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FURTHER  
TESTING OF THE  
HUMAN-CAPITAL  
AUGMENTED  
SOLOW MODEL

Pasi Ikonen

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Valtion taloudellinen tutkimuskeskus

Government Institute for Economic Research

Hämeentie 3, 00530 Helsinki, Finland

Email: first name.surname@vatt.fi

J-Paino Oy

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**Abstract:** This paper examines further the human-capital augmented Solow growth model, that is, it continues the work of Mankiw, Romer and Weil (1992) and Islam (1995). A panel-data approach with yearly observations is adopted. The main contribution is letting technological progress vary across countries. Additionally, different measures of human capital are used. The paper concludes that letting the technological progress vary does not eliminate the inconsistencies between the human-capital augmented Solow growth model and reality. In the growth equation the sign of the coefficient for every human capital variable is found to be positive and the variable statistically very significant. However, the conclusions related to this fact are only suggestive.

**Key words:** Human-capital augmented Solow growth model, Human capital, Technological progress, Neoclassical growth theory

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**Tiivistelmä:** Tutkimuksessa selvitetään inhimillisellä pääomalla täydennetyt Solow'n kasvumallin kykyä kuvata todellisuutta ja se on jatkoa Mankiw'n, Romerin ja Weilin (1992) sekä Islamin (1995) tutkimuksille samasta aiheesta. Mallin toimivuutta on selvitetty käyttäen paneeliaineistoa. Tärkein malliin tuotu uusi piirre on teknisen kehityksen maittaisen vaihtelun salliminen. Lisäksi tutkimuksessa on käytetty useita inhimillisen pääoman muuttujia. Johtopäätöksenä esitetään, ettei edes teknisen kehityksen maittaisen vaihtelun salliminen ei poista ristiriitaa inhimillisellä pääomalla täydennetyt Solow'n kasvumallin ja todellisuuden väliltä. Kasvuyhtälössä jokaisen inhimillisen pääoman muuttujan kertoimen etumerkki on positiivinen ja itse muuttuja tilastollisesti erittäin merkitsevä. Tästä tehtävät johtopäätökset ovat kuitenkin vain suuntaa-antavia.

**Asiasanat:** Inhimillisellä pääomalla täydennetty Solow'n kasvumalli, inhimillinen pääoma, tekninen kehitys, uusklassinen kasvuteoria.

## Summary

The starting point of this paper lies at the so-called human-capital augmented Solow growth model developed by Mankiw, Romer and Weil (1992) and Islam (1995) using regression analysis. This model has shown better performance than the basic Solow growth model but particularly the panel-data approach adopted by Islam (1995) has shown serious inconsistencies in the assumptions of the model.

The first goal of this paper is to continue from where Islam left off and examine whether the inconsistencies of the model hold by giving it even more possibilities to show its validity, that is by further relaxing the assumptions. Specifically, technological progress is allowed to vary across countries.

Other differences compared to Islam are a different sample of countries and points of time. The reference to points of time is related to the fact that here the time period is longer – which in practice means a time period of 1960 - 1993 – while Islam uses the so-called Summers-Heston data set, which covers only the years 1960 - 1985. Additionally, there are more observations here within the same time period than in Islam's regressions. His data has an observation for every five years while here an observation exists for each year. Moreover, the set of used human capital variables has been supplemented.

As a second goal, the role of human capital will be of special interest in the regressions. This is because education has become more and more important in today's society. It would be useful to know how investments in it affect economic growth. However, since the econometric model may be misspecified, the conclusions concerning this effect will only be suggestive.

The data covers 29 countries and 34 time periods. It is from the World Bank files with the exception of human capital variables, which are from the UNESCO yearbooks 1965 - 1995. The other variables used in the regressions are: income per capita, which is estimated by GDP per capita; the saving rate which is estimated by gross investment per GDP; and the working population, which is estimated by the population between 15 and 64 years of age. Additionally, the regressions include technological progress and the depreciation rate for which there is no direct data.

According to the theory, human capital should be included in the regressions in the form of human capital accumulation or in the form of the level of human capital. Different estimates for these are used. The variable *studter* means the proportion of working-age (15 - 64) people enrolled in tertiary education, *studsec* means the proportion of working-age people enrolled in secondary education and

teratt the proportion of people over 25 years old having tertiary education. None of these variables is perfect, but the combination of regressions with these different variables provides us with an overview of how different measures of human capital affect the results. According to the results it can be pointed out that the general conclusions are not affected.

The possibility of testing of the model lies in the fact that the Solow growth model predicts the values for some estimable parameters in quite a restrictive way since the theory gives a specific meaning to these parameters. If the values for the parameters estimated by the regressions are different from the values given by the meaning of the parameters according to the model it is natural to conclude that the model is not consistent with the real world.

Considering the first goal of this paper the results give us reason to conclude that the augmented textbook Solow growth model is not consistent with reality, at least as far as this data is concerned, even with further relaxation of some of the assumptions. Particularly the estimates for the rate of convergence – which is an estimable parameter – provided by the regressions are too far from the value predicted by the model. It is clear that the model has to be further modified or a totally new model has to be developed.

The second goal was to examine the relation between human capital and economic growth. Table 2 shows that in each regression the sign of the coefficient for the human capital variable is positive and the variable is statistically very significant. This indicates that the relation between economic growth and human capital is positive. The greater the investments in human capital or the greater the level of human capital, the greater the economic growth. However – as stated before – this result is only suggestive.

# Contents

<b>1. Introduction</b>	<b>1</b>
<b>2. Testing the Human-Capital Augmented Solow Model</b>	<b>3</b>
2.1 Model	3
2.2 Mankiw, Romer and Weil Specification	4
2.3 A Panel-Data Approach	6
2.4 Specification	7
2.5 Data	10
2.6 Results	11
<b>3. Conclusions</b>	<b>15</b>
<b>References</b>	<b>16</b>
<b>Appendix 1</b>	<b>17</b>

# 1. Introduction

The Solow growth model is the basic model of growth theory (Barro 1995). However, a new approach – so-called endogenous growth theory – has emerged lately (McCallum 1997).

The problem with the basic Solow model is the fact that it cannot explain large income differences without large values of elasticities of output, which are rejected by international statistics. In response, the Solow growth model has been augmented with a human capital variable. (Mankiw, Romer and Weil 1992). Nonneman and Vanhoudt (1996) have even introduced a general augmented Solow model where the number of augmenting variables is dependent on the desires of the researcher.

The basic property of endogenous growth theory is its unwillingness to explain growth with exogenous variables such as exogenous technological progress while exogenous growth theory – of which the (augmented) Solow growth model is one very good example – has this property. A further criticism to the Solow type growth model is given by Quah (1995), who sees a statistical relation as a principal factor in explaining the results given by the model.

The starting point of this paper lies with the human-capital augmented Solow growth model developed by Mankiw, Romer and Weil (1992) and Islam (1995). This model has shown better performance than the basic Solow growth model, but particularly the panel-data approach adopted by Islam (1995) has shown serious inconsistencies in the assumptions of the model.

The first goal of this paper is to continue from where Islam left off and examine whether the inconsistencies of the model hold by giving it even more possibilities to show its validity, that is by further relaxing the assumptions. Specifically, technological progress is allowed to vary across countries. If even this modification does not help, we have to reject the model as an insufficient explanation of reality.

Other differences compared to Islam are a different sample of countries and points of time. The reference to points of time is related to the fact that here the time period is longer – which in practice means a time period of 1960 - 1993 – while Islam uses the so-called Summers-Heston data set, which covers only the years 1960 - 1985. Additionally, there are more observations here within the same time period than in Islam's regressions. His data has an observation for every five years while here an observation exists for each year. What is more, the set of used human capital variables has been supplemented.

As a second goal, the role of human capital will be of special interest in the regressions. This is because education has become more and more important in today's society. It would be useful to know how investments in education affect economic growth. However, since the econometric model may be misspecified, the conclusions concerning this effect will be only suggestive.

As a background for the regression equation (2.1) the production function has been assumed to take the Cobb-Douglas form

$$(1.1) \quad Y(t) = K(t)^\alpha H(t)^\beta [A(t)L(t)]^{1-\alpha-\beta}, \quad \alpha > 0, \beta > 0, \alpha + \beta < 1,$$

where  $Y$  denotes output,  $K$  capital,  $A$  "knowledge" or "effectiveness of labour",  $L$  labour,  $t$  time and  $H$  is the stock of human capital. The growth rates take the forms

$$(1.2) \quad \dot{K}(t) = s_K Y(t) - \delta K(t),$$

$$(1.3) \quad \dot{L}(t) = nL(t),$$

$$(1.4) \quad \dot{A}(t) = gA(t),$$

$$(1.5) \quad \dot{H}(t) = s_H Y(t) - \delta H(t),$$

where  $s_K$  denotes the saving rate (=output devoted to investment),  $\delta$  depreciation rate,  $n$  population growth,  $g$  technological progress and  $s_H$  is the fraction of resources devoted to human capital accumulation. All of these five parameters are exogenous constants. Additionally,  $n + g + \delta$  is assumed to be constant in order to be able to take its logarithm. A dot over a variable means the derivative with respect to time.

Both Mankiw, Romer and Weil (1992) and Islam (1995) have used the same assumptions concerning the production function and also other features unless stated otherwise. As a point of clarification, the model is always considered here with all these assumptions. This implies that the conclusions are valid with certainty only when both the model and the assumptions used here are taken into account.



## 2. Testing the Human-Capital Augmented Solow Model

### 2.1 Model

The steady state equation of the human-capital augmented Solow growth model (Romer 1995) can be written in the form of

(2.1)

$$\ln \left[ \frac{Y(t)}{L(t)} \right] = \ln A(0) + gt + \frac{\alpha}{1-\alpha-\beta} \ln s_K + \frac{\beta}{1-\alpha-\beta} \ln s_H - \frac{\alpha+\beta}{1-\alpha-\beta} \ln(n+g+\delta)$$

when taken into account that the log income per effective worker  $y(t)$  – denoted by  $\ln y^*$  at the steady state – is defined as

$$(2.1b) \quad y(t) = \ln \frac{Y(t)}{A(t)L(t)} = \ln \frac{Y(t)}{[A(0)e^{gt}]L(t)} = \ln \frac{Y(t)}{L(t)} - \ln A(0) - gt \quad \text{since } A(t)$$

is assumed to grow exponentially (Romer 1995). In (2.1)

$t$  is time (here for the years 1960 - 1993 such that the year 1960 is defined as the starting point of the time index, that is  $t = 0$  in 1960),

$Y(t)$  income volume,

$L(t)$  labour input volume,

$A(t)$  level of technology,

$A(0)$  the initial level of technology (at  $t = 0$ ),

$g$  equals constant (yearly) change of the level of technology,

$\alpha$  elasticity of output with respect to physical capital,

$\beta$  elasticity of output with respect to human capital,

$s_K$  fraction of income invested in physical capital (= saving rate),

$s_H$  fraction of income invested in human capital,

$n$  (working) population growth and

$\delta$  depreciation rate.

What is more, the parameters  $\alpha$  and  $\beta$  have the following properties:

$$\alpha > 0, \beta > 0, \alpha + \beta < 1.$$

For example Romer (1995) has discussed that the neo-classical growth theory predicts the values for these parameters (at least for  $\alpha$ ) in quite a restrictive way. According to the model it is natural to think  $\alpha$  is equal to physical capital's share of income, which is roughly one third globally.

In contrast, it is far more difficult to predict the value for  $\beta$ . In the United States the minimum wage (= the return to labour without human capital) has averaged between 30 to 50 percent of the average wage in manufacturing. Thus 50 to 70 percent of total labour income would represent the return to human capital. Because capital's share of income is one third, there remains two thirds for labour. By multiplying labour's share of income with human capital's share of labour income we get one third to one half as labour's share of total income (Mankiw, Romer and Weil 1992, 417). This prediction cannot be considered very reliable since it has so many approximating elements.

## 2.2 Mankiw, Romer and Weil Specification

Mankiw, Romer and Weil (1992) (hereinafter M-R-W) used (2.1) to see how the saving rate, the rate of human capital accumulation and labour growth differences can explain cross-country differences in income per capita. To get an answer to precisely this question they assumed the technological progress  $g$  to be the same across countries and removed the time dimension by using only one cross section of over-time averages. This way the term  $gt$  melts as a part of the constant term. However, the assumption of the constantness of  $\ln A(0)$  was not made since the cross-country differences related to it can be assumed to lie in the error term  $\varepsilon$ . In other words, M-R-W defined  $\ln A(0)$  as  $a + \varepsilon$ , where  $a$  is a constant. Additionally, M-R-W demand  $s_K$ ,  $s_H$  and  $n$  to be independent of  $\varepsilon$ .

Moreover,  $g + \delta$  in  $(n + g + \delta)$  has no influence, since they assumed  $g + \delta$  is a constant. By making these assumptions the OLS regressions based on (2.1) tell how cross-country differences in the saving rate, the rate of human capital accumulation and labour growth rate explain the cross-country differences in income per capita. Then  $R^2$  informs how large a proportion of total variation in income is explained by these factors.

M-R-W found  $R^2$  to be a little under 0.80 in two of their three samples of countries. It implies that the three observable factors explain 80 % of cross-country variation in income. This is an important result since the criticism towards the Solow model has claimed that it explains the variation in income largely by differences in some unobservable technology, which, in fact, means

that the explanation provided by the model would be quite abstract (Mankiw, Romer, Weil 1992, 414-415). Furthermore, the magnitude of  $\alpha$  and  $\beta$  in these two samples is what the model predicts, that is,  $\alpha$  is about one third, as well as  $\beta$ . Even the restriction that the sum of the coefficients of  $s_K$  and  $s_H$  should equal the negative of the coefficient of  $(n + g + \delta)$  cannot be rejected, although the reliability of the test is not clear.

The third sample of M-R-W consisted of 22 OECD countries. For this sample, their estimate for  $\alpha$  was not the one predicted by the model. They reckoned this to be due to the very probable possibility according to which particularly the OECD countries were not in a steady state. That is why they also ran regressions that allowed out-of-steady-state dynamics. The form of the dynamics is described in more detail in the next section. The regressions brought the OECD sample in line with the other samples. However, the estimates for  $\alpha$  became slightly too high in all the samples. Furthermore, the estimates for  $\beta$  were too low, which is perhaps less dangerous. Instead, the rate of convergence – an additional parameter included in out-of-steady-state regressions – was of the predicted size.

In an econometric sense the most questionable assumption of M-R-W is the assumption of independence of  $\varepsilon$  concerning  $s_K$ ,  $s_H$  and  $n$ . This implies that the initial level of technology – probably greater in richer countries – would be uncorrelated with the saving rate, the rate of human capital accumulation and working population growth. This sounds very unrealistic. The arguments of M-R-W for this assumption are not econometrical except in the sense that this assumption justifies their specification, which makes the situation even more serious. Criticism toward this assumption is also uttered by Islam (1995, 1134) and McCallum (1997, 63), who is worried about the possible correlation between  $s_K$  and the error term due to the fact that since  $gt$  is subsumed in the constant term, possible cross-country variation in  $g$  would remain in the error term, and due to the fact that  $g$  is likely to be correlated with  $s_K$ .

A panel data approach would solve this problem. Besides, it would use the information more efficiently, since there would be several cross sections (over time) instead of only one cross section. However, a panel data approach is more appropriate for the out-of-steady-state dynamics model than for the steady-state model. This is because if the saving rate, the rate of human capital accumulation and working population growth are allowed to change over time, the economies cannot be at their steady states since these kinds of changes create out-of-steady-state dynamics by shifting the steady-state value of income.

Islam (1995) has implicitly thought in a similar way as he has started directly at the out-of-steady-state-dynamics model when applying a panel data framework for the regressions of M-R-W. Besides, the assumption of the economies being at

their steady states is more restricting. The steady-state regressions will not be run even in the one-cross-section form here since they will not provide any essential further information compared to the regressions of M-R-W. The differences between the data sets favour M-R-W in this context, with the exception of the human capital variables, the differences between which, however, will be controlled for by comparing the results of a out-of-steady-state model to the ones of Islam (1995) concerning the same model.

### 2.3 A Panel-Data Approach

Let  $y^*$  denote income per effective worker at the steady state and let  $y(t)$  be its actual value at any time. Approximating by a Taylor-series around the steady state we get

$$(2.2) \quad \frac{d(\ln(y(t)))}{dt} = -\lambda[\ln y(t) - \ln(y^*)]$$

– where  $\lambda = (n + g + \delta)(1 - \alpha - \beta)$  – as the pace of convergence.

This first-order differential equation implies that

$$(2.3) \quad \ln y(t_2) = (1 - e^{-\lambda\tau}) \ln y^* + e^{-\lambda\tau} \ln y(t_1),$$

where  $y(t_1)$  is income per effective worker at some initial point of time and  $\tau = (t_2 - t_1)$ . Subtracting  $y(t_1)$  from both sides gives

$$(2.4) \quad \ln y(t_2) - \ln y(t_1) = (1 - e^{-\lambda\tau}) \ln y^* - (1 - e^{-\lambda\tau}) \ln y(t_1).$$

This can be written in the form of

$$(2.5) \quad \ln y(t_2) - \ln y(t_1) = (1 - e^{-\lambda\tau})(\ln y^* - \ln y(t_1))$$

which is a standard partial adjustment model (Islam 1995, 1135).

Substituting the target value – that is the steady-state value  $y^*$  – yields

$$(2.6) \quad \ln y(t_2) - \ln y(t_1) = (1 - e^{-\lambda\tau}) \frac{\alpha}{1 - \alpha - \beta} \ln s_K + (1 - e^{-\lambda\tau}) \frac{\beta}{1 - \alpha - \beta} \ln s_H \\ - (1 - e^{-\lambda\tau}) \frac{\alpha + \beta}{1 - \alpha - \beta} \ln(n + g + \delta) - (1 - e^{-\lambda\tau}) \ln y(t_1).$$

Since income per effective worker is not observable, (2.6) has to be modified to be defined in terms of income per capita. Using (2.1b) to substitute for  $y(t)$  in (2.6) and collecting terms with  $\ln y(t)$  on the right-hand side gives

$$(2.7) \quad \ln \left[ \frac{Y(t_2)}{L(t_2)} \right] = (1 - e^{-\lambda\tau}) \frac{\alpha}{1 - \alpha - \beta} \ln s_K + (1 - e^{-\lambda\tau}) \frac{\beta}{1 - \alpha - \beta} \ln s_H \\ - (1 - e^{-\lambda\tau}) \frac{\alpha + \beta}{1 - \alpha - \beta} \ln(n + g + \delta) + e^{-\lambda\tau} \left[ \frac{Y(t_1)}{L(t_1)} \right] \\ + (1 - e^{-\lambda\tau}) \ln A(0) + g(t_2 - e^{-\lambda\tau} t_1).$$

There exists another possibility to express the human capital's role in the model. This alternative method replaces the rate of human capital accumulation  $s_H$  by the level of human capital  $h^*$ , which transforms (2.7) into (Romer 1995)

$$(2.8) \quad \ln \left[ \frac{Y(t_2)}{L(t_2)} \right] = (1 - e^{-\lambda\tau}) \frac{\alpha}{1 - \alpha} \ln s_K + (1 - e^{-\lambda\tau}) \frac{\beta}{1 - \alpha} \ln h^* \\ - (1 - e^{-\lambda\tau}) \frac{\alpha}{1 - \alpha} \ln(n + g + \delta) + e^{-\lambda\tau} \left[ \frac{Y(t_1)}{L(t_1)} \right] \\ + (1 - e^{-\lambda\tau}) \ln A(0) + g(t_2 - e^{-\lambda\tau} t_1).$$

Thus there are two alternative ways of describing the role of human capital. Later on we refer to the form of (2.7) as the basic equation. When (2.8) is referred to, this is indicated explicitly or it is obvious in the context. Thus the theory is specified in the form of (2.7) but the specification applies with obvious modifications for (2.8), too.

## 2.4 Specification

The empirical specification of (2.7) to be applied here is:

$$(2.9) \quad \ln \left[ \frac{Y_{it}}{L_{it}} \right] = (1 - e^{-\lambda}) \frac{\alpha}{1 - \alpha - \beta} \ln s_{Kit} + (1 - e^{-\lambda}) \frac{\beta}{1 - \alpha - \beta} \ln s_{Hit}, \\ - (1 - e^{-\lambda}) \frac{\alpha + \beta}{1 - \alpha - \beta} \ln(n_{it} + g_i + \delta) + e^{-\lambda} \left[ \frac{Y_{i,t-1}}{L_{i,t-1}} \right]$$

$$+(1 - e^{-\lambda}) \ln A_{i0} + g_i (t - e^{-\lambda} (t - 1)) + \varepsilon_{it},$$

where  $i = 1, \dots, 29$  and  $t = 1, \dots, 33$ . Each  $i$  denotes a country and each  $t$  a year (1960 - 1993).

A few remarks are of relevance. The task of the disturbance  $\varepsilon_{it}$  is to reflect a time- (and country-) specific shock. The variables  $n$ ,  $s_H$  and  $s_K$  for each country are allowed to change across time. Actually, the depreciation rate  $\delta$  could also be country-specific and vary across time. Its constantness is merely due to deficient data. This assumption will be discussed in more detail in the next section. Furthermore,  $\ln A(0)$ , the initial level of technology, can now vary across countries, which implies that the cross-country differences related to it are no longer a part of the error term.

The main difference between the regressions of Islam (1995) and the regression here is the fact that Islam assumed  $g$  – technological progress – to be constant across countries, which is not required here. Even M-R-W assumed it to be constant and allowed  $\ln A(0)$  to vary. I see a logical conflict here: how can the initial levels be different if the progress has not been able to vary during some periods. However, M-R-W argue that  $\ln A(0)$  reflects not only technology, but resource endowments, climate, institutions and so on. I cannot see why these factors could not change across time as well.

Additionally, McCallum (1997, 63) has presented a serious econometrical critique to this assumption (see Section 2.2). What is more, technological progress is not the same in all the countries in the real world. For example, the United States has superior military technology which is progressing all the time and which is not allowed to be exported with perhaps the exception of the United Kingdom. Due to these reasons and to the desire to allow the theory all the possibilities to show its consistency with reality,  $g$  is not fixed as a constant for the different countries here.

An interesting discussion arises when the universality of  $\alpha$  and  $\beta$  is concerned. They are roughly constant globally but only roughly. A natural question is that when almost all the other parameters are relaxed why could not  $\alpha$  and  $\beta$  be relaxed, too. The explanation lies in the fact that most of the variation in  $s_H$  and  $s_K$  is between cross sections, although human capital has relatively a lot of variation in time, too. Besides, as we have the work of M-R-W and Islam (1995) as our starting point, we would have a very different framework – that is, almost a different model.

The initial regression model used is the so-called covariance model (Kmenta 1986, 630-635). In practice this means pooling by OLS with fixed country-

specific dummy variable, for  $\ln A(0)$ . Therefore it is sometimes called the LSDV model (= least squares dummy variables). (2.9) represents this model augmented by the terms  $g_i(t - e^{-\lambda}(t-1))$ , which are slightly increasing trend terms. Because the method is OLS, there could easily arise specification errors (such as correlated error terms, heteroskedasticity or non-normality) but they do not affect the unbiasedness or consistency of the coefficients.

However, due to the existence of a lagged dependent variable in the regression equation the OLS estimates will only be asymptotically unbiased and consistent. The existence of a lagged dependent variable causes another difficulty, too: (time-wise) autocorrelation cannot be allowed since otherwise the OLS estimates would be inconsistent as the error term and the lagged dependent variable would be contemporaneously correlated. That is why it is important to consider the value of the Durbin h-statistic (Kmenta 1986, 333-334), which will be used instead of the Durbin-Watson statistic (Kmenta 1986, 329-332) when there is a lagged dependent variable in the regression.

Since panel-data estimation can produce Durbin h-statistics showing correlation also because of cross-country correlation, the autocorrelation tests should be done for each country separately. These tests did not reject the hypothesis of non-autocorrelation for almost every country. This is not very unnatural as a lagged dependent variable is likely to catch the effects of the previous period(s). Cross-sectional correlation is not dangerous since it does not make the error term contemporaneously correlated with any of the explanatory variables even in the presence of lagged dependent variable in the regression. Thus we can accept that no autocorrelation exists but we must keep an eye on the value of Durbin h.

In addition to the LSDV model Islam (1995) used the so-called MD model (= minimum distance). Its properties are also asymptotical. The results acquired by both of the models were quite similar. The reason for using the MD model is the fact that with the presence of a lagged dependent variable the asymptotics of the LSDV model can only be considered in the direction of  $T \rightarrow \infty$  (Islam 1995, 1138). Since we have more time periods than cross-sections, the relevance of this consideration is zero.

On the whole, the LSDV model can tell us the point estimates of the coefficients relatively reliably. Instead, in the presence of heteroskedasticity or a cross-country correlation – which are both very likely – the standard errors will be biased and inconsistent. A cross-country correlation can be viewed as (business cycle) shocks affecting each country. If the standard errors are not correct, confidence intervals will not be correct. Confidence intervals would be practical – although not indispensable – in statistical inference. Furthermore, the Wald test statistic (White 1997, 103-104) for testing whether the sum of the coefficients

of  $s_H$  and  $s_K$  is equal to the negative of  $(n + g + \delta)$  will not be appropriate either under such conditions. Instead, the assumption of normality is not so crucial since the distribution of the OLS estimators will be asymptotically normal, anyway.

Time-wise run B-P-G and ARCH tests (White 1997, 182) for each country reveal that the error terms are time-wise homoskedastic. This with (time-wise) nonautocorrelation allow us to use a special GLS-model with cross-sectional heteroskedasticity and correlation. This type of model has some problems (Spanos (1996) but they are not relevant here. The properties of the feasible GLS estimators are asymptotically desirable, that is they are asymptotically unbiased, consistent and asymptotically efficient. The last property makes the standard errors asymptotically unbiased and consistent, but even the feasible GLS estimators demand (time-wise) nonautocorrelation in the presence of a lagged dependent variable in the regression (Kmenta 1986, 607-625). Thus we can get relatively reliable estimates for the confidence interval and the Wald test statistic. Additionally, the heteroskedasticity-and-cross-sectional-correlation-cleared error terms provide the possibility to further check the assumption of (time-wise) nonautocorrelation with the Durbin h statistic in the panel data.

## 2.5 Data

The data is from the World Bank files with the exception of human capital variables, which are from the UNESCO yearbooks 1965 - 1995. The data covers the years 1960 - 1993. The variable *studter* means the proportion of working-age (15 - 64) people enrolled in tertiary education, *studsec* means the proportion of working-age people enrolled in secondary education and *teratt* the proportion of people over 25 years old having tertiary education. *Teratt* is applied for (2.8).

None of these variables is a perfect estimate of human capital but if they are proportional to  $s_H$ , then the factor of proportionality will affect only the constant term. Furthermore, the combination of regressions with these different variables provides us with an overview in how different measures of human capital affect the results.

Income per capita is estimated by GDP per capita, the saving rate by gross investment per GDP and the working population by the population between 15 and 64 years. These estimates are relatively reliable. The term  $\lambda$  in the quantity  $(t - e^{-\lambda}(t-1))$  is estimated by iteration until it is no longer different from the directly estimable parameter  $\lambda$ . During this iteration  $g + \delta$  in the quantity  $(n + g + \delta)$  is assumed to be 0.05 everywhere. Next we replace  $g$  in  $(n + g + \delta)$  with the iterated  $g_i$  for each country, assume  $\delta$  to be 0.03 and continue the iteration process for  $g_i$  and  $\lambda$  in  $(t - e^{-\lambda}(t-1))$  until  $\lambda$  in  $(t - e^{-\lambda}(t-1))$  is no longer



different from the estimable parameter  $\lambda$  again. The remaining errors are not visible in third significant numbers.

The regressions are also completed when  $g + \delta$  in  $(n + g + \delta)$  remain to be 0.05. The results are very similar, which implies that the conclusions would be the same. Thus the assumption of  $g + \delta$  being 0.05 (although this assumption is not made here) would be a harmless simplification. Actually, the assumption of normality of the error term is a harmless simplification, too (Vartia 1988, 307). However, this allows us to justify the assumption of  $\delta$  being 0.03 which has to be made – as stated before – due to deficient data. The value 0.03 is a rough – but often used – approximation based on US data (Mankiw, Romer, Weil 1992, 413).

The data contains 29 countries and 34 time periods. The list of countries is to be found in Appendix 1. Augmented Dickey-Fuller unit root tests (White 1997, 165-171) are run for each GDP series in the panel data, and in most time series the hypothesis of a unit root could be rejected. This is needed to ensure stationarity.

M-R-W and Islam (1995) use both the so-called Summers-Heston data set (Summers-Heston 1988). From this set they make three samples. For example, Islam has 79 countries in the NONOIL sample, 67 in the INTER sample and 22 in the OECD sample. The Summers-Heston data set covers the period 1960-1985 but Islam's panel data has observation points only every 5 years. They both use 0.05 as an approximation for  $g + \delta$ . M-R-W used a human capital variable very close to *studsec*. Islam used the Barro-Lee human capital variable (Barro-Lee 1993), which corresponds to *studatt* to some extent. Otherwise the variables were the same as here. In order to control for the differences in data regressions were made also assuming a constant  $g$ . The results were in line with the ones achieved by Islam: the coefficient  $\alpha$  became lower, the rate of convergence  $\lambda$  higher and the coefficient  $\beta$  obtained the wrong sign and a minimum absolute value. The first two observations can be explained by the inclusion of  $\ln A(0)$ . The last one is much harder to explain. Actually, Islam is currently working on it in one way. On the whole, before allowing  $g$  to vary across countries, our framework is comparable to that of Islam.

## 2.6 Results

The results of the LSDV regressions are presented in Table 1. They show very precise values for  $\alpha$  and tolerable values for  $\beta$  (the prediction for  $\beta$  is not obvious). Particularly in regressions with *studter* and *studatt* as human capital variables the implied values of 0.303 and 0.305 for  $\alpha$  are very close to one third, as the model predicts. Additionally,  $R^2$  is very high. What is more, varying the

measure of human capital seems to have no effect on general results. Allowing  $g$  to vary seems to make the augmented Solow model valid at first glance. A high Durbin  $h$  is (hopefully) due to cross-country correlation rather than (time-wise) autocorrelation.

*Table 1 OLS Estimation, Cross-Sectional Fixed Effects and Trends*

Dependent variable: log GDP per working-age person during the period 1960 - 1993 in 29 countries

Human capital	Studsec	Studter	Studatt
Unrestricted Coefficient:			
lagged $\ln(\text{GDP}/\text{wpop})$	0.880 (0.015)	0.899 (0.015)	0.888 (0.0151)
$\ln(I/\text{GDP})$	0.0527 (0.0081)	0.0486 (0.0082)	0.0508 (0.0081)
$\ln(n + g + \delta)$	-0.0637 (0.0173)	-0.0537 (0.0177)	-0.0373 (0.0138)
$\ln(\text{human capital})$	0.0455 (0.0082)	0.0109 (0.0054)	0.0298 (0.0089)
$R^2$	0.9995	0.9994	0.9994
Durbin $h$	8.02	8.38	8.60
Implied $\lambda$	0.128	0.106	0.119
Restricted Coefficient:			
lagged $\ln(\text{GDP}/\text{wpop})$	0.888 (0.014)	0.901 (0.014)	0.891 (0.015)
$\ln(I/\text{GDP}) - \ln(n + g + \delta)$	0.0470 (0.0074)	0.0478 (0.0078)	0.0477 (0.0073)
$\ln(\text{human capital}) - \ln(n + g + \delta)$	0.0420 (0.0080)	0.0107 (0.0053)	0.0304 (0.0088)
$R^2$	0.9995	0.9994	0.9994
Durbin $h$	7.91	8.28	8.50
Implied $\lambda$	0.119	0.105	0.115
Implied $\alpha$	0.233	0.303	0.305
Implied $\beta$	0.209	0.0678	0.194

studter means the proportion of working-age (15 - 64) people enrolled in tertiary education, studsec means the proportion of working-age people enrolled in secondary education and teratt the proportion of people over 25 years old having tertiary education. Figures in parentheses are standard errors.

However, further investigation shows that the model is all but valid. If we remember the definition of  $\lambda$  as  $(n + g + \delta)(1 - \alpha - \beta)$  and approximate  $(n + g + \delta)$  by 0.06 (a harmless simplification again) we should have at the highest  $\lambda = 0.04$ . This value is calculated by using studter as a human capital variable since the sum of  $\alpha$  and  $\beta$  becomes lowest in this way. This is very far from the estimates in

Table 1, where  $\lambda = 0.105$  at the lowest (for studter). This implies a serious conflict between the model and reality.

B-P-G and ARCH tests showed heteroskedasticity as suspected. Additionally, there exists cross-section correlation showed by Durbin h. Thus we have to perform the GLS estimation described above in order to calculate the correct standard errors. The results are presented in Table 2.

The results are convincingly in line with those of the LSDV model.  $R^2$  is even higher than with the LSDV model. Furthermore, all the variables are statistically significant. As suspected, there is no (time-wise) autocorrelation showed by the low (absolute) value of Durbin h. However, there is even more evidence against the augmented Solow model. The Wald test statistics reject all the hypotheses of restriction. Moreover, the value of 0.04 for  $\lambda$  – calculated in the same way as with OLS regressions – or even the value of 0.33 for  $\alpha$  are not included in any of their respected 95 % confidence intervals.

*Table 2 GLS Estimation with Cross-Sectional Correlation and Heteroskedasticity, Cross-Sectional Fixed Effects and Trends*

Dependent variable: log GDP per working-age person during the period 1960 - 1993 in 29 countries

Human capital:	Studsec	Studter	Studatt
Unrestricted Coefficient:			
lagged ln(GDP/wpop)	0.870 (0.005)	0.887 (0.007)	0.875 (0.007)
ln(I/GDP)	0.0537 (0.0015)	0.0487 (0.0020)	0.0502 (0.0019)
ln(n + g + $\delta$ )	-0.0563 (0.0048)	-0.0478 (0.0053)	-0.0373 (0.0047)
ln(human capital)	0.0401 (0.0023)	0.0107 (0.0013)	0.0275 (0.0020)
Buse R <sup>2</sup>	0.99995	0.99996	0.99997
Durbin h	-1.00	-1.04	-1.01
Implied $\lambda$	0.139 (0.006)	0.120 (0.007)	0.134 (0.009)
Restricted Coefficient:			
lagged ln(GDP/wpop)	0.878 (0.005)	0.889 (0.007)	0.878 (0.007)
ln(I/GDP) – ln(n + g + $\delta$ )	0.0483 (0.0014)	0.0474 (0.0019)	0.0479 (0.0017)
ln(human capital) – ln(n + g + $\delta$ )	0.0373 (0.0023)	0.0101 (0.0013)	0.0287 (0.0019)
Buse R <sup>2</sup>	0.99995	0.99996	0.99996
Durbin h	-1.06	-1.14	-1.09
Implied $\lambda$	0.130 (0.006)	0.117 (0.007)	0.130 (0.008)
Implied $\alpha$	0.233 (0.008)	0.281 (0.014)	0.282 (0.014)
Implied $\beta$	0.180 (0.010)	0.060 (0.007)	0.169 (0.014)
Wald test for restriction:			
p-value	0.000	0.048	0.016

studter means the proportion of working-age (15 - 64) people enrolled in tertiary education, studsec means the proportion of working-age people enrolled in secondary education and teratt the proportion of people over 25 years old having tertiary education. Figures in parentheses are standard errors.

### 3. Conclusions

Considering the first goal of this paper we can conclude that the augmented textbook Solow growth model is not consistent with reality even with some further relaxation of assumptions, at least as far as this data is concerned. Particularly the estimates for the rate of convergence provided by the regressions are too far from the value predicted by the model. It is clear that the model has to be further modified or a totally new model has to be developed.

The second goal was to examine the relation between human capital and economic growth. Table 2 shows that in each regression the sign of the coefficient for the human capital variable is positive and the variable is statistically very significant. This indicates that the relation between economic growth and human capital is positive. The greater the investments in human capital or the greater the level of human capital, the greater the economic growth.

However, the conclusions concerning the second goal are only suggestive. This is due to the fact that the econometric model constructed according to the Solow growth model may be misspecified. This means that it may not describe reality correctly. However, even a model that is only slightly incorrect or incorrect in unessential parts can lead to the right conclusions. This requires that the simplifications of the model are harmless. Thus, the central question is whether the simplifications are harmless (Vartia 1988). This could be evaluated by comparing the results with the results of a correct model or if and when it is not known, with the results of several misspecified models (Leamer 1983). These considerations are beyond of the scope of this paper.

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# Appendix 1

## Countries:

Austria  
Belgium  
Cameroon  
Chile  
Denmark  
Egypt  
Finland  
France  
Greece  
Haiti  
India  
Indonesia  
Ireland  
Israel  
Italy  
Malaysia  
Malta  
Mexico  
Morocco  
Netherlands  
New Zealand  
Norway  
Paraguay  
Philippines  
Singapore  
Spain  
Turkey  
UK  
Venezuela