D^{-1}\{-(1+\delta)^{-1}(1-t)(F''E[U'(C_2)\hat{p}_2] + (1+F')^2E[U''(C_2)\hat{p}_2^2])\} > 0$$

$$C_{T2} = D^{-1}\{-(1+\delta)^{-1}(1-t)(1+F')\hat{p}_1E[U''(C_2)\hat{p}_2]\} > 0$$

APPENDIX 2: THE LAFFER-CURVE AND THE COMPENSATED CHANGE IN TAXES

It will be shown that assumptions about the upward-sloping Laffer-curve imply positive denominator [Equation (2.22)]. The procedure will be the same in other cases. In the text, the following form was obtained for the *compensated* change between T and t_r :

$$dX^*/dt_r = \Phi^{-1}[(1 + mt_r C_{I1})X_{I1}^c - mt_r X_{I2} C_{I2}^c] < 0.$$

Recalling that $S = (p_1 x + (1 - t)I_1 - T - C_1)$ in the government tax revenue function [Equation (2.14)] and differentiating it with respect to T , x , and C_1 , produces an *uncompensated effect* of the change of T on the tax revenue function.

$$d\Gamma = \Gamma_T dT + \Gamma_c dC + \Gamma_x dX$$

Next the equation is divided by dT . Because interest now focuses only on the effects of a change in one tax, c_t can be substituted for dC/dt and x_t for dX/dt . Then, Equation (2.14) gives $\Gamma_c = -R^{-1}t_r$ and $\Gamma_x = R^{-1}t_r p_1$. The uncompensated effect of a change in the lump sum tax on the tax revenue function is the following.

$$\left. \frac{d\Gamma}{dt} \right|_{dt_r = 0} = \Gamma_T [1 + R^{-1}t_r(p_1 x_t - c_t)] / \Gamma_T \quad (*)$$

The assumption of the upward-sloping Laffer-curve implies that Equation (*) has to be positive. Differentiating Equation (2.14) with respect to T shows that $\Gamma_T = (1 + R)$ is positive. Therefore the terms in brackets also have to be positive. Let us use the above definition of Γ_T and calculate the terms in brackets. It is then clear that the following expression has to be positive by assumption $[1 + t_r(p_1 x_T - c_T)/(1 + R)] > 0$. However, if the definitions of α_2 and α_3 are used, the expression can be written as $[1 + \alpha_2 x_T - \alpha_3 c_T] = \Phi > 0$.

4

Markku Ollikainen*

**THE ANALYTICS OF TIMBER SUPPLY AND FOREST TAXES
UNDER ENDOGENOUS CREDIT RATIONING
- SEPARABILITY AFTER ALL****

ABSTRACT

In his celebrated essay, Paul Samuelson defined the "heroic" assumptions of perfect foresight and perfect capital and land markets, under which the harvesting decision is separable from forest owners' preferences, and under which the Faustmann rotation model offers an adequate model of harvesting behavior. By exogenously assuming the existence of imperfect capital markets, some analysts have concluded that the separability, Faustmann analysis and conventional wisdom about forest taxation is not valid under imperfect capital markets. In this paper credit rationing is endogenized to analyze the harvesting behavior of nonindustrial private forest owners and the effects of forest taxation on timber supply in a two-period model. It turns out that, contrary to exogenous credit rationing models, the cutting decision is separable from the forest owner's preferences under endogenous credit rationing. The liquidity effects of forest taxes are also absent undermining the ceteris paribus effects derived in exogenous credit rationing models. The paper defines the preferable tax base subject to the government budget constraint in terms of harvesting incentives and derives the optimal forest tax design in terms of the welfare of forest owners. It is shown that, under credit rationing, it is optimal to introduce yield tax at the margin even though the land productivity tax has been chosen optimally. The optimal yield tax rate is greater than zero but less than 100%, which differs from the perfect capital market case, where the 100% yield tax rate is optimal.

Key words: credit rationing, default risk, optimal forest taxation

JEL classification: Q23, H21, Q28

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1. INTRODUCTION

In his seminal article in this journal, Paul Samuelson defined the "heroic assumptions" behind the Faustmann rotation model. If capital and land markets are perfect and all economic variables known for certain, then the timing of harvests can be solved from the sum of the present value of an infinite series of rotations. The rotation length depends on timber price, the interest rate, planting costs, the growth function of trees and other purely economic factors, not on the owner's preferences (Samuelson 1976, Chang 1982, Hyde 1980 and, Johansson and Löfgren 1985). Conventional theory of the effects of forest taxes has been derived in this framework: a site productivity tax (as a lump-sum type tax) is neutral, but other forest taxes are distortionary and affect the rotation time (see, e.g., Chang 1982 and Jackson 1980).

The assumptions behind the Faustmann model imply that the harvesting decision is separable from the forest owner's preferences. Separability implies that the owner's consumption plans and other preference related factors do not affect cutting decisions. Samuelson regarded his assumptions as first approximations which can be relaxed without major changes in the analysis. Recently, this view has been challenged. The assumption of perfect capital markets has been relaxed by assuming that they are imperfect because of credit rationing. This gives a raise the question of how a loan ceiling on borrowing possibilities affects cutting (see Koskela 1989a and also Kuuluvainen 1990). Koskela argues that, under credit rationing, the owners cannot necessarily harvest their forest stands at the optimal point of time (as defined by perfect capital market conditions), because they must finance their consumption. Thus, harvesting is not separable from preferences, and the Faustmann analysis is invalid. What is more, he also argues that the effects of forest taxation will differ from those of conventional wisdom.

These new results seem intuitively appealing. When forest owners have an exogenous upper limit on borrowing, which they would like to exceed but cannot, in the absence of other income sources, they can adjust their behavior only through cutting. Thus, they can no longer necessarily guarantee equality between the forest growth rate and the interest rate, as they could under certainty and perfect capital markets. For these reasons the standard results of forest taxation must be corrected to take liquidity effects into account, but the question remains, how seriously should this message be taken? Do we really have to believe, for example, that a site productivity tax is non-neutral in terms of timber supply? The answer is, not necessarily. Credit rationing in Koskela's model was an exogenous assumption, not explained through the behavior of the banks and forest owners. It is important to understand whether these results change if credit rationing is endogenized in the model. Does the separability behind the Faustmann analysis still prevail and, more interestingly, what should the proper design of forest taxation be under credit rationing? Answering these questions is the task of the present paper.

The credit rationing literature offers many possibilities for this kind of examination. Credit rationing is broadly defined as a situation in which there is an excess demand for loans, because quoted loan rates are below the Walrasian market-clearing level (Jaffee and Stiglitz 1990). There are three different hypotheses concerning the way credit rationing appears in the capital markets. They are the following: credit rationing as the quantitative limit on the amount of borrowing, (called loan ceiling in this paper) (Stiglitz and Weiss 1981), an endogenously determined wedge between the borrowing and lending rate (King 1986), and a nonlinear interest rate, i.e., an interest rate that increases as a function of the amount borrowed (Jaffee and Russel 1976, Keeton 1979).

In this paper, credit rationing is endogenized by using the nonlinear interest rate hypothesis, because the harvesting problem can be quite conveniently included in the framework. The basic features of the model to be developed here are the following and similar in some respects to those of Webb (1984) and de Meza and Webb (1992) (for further applications of the two-period model, see Montgomery and Adams (1995)). Forest owners are assumed to be identical in that among other things, they have similar attitudes towards risk-taking. A forest owner asks for a loan during the first period in order to finance consumption plans while intending to pay it back with future timber sale revenues. Future timber price is uncertain but the owner and the bank know the probability distribution of the timber price. Information in the model is thus imperfect but symmetrical.¹ If the timber price realization is high enough, then the owner is able to pay back the loan, but under sufficiently low realizations he defaults. Banks can determine the probability of default as a function of the probability distribution. The default risk affects the profits of banks and is the source of credit rationing. The study is organized as follows. The basic model is developed and the equilibrium solution analyzed in section 2. The effects of forest taxation are analyzed from various viewpoints in section 3. After having calculated the comparative statics of forest taxes, the incentive effects in terms of timber supply are solved. The focus is then extended towards a novel analysis, the welfare effects of forest taxes.

2. THE MODEL

2.1 The representative forest owner as a borrower

The cutting possibilities of the owner are defined over periods one and two. Given cutting during the first period (x), the growth function of forest ($g(Q)$) and the original volume of timber (Q), then future cutting (z), is uniquely determined. Equation (1) shows that the more the forest owner cuts today, the less is available tomorrow, i.e. $dz / dx = -(1 + g')$.

¹ Symmetrical information is enough for the emergence of credit rationing. It is also a very plausible assumption in the Scandinavian countries and the USA, which have well-functioning roundwood markets and established procedures for forecasting future timber prices.

$$(1) \quad z = (Q - x) + g(Q - x), \text{ with } g'(Q - x) > 0, g''(Q - x) < 0.$$

During the first period, the representative forest owner receives timber income from cutting. If p_i , $i=1,2$ denotes pre-tax timber prices, this pre-tax income is p_1x . The owner faces two different forms of forest tax: site productivity and yield taxes (T_i, τ_i) , the rates of which differ between periods 1 and 2. Site productivity tax is a lump-sum tax, which is the benchmark case in the forest taxation literature. It can be regarded as a simple property tax, which is independent of the level of timber selling or silvicultural activities. Yield tax is chosen as a representative of general harvest taxes and is levied on timber sale revenue.² After-tax timber prices are therefore defined as $p_i^* = p_i(1 - \tau_i)$, $i=1,2$.

The owner demands a bank loan B at the quoted interest rate $R = (1 + r)$. Consumption during the first period can be written as equation (2).

$$(2) \quad c_1 = p_1^*x + B - T_1$$

During the second period, the owner receives after-tax timber-selling income p_2^*z , and pays site productivity tax T_2 . Let $K = \tilde{p}_2^*$ denote the highest possible (finite) realization of p_2^* . Then the expected future timber price $E[p_2^*] = \int_0^K p_2^* f(p_2^*) dp_2^*$. Under a favourable realization of the future

timber price, the forest owner earns a positive income over interest payments. Assuming limited liability, under low realization, the owner defaults on the bank loan. It is assumed that he is guaranteed a minimum security income by the state, which can be omitted from the calculations without losing generality.

Let \underline{p}_2^* be that critical price which is just high enough to allow the forest owner to pay back the loan and interest on it according to equation (3). This equation shows that by assumption, under a default, the government has the right to take its taxes before the bank has the right to reclaim its money. The critical $\underline{p}_2^* = [RB + T_2] / z$ depends on B , which is endogenous, making the critical price endogenous too. The higher the interest rate, the size of the loan, the current harvest and the site productivity tax, the higher the critical value, \underline{p}_2^* and the probability of default, other things being equal. This is counter-affected by future cutting so that the higher the z , the lower the value of \underline{p}_2^* .

$$(3) \quad \underline{p}_2^* z - T_2 - RB = 0$$

Given the critical value of \underline{p}_2^* , equation (4) defines the owner's expected consumption during

² The assumption of differing tax rates come from exogenous credit rationing models, but the comparative statics of constant forest tax rates will also be analyzed.

the second period.

$$(4) \quad c_2 = \int_{p_2^*}^K [p_2^* z - T_2 - (1+r)B] f(p_2^*) dp_2^*$$

Assume that the owner's preferences are described by an additive utility function $u(c_1) + \beta u(c_2)$, where $\beta = (1+\rho)^{-1}$ is the time preference factor. Substituting equations (2) and (4) for c_1 and c_2 into the target function indicates that the owner's problem is to choose the size of bank loan and timber cutting so as to maximize his utility from consumption in equation (5).³

$$(5) \quad \text{MAX } E[U] = u(p_1^* x + B - T_1) + \beta \int_{p_2^*}^K u(p_2^* z - T_2 - (1+r)B) f(p_2^*) dp_2^*$$

Assume, first, that the interest rate is fixed, i.e. that the owner can freely borrow at a constant interest rate. This case will help to later trace out how the existence of credit rationing changes the owner's cutting behavior. In what follows, the partial derivatives are denoted by primes for functions with one argument and by subscripts for functions with many arguments. The optimal choice of current cutting and borrowing satisfies the following first-order conditions.

$$(6a) \quad E[U]_x = p_1^* u'(c_1) - \beta \int_{p_2^*}^K u'(c_2) p_2^* (1+g') f(p_2^*) dp_2^* = 0$$

$$(6b) \quad E[U]_B = u'(c_1) - \beta \int_{p_2^*}^K u'(c_2) R f(p_2^*) dp_2^* = 0$$

$E[U]_B = 0$ implies that $u'(c_1) = \beta \int_{p_2^*}^K u'(c_2) R f(p_2^*) dp_2^*$. Using this in $E[U]_x = 0$ gives the familiar

cutting rule under price uncertainty: the marginal return on cutting $R p_1^*$, must be equal to the opportunity cost of cutting $(1+g') p_2^*$ over all favorable realizations of p_2^* .

$$(7) \quad \beta \int_{p_2^*}^K u'(c_2) [R p_1^* - p_2^* (1+g')] f(p_2^*) dp_2^* = 0$$

³ Note that (5) could equivalently be written as $u(p_1^* x + B - T_1) + \beta E[u(p_2^* z - T_2 - (1+r)B)]$ by defining

$E[u(c_2)] = \int_{p_2^*}^K u(c_2) f(p_2^*) dp_2^*$. However, because the definition of the default risk through p_2^* is a crucial part of the

later analysis, the density function formulation is used throughout the paper.

This rule is analyzed in detail in Koskela 1989b and related to various other modifications in Ollikainen 1993. Two properties of rule (7) will be important for later analysis. First, price uncertainty means that the cutting decision does not separate from preferences. Second, because of risk-aversion, the owner cuts more timber today relative to conditions under certainty. Both properties become apparent by comparing (7) with the corresponding rule under certainty, where the probability distribution is absent, and $Rp_1^* = p_2^*(1 + g')$ holds at the margin.

Finally, it is useful to derive the isoutility curves of the forest owner. These curves will be employed in the graphic illustration of equilibrium in the credit market and the comparative statics of timber supply in the following sections. Isoutility curves show the combinations of r and B that keep the forest owner's utility constant for a given level of cutting, x and given exogenous parameters. They have the properties described in Remark 1, which is proved in Appendix 1. Figure 1 on page 9 describes the isoutility curves (denoted by k_1 and k_2) and also illustrates the relationship between isoutility curves and the demand-for-loans curve, $d(B)$, which cuts the isoutility curves at their top.

REMARK 1: *The isoutility curves of the representative forest owner in $\{r, B\}$ -space increase up to the point where they cut the demand curve for loans, and decrease thereafter. Moreover, as a low interest rate is in the owner's interests, the lower the isoutility curves, the higher the utility with which they are associated.*

2.2 The banking sector

By assumption, perfect competition and free entry prevails in the banking sector. Risk-neutral banks maximize their expected profits given that it is uncertain whether the forest owner will be able to pay back the loan. Banks get their funds from competitive markets at some constant cost, at the deposit rate $I = (1+i)$. They will get all their loans back with interest (RB) if the realization of p_2^* is equal to or higher than \underline{p}_2^* . If p_2^* is smaller than \underline{p}_2^* , banks only receive a part of their expected income. It is assumed that the government has the right to collect its taxes from the defaulting owner's income before banks get their share. The expected profit function of a representative risk-neutral bank can thus be written as follows

$$E[\pi] = RB \int_{\underline{p}_2^*}^{\bar{p}_2^*} f(p_2^*) dp_2^* + \int_0^{\underline{p}_2^*} [p_2^* z - T_2] f(p_2^*) dp_2^* - IB.$$

This expression can be manipulated into the more convenient form given by equation (8).⁴ The target function indicates that the bank will get its money back at the quoted rate R with the probability of $1 - F(p_2^*)$, and will suffer some losses with the probability of $F(p_2^*)$.

$$(8) \quad E[\pi] = RB \left[1 - F(p_2^*) \right] + \int_0^{p_2^*} [p_2^* z - T_2] f(p_2^*) dp_2^* - IB$$

Under perfect competition and free entry, bank profits will be zero and banks are obliged to behave under the zero-profit condition, $E[\pi] = 0$. The implications of this condition become apparent through the zero-profit curve, the properties of which are given in remark 2. For proof, see appendix 1. The bank's zero-profit curve $E[\pi] = 0$ is the loan supply curve and is shown in Figure 1 on page 9.

REMARK 2: *The bank zero-profit curve, $E[\pi] = 0$ in $\{r, B\}$ -space is convex, indicating that when the loan size increases, the interest rate has to be raised in order to keep the expected profits at zero.*

Differentiating (8) with respect to the interest rate, cutting and taxes reveals how the interest rate charged by banks changes with changes in loan size cutting and forest taxes. An increase in future (current) forest taxes increases (has no effect on) the quoted rate. Increasing harvesting and the size of the loan tends to increase the quoted rate as suggested by equation (9), where $\alpha = B(1 - F(p_2^*))$

$$(9) \quad \begin{aligned} \frac{dr}{dB} = r_B &= - \frac{R(1 - F(p_2^*)) - I}{\alpha} > 0 \\ \frac{dr}{dx} = r_x &= \frac{\int_0^{p_2^*} (1 + g') p_2^* f(p_2^*) dp_2^*}{\alpha} > 0 \end{aligned}$$

⁴ To get equation (8), add to and at the same time subtract the term $RB \int_0^{p_2^*} f(p_2^*) dp_2^*$ in $E[\pi]$. This produces

$$E[\pi] = RB \left(\int_0^K f(p_2^*) dp_2^* - \int_0^{p_2^*} f(p_2^*) dp_2^* \right) + \int_0^{p_2^*} (p_2^* z - T_2) f(p_2^*) dp_2^* - IB.$$

Integrating the terms in braces produces the cumulative distribution function of p_2^* , denoted by $F(p_2^*)$ in equation (8).

$$\frac{dr}{dT_2} = r_{T_2} = \frac{\int_0^{p_2^*} f(p_2^*) dp_2^*}{\alpha} > 0$$

$$\frac{dr}{d\tau_2} = r_{\tau_2} = \frac{\int_0^{p_2^*} p_2 z f(p_2^*) dp_2^*}{\alpha} > 0.$$

2.3. Loan contract and equilibrium in the credit market: cutting, borrowing and the quoted rate

Borrowers are assumed to be identical and the banking sector competitive with free entry. Since the bank knows the probability of default, and free competition in the banking sector prevails, the optimal contract between the bank and the owner can be defined by maximizing the objective function of the representative forest owner subject to the zero-profit condition of the bank. Thus, in this case, the social planner problem and the determination of equilibrium in the credit market have the same form. An equivalent way to determine the optimal contract and market equilibrium is to write the interest rate as a function of the zero-profit condition in problem (4), as is done in (10).

$$(10) \text{ MAX } P = u(p_1^* x + B - T_1) + \beta \int_{p_2^*}^K u[p_2 z - T_2 - (1 + r(x, B, T_2, \tau_2))B] f(p_2^*) dp_2^*$$

Differentiating (10) with respect to x and B by applying the Leibnitz rule of differentiation of integral functions, recalling the endogeneity of the critical price p_2^* , and setting the resulting derivatives equal to zero produces

$$P_x = p_1^* u'(c_1) - \beta \int_{p_2^*}^K u'(c_2) [(1 + g') p_2^* + r_x B] f(p_2^*) dp_2^* = 0 \text{ and}$$

$$P_B = u'(c_1) - \beta \int_{p_2^*}^K u'(c_2) [R + r_B B] f(p_2^*) dp_2^* = 0,$$

where the fact that the derivative of P with respect to critical price p_2^* is zero has been utilized. Using expressions (9) for r_x and r_B in the first-order conditions (10) yields

$$P_x = p_1^* u'(c_1) - \beta \int_{p_2^*}^K u'(c_2) \left((1+g') p_2^* + \frac{\int_{p_2^*}^{p_2^*} (1+g') p_2^* f(p_2^*) dp_2}{1-F(p_2^*)} \right) f(p_2^*) dp_2^* = 0 \text{ and}$$

$$P_B = u'(c_1) - \beta \int_{p_2^*}^K u'(c_2) \left(R - \frac{R(1-F(p_2^*)) - I}{1-F(p_2^*)} \right) f(p_2^*) dp_2^* = 0.$$

Adding (subtracting) the terms in $P_x = 0$ (in $P_x = 0$) and noting that $(1-F(p_2^*)) = \int_{p_2^*}^K f(p_2^*) dp_2^*$

produces the first-order conditions (11).

$$(11a) \quad P_x = p_1^* u'(c_1) - \beta \int_{p_2^*}^K u'(c_2) (1+g') E[p_2^*] f(p_2^*) dp_2^* \left[1 - F(p_2^*) \right]^{-1} = 0$$

$$(11b) \quad P_B = u'(c_1) - \beta \int_{p_2^*}^K u'(c_2) I f(p_2^*) dp_2^* \left[1 - F(p_2^*) \right]^{-1} = 0,$$

where $E[p_2^*] = \int_0^K p_2^* f(p_2^*) dp_2^*$.

The second-order conditions for the maximization problem are stated in equation (12). They hold because of assumptions of the concavity of the forest growth function and the utility function of the forest owner.

$$(12a) \quad P_{xx} < 0, P_{BB} < 0, P_{xB} < 0$$

$$(12b) \quad \Delta = P_{xx} P_{BB} - (P_{xB})^2 = \beta g'' P_{BB} \left[1 - F(p_2^*) \right]^{-1} \int_{p_2^*}^K u'(c_2) E[p_2^*] f(p_2^*) dp_2^* > 0$$

The first-order conditions are worth closer analysis. Solving $u'(c_1)$ from $P_B = 0$ and using it in condition $P_x = 0$ produces $\int_{p_2^*}^K u'(c_2) \left\{ p_1^* I - (1+g') E[p_2^*] \right\} f(p_2^*) dp_2^* \left[1 - F(p_2^*) \right]^{-1} = 0$. Given that

the probability of success is positive and $u'(c_2)$ is always positive, the terms in braces must, therefore, be zero for the condition to hold. The following new cutting rule now emerges.

$$(13) \quad p_1^* I = (1 + g') \int_0^K p_2^* f(p_2^*) dp_2^* = (1 + g') E[p_2^*] = 0$$

Equation (13) suggests that the cutting decision separates from the preferences of the owner. This result is really surprising when contrasted to earlier results. Section 2.1 revealed that under perfect capital market conditions and price uncertainty, preferences affect the owner's cutting decision. Moreover, nonseparability also occurs in loan ceiling models. The questions to be answered are, then, why does endogenous credit rationing lead to separability and why does the amount of cutting still differ from that of perfect capital market models? Let us start with the first question. Why does the level of cutting depend on the deposit rate i , and the whole distribution of future timber price $E[p_2^*]$, although they do not even belong to the owner's target function? Why does risk-aversion have no role in optimal cutting? The answer to the first question is as follows. The risk-neutral bank's zero-profit curve determines the size of the cut through the loan contract. This suggests that the difference from exogenous loan ceiling models must lie in the treatment of the default risk. In loan ceiling models, the upper limit of borrowing is used as a means of rationing and of preventing the default risk from going too high. In the nonlinear interest rate model, higher loans are given at a higher quoted rate. The quoted rate the bank charges, reflects the default risk, which depends on the size of the loan, the amount of timber left for the second period, and on the distribution of future timber price. The price distribution is exogenous, but timber volume can be affected. By fixing current cutting in the loan contract, the bank ensures that there will be enough timber for the repayment during the second period. As the bank is risk-neutral, it uses the deposit rate and the expectation value of the future timber price in determining the size of the harvest.

The answer to the second question is straightforward. By assumption the deposit rate reflects the competitive interest rate. Thus, the marginal revenue of cutting in (13) is the same as in perfect capital market models ($i = r$), but there is a difference in the opportunity cost of harvesting. As the bank takes into account the whole distribution of the future timber price, the opportunity cost of cutting decreases relative to perfect capital market conditions and conventional price uncertainty under risk-neutrality. The reason for this lies in the fact that yield tax cuts the price distribution at the upper integration level in (13) so that $\int_0^{\bar{p}_2^*} p_2^* f(p_2^*) dp_2^* < \int_0^{\bar{p}_2} p_2^* f(p_2) dp_2$.

Therefore, current cutting will be higher under equilibrium credit rationing relative to exogenous credit rationing. If institutional arrangements allow the banks to get their money first in the case of default, then the yield tax level would not affect the expected price distribution, and cutting would be the same as in perfect capital markets. Notice finally, that the site productivity tax does not affect cutting at the margin, i.e. it is neutral, while the yield tax is distortionary. Only if banks were allowed to get their money first in the default, would a constant yield tax be neutral in terms of timber supply. Thus the institutional arrangements in

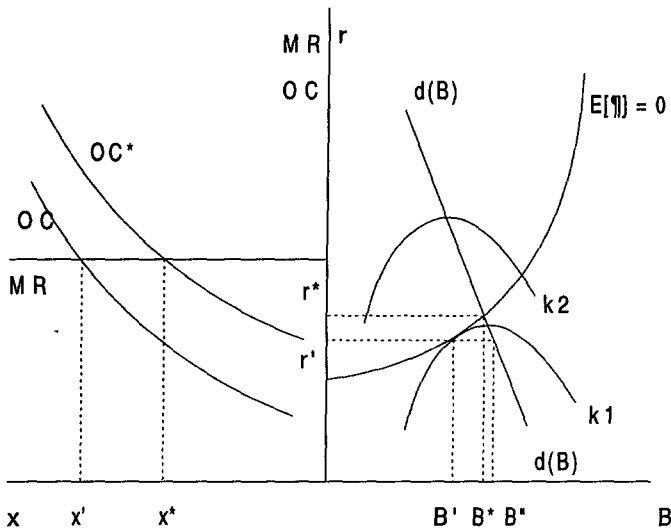
the case of default crucially affect timber supply under (endogenous) credit rationing.

These observations have been summarized as proposition 1.

Proposition 1: *Under endogeneous credit rationing modelled on the nonlinear interest hypothesis, the cutting decision is separable from the forest owner's preferences and is determined only by relative prices, the deposit rate and the tree growth function. If in the case of default, the government collects taxes before (after) the banks get their money, current timber supply is higher (the same) than (as) under certainty or risk-neutrality.*

The contract between the bank and the representative owner can be interpreted as representing the credit market equilibrium described in the north-east part of Figure 1. The loan (i.e., zero-profit) supply and demand curves intersect each other at point (r^*, B^*) . This, however, is not the equilibrium of the model, which occurs at point (r', B') , where the isoutility curve k_1 is tangential to the isoprofit curve. At interest rate r' , the forest owner would be willing to demand B'' , which is his optimum, but he is not allowed to as the banks would ask for a higher quoted rate for B'' . For this reason, Jaffee and Russel (1976) call this equilibrium (r', B') a rationing equilibrium. It is interesting to ask why borrowers are satisfied with equilibrium. The answer lies in the trade-off between loan size and loan rate. Increasing the loan size is connected with an increasing loan rate. Borrowers thus seek the optimal trade-off between the two and find it at the tangent point of the curve and highest possible isoutility curve, point (r', B') , even though they are moving away from the demand curve. The north-west part of Figure 1 describes how the short-term timber supply is related to the capital market conditions according to the harvesting rule (13). The marginal revenue of harvesting, $p_1^* I$, is described by a straight line MR and the opportunity cost of harvesting, $(1+g')E[p_2^*]$ by a convex curve OC. Optimal harvesting under endogenous credit rationing is determined by their intersection leading to the supply of x' . If capital markets were perfect, the opportunity cost curve would be the higher OC^* , and the supply correspondingly the smaller x^* , as was proved above.

FIGURE 1: Equilibrium interest rate, loan size and harvesting: perfect and imperfect capital markets



3. COMPARATIVE STATICS OF FOREST TAXATION UNDER ENDOGENOUS CREDIT RATIONING

In this section, the effects of forest taxes on timber supply, borrowing and the interest rate are analyzed in more detail. This is interesting per se -- e.g. to see whether liquidity effects exogenous credit rationing models do emerge -- and the results will be used later to examine the incentive and welfare effects of government tax policy. The comparative statics will be developed for both transitory (differing tax rates) and steady-state (constant tax rates) cases. The former facilitates comparisons with the results of exogenous credit rationing models and the latter with those of the traditional rotation models.

3.1 Transitory effects of forest taxes

Assume that the bank and the owner can and do renegotiate a new contract without any cost whenever the exogenous parameters of the model change. This allows one to solve the conventional ceteris paribus effects of forest taxes on cutting, borrowing and interest rate. Note that the comparative static results of the interest rate can be derived directly from the zero-profit condition. Denote taxes generally by θ . Then, differentiating $E[\pi]$ with respect to x , B and θ produce $dr/d\theta = r_\theta + r_B B_\theta + r_x x_\theta$ as the very formulation from which the results are easily calculated. The first term, r_θ is the direct effect of a change in θ on the quoted rate. The next

two are indirect effects arising from changes in optimal cutting and borrowing.

The wealth effect of site productivity tax

A change in site productivity tax during the first period will not change current cutting, i.e. the wealth effect is zero in (14), as was expected because of separability. It is more profitable for the owner to increase his borrowing and pay a higher quoted rate. The quoted rate goes up because of increased borrowing $r_B B_{T_1} > 0$ (as $r_x x_{T_1} = r_{T_1} = 0$).

$$x_{T_1} = 0$$

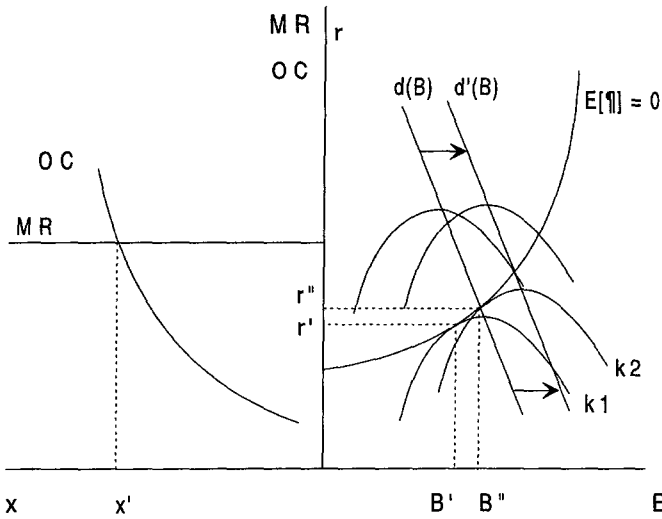
$$(14) \quad B_{T_1} = \frac{u''(c_1)}{P_{BB}} > 0$$

$$\frac{dr}{dT_1} = - \frac{[R(1 - F(p_2^*)) - I] u''(c_2)}{B[1 - F(p_2^*)] P_{BB}} > 0$$

The outcome is in sharp contrast with those derived from exogenous credit rationing models, in which the owner increases his cutting due to the liquidity effect. The difference between this and exogenous credit rationing models lies in the possibility of increasing the size of the bank loan by paying a higher quoted rate. Although a higher loan is associated with a higher interest rate, it pays both parties of the contract to adjust through borrowing, because the marginal conditions of cutting have not been changed. Figure 2 illustrates the results. The original equilibrium is a loan contract $L = L(r', B', x')$. As a result of the increase in the current site productivity tax, the loan demand curve $d(B)$ shifts to the right. A new tangency point of loan supply function and isoutility curve k_1 leads to an increased borrowing and a higher quoted rate. As the site productivity tax is neutral, no change takes place in the timber supply.⁵

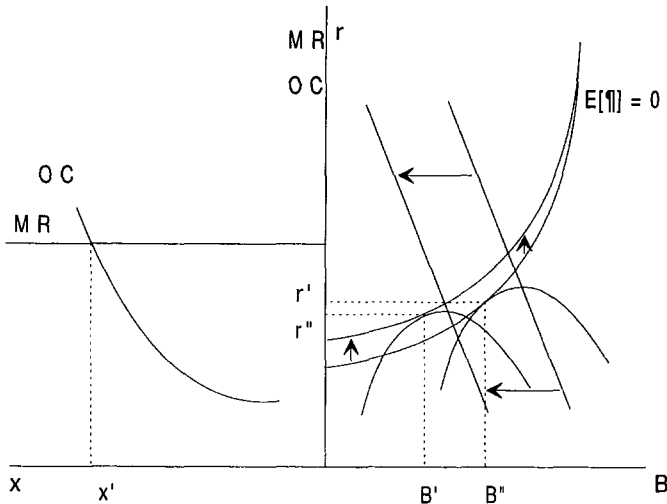
⁵ The shift of the demand for loans curve and of isoutility curves reflects the fact that, under **perfect** capital markets, $B_{T_1} > 0$. The new equilibrium under **credit rationing** is obtained at the tangency point of isoutility curves and zero-profit function. The zero-profit function depends on the second period's variables, whose change shift the zero-profit function.

FIGURE 2: Comparative statics of current site productivity tax



A change in the future site productivity tax has no effect on current cutting. However, a higher tax causes the owner to decrease the size of the bank loan. The change in the interest rate is given by equation $dr/dT_2 = r_{T_2} + r_B B_{T_1}$ (as $r_x x_{T_2} = 0$). The first term is positive and reflects increased default risk. The second one is negative owing to decreased borrowing. It plausibly dominates so that the quoted rate decreases. This is illustrated in Figure 3 where the initial loan contract is given by the tangency point between the loan supply and isoutility curves. As the site productivity tax increases, the loan supply function shifts upwards, because a higher site productivity tax increases the bank's default risk. The demand function for loans shifts downwards leading to a new credit market equilibrium (r'', B'') . As the site productivity tax does not affect the harvesting decision at the margin, timber supply will not change.

$$(15) \quad \begin{aligned} x_{T_2} &= 0 \\ B_{T_2} &= -\beta \int_{p_2^*}^k I \left[u''(c_2) [1 - F(p_2^*)] - u'(c_2) z^{-1} f(p_2^*) \right] f(p_2^*) dp_2^* [1 - F(p_2^*)]^{-2} (P_{BB})^{-1} < 0 \\ \frac{dr}{dT_2} &< 0 \end{aligned}$$

FIGURE 3: Comparative statics of future site productivity tax**The substitution effect of yield tax**

A higher yield tax decreases the marginal return of cutting, shifting some of it to the second period, the wealth effect being zero. The deficit in the income flow is compensated for by increasing the size of the bank loan. To opposing effects will change the quoted rate (as $r_{\tau_1} = 0$).

The lower level of cutting means that the default risk decreases as a greater quantity of timber is reserved for the second period ($r_x x_{\tau_1} < 0$). Increased borrowing, however, tends to increase the default risk ($r_B B_{\tau_1} > 0$). The final sign of the quoted rate depends on the relative magnitudes of $r_x x_{\tau_1}$ and $r_B B_{\tau_1}$.

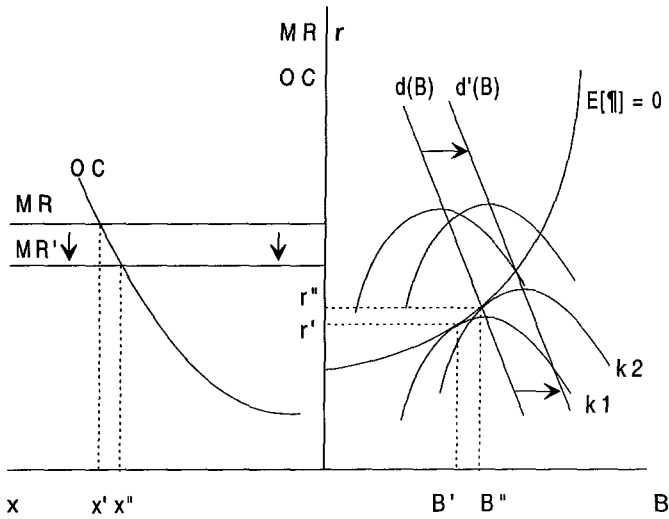
$$x_{\tau} = x_{\tau}^c = \frac{I p_1^*}{g'' E[p_2^*]} < 0$$

$$(16) \quad B_{\tau_1} = p_1 x B_{\tau_1} > 0$$

$$dr / d\tau_1 = r_x x_{\tau_1} + r_B B_{\tau_1}$$

Equation (16) is illustrated in Figure 4 for the case of $dr / d\tau_1 > 0$. The loan demand curve shifts upwards and a new equilibrium in the credit market is described by (r'', B'') . A higher current yield tax decreases the marginal return of harvesting making it profitable to shift some of the cutting to the future, thus decreasing current harvesting, i.e. $x'' < x'$.

FIGURE 4: Comparative statics of current yield tax



A higher future yield tax boosts current timber supply and decreases the size of the bank loan tending to decrease the quoted rate. On the other hand, a higher timber supply and yield tax level tend to increase it, so that the overall effect is ambiguous. If the loan size effect is dominant, the quoted rate goes down.

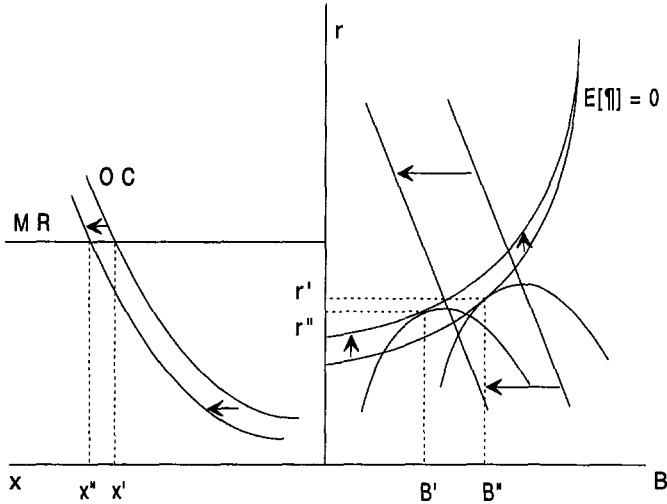
$$x_{\tau} = x_{\tau}^c = -\frac{(1+g')\{\tilde{p}_2 f(\tilde{p}_2) + E[p_2^*]\}}{g'' E[p_2^*]} > 0$$

$$(17) \quad B_{\tau_2} = E[p_2]x(1-\tau_2)^{-1}B_{\tau_2} < 0$$

$$\frac{dr}{d\tau_2} = r_{\tau_2} + r_x x_{\tau_2} + r_B B_{\tau_2}$$

Figure 5 illustrates this result for the case where the loan size effect is so great that the quoted rate decreases. The starting point is the original loan contract $L = L(r', B', x')$ and the new credit market equilibrium is (r'', B'') . A higher future yield tax decreases the opportunity cost of cutting which tends to increase current timber supply, $x'' > x'$.

FIGURE 5: Comparative statics of future yield tax



3.2 The permanent effects of forest taxes

Since a credible tax policy should not be based on changing tax rates, one must analyze the comparative statics of the model under constant tax rates. Under this assumption, the optimal conditions of credit market equilibrium defined by equations (11) - (13) still hold with the provision that one has to substitute the transitory tax rates for constant ones in the arguments of the utility function (10) and the bank's expected profit function (8). The effects of constant forest taxes on endogenous variables can be expressed as a sum of transitory effects. For the site productivity tax one obtains

$$\begin{aligned}
 x_T &= 0 \\
 (18) \quad B_T &= B_{T_1} + B_{T_2} \\
 dr / dT &= r_T + r_B B_T.
 \end{aligned}$$

The site productivity tax is neutral. The sign of borrowing, however, is ambiguous a priori. Its sign depends on the

$$\text{sgn} B_T = \text{sgn} \left(u''(c_1) - \beta \left[1 - F(p_2^*) \right]^{-2} I \int_{p_2^*}^K \left[u''(c_2) - u'(c_2)(1 - F(p_2^*))^{-1} z^{-1} \right] f(p_2^*) dp_2^* \right).$$

This expression reveals that the greater the amount of timber left to the second period and the smaller the probability of default, the more probable it is that the owner will ask for larger loan. If $B_T > 0$ the quoted rate goes up, because both the direct effects r_T and the indirect effect $r_B B_T$ are positive. A higher yield tax boosts current timber supply, because it affects the bank's expected profits negatively, as the upper level of the expected timber price goes down. Had the banks the right to take their money before taxes in the case of default, the yield tax would be neutral. Now the case for decreasing borrowing is stronger than in the case of the site productivity tax, as there is an additional negative term in B_T . The effect on the interest rate is ambiguous a priori, because the direct default risk r_T and increased cutting tend to increase the quoted interest rate but decreased borrowing tends to lower it.

$$x_\tau = x_\tau^c = -\frac{(1+g')\tilde{p}_2 f(\tilde{p}_2)}{g''E[p_2^*]} > 0$$

$$(19) \quad B_\tau = p_1 x B_{T_1} + E[p_2] z B_{T_2} + \frac{\beta(1+F(p_2^*))u'(c_2)If(\tilde{p}_2^*)}{P_{BB}}$$

$$+ \quad - \quad -$$

$$dr / d\tau = r_\tau + r_x x_\tau + r_B B_\tau$$

Proposition 2 sums up the findings under steady-state forest taxes.

Proposition 2: *Under endogenous credit rationing with nonlinear interest rate, (a) the site productivity tax is neutral in terms of timber supply, (b) the effect of the yield tax depends on the institutional arrangements; if the government (bank) first collects the taxes (its money), then in the case of default, the yield tax is distortionary (neutral) and increases (does not change) timber supply.*

3.3 The implications of the endogenous credit rationing model

Equations (18)-(19) show that the site productivity tax is neutral and the yield tax distortionary. How does this relate to the corresponding results derived in the Faustmann model? Chang (1982) observed that the site productivity tax is neutral while the yield tax is distortionary in terms of the rotation period. Given that the rotation period implicitly determines the steady-state timber supply, the immediate conclusion from Chang's analysis is that the site productivity tax does not affect steady-state timber supply. In the two-period model the overall timber supply is

given by the sum of current and future cutting according to equation (1). As the site productivity tax is neutral, it does not affect current timber supply and future cutting will not change either ($z_T = -(1+g')x_T = 0$). Consequently, the site productivity tax does not change steady-state timber supply in a two period model. The results are, thus, qualitatively the same. The yield tax increases the rotation period, but how is a longer rotation period and steady-state timber supply related in the Faustmann model?⁶ As shown, in Jackson (1980), a longer rotation period increases steady-state timber supply. In the two-period credit rationing model, the yield tax increases current timber supply and decreases future supply according to equation $z_T = -(1+g')x_T$. Thus the discounted sum of changes in the value of current and future supply is given by $x_T - I^{-1}(1+g')x_T = (i-g')I^{-1}x_T$. It is positive if $i > g'$, which is found to be a necessary condition for dynamic efficiency in OLG models applied to forestry (see Löfgren 1991). Summing up, endogenous credit rationing -- at least in this model type -- does not change the taxation results derived in the Faustmann model. Thus, the challenge posed by exogenous credit rationing models is not as demanding as it might have seemed at first glance.

4. THE CHOICE OF THE TAX BASE: INCENTIVE AND WELFARE ANALYSIS

The above analysis of credit market equilibrium and its comparative static properties provide a basis for analyzing further forest taxation issues: the incentive effects of forest taxation in terms of timber supply, and the properties of optimal forest taxation from the viewpoint of society. Solving incentive effects supplies the government with an answer to the question of which forest tax base will produce a greater or a smaller timber supply? The most interesting is, of course, to discover the properties of the optimal forest taxation under credit rationing. Before going into the detailed analysis three assumptions are needed. First, in line with optimal taxation literature, it is assumed that the tax rates are chosen subject to an exogenously given forest tax revenue requirement. Second, it is assumed that before any private decisions are made, the government announces a tax policy and commits itself to it. Third, it is assumed that the risk is idiosyncratic, i.e., that it is identically and independently distributed among individual forest owners so that the government can be regarded as risk-neutral and the tax revenue requirement is deterministic (see, e.g. Varian 1980). This implies that the government uses the competitive interest rate i , which has an important impact on the optimal forest taxation.⁷ The government budget constraint is given in equation (20) where $\bar{p} = E[p_2]$, i.e., the expected pre-tax timber price.

⁶ Note that the long run timber supply function defined by the Faustmann model is not a supply function in the usual meaning of the word. Owing to the assumption of one price independent of time, it does not tell us what happens to the quantity supplied when the price increases in a particular period. This fact explains the curious supply results of the Faustmann model - e.g. a higher timber price lowers long-term timber supply (see Johansson and Löfgren 1985, 110-111).

⁷ According to Arrow and Lind (1970), the use of perfect capital market's interest rate in government decisions is also legitimate under aggregate risk, if the risk is small.

$$(20) \quad G = T_1 + I^{-1}T_2 + \tau_1 p_1 x + I^{-1} \tau_2 \bar{p}_2 z$$

4.1 Incentive effects of forest taxes

As the comparative static results reveal, changes in forest taxes only cause substitution effects on cutting. Therefore, the incentive effects of the switches in the forest tax base in terms of timber supply can be quickly derived. Assume first, that government decreases the yield tax and increases the site productivity tax during the first period, while keeping the tax revenue constant. Differentiating (20) with respect to T_1 , τ_1 and x produces $0 = dT_1 + x d\tau_1 + m dx$, where $m = [\tau_1 - (1+i)^{-1}(1+g')\tau_2]$. The resulting change in timber supply can be defined as $dx = x_{T_1} dT_1 + x_{\tau_1} d\tau_1$. Solving the government budget constraint for $d\tau_1$ and substituting in the second equation produces

$$(21) \quad \frac{dx}{dT_1} = \frac{x_{T_1} - x^{-1} x_{\tau_1}}{1 + x^{-1} m x_{\tau_1}} > 0.$$

Comparative static results (14) and (16) indicate that $x_{T_1} = 0$ and $x_{\tau_1} < 0$; thus the denominator is positive and the numerator can also be shown to be positive if we assume that tax revenue increases when tax rates increase, i.e., under a positive Laffer-curve. Shifting the tax base towards the site productivity taxation during the first period thus increases current timber supply.

Assume next that the same switch takes place during the second period. Differentiating the budget constraint with respect to T_2 , τ_2 and x produces $0 = I^{-1} dT_2 + I^{-1} \bar{p}_2 z d\tau_2 + m dx$, with m is as above. The change in timber supply is given by $dx = x_{T_2} dT_2 + x_{\tau_2} d\tau_2$. Solving the two equations for dT_2 and dx produces (22). Because $x_{T_2} = 0$ and $x_{\tau_2} > 0$, timber supply will decrease as a result of the tax switch.

$$(22) \quad \frac{dx}{dT_2} = \frac{x_{T_2} - \bar{p}_2 z x_{\tau_2}}{1 + z^{-1} m x_{\tau_2}} < 0$$

Finally, by applying the same procedure one can show that a tax switch from yield tax towards site productivity tax under constant tax rates decreases current timber supply, as equation (23) suggests.

$$(23) \quad \frac{dx}{dT} = \frac{x_T - \bar{p}_2 z x_\tau}{1 + z^{-1} m x_\tau} < 0$$

To sum up, results (21) - (23) have established the following proposition.

Proposition 3: *Under endogenous credit rationing and an upward-sloping Laffer-curve, changing the tax base while keeping government tax revenue constant will a) under differing tax rates, increase current timber supply in a switch from yield tax to site productivity tax during the first period, and decrease current timber supply in a switch from yield tax to site productivity tax during the second period, b) under constant tax rates decrease current timber supply in a tax switch from yield taxation to site productivity tax.*

4.2 Optimal forest taxation

Optimal forest taxation under endogenous credit rationing from the viewpoint of society is to be obtained by assuming a social planner who maximizes the social welfare function by choosing the yield and the site productivity tax subject to both the government budget constraint and the behavioral and credit market constraints analyzed in the previous sections. The social welfare function consists of the sum of the expected indirect utility function of the forest owner $EU^*(T, \tau)$ and of the expected indirect profit function of the bank $E\pi^*(T, \tau)$. These functions define the maximum expected utility and the maximum expected profits given by the exogenous parameters, the constant forest taxes. However, as the competitive banking sector operates under zero-profit condition, $E\pi^*(T, \tau)$ will always be zero. The social welfare function can therefore be written only in terms of the indirect utility function of the owner, which includes the endogenous determination of cutting, borrowing and the quoted rate.

$$(24) \quad W = EU^*(T, \tau)$$

To solve the social welfare maximization problem, form the Lagrangian function $L = W + \lambda G$, where G is written in terms of constant tax rates, i.e., $G = T(1 + I^{-1}) + \tau(p_1 x + I^{-1} \bar{p}_2 z)$. The first-order conditions for the social welfare maximization under a given tax revenue requirement can be obtained by setting the partial derivatives of the Lagrangian with respect to T and τ equal to zero. Equation (25) gives the optimal condition of the site productivity tax.

$$(25) \quad L_\tau = EU_\tau^* + \lambda(1 + I^{-1}) = 0,$$

where $EU_T^* = -u'(c_1) - \beta \int_{\underline{p}_2}^{\bar{p}_2} u'(c_2) f(p_2^*) dp_2^* = -(1 + I \left[1 - F(\underline{p}_2) \right] - 1) \beta \int_{\underline{p}_2}^{\bar{p}_2} u'(c_2) f(p_2^*) dp_2^* < 0$.

As (25) shows the optimal site productivity tax has to be chosen so that the present value of the marginal utility of consumption is equal to the cost of tax, $-(1 + I)EU_T^* = \lambda$. Solving the optimal yield tax rate is more complicated, as the timber supply effects of the yield tax on the government tax revenue requirement have to be taken into account.

$$(26) \quad L_\tau = EU_T^* + \lambda \left[(p_1 x + \bar{p}_2 z I^{-1}) + (I p_1 - (1 + g') \bar{p}_2) I^{-1} \tau x_\tau^c \right] = 0,$$

with $EU_T^* = \alpha EU_T^* - \beta z \text{cov}(u'(c_2), p_2) - \beta \bar{p}_2 u(\cdot) f(\bar{p}_2^*) p_1 x u'(c_1) - \int_{\underline{p}_2}^{\bar{p}_2} p_2 z f(p_2^*) dp_2^* \beta \int_{\underline{p}_2}^{\bar{p}_2} u'(c_2) f(p_2^*) dp_2^*$,

where the rule $E(ab) = E(a)E(b) + \text{cov}(a, b)$ has been used and α is defined as follows:

$$\alpha = (1 + I \left[1 - F(\underline{p}_2) \right]^{-1})^{-1} (p_1 x I \left[1 - F(\underline{p}_2) \right]^{-1} + \int_{\underline{p}_2}^{\bar{p}_2} p_2 z f(p_2^*) dp_2^* \beta \int_{\underline{p}_2}^{\bar{p}_2} u'(c_2) f(p_2^*) dp_2^*)^{-1} \cdot^8$$

Equation (26) implicitly defines the optimal yield tax rate. (Notice that the third term in equation (26) is not the cutting rule (13) because the distribution of the future timber price is given here in terms of the pre-tax price.) If the site productivity tax has been set optimally, (26) can be written as

$$L_\tau = \alpha EU_T^* - \text{cov}(u'(c_2), p_2) - \beta \bar{p}_2 u(\cdot) f(\bar{p}_2^*) - \alpha \lambda (1 + I^{-1}) + \lambda \left[(p_1 x + \bar{p}_2 z I^{-1}) + (I p_1 - \bar{p}_2 (1 + g') I^{-1} \tau x_\tau^c \right] = 0$$

which reduces to

$$(27) \quad L_{\tau|T=\tau^*} = -\text{cov}(u'(c_2), p_2) - \beta u(\cdot) \bar{p}_2 f(\bar{p}_2^*) + \lambda \left[(I p_1 - (1 + g') \bar{p}_2) I^{-1} \right] \tau x_\tau^c + \Psi = 0.$$

where $\Psi = \lambda(p_1 x + I^{-1} \bar{p}_2 z) - \alpha \lambda (1 + I) > 0$ and reflects the special features of this endogenous credit rationing model. It emerges because the government uses the pre-tax expectation value of the future timber price but the owner (and the banks) use just the truncated price distribution (in the perfect capital market case the term would be zero as can be seen in appendix 2).

In order to see whether the yield tax is needed at all under endogenous credit rationing, it is necessary to look at the corner solutions, $\tau = 0$ and $\tau = 1$. If the yield tax rate approaches zero, it

⁸ Notice that the derivative of EU_T^* with respect to the lower intergration factor, the critical timber price, is zero.

no longer affects the integration limits of the utility function, so that the second and third terms in (27) vanish yielding

$$(28) \quad L_{\tau=0|T=T^*} = -\text{cov}(u'(c_2), p_2) + \Psi > 0.$$

The positivity of equation (28) reveals that the zero yield tax rate cannot be optimal. Thus given the optimal site productivity tax, it is welfare-increasing to introduce the yield tax at the margin. The yield tax reduces the volatility of timber prices by limiting the price distribution from below and above. Even though it raises the critical price, it increases cutting and decreases borrowing, working as a risk decreasing device. But how far should one go in increasing the yield tax rate? Not to 100%, because $\tau=1$ contradicts the interior solution. The reason for this is as follows. If $\tau=1$ and the government collects all timber-selling revenues as taxes, the optimal solution for the banks is to stop giving loans to forest owners. This, however, does not reflect the optimum conditions (11a) and (11b) which indicate a positive bank loan size. Therefore, it is not optimal to tax away all the uncertainty. Tax rate can be solved from equation (27) to yield equation (29), where $\Phi = -\beta u(\cdot) \bar{p}_2 f(\bar{p}_2^*) + \Psi$.

$$(29) \quad \tau^* = \frac{\text{cov}(u'(c_2), p_2) + \Phi}{\lambda I^{-1} [I p_1 - (1+g') \bar{p}_2] x_T^c} > 0$$

It is interesting to contrast these findings with the optimal forest taxation under perfect capital market conditions. The optimal forest tax formulas are derived in appendix 2. Under perfect capital markets (27) can be expressed as follows (the upper index denoting perfect capital markets).

$$(30) \quad L_{\tau|T=T^*}^0 = -\beta z \text{cov}(u'(c_2), p_2) - \frac{\beta z \text{cov}(u'(c_2), p_2)}{(1+I)\beta E[u'(c_2)]} x_T + \lambda \tau [I p_1 - (1+g') \bar{p}_2] I^{-1} x_T^c = 0$$

Evaluating (30) at the corners, $\tau=0$ and $\tau=1$ (and, noting that at $\tau=1$, the covariance terms vanish and $I p_1 - \bar{p}_2(1+g')=0$) one obtains

$$(31) \quad \begin{aligned} \text{a) } L_{\tau=0|T=T^*}^0 &= -\beta z \text{cov}(u'(c_2), p_2) - \frac{\beta z \text{cov}(u'(c_2), p_2)}{(1+I)E[u'(c_2)]} x_T > 0 \\ \text{b) } L_{\tau=1|T=T^*}^0 &= 0 \end{aligned}$$

It is thus immediately clear that, under perfect capital markets, it is beneficial to introduce the yield tax after the site productivity tax has been chosen optimally. The optimal tax rate is 100%. The reason for this is the fact that the yield tax functions as an insurance device against timber price uncertainty. The government simply taxes away uncertainty caused by stochastic future timber price -- the covariance term in (27) -- and redistributes the money as lump sum subsidies. This seemingly differs from solution (28) for credit rationing, under which it is not optimal to tax away all the uncertainty because of the banking sector. Notice, however, that this result is mainly of theoretical importance and without practical relevance. It is unrealistic to imagine that the 100% yield tax scheme would be implemented in practice given all the imperfections in the economy and the possible problems of moral hazard that the scheme with lump-sum transfers would cause.

The properties of optimal forest taxation under credit rationing and perfect capital markets are collected as the proposition 4.

Proposition 4: *Under endogenous credit rationing, the optimal forest tax structure consists of a combination of site productivity and yield taxes. Given the optimal site productivity tax, it is desirable to introduce the yield tax at the margin as an insurance device. The optimal yield tax rate is less than 100% under credit rationing indicating that it is not optimal to eliminate uncertainty altogether. Instead, under perfect capital markets it is optimal to set the yield tax rate to 100% and redistribute the tax revenue as lump-sum subsidies to forest owners so that the uncertainty caused by future timber price is eliminated.*

4. DISCUSSION

This paper has presented an initial application of the equilibrium credit rationing hypothesis to the analysis of timber supply and forest taxation. The credit rationing in the model was caused by timber price uncertainty and the forest owner's borrowing, which generate a default risk for the risk-neutral banks. The purpose of the paper was to study cutting behavior under credit rationing and the effects of forest taxation in terms of incentives and optimality especially. The central finding was that endogenous credit rationing implies separability and thus does not invalidate the rotation results, as was suggested by exogenous credit rationing models. This result also has clear consequences for the comparative statics. The site productivity tax turned out to be neutral and the yield tax distortionary, similarly to the rotation models. Thus the challenge of exogenous credit rationing models to traditional analysis was not as demanding as was proposed. Why was this so? The difference between endogenous and exogenous credit rationing models lies in the renegotiation of the loan contract. This makes timber supply a function of loan availability, implying that the dependence of the timber supply function on loan

availability should be explicitly taken into account in empirical analyses of imperfect capital markets. This is absent in the analyses undertaken so far.

The result of separability under endogenous credit rationing derived in this paper is not just an occasion, but an important and, in many cases, a robust result. A similar outcome has been derived in the debate about Ricardian Equivalence by Toshiki Yotsuzuka (1987). In his famous article, Barro demonstrated that whether government spending is financed by bonds or taxes has no effects on the economy under some assumptions, which include perfect capital markets (Barro 1974). The Keynesian critics of Ricardian Equivalence argue that under imperfect capital markets, liquidity constraints prevent the agent's optimal intertemporal adjustment and Barro's neutrality result will not hold. Yotsuzuka proved that endogenous credit rationing -- at least in certain types of models-- leads to optimal intertemporal adjustments and results in the neutrality of money, as was stated by Barro.

The final conclusion of this paper is that the separability of the harvesting decision from preferences is a more general feature than has been thought so far. It is even more general than Paul Samuelson assumed in his article. Credit rationing does not then necessarily lead to the rejection of rotation frameworks and the conventional results of forest taxation. The theoretical analysis of credit rationing is, however, important if we want to understand its relationship with harvesting behavior and to find a plausible hypothesis for empirical research concerning harvesting behavior under credit rationing.

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APPENDIX 1: ISOUTILITY AND ISOPROFIT CURVES

A. To prove the remark 1, fix the utility function at some level $k = u(c_1) + \beta \int_{p_2^*}^K u(c_2) f(p_2^*) dp_2^*$.

Differentiating it with respect to B and r produces

$$\frac{dr}{dB} = \frac{u'(c_1) - \beta \int_{p_2^*}^K u'(c_2) R f(p_2^*) dp_2^*}{\beta \int_{p_2^*}^K u'(c_2) B f(p_2^*) dp_2^*}$$

If the loan size chosen is optimal B^* , then $dr/dB = 0$ due to the first-order conditions. For any value of $B < B^*$, $dr/dB > 0$ and for any value of $B > B^*$, $dr/dB < 0$. Thus, around the optimum, the isoutility curves are first increasing and then decreasing. To prove the second part of the proposition, define the owner's expected indirect utility function EU^* by substituting the optimum values $x^* = x^*(B, r, \dots)$ and $B^* = B^*(x, r, \dots)$ implicitly defined by the first-order conditions (6) into the direct utility function. Applying the envelope theorem to EU^* and differentiating it with respect to r produces $EU_r^* = -\beta \int_{p_2^*}^K u'(c_2) B f(p_2^*) dp_2^*$, which is negative,

indicating that a lower interest rate leads to higher utility and showing that lower isoutility curves in $\{r, B\}$ -space are associated with higher utility.

B. To prove remark 2 set (8) equal to zero. Recalling that $p_2^* = [RB + T_2]/z$, and, $\partial p_2^*/\partial B = R/z$ and $\partial p_2^*/\partial r = B/z$, produces $dr/dB = -\left[R(1 - F(p_2^*)) - I \right] / B \left[1 - F(p_2^*) \right] > 0$.

The denominator is always positive. The numerator can also be shown to be positive as follows:

dividing (8) by $-B$ reveals that $-\left[R(1 - F(p_2^*)) - I \right] = B^{-1} \int_0^{p_2^*} [p_2^* z - T_2] f(p_2^*) dp_2^* > 0$. Further,

d^2r/dB^2 is also positive by

$$\frac{d^2r}{dB^2} = \frac{[R^2 + B^2(dr/dB)]f(p_2^*)}{zB[1 - F(p_2^*)]} > 0$$

indicating that a zero-profit curve is convex. The curve emanates from the r -axis. The loan supply is zero, when $r < i$. Under certainty, the quoted rate $r = i$ and the loan supply is infinite. Under default risk, the bank charges a higher quoted rate for larger loans. For the second part of remark 2, notice that the derivative of the bank's indirect profit function is $EP[\pi]_r = B[1 - F(p_2^*)] > 0$, indicating that a higher expected profit is associated with a higher interest rate.

* * * * *

APPENDIX 2: OPTIMAL FOREST TAXATION UNDER PERFECT CAPITAL MARKETS

Under perfect capital markets, the forest owner can borrow freely at a constant interest rate i . Therefore his maximization problem can be expressed in the way familiar from the standard version of the two-period model.

$$(1) \quad \text{Max } EU = u(c_1) + \beta E[u(c_2)],$$

where $\int_0^{\bar{p}_2} u(c_2) f(p_2) dp_2 = E[u(c_2)]$ and future consumption $c_2 = p_2^* z - T - I(c_1 - p_1^* x + T)$.

Drawing on results from previous literature, it can be shown that the site productivity tax is not neutral, but causes a positive wealth effect denoted by $x_\tau > 0$ and the influence of yield tax is given by the sum of wealth and substitution effects as follows $x_\tau = (p_1 x + p_2 z)(1+I)^{-1} x_\tau - \frac{\beta z \text{cov}(u'(c_2), p_2)}{(1+I)\beta E[u'(c_2)]} x_\tau + x_\tau^c$, where the second term refers to the risk effect and the third term to the substitution effect, (see, Koskela 1989b).

The planner chooses T and τ so as to maximize the Lagrangian function $L = W + \lambda G$, where $W = EU^*$ and G denotes the tax revenue requirement given in the text, yielding

$$(1) \quad \begin{aligned} \text{a) } L_\tau &= EU_\tau^* + \lambda(1+I^{-1}) + \lambda\tau(Ip_1 - (1+g')\bar{p}_2)I^{-1}x_\tau = 0 \\ \text{b) } L_\tau &= EU_\tau^* + \lambda[(p_1 x + \bar{p}_2 z I^{-1}) + (Ip_1 - (1+g')\bar{p}_2)I^{-1}\tau x_\tau] = 0. \end{aligned}$$

For the envelopes it holds that $EU_\tau^* = (1+I)^{-1}(p_1x + I^{-1}\bar{p}_2z)EU_T^* - \beta z \text{cov}(u'(c_2)p_2)$, where the covariance term, $\text{cov}(u'(c_2)p_2)$ is negative. Utilizing this and the Slutsky equation of x_τ leads to

$$(2) \quad L_\tau = -\beta z \text{cov}(u'(c_2)p_2) - \frac{\beta z \text{cov}(u'(c_2)p_2)}{(1+I)E[u'(c_2)]} x_T + \lambda(Ip_1x - (1+g')\bar{p}_2)I^{-1}\alpha x_\tau^c = 0,$$

where the two first terms reflect the effects of an uncertain future timber price and the third term is the substitution effect. Evaluating (2) at the corners ($\tau = 0, \tau = 1$) yields equation (3)

$$(3) \quad \begin{aligned} \text{a) } L_{\tau=0|\tau=T^*} &= -\text{cov}(u'(c_2), p_2) > 0 \\ \text{b) } L_{\tau=1|\tau=T^*} &= 0. \end{aligned}$$

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