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ESTIMATING EQUILIBRIUM SEARCH MODELS FROM FINNISH DATA

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ABSTRACT: Empirical equilibrium search models have attracted a growing interest in recent years. Estimation of such models has not been completely successful, however. This has led researchers to develop more sophisticated versions of the models, in an attempt to get a better fit to the data. This study investigates whether various proposed specifications are able to explain the labour market histories observed in Finnish panel data. We begin with a pure search model in which all wage dispersion results from search frictions. Then we proceed to more complex specifications by introducing measurement error in wages and unobserved employer heterogeneity.

Keywords: Search theory, wage dispersion, labour market

TIIVISTELMÄ: Työmarkkinoiden etsintäteoreettiset tasapainomallit ovat olleet kasvavan kiinnostuksen kohteena viime vuosina. Näiden mallien sovittaminen empiiriseen aineistoon on kuitenkin osoittautunut ongelmalliseksi. Siksi tutkijat ovat kilvan kehitelleet entistä hienostuneempia malliversiota pyrkiessään paikkailemaan teorian ja empiirisen aineiston välisiä ristiriitoja. Tässä tutkimuksessa testataan, missä määrin kirjallisuudessa esiintyvät empiiriset etsintäteoreettiset tasapainomallit kykenevät selittämään Suomen aineistossa havaittuja työmarkkinakokemuksia. Aluksi tarkastellaan yksinkertaista perusmallia, jossa kaiken palkkavariaation oletetaan seuraavan työmarkkinoiden epätäydellisestä informaatiosta. Tämän jälkeen analysoidaan mutkikkaampia malliversioita, jotka sisältävät mittavirhettä palkkamuuttujissa sekä havaitsematonta heterogeenisuutta yrityspopulaatiossa.

Asiasanat: Etsintäteoria, palkkaerot, työmarkkinat

Yhteenveto

Etsintäteoriassa yksilön työpaikan etsintää kuvataan yleensä olettamalla, että yksilö saa satunnaisin väliajoin palkkatarjouksia ja tarjouksen saadessaan hän päättää hyväksyykö vai hylkääkö sen. Ominaista 'perinteiselle' etsintäteorialla on, että palkkatarjousten jakauma oletetaan eksogeenisesti annetuksi mallin ulkopuolelta. Kysyntäpuolen huomiotta jättäminen rajoittaa näiden mallien käyttöaluetta oleellisesti; jos jokin muutos (esim. työttömyysturvan nousu) vaikuttaa työntekijöiden optimaaliseen käyttäytymiseen, on epätodennäköistä, että rationaaliset yritykset jättäisivät reagoimatta siihen. Siksi perinteisen etsintäteorian puitteissa ei voida analysoida tekijöitä, joilla on vaikutuksia palkanmuodostukseen tai työntekijöiden ja yritysten välisiin interaktioihin yleisemmin.

Etsintäteoreettisissa tasapainomalleissa sen sijaan tarkastellaan sekä työntekijöiden että yritysten käyttäytymistä. Valitessaan omaa palkkatarjoustaan yrityksen oletetaan huomioivan työntekijöiden optimaalisen käyttäytymisen sekä toisten yritysten palkkatarjoukset. Seurauksena on, että palkkatarjousten jakauma, jonka työntekijät kohtaavat markkinoilla, määräytyy endogeenisesti mallissa. Tällaisessa mallikehikossa voidaan tarkastella luontevammin esimerkiksi erilaisia politiikkamuutoksia. Siksi työmarkkinoiden etsintäteoreettiset tasapainomallit tarjoavat hyödyllisen kehikon kiinnostavien työmarkkinailmiöiden analysoinnille.

Etsintäteoreettisten tasapainomallien kehittely ja estimointi ovat yleistyneet viime vuosina nopeasti. Tosin alan empiirinen kirjallisuus on verraten nuorta, minkä vuoksi mallien empiiriset sovellutukset ovat suhteellisen karkeita. Tämän tutkimuksen tarkoituksena on estimoida useita viime vuosina kehiteltyjä työmarkkinoiden etsintäteoreettisia malleja Suomen aineistosta (linkitetyn työntekijä-yritysaineiston työntekijäpaneelista). Tavoitteena on testata eri mallispesifikaatioita ja tutkia, miten robusteja mallin rakenneparametrit ovat eri spesifikaatioiden suhteen. Estimointitulokset tarjoavat myös informaatiota eri työntekijäryhmien kohtaamista työmarkkinajäykkyyksistä (esim. työttömien työllistymistodennäköisyydestä ja työllisten todennäköisyydestä saada kilpailevia palkkatarjouksia) ja niiden vaikutuksesta palkkaeroihin.

Teoreettinen analyysi rakentuu Burdettin ja Mortensenin (1998) etsintäteoreettisen tasapainomallin varaan. Mallin perusversiossa sekä työntekijät että yritykset ovat keskenään identtisiä. Työntekijät etsivät työpaikkoja jatkuvasti sekä työttömänä että työllisenä ollessaan. Yritykset asettavat palkkatarjoukset, jotka muodostavat palkkatarjousten jakauman. Työntekijät poimivat tarjouksia palkkatarjousten jakaumasta satunnaisin väliajoin. Tasapainossa kaikki palkkatarjoukset ovat työttömien hyväksyttävissä, mutta työlliset hyväksyvät saamansa palkkatarjouksen ainoastaan, jos se ylittää nykyisen palkan. Informaatio työmarkkinoilla on puutteellista, minkä vuoksi työntekijöiltä kuluu aikaa vaihtoehtoisia työpaikkoja etsiessä. Tämä taas antaa palkat asettaville yrityksille dynaamista monopsonivoimaa: yritys voi tarjota työntekijöilleen palkkaa, joka alittaa näiden työpanoksen arvon, koska työntekijät eivät voi välittömästi siirtyä parempaa palkkaa maksavaan yritykseen.

Tarjoamalla korkeampaa palkkaa yrityksen voitto työntekijää kohden pienenee. Toisaalta korkeampi palkka houkuttelee enemmän uusia työntekijöitä kilpailevista yrityksistä sekä alentaa todennäköisyyttä, että vanhat työntekijät lopettavat ja vaihtavat työnantajaa paremman palkkatarjouksen vuoksi. Siksi yritykset, jotka tarjoavat korkeampaa palkkaa, kasvavat keskimääräistä suuremmiksi, mikä taas kompensoi niiden heikompaa työntekijäkohtaista nettotulosta. Identtisten yritysten voittojen on luonnollisesti oltava yhtä suuret tasapainossa, mutta sama voittotaso voidaan saavuttaa eri palkkastrategioilla. Teorian päätulos siis on, että tasapainossa esiintyy palkkaeroja identtisten työntekijöiden välillä, vaikka kaikki työpaikat ovat yhtä tuottavia. Mallista voidaan johtaa myös muita vahvoja ennusteita koskien palkkojen, työsuhteiden pituuden ja yritysten koon välisiä relaatioita.

Mallin rakenneparametrit voidaan estimoida paneeliaineistosta, joka sisältää tietoja työntekijöiden työttömyys- ja työjaksojen pituuksista sekä palkoista. Burdettin ja Mortensenin (1998) mallin perusversio oletuksineen työntekijöiden ja yritysten homogeenisuudesta on ilmeisen pelkistetty. Empiirisen työn kannalta erityisen ongelmallista on se, että mallin ennustaman palkkatarjousten jakauman tiheysfunktio on nouseva koko arvoalueellaan. Mallin empiirinen analyysi edellyttääkin jonkin asteisen heterogeenisuuden sallimista. Yksinkertaisin tapa lienee luokitella aineisto työntekijöiden havaittujen taustaominaisuuksien mukaan erillisiin ryhmiin ja estimoida malli erikseen kullekin ryhmälle. Tämä ei kuitenkaan vielä riitä, vaan kunnollisen sovitteen saaminen ryhmien sisäisille palkkajakaumille edellyttää lisälähteitä heterogeenisuudelle.

Ad hoc -lähestymistapa on olettaa, että palkkamuuttujat sisältävät mittavirhettä, jolloin itse teoreettinen kehikko pysyy muuttumattomana, mutta tilastollisen mallin on huomioitava mittavirheen olemassaolo. Kehittyneempi lähestymistapa on kuitenkin yleistää teoreettista mallia olettamalla, että yritykset ovat heterogeenisia tuottavuusparametriensa suhteen eri työmarkkinasegmenttien sisällä. Estimoinnissa näiden tuottavuusparametrien jakauma voidaan estimoida ei-parametrisesti olettamalla niille joko diskreetti tai jatkuva jakauma.

Tulosten mukaan mallispesifikaatiot ilman yritysheterogeenisuutta eivät kykene selittämään empiirisessä aineistossa havaittua palkkavariaatiota. Sen sijaan spesifikaatiot, jotka sisälsivät havaitsematonta heterogeenisuutta tuottavuusparametreissa, antoivat kohtuullisen hyvän sovitteen palkkatarjousten jakaumalle. Tosin näilläkin oli ongelmia palkkatarjousten jakauman alapään muodon selittämisessä ja spesifikaation, jossa tuottavuuden oletettiin olevan jatkuvasti jakautunut, teoreettiset rajoitukset osittain hylättiin. Eräiden keskeisten rakenneparametrien estimaatit olivat lähes identtiset kaikissa spesifikaatioissa, jotka onnistuivat jäljittelemään palkkatarjousten jakauman muotoa riittävän hyvin (perusmalli mittavirheellä sekä molemmat spesifikaatiot, jotka sisälsivät yritysheterogeenisuutta).

Etsintäteoreettiset tasapainomallit yritysheterogeenisuudella höystettynä näyttäisivät antavan kohtuullisen kuvauksen työmarkkinoiden eräistä mekanismeista, joskin puutteitakin löytyy. Toisaalta tarkastellut mallit olivat varsin tyyliteltyjä monessakin suhteessa ja erityisesti mallien kysyntäpuoli oli äärimmilleen yksinkertaistettu. Onkin odetettavissa, että etsintäteoreettisten tasapainomallien saralla tulee tapahtumaan vielä paljon kehitystä lähitulevaisuudessa. Etenkin yhdistettyjen yritys- ja työntekijäaineistojen lisääntynyt saatavuus avaa uusia mahdollisuuksia empiiristen työmarkkinoiden tasapainomallien kehittelytyölle.

Contents

1 Introduction

The focus of *partial* job search models is on the behaviour of workers in the labour market characterised by search frictions in the form of time it takes for workers to collect information about alternative jobs. In such models the optimal search strategy of workers is typically characterised by the reservation wage but the wage offer distribution the workers face when searching is taken as exogenously given. The analysis of partial job search models has been the focus of much empirical work and such models have proved to be able to explain many stylized facts of the labour market (see, for example, surveys by Devine and Kiefer, 1991, and by Wolpin, 1995). However, in ignoring the demand side of the story the partial job search approach rules out the analysis of several important issues. Among the issues which cannot be analysed within the partial framework are all those which are related to wage determination, firm behaviour, interactions between worker and firm behaviour as well as the effects of policy reforms which affect wages (Bontemps et al., 1999).

In *equilibrium* models of job search, labour market phenomena are modelled as the outcome of optimal choices by both sides of the labour market. When firms are assumed to take the search behaviour of workers and wages set by other firms into account when setting wages, the wage offer distribution becomes essentially endogenous. The result that the wage offer distribution is determined by the model implies that policy reforms, such as changes in the minimum wage or unemployment benefits, may have quite a different effect than what is predicted by the partial job search models. Since the equilibrium search approach provides a natural framework for analysing different labour market phenomena, it is not very surprising that the estimation of such models has attracted a growing interest in recent years (see surveys by Mortensen and Pissarides, 1999, and by Van den Berg, 1999).

The first empirical application of equilibrium search theory is Eckstein and Wolpin's (1990) empirical analysis of the model of Albrecht and Axell (1984). In the Albrecht-Axell model firms set their wages so as to maximise profits taking the responses of workers and other firms into account. Only unemployed workers are assumed to be searching for jobs, and a worker who accepts a job is expected to hold it as long as he remains in the labour market. In this setting each wage offer in the market must be equal to the reservation wage of some group of searching workers. This is because offering a higher wage for a given type of workers would not attract any more workers from that group. Specifically, if all workers are homogeneous, then it is optimal to all firms to offer a single wage equal to the common reservation wage of the unemployed. From the viewpoint of workers, this corresponds to the case where one firm is monopolist in the labour market.[1]

Obviously the equilibrium search model should produce wage dispersion to be useful for empirical purposes. In the Albrecht-Axell model wage dispersion can arise only in the presence of exogenous worker heterogeneity. Eckstein and Wolpin (1990) in particular assume a finite number of worker types who differ from each others according to their value of non-market time. Since the computational complexity of the equilibrium solution increases rapidly with the number of different types, only a small number of worker types could be considered in practice.[2] This results in a discrete distribution of wage offers with only a few support points. To make this consistent with the observed smooth distribution, Eckstein and Wolpin (1990) assume that their wage data are measured with error. The model gives an acceptable fit to the duration data but does not fit to the wage data well as the measurement errors account for almost all of observed variation in wages. The latter result is what was expected as each point of the wage offer distribution necessarily corresponds to the reservation wage of unemployed workers of a given type.

Burdett and Mortensen (1998) (see also Burdett, 1990, and Mortensen, 1990) generalise the Albrecht-Axell model by allowing workers to search for jobs both on and off the job. Since for workers jobs are identical apart from the wage associated with them, employed workers are willing to move into higher-paying jobs whenever the opportunity arises. The fact that the current wage serves as the reservation wage for employed workers extends the range of reservation wages. For the wage-setting firms this means that the labour supply curve is upward-sloping. By offering a higher wage the firm makes a lower profit per worker but attracts more workers from other firms and retains them longer as high-paid workers are less likely to receive an acceptable offer elsewhere. It follows that the wage offer distribution which emerges as the equilibrium of a non-cooperative job search and wage posting game between labour market participants is dispersed even when all workers and firms are

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[1] This solution is known as the Diamond's (1971) paradox as he was the first person who acknowledged it.

[2] To be specific, Eckstein and Wolpin (1990) allow for employer heterogeity as well by assuming a finite number of firm types which differ in their labour productivity. It follows that in equilibrium more productive firms offer higher wages and therefore attract more workers. However, employer heterogeneity is not the key source of wage dispersion as it is not a sufficient, nor necessary, condition for wage dispersion.

respectively identical.[3] Thus persistent wage differentials across identical workers can exist as an equilibrium outcome in an environment characterised by search frictions. This main prediction of the model is consistent with empirical evidence on the existence of considerable wage differentials which cannot be explained by worker and job characteristics (see e.g. Bowlus et al., 1995*a*).[4] Other strong predictions concerning the relationships between wages, job durations and the size of firms follow from this simple model as well. Since many of these predictions are consistent with empirical observations, it is not surprising that much of research effort has been directed to estimating equilibrium search models that builds on the framework of Burdett and Mortensen (1998).

However, in the absence of exogenous worker or firm heterogeneity, the equilibrium distribution of wages generated by the Burdett-Mortensen model has an increasing density over its whole support which contradicts the shape of wage densities usually observed in the data. This results in a poor fit between the theoretical wage distribution of the Burdett-Mortensen model and the wage data. Stated differently, the simplest version of the model is unable to explain the *shape* of the wage distribution. When the populations of workers and firms are taken to be homogeneous, wage dispersion is fully explained by the degree of search frictions and the common values of productivity and non-market time. Of course, allowing for heterogeneity on either side of the labour market can be expected to have implications for wage dispersion. If workers are heterogeneous with respect to their value of non-market time, they apply different reservation wage strategies when unemployed, which in turn has an effect on the optimal wage-setting strategies of firms. Alternatively, when firms use different production technologies, the labour productivity of identical workers varies across different employers. It follows that more productive firms may offer higher wages than less productive ones as the wages set by the former can lie in a broader interval.

It seems obvious that some sort of heterogeneity has to be allowed for in the empirical analysis of the Burdett-Mortensen model to obtain an acceptable fit to the wage data. A simple way of making the model more realistic is to assume that the labour market consists of a large number of segments which differ from each others

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[3] Bunzel et al. (2001) refer to the homogeneous version of the Burdett-Mortensen as a 'pure' equilibrium search model because no heterogeneity is needed on either side of the labour market to produce a dispersed wage offer distribution.

[4] Of course, this result contradics with the prediction of competitive market analysis that all workers of a given type should receive the same wage in equilibrium.

according to observable characteristics of workers and jobs, such as education, age and industry. Assuming workers and firms to be identical within the segments one can then apply the homogeneous model separately to each segment of the labour market (see Kiefer and Neumann, 1993, Koning et al., 1995, Van den Berg and Ridder, 1998, and Bunzel et al., 2001).

Since empirical wage distributions do not usually exhibit increasing densities even within narrowly defined worker categories, this kind of *between-market* heterogeneity across the labour market segments is perhaps not sufficient to guarantee an accurate fit to the wage data. An *ad hoc* way of accounting for the discrepancy between the empirical and theoretical wage distributions within the segments is to assume that the wage data are subject to measurement error (see Christensen and Kiefer, 1994*a*). In this case the underlying theoretical model remains unchanged, but the estimation procedure must deal with a more complex measurement process. With an appropriate distribution for the measurement error, the discrepancy between the empirical and theoretical distributions can be attributed to the presence of measurement error in wages.

Of course, in reality workers and jobs are different also within the narrowly defined labour market segments. In this sense a more sophisticated approach is perhaps to introduce *within-market* heterogeneity in terms of unobserved differences across workers and/or firms operating in the same market. With an exception of Bontemps et al. (1999), the empirical applications of the Burdett-Mortensen model have focused entirely on allowing for employer heterogeneity rather than worker heterogeneity. This is mainly due to the difficulties of accounting for heterogeneity simultaneously on both sides of the market in the empirical analysis of the model,[5] while employer heterogeneity is expected to be a more important source of wage dispersion.[6]

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[5] Bontemps et al. (1999) estimate a version of the Burdett-Mortensen model with a continuous distribution for labour productivity (across firms) as well as for the value of non-market time (across workers). However, this makes the model intractable which enforces them to restrict the arrival rate of wage offers to be the same for unemployed and employed workers. This is a problematic assumption as the empirical evidence supports the view that unemployed workers receive wage offers more frequently (see, for example, our empirical results below).

[6] On the other hand, it is well known that unobservable differences across workers generate negative unemployment duration dependence. Thus, in the presence of negative duration dependence in the data, allowing for unobservable worker heterogeneity provides a way of improving the model's fit to the unemployment duration data. However, the poor fit to the duration data is unlikely to be the main concern when estimating equilibrium search models. In addition, Rantala (1998) does not find statistically significant unemployment duration dependence when estimating semiparametric proportional hazard models from the Finnish data.

A theoretical extension of the Burdett-Mortensen model which allows for a discrete distribution of labour productivity across firms is outlined in Mortensen (1990) and Burdett and Mortensen (1998). Bowlus et al. (1995*a*) develop an estimation method for this model which is able to deal with its ill-behaved likelihood function. As the method relies on numerical algorithms which do not require differentiation of the optimisation criterion, the computational burden increases rapidly with the number of firm types. Consequently, only a small number of firm types can be considered in practice which in turn does not necessarily guarantee a good fit to the wage data. Bontemps et al. (2000) introduce an alternative version of the Burdett-Mortensen model which allows for a continuous productivity distribution. They also propose a structural nonparametric estimation procedure for the model that does not restrict the productivity distribution to belong to any parametric family. An advantage of the continuous specification over that of Bowlus et al. (1995*a*) is its computational simplicity. On the other hand, the equilibrium model with continuous productivity dispersion imposes some strong restrictions on the shapes of productivity and wage distributions.

This paper contributes to the expanding literature in which equilibrium search models are estimated from panel data. As search on the job is by now regarded to be an important source of wage dispersion, we build our analysis on the framework of Burdett and Mortensen (1998). In particular we investigate whether the various specifications of the Burdett-Mortensen model are able to explain labour market histories observed in the Finnish panel data. Although the fundamental structure of the model remains unchanged, the source and interpretation of observed wage dispersion differ across the different specifications. These differences have potentially an effect on the estimates of the fundamental parameters as well. In this sense it seems meaningful to test which of the specifications fits to the data best and how the implications of the Burdett-Mortensen model differ across the specifications. This kind of exercise has been done by Bunzel et al. (2001) with the Danish data. Thus this paper is closely related to their study in which the results obtained from a number of different equilibrium search models are compared. However, our set of equilibrium search models to be compared is somewhat different and we apply the models to the Finnish data instead of the Danish data.

Structural parameters of the Burdett-Mortensen model can be identified from data on wages, job durations, unemployment durations and transitions between employment and unemployment. This study uses a sample of workers drawn from those who entered the unemployment register in Finland during 1992. We distinguish between separate segments of the labour market by stratifying the data according to education, sex and age. All structural parameters of the model are allowed to vary freely across the different segments. We begin with the simplest version of the model in which all workers and firms are assumed to be respectively identical. This analysis is followed by the estimation of specifications involving the measurement error in wages and unobserved employer heterogeneity within the labour market segments. Both the discrete and continuous distributions for labour productivity are considered. We do not allow for unobserved worker heterogeneity in any of specifications but rather focus on employer heterogeneity. We discuss the variation in the parameter estimates across the worker groups as well as across the model specifications. Different specifications of the model are compared in terms of their fit to the data.

The rest of the paper is organised as follows. In the next section the concepts of the equilibrium search theory are briefly discussed. Section 3 describes the data and provides some summary statistics. Empirical specifications and estimation results are given in Section 4. The final section concludes.

2 Equilibrium search theory

In this section we briefly describe equilibrium search theory along the lines of Burdett and Mortensen (1998) and Bontemps et al. (2000). We begin with a pure search model in which all jobs are equally productive and all workers are identical. Then we introduce employer heterogeneity in the model but retain the assumption of homogeneity of the worker population throughout the paper.

2.1 Equilibrium search with identical agents

2.1.1 Worker behaviour

The supply side of the labour market is populated by a continuum of *ex ante* identical workers. Behaviour of workers is characterised with the standard job search model with search on the job. In particular workers are assumed to be risk-neutral agents who are maximising the expected present value of future income stream with infinite horizon. Each worker in the labour market is either employed or unemployed. Events the worker faces in the labour market arrive with random time intervals. Each worker is facing a know distribution of wage offers F with associated jobs, from which he randomly samples wage offers both on and off the job. Wage offers arrive at the Poisson rate λ_0 when unemployed and at the Poisson rate λ_1 when employed. Unemployed workers search for an acceptable job and employed workers for a better job. Demand shocks arrive at the Poisson rate δ, destroying jobs and throwing workers who hold them into unemployment.

Given this framework, the present value of being unemployed, V, solves the continuous time asset pricing equation

$$\rho V = b + \lambda_0 \left(E_F \left(\max \left\{ W \left(\widetilde{w} \right), V \right\} \right) - V \right), \tag{1}$$

where ρ is the common discount rate, $W(w)$ is the present value of a job paying wage w, and b is the value of non-market time, including unemployment benefits net of search costs. The expectation above is taken over the support of F, and the tilde above w refers to a random draw from F. The equation (1) simply states that the opportunity cost of unemployment, the left-hand side of (1), is equal to the sum of the value of non-market time and the expected capital gain of finding an acceptable job, the right-hand side of (1). Analogously, the present value of being employed at wage w, $W(w)$, solves

$$\rho W(w) = w + \lambda_1 \left(E_F \left(\max \left\{ W \left(\widetilde{w} \right), W(w) \right\} \right) - W(w) \right) + \delta \left(V - W(w) \right), \tag{2}$$

which consists of the current wage, the likelihood and value of receiving an alternative job offer, and the likelihood and value of becoming unemployed. Note that the utility flow of an employed worker is assumed to be equal to his current wage.

Since $W(w)$ increases with w and V is independent of it, there exists a reservation wage r such that $W(r) = V$. By virtue of (1) and (2), it then holds that

$$\begin{aligned} r &= b + (\lambda_0 - \lambda_1)\left(E_F\left(\max\left\{W\left(\widetilde{w}\right), V\right\}\right) - V\right) \qquad (3) \\ &= b + (\lambda_0 - \lambda_1)\int_r^h \left(W(z) - V\right) dF(z), \end{aligned}$$

where h is the upper bound of the support of F. To put this expression into a more convenient form, we integrate by parts to obtain

$$\begin{aligned} r &= b + (\lambda_0 - \lambda_1)\int_r^h \left[1 - F(z)\right] dW(z) \qquad (4) \\ &= b + (\lambda_0 - \lambda_1)\int_r^h \frac{1 - F(z)}{\rho + \delta + \lambda_1\left[1 - F(z)\right]} dz. \end{aligned}$$

Following Burdett and Mortensen (1998), we focus on the limiting case of zero discounting and set $\rho = 0$. This allows us to rewrite (4) in the simpler form:

$$r = b + (\kappa_0 - \kappa_1)\int_r^h \frac{1 - F(z)}{1 + \kappa_1\left[1 - F(z)\right]} dz, \qquad (5)$$

where $\kappa_0 = \lambda_0/\delta$ and $\kappa_1 = \lambda_1/\delta$. This equation defines the reservation wage r as a function of the structural parameters of the model.

From (5) one can see how the possibility of search on the job affects the optimal search strategy of an unemployed worker. If wage offers arrive more frequently when unemployed than when employed $(\lambda_0 > \lambda_1)$, the reservation wage r exceeds the value of non-market time b. In that case it is more rewarding to search while unemployed and the worker rejects wage offers in the interval (b, r), even though this causes an utility loss in a short run. When the arrival rate is independent of employment status $(\lambda_0 = \lambda_1)$, the worker is indifferent between searching while employed and while unemployed. Any job that compensates for the foregone value of non-market time is acceptable in this case and thus $r = b$. If search on the job is not possible $(\lambda_1 = 0)$, the expression in (5) reduces to the standard optimality condition.

2.1.2 Firm behaviour

The demand side of the labour market consists of a continuum of identical firms. The firms are assumed to use only labour inputs in production. Each worker generates a flow of revenue p to his employer. We assume that p is independent of the size of the workforce and refer to p as the (labour) productivity of the firm. The firm sets its wage so as to maximise the steady-state profit flow taking the optimal search behaviour of workers and wages set by other firms as given. To attract workers the firm posts wage offers, among which workers randomly search using a uniform sampling scheme. Contrary to the competitive setting, the presence of search frictions in the labour market generates dynamic monopsony power for wage-setting firms. As workers cannot find a higher-paying job instantaneously, firms can offer wages strictly smaller than marginal labour productivity.

The steady-state profit flow of a firm paying wage w is given by

$$\pi(p, w) = (p - w)\, l(w), \tag{6}$$

where $l(w)$ is the size of the steady-state workforce (associated with a given F). The firm would employ as many workers as possible to maximise its profit flow as long as $p > w$. Since the current wage serves as the reservation wage for employed workers, the number of workers available to the firm in equilibrium increases with the wage offered, i.e. the labour supply curve the firm is facing with is upward-sloping. The firm takes the function l as given and offers a wage that maximises its steady-state profit flow. Obviously, a firm never offers a wage above p as the profits would be negative, nor it offers a wage less than r as such a wage would not attract any workers. The optimal wage offer of a firm with productivity p is a point in a set K_p of wages that maximise the steady-state profits,

$$K_p = \arg\max_{w} \left\{ \pi(p, w) \,|\, r \leq w \leq p \right\}. \tag{7}$$

When K_p is not a singleton the firm is indifferent between alternative wage strategies.

Firms which are equally productive must receive the same profit flow in equilibrium. This does not mean that wage offers need to be equal, however. A firm paying a higher wage makes a lower profit per worker but makes it up in volume as the higher wage attracts more workers from other firms and enables the firm to retain them for a longer time. It follows that some firms choose to offer low wages with a cost of high labour turnover, while others pay higher wages and experience lower labour turnover. Due to this trade-off between the wage offered and labour turnover, the same profit level can be attained by paying different wages.

2.1.3 Steady-state outcomes

Denote the fixed size of the labour force with m and the steady-state number of unemployed workers with u $(\leq m)$. In a short time interval dt a fraction δdt of employed workers, $m-u$, lose their jobs and become unemployed, so the flow from employment into unemployment is $\delta\,(m-u)\,dt$. The corresponding flow out of unemployment into employment is given by $\lambda_0\,[1-F(r)]\,udt$. Since firms offering a wage below r do not attract any workers and cannot therefore survive, we know that in equilibrium $F(r)=0$. In a steady state the flows into and out of unemployment are equal which implies that the steady-state unemployment rate is

$$\frac{u}{m}=\frac{\delta}{\delta+\lambda_0}=\frac{1}{1+\kappa_0}. \tag{8}$$

Using an analogous argument we can derive the steady-state *earnings* distribution G, the cross-section wage distribution of currently employed workers, associated with a given wage offer distribution F. Given the initial allocation of workers to firms, the number of workers employed at a wage no greater than w is given by $G(w)(m-u)$. The flow out of jobs paying w or less in a short time interval dt is $\delta G(w)(m-u)dt+\lambda_1\,[1-F(w)]\,G(w)(m-u)dt$, while the flow into such jobs is $\lambda_0 F(w)udt$. The outflow is equal to the number of workers who lose their jobs due to a demand shock plus the number of those who receive an offer greater than w. The inflow consists of those unemployed who receive an offer no greater than w. By equating these flows, we find the following steady-state relationship between the wage offer and earnings distribution:

$$\begin{aligned} G(w) &= \frac{F(w)}{\delta+\lambda_1\,[1-F(w)]}\cdot\frac{\lambda_0 u}{m-u} \\ &= \frac{F(w)}{1+\kappa_1\,[1-F(w)]} \end{aligned} \tag{9}$$

for all w on the common support of F and G. Since workers tend to move up the wage range over time, the earnings distribution lies to the right of the wage offer distribution, or more formally, G first-order stochastically dominates F as $F(w)-G(w)\geq 0$ for all w and $\kappa_1\geq 0$. The discrepancy between the earnings and wage offer distributions depends on κ_1 which is equal to the expected number of wage offers during a spell of *employment* (which may consist of several consecutive job spells) and can be thought of a relative measure of competition among firms for workers.

Using the relationship outlined in (9) we can derive the expression for the steady-state workforce. The number of workers receiving a wage on the interval $(w-\epsilon, w]$, where $\epsilon>0$, can be expressed as $[G(w)-G(w-\epsilon)](m-u)$. Given that the size of the firm population is normalised to one, there are $F(w)-F(w-\epsilon)$ firms offering wages on this interval. The steady-state workforce of the firm offering a wage w on the support of F can be expressed as

$$\begin{aligned} l(w) &= \lim_{\epsilon\rightarrow 0}\frac{[G(w)-G(w-\epsilon)](m-u)}{F(w)-F(w-\epsilon)} \\ &= \frac{\kappa_0(1+\kappa_1)m}{(1+\kappa_0)(1+\kappa_1[1-F(w)])^2} \end{aligned} \tag{10}$$

given that $F(w)-F(w-\epsilon)>0$ for all sufficiently small $\epsilon>0$. From (10) it is clear that l is increasing in w and continuous where F is continuous. Note that no assumptions on the shape of F have been made so far.

Burdett and Mortensen (1998) prove that there exists a unique non-cooperative equilibrium which consists of a triple $(r, F, \overline{\pi})$, such that (i) r satisfies (5) given F and (ii) each w on the support of F maximises $\pi(p, w)$, yielding the steady-state profit flow equal to $\overline{\pi}$.[7] Furthermore, they show that the equilibrium solutions for F and G are absolutely continuous with the common support $[r, h]$.

To see that there cannot be mass points in the equilibrium wage distributions, suppose that there is a mass point at $w^* \in [r,h]$. This induces the firm offering w^* to increase its offer slightly to increase its steady-state workforce substantially at the cost of only a second-order decrease in the profit per worker. It follows that a wage offer equal to a mass point cannot be profit-maximising for any firm. Secondly, to illustrate that there cannot be gaps in the support $[r,h]$, let us suppose that no firm offers a wage on the interval $(\underline{w}, \overline{w}) \subset [r,h]$. This cannot be the case in equilibrium as the firm offering wage $\overline{w}$ could increase its profits by reducing its wage offer to $\underline{w}$. The same argument implies that the firms offering the lowest wage in the market must offer a wage equal to r.

Since the profit flow is $\overline{\pi}$ across all firms in equilibrium, it holds in particular that $\pi(r)=\overline{\pi}=\pi(w)$ for all w on the support of F. Taking this together with (10)

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[7] Trivial solutions are ruled out by making the natural assumptions that $\infty > p > b$ and $\infty > \kappa_i > 0$ for $i=0,1$.

gives the equilibrium wage offer distribution[8]

$$F(w) = \frac{1+\kappa_1}{\kappa_1}\left(1 - \sqrt{\frac{p-w}{p-r}}\right), \quad w \in [r,h], \tag{11}$$

with the associated density

$$f(w) = \frac{1+\kappa_1}{2\kappa_1\sqrt{(p-r)(p-w)}}, \quad w \in [r,h]. \tag{12}$$

Moreover, by substituting (11) into (5) and recognising that $F(h) = 1$, we can write the bounds of support of F as

$$r = \gamma b + (1-\gamma)p, \tag{13}$$

$$h = \beta b + (1-\beta)p, \tag{14}$$

where the weights are given by

$$\gamma = \frac{(1+\kappa_1)^2}{(1+\kappa_1)^2 + (\kappa_0 - \kappa_1)\kappa_1}, \tag{15}$$

$$\beta = \frac{1}{(1+\kappa_1)^2 + (\kappa_0 - \kappa_1)\kappa_1}. \tag{16}$$

In other words, both the support and functional form of the wage offer distribution depends only on the structural parameters of the model.[9] The fact that h is a weighed average of b and p further implies that the highest wage offered in the market is strictly smaller than p.

The main outcome of equilibrium search theory with on-the-job search is that wages are dispersed in equilibrium even when all workers and firms are respectively identical. However, when the arrival rates of job offers, λ_0 and λ_1, tend to infinity, the equilibrium earnings distribution G converges to a mass point at p, and both the steady-state unemployment rate and the equilibrium profit rate tends to zero. Thus the competitive solution emerges as a liming case when search frictions disappear. As a second extreme, if only unemployed workers receive offers ($0 < \lambda_0 < \infty$ and $\lambda_1 = 0$), the firms cannot increase their workforce by offering higher wages. Thus all firms offer the same wage equal to r which in turn converges to b. In this case the equilibrium earnings distribution G limits to a mass point at b, and the Diamond's

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[8] The associated equilibrium solutions for the earnings distribution follows directly from the steady-state relationship outlined in (9).

[9] Obviously, $\beta = \gamma/(1+\kappa_1)^2$ which implies that $0 < \beta < \gamma < 1$ for $\kappa_1 > 0$, so $h > r$ provided that $p > b$.

(1971) paradoxical monopsony solution emerges. Moreover, all employment would be uniformly distributed across the firms as $l(w) = m - u$ by virtue of (10) and (8).

Other strong predictions follow from the simple model outlined above. Firstly, workers with longer employment history are predicted to be more likely to be located at the upper end of the wage distribution. This is because wage growth in the model results from job-to-job transitions. Secondly, the model implies a positive relationship between the size of workforce and the wage paid by the firm. Firms offering higher wages grow at a larger size because a higher wage attracts more workers to a firm from other firms and reduces the quit rate, $\lambda_1 [1 - F(w)]$. This results is driven by on-the-job search.

An interesting prediction of the model is that a change in the unemployment benefit b does not affect equilibrium unemployment as long as $b < p$. For example, an increase in b increases the reservation wage r by virtue of (5). However, to retain a positive workforce, firms offering wages below the new value of r must react by increasing their wage offers which in turn affects the wage offers of other firms. The net result is that the exit rate out of unemployment and thus the unemployment level remain unchanged. Using a similar reasoning one can see that a decrease in b does not affect unemployment either.

2.2 Employer heterogeneity

The model of the previous section makes several predictions which can be expected to be consistent with empirical data. However, a closer look at (12) reveals that the density of the wage offer distribution (and, consequently, that of the earnings distribution) is strictly increasing and concave on its whole support. This contradicts with the shape of wage distributions usually observed in the data as empirical wage densities are typically unimodal and skewed with a long right tail. This calls to doubt whether the simple equilibrium search model with identical agents can provide an acceptable fit to the wage data. To make the model more realistic, we extend the basic model by allowing for employer heterogeneity. In the first case we introduce heterogeneity assuming a discrete distribution for productivity across firms along the lines of Mortensen (1990) and Burdett and Mortensen (1998). Then we proceed to an extended model of Bontemps et al. (2000) which allows for continuous productivity dispersion.

2.2.1 A finite number of firm types

Assume that there are q types of firms which differ in their labour productivity such that $p_1 < p_2 < ... < p_q$. Keeping all other aspects of the model unchanged, Burdett and Mortensen (1998) show that an equilibrium in this case is characterised by $(r, F_1, ..., F_q, \overline{\pi}_1, ..., \overline{\pi}_q)$, where (i) r is the common reservation wage satisfying (5), (ii) F_i is a wage offer distribution of firms with productivity p_i, and (iii) $\overline{\pi}_i = (p_i - w)\, l(w)$ is the steady-state profit flow of firms with productivity p_i offering wages on the support of F_i, $i = 1, 2, ..., q$. Thus in equilibrium all firms of type i receive the same profit flow $\overline{\pi}_i$ and offer wages on the given interval $[\underline{w}_i, \overline{w}_i)$. Optimal wage setting implies that the wage offered increases with productivity and the bounds of intervals are such that $\overline{w}_i = \underline{w}_{i+1}$ for $i = 1, 2, ..., q-1$. Thus the solution results in a complete segmentation of the wage offer range among firm types. In addition, the lowest wage offered is equal to the reservation wage, so that $\underline{w}_1 = r$. To be consistent with our previous notations we also define that $\overline{w}_q = h$.

Analogously to (11), the offer distribution of type i firms is given by

$$F_i(w) = \frac{1+\kappa_1}{\kappa_1}\left(1 - \sqrt{\frac{p_i - w}{p_i - \underline{w}_i}}\right), \quad w \in [\underline{w}_i, \overline{w}_i)\,. \tag{17}$$

The equilibrium distribution of wage offers faced by workers emerges as a mixture of underlying wage offer distributions of different firm types and is given by

$$F(w) = \sum_{i=1}^{q} \left(\gamma_i - \gamma_{i-1}\right) F_i(w), \quad w \in [r, h]\,, \tag{18}$$

where $\gamma_i = F(\overline{w}_i)$ is the fraction of firms with productivity p_i or less, with the convention that $\gamma_0 = 0$. Using the equal profit condition which implies that in equilibrium $(p_i - w)\, l(w) = (p_i - \underline{w}_i)\, l(\underline{w}_i)$, the wage offer distribution on the interval $[\underline{w}_i, \overline{w}_i)$ can be expressed as

$$F(w) = \frac{1+\kappa_1}{\kappa_1}\left(1 - \frac{1+\kappa_1\left(1-\gamma_{i-1}\right)}{1+\kappa_1}\sqrt{\frac{p_i - w}{p_i - \underline{w}_i}}\right), \quad w \in [\underline{w}_i, \overline{w}_i)\,, \tag{19}$$

with the associated density

$$f(w) = \frac{1+\kappa_1\left(1-\gamma_{i-1}\right)}{2\kappa_1\sqrt{(p_i - w)\,(p_i - \underline{w}_i)}}, \quad w \in [\underline{w}_i, \overline{w}_i)\,. \tag{20}$$

In general, when there are q types of firms, the resulting distribution of wage offers F is absolutely continuous with the support $[r, h]$ and has $q-1$ 'kinks' corresponding to the wage cuts $(\overline{w}_1, \overline{w}_2, ..., \overline{w}_{q-1})$.

Recall from the previous section that the equilibrium wage distributions have increasing densities over their whole support when all jobs are equally productive. This property is at odds with the long flat right tail commonly observed in the wage data. In contrast, the model of this section with productivity heterogeneity implies that the wage density f is discontinuous at the wage cuts, between which it exhibits locally increasing patterns. This theoretical distribution can indeed take the functional form consistent with the observed data.

Allowing for employer heterogeneity has some new implications. Since more productive firms offer higher wages, they attract more workers, face lower quit rates and, consequently, are larger on average. In addition, more productive firms make more profit in equilibrium than less productive firms.

2.2.2 A continuum of firm types

Bontemps et al. (2000) (see also Bontemps et al., 1999) propose an alternative extension of the Burdett-Mortensen model which allows for continuous productivity dispersion. For this specification we suppose that productivity p is continuously distributed across *active* firms according to the distribution function Γ.[10] Contrary to the previous cases, Bontemps et al. (2000) show that in equilibrium only one wage can be profit-maximising for each firm in this case (that is, the set K_p of profit-maximising wages for a firm with productivity p is a singleton). In particular there exists a direct map between the productivity distribution Γ and wage offer distribution F. More formally, the wage offered by a firm with productivity p can be expressed as $w = K(p)$, where K is an increasing and continuous function over the support $\left[\underline{p}, \overline{p}\right]$, where $\underline{p} \geq r$.

Given Γ an equilibrium is characterised by (r, F) , where (i) r is the common reservation wage satisfying (5) and (ii) the wage offer distribution the workers face while searching is given by $F(w) = \Gamma\left(K^{-1}(w)\right)$.[11] As before the equilibrium dis-

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[10] The number of active firms in the market can be viewed to be endogenous in this setting. To see this, suppose that there are N_0 firms willing to participate in the market, across which productivity is distributed according to the distribution function Γ_0, with the lower and upper bound of support $\underline{p}_0$ and $\overline{p}$ respectively. However, only firms making a non-negative profit are active and hence participating in the market. The distribution of productivities across these firms is obviously $\Gamma(p) = \Gamma_0\left(p \,|p \geq r\right)$, where the threshold value for participation is the reservation wage r. Stated differently, only those firms which are able to pay at least r can participate in the market. If the reservation wage is high enough to drop some firms out of the market (i.e. when $r > \underline{p}_0$), the measure of active firms in the market is $N_0\left[1 - \Gamma_0(r)\right]$ (this figure is normalize to one in the text to be consistent with our previous analysis). See Bontemps et al. (2000) for further discussion.

[11] To be specific, there can be a single equilibrium, multiple equilibria or equilibrium may not

tribution of wage offers F is absolutely continuous over its support $[r,h]$. As K is strictly increasing in p it follows that firms with the lowest productivity $\underline{p}$ offer a wage equal to the reservation wage r and the highest wage in the market h is offered by firms with the highest productivity $\overline{p}$, whereas the wage offers of other firms lies between r and h.

Like previously, the profit function of a firm with productivity p is given by $\pi(p,w) = (p-w)\,l(w)$, where $l(w)$ is as defined in (10). The optimal wage offer $w = K(p)$ solves the first-order condition, $\partial\pi(p,w)/\partial w = 0$. For given F and p this writes as

$$2\kappa_1 f(w)(p-w) - (1+\kappa_1[1-F(w)]) = 0, \tag{21}$$

and the associated second-order condition can be expressed as

$$f'(w)(1+\kappa_1[1-F(w)]) - \kappa_1 f(w)^2 < 0, \tag{22}$$

provided that $w = K(p) \geq r$. The first-order condition (21) gives an implicit function of the wage offer of a firm with productivity p given κ_1 and the distribution of wages offered by other firms.

Solving the profit function for $w = K(p)$ yields

$$K(p) = p - \frac{\pi(p)}{l(K(p))}, \tag{23}$$

where $\pi(p) \equiv \pi(p, K(p))$. Bontemps et al. (2000) show that one can use (23) to derive the expression for K. To pursue this route, we have to express $\pi(p)$ and $l(K(p))$ in terms of the primitives of the model. The latter is easy one as substitution of $\Gamma(p)$ for $F(K(p))$ into (10) yields

$$l(K(p)) = \frac{\kappa_0(1+\kappa_1)m}{(1+\kappa_0)(1+\kappa_1[1-\Gamma(p)])^2}. \tag{24}$$

Using the envelope theorem we find that $\pi'(p) = l(K(p))$ which implies

$$\int_{\underline{p}}^{p} \pi'(z)dz = \pi(p) - \pi(\underline{p}) = \int_{\underline{p}}^{p} l(K(z))\,dz. \tag{25}$$

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exist at all, depending on the values of structural parameters of the model. Bontemps et al. (2000) point out that it is in general hard to differentiate between alternative cases. In the text we arbitrarily assume that an unique equilibrium exists.

Taking together with (24) and the boundary condition $\pi(\underline{p}) = \big(\underline{p} - r\big)\, l\,(r)$ this suggests

$$\begin{aligned}
\pi(p) &= \big(\underline{p} - r\big)\, l\,(r) + \int_{\underline{p}}^{p} l\,(K(z))\, dz \\
&= \frac{\kappa_0\,(1+\kappa_1)\, m}{1+\kappa_0} \left(\frac{\underline{p} - r}{(1+\kappa_1)^2} + \int_{\underline{p}}^{p} \frac{dz}{(1+\kappa_1\,[1-\Gamma(z)])^2} \right) \\
&= \frac{\kappa_0\,(1+\kappa_1)\, m}{1+\kappa_0} \int_{r}^{p} \frac{dz}{(1+\kappa_1\,[1-\Gamma(z)])^2}.
\end{aligned} \tag{26}$$

Substituting (24) and (26) into (23) yields

$$K(p) = p - (1+\kappa_1\,[1-\Gamma(p)])^2 \int_{r}^{p} \frac{dz}{(1+\kappa_1\,[1-\Gamma(z)])^2}, \tag{27}$$

which defines the optimal wage offer $w = K(p)$ in terms of the primitives of the model.[12] Differentiation of (27) with respect to p gives

$$K'(p) = \frac{2\kappa_1 \gamma(p)}{1+\kappa_1\,[1-\Gamma(p)]}\,(p - K(p))\,, \tag{28}$$

where γ is the productivity density associated with Γ. From $\Gamma(p) = F\,(K(p))$ it follows that $\gamma(p) = f(K(p))K'(p)$. Substitution of these reveals that (28) is equivalent to the first-order condition (21). Furthermore, it is not hard to see that the second-order condition (22) is equivalent to the condition $K'(p) > 0$.

The expression for the equilibrium wage offer distribution F follows directly from (27) and the relationship $F(w) = \Gamma\left(K^{-1}(w)\right)$. However, neither K nor F has a closed-form expression in general. Despite this the model outlined above makes some strong restrictions on the shape of the productivity and wage distributions, excluding certain shapes for Γ and F (and hence for G). These impose testable restrictions which can used as a specification test in the empirical analysis of the model. We will come back to this later on.

Bontemps et al. (2000) show that the set of wage offer distributions that can be generated by the model is characterised by the following conditions:[13] (i) the upper bound of support of F is finite (i.e. $h < \infty$), (ii) the density f has an infinitely high peak at the lowest wage if and only if the lowest wage equals the lower bound

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[12] One should substitute r from (5) into (27) to express the wage offer $w = K(p)$ in terms of $(b, \lambda_0, \lambda_1, \delta, p, \Gamma)$.

[13] Bontemps et al. (2000) derive the corresponding conditions for the earnings distribution G as well. Since F and G are directly related through (9), as imposed by the equilibrium flow conditions, we focus here on the conditions for F only.

of the support of the productivity distribution (i.e. if $r = \underline{p}$), and (*iii*) for given κ_1, $f(w)\left(1+\kappa_1\left[1-F(w)\right]\right)$ decreases over the whole support of F (that is, the second-order condition (22) holds).[14]

In ruling out wage distributions with unbounded support the first condition implies that firms with high productivity may have very high monopsony power and, consequently, receive very high profits. The second condition reflects the fact that if the reservation wage is high enough it destroys monopsony power of firms located at the lower end of the productivity distribution. Since the wage offers of these firms must lie in a narrow interval between the reservation wage and their productivity, there is a congestion at the lowest wages, resulting in a huge peak. In contrast, when the lower bound of productivity support exceeds the reservation wage notably, low-productivity firms can also choose their wages from a wide range. In such a case a wage offer equal to a mass point cannot be profit-maximising for any firm, as pointed out previously. The third condition imposes the restriction how steeply f can increase given κ_1. Where the density f is decreasing the condition is obviously met regardless of the value of κ_1. For large κ_1 the condition allows f to increase quite steeply, but for small κ_1 the condition is violated even if f increases only slightly.

Moreover, Bontemps et al. (2000) derive a condition concerning the relationship between the right tails of productivity and wage distributions. Given that the productivity density γ is monotone for all sufficiently large $p < \underline{p}$, they show that the wage density f has a flat right tail if and only if the right tail of γ goes to zero at a rate slower than p^{-3}. Since empirical wage distributions usually have a flat right tail, this condition suggests that we should expect a very flat right tail for the productivity distribution.

Turning to the properties of the equilibrium in the case of continuous productivity dispersion. The result that $\pi'(p) = l\left(K(p)\right) > 0$ implies that the profit level is an increasing function of productivity p. Moreover, since both l and K are increasing functions the profit function $\pi(p)$ is convex in p. This suggests that the monopsony power of the firm increases with productivity. Secondly, the degree of wage dispersion can be decomposed into two parts: one which is due to search frictions analogously to the homogeneous case and another which results from variation in productivity across firms. In particular it can be shown that the equilibrium of the

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[14] Note that $f(w)\left(1+\kappa_1\left[1-F(w)\right]\right)$ is constant for the homogeneous version of the Burdett-Mortensen model.

homogeneous version of the Burdett-Mortensen models emerges as a limiting case when the degree of productivity dispersion goes to zero (see Bontemps et al., 2000, for details).

3 Data

Christensen and Kiefer (1997) discuss data requirements for identification of the structural parameters of the Burdett-Mortensen model using the maximum likelihood method. They show that the model can be estimated from data on individual labour market histories where at least some of the workers are observed with both unemployment duration and job duration with the associated wage.

Empirical analysis of this study is based on a sample of individuals drawn from the worker data of the Integrated Panel of Finnish Companies and Workers (the IP data).[15] Underlying source of information on workers in the IP data is the Employment Statistics (ES) database of Statistics Finland. The ES database is a longitudinal database which combines information from over 20 administrative registers. Since 1987 the ES database has been updated regularly, and it covers effectively all people with a permanent residence in Finland.

Each individual in the ES database who holds a job at the last week of the year is associated to his or her employer with a company and establishment identifier. The worker panel of the IP data covers all people from the ES database with an identifier of the private-sector employer at least in one of the years between 1988 and 1996. As a result, the underlying worker panel covers practically *all* persons who have been employed in the private sector during the period 1988-1996 (at least at the end of one year). The total number of persons in the IP data is slightly below two million. For these people a set of variables, collected by combining the annual records of the ES database, is available over the period 1988-1996.

In this paper we focus on a certain subsample of the worker panel of the IP data. As a first step we select all individuals between the ages of 16 and 65 who entered unemployment during 1992. Due to the coverage of the IP data, this set of workers accounts for a large proportion of the total flow into unemployment in 1992.[16] We exclude workers who have been self-employed as well as those have been employed

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[15] A detailed description of the IP data can be found from Korkeamäki and Kyyrä (2000).

[16] A few points concerning the sampling scheme should be stressed. Firstly, the sample drawn from the inflow of unemployment is certainly not a representative sample of the labour force. In such a sample workers with poor employment prospects are likely to be over represented. As an example, the job destruction rate among workers recently hired from unemployment can be expected to be higher than that among the employed in general. Secondly, the sampled workers are followed from 1992 to 1996, which is the period following a deep recession and is, consequently, characterised by a record high unemployment level (see the discussion below). This sub-period however exhibits the most stable labour market conditions in the period 1988-1996 potentially available for the analysis.

by the public sector or non-profit organisation during the period 1990-1996. These groups of workers are excluded as the underlying equilibrium search model does not describe their labour market experiences. Workers of the former group set their own wages and their behaviour in the labour market can be expected to differ from that of other workers in other respect as well. The latter group is omitted because their employers do not set wages so as to maximise profits.[17]

For all individuals selected in the sample, we record the duration of the unemployment spell (d) and information on whether unemployment ended because of finding a job ($c_d = 0$) or for some another reason ($c_d = 1$).[18] Unemployment spells not followed by a job are treated as right-censored in the empirical analysis. This may occur due to a drop out of the labour force, participation in the active labour market programme or the spell continuing beyond the observation period. It should be stressed that we treat unemployment spells ended in a job placement programme as right-censored as well. Thus we make a difference between finding a job from the open labour market and becoming employed by labour administrative measures.

For those workers who found a job, we further record the accepted wage (w) and the duration of the subsequent job spell (j) along with the reason for termination. The wage rate is computed using information on annual earnings and the days worked. A job spell may end in a layoff ($a = 0$, $c_j = 0$), a quit ($a = 1$, $c_j = 0$) or be right-censored ($c_j = 1$). Job spells followed by unemployment are classified to be ended in a layoff, whereas job spells consecutively followed by another job spell with a new employer are interpreted to be ended in a quit for a better job. We identify changes in employer by comparing establishment identifiers attached to workers on the basis of the employer.[19] Job spells terminated due to a drop out of the labour force and those continuing beyond the observation period are treated as right-censored. All durations are measured in months, and wages are converted into monthly rates to match the duration measures.

Recall that our theoretical model is concerned with the population of homogeneous workers. While all workers are different in practice, we cannot allow for the parameters to be different for each individual as the model will be of no use at all in

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[17] Moreover, the underlying sampling scheme for the worker panel of the IP data implies that the data do not provide a representative sample of public-sector employees.

[18] If the worker has several unemployment spells started in 1992, we choose the first one.

[19] Alternatively, we could use company identifiers in detecting job changes. We prefer establishment identifiers since they are not so sensitive to change due to changes in the ownership, legal status or sector of the employer.

such a case. Instead we assume that the labour market consists of a large number of segments, each of which forms a single market of its own. These segments are assumed to differ from each others according to observed characteristics of workers. To deal with this kind of heterogeneity, the model can be applied separately to each group of workers, allowing for all parameters to vary freely across the groups. This approach corresponds to controlling observed heterogeneity with a complete set of discrete regressors (Christensen and Kiefer, 1994*a*).[20] To pursue this approach, we stratify the data by education, sex and age. We categorise education as follows: lower vocational education or less (11 years or less), upper vocational education (12-13 years), lower university (a Bachelor degree or the lowest level of university education, 13-15 years), and upper university (a Master degree or higher, 16 years or more). The age groups considered are: 16-21 years, 22-30 years, 31-50 years, and 51-65 years. As only few workers with lower or upper university education are aged below 22 or above 50, we combine the two lowest age groups as well as the two highest age groups for these education groups.

As some of the estimation procedures used are sensitive to outliers in the wage data, some concern needs to be taken with our wage measures. Since the monthly wages are computed from annual earnings without information on hours worked, the wage data can be expected to contain some measurement error. To deal with outliers in the wage data, we first require that all wages must be at least 80% of the lowest salary grade of the central government, after which we trim the lowest and highest 3% of wage observations in each subgroup.

Table 1 gives some descriptive statistics for the worker groups to be analysed. The number of observations in the underlying group (N) is given in the first column of the table. Since the computational burden of the estimation method for the model with a discrete distribution of productivity increases rapidly with the sample size and the number of firm types, the maximum size of the estimation sample is restricted to 3,000 observations. Thus, where N exceeds 3,000, we have drawn a random subsample of 3,000 workers from the underlying group (after trimming the wage data). All sample statistics in the table are computed from this subsample, describing the sample to be used in the estimations.

It is worth emphasising that the period under investigation is exceptional one.

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[20] Although the segmentation assumption provides a simple and flexible way to bring heterogeneity in the model, it can be viewed be quite restrictive as it implies that workers do not move from one segment to another and firms in different segments do not compete.

Table 1: Summary statistics

	N	$\overline{w}$	$w_{\min}$	$w_{\max}$	$\overline{d}$	c_d	$\overline{j}$	c_j	a
Lower vocational and less									
Men, 16-21	17,051	7,819	4,625	16,713	7.56	.78	12.54	.32	.20
Men, 22-30	40,145	9,526	4,845	23,464	10.49	.58	12.28	.20	.22
Men, 31-50	73,142	10,483	5,067	26,669	11.58	.57	12.52	.22	.20
Men, 51-65	20,129	10,320	4,940	27,002	16.43	.73	9.94	.20	.19
Women, 16-21	7,604	7,071	4,546	18,325	7.07	.80	12.24	.33	.17
Women, 22-30	13,721	7,638	4,607	18,858	10.06	.71	14.63	.31	.22
Women, 31-50	31,212	7,932	4,685	18,172	11.94	.66	16.71	.29	.14
Women, 51-65	13,724	7,871	4,647	17,741	19.17	.79	13.95	.22	.10
Upper vocational									
Men, 16-21	9,544	7,843	4,659	15,324	5.83	.78	11.91	.41	.27
Men, 22-30	11,233	9,298	4,834	22,003	8.81	.57	14.92	.28	.27
Men, 31-50	8,993	11,742	5,278	28,770	11.22	.60	17.82	.31	.22
Men, 51-65	1,811	13,645	5,724	27,963	18.08	.79	14.02	.30	.12
Women, 16-21	6,769	6,919	4,535	14,799	5.41	.73	11.45	.37	.25
Women, 22-30	11,002	7,734	4,680	16,232	7.51	.60	15.72	.31	.26
Women, 31-50	6,273	9,298	4,837	22,809	10.35	.65	17.60	.32	.23
Women, 51-65	695	10,284	5,028	23,591	18.98	.82	13.98	.26	.07
Lower university									
Men, 16-30	5,042	10,531	5,185	23,859	7.36	.59	17.77	.35	.28
Men, 31-65	4,495	13,755	5,338	37,047	9.28	.54	18.88	.34	.24
Women, 16-30	1,831	8,541	4,725	16,842	6.55	.56	17.31	.36	.32
Women, 31-65	1,512	11,241	4,938	28,597	10.83	.61	17.97	.36	.25
Upper university									
Men, 16-30	502	12,883	5,636	27,255	7.21	.47	20.87	.35	.55
Men, 31-65	1,321	17,874	5,667	53,488	12.12	.56	22.18	.38	.32
Women, 16-30	526	11,756	5,683	34,397	6.33	.51	18.09	.40	.38
Women, 31-65	543	12,887	4,831	34,092	10.32	.59	16.64	.34	.32

Notes: N is the number of observations in the underlying population from which the estimation sample was drawn. $\overline{w}$ is the average accepted wage in the estimation sample. $w_{\min}$ and $w_{\max}$ are the minimum and maximum of observed wages in the estimation sample respectively. $\overline{d}$ and $\overline{j}$ are the average durations of unemployment and job spells respectively. c_d is the share of censored unemployment spells, and c_j is the share of censored job spells in the estimation sample. a is the share of uncensored job spells ending in a quit for a better job.

An overheating period of the Finnish economy in the last years of the 1980s was followed by a deep recession in the early 1990s. The annual change in the GDP was negative during the period 1991-1993, and in the worst year, 1991, the GDP decreased by over 7%. According to the Labour Force Survey, the unemployment rate rose from 3.2% in 1990 to over 16% in 1993, remaining at the level beyond 14.5% until 1996. Labour market experiences of the sampled workers thus took place in a period of record high and stable unemployment which should be kept in mind when interpreting the results.

From the first column of the table it appears that the size of the underlying worker group is much lower for highly educated groups. This does not reflect only the education structure of the labour force but also a lower incidence of unemployment among more educated workers. The fact that the period under investigation is characterised by high unemployment levels is reflected to the figures in the table. Unemployment durations are relatively long with a high rate of censoring, and most of subsequent job spells are ended in a layoff.

Unemployment duration increases with age and is exceptionally high among low educated workers aged over 50. There are no clear differences in the average duration of unemployment by sex. The rate of censoring in the unemployment data is found to be very high. It is also worth emphasising that the average duration of censored spells is over two times higher than that of uncensored spells (not shown in the table). This is because long-duration spells of unemployment are often terminated by labour administrative measures. This explains partly the higher censoring rates for the groups with the longest unemployment durations. Young job seekers are often regarded as a special target group of the labour administrative measures, resulting in a relatively high censoring rate for workers aged under 22. There are no large differences in job duration across age groups. Highly educated workers experience slightly longer job spells and are more likely to quit for a better job. Compared to unemployment spells, job spells are longer on average and the rate of censoring in the job duration data is much lower.

Wages increase with age at least up until the interval 31-50 years of age. There are no clear wage differentials between workers at the two lowest levels of education, whereas higher education yields slightly higher return. Despite the trimming procedure there are still wage observations which are relatively low compared to minimum requirements, reflecting some measurement problems in the wage data. Empirical wage densities for each worker group are shown in Figures 1 to 3, where the solid lines represent the kernel density estimates obtained using Gaussian kernels with the bandwidth chosen by a rule of thumb. Other lines depict the predicted densities obtained from the different specifications of the equilibrium search model and they will be discussed later on. The empirical wage densities are generally unimodal and skewed with a long right hand.

4 Empirical application

We derived previously the explicit solutions for the equilibrium wage offer distribution with and without employer heterogeneity. From the assumptions underlying the theoretical model it is straightforward to derive distributions for unemployment and job durations as well. Knowledge about these distributions allows us to write down likelihood functions for various specifications of the model. In the next section we derive a general form of the likelihood function without specifying the functional form for the wage offer distribution. In the subsequent sections we then discuss the estimation of various specifications of the model and represent the empirical results.

4.1 The likelihood function

The structural parameters of interest to be estimated are $(\lambda_0, \lambda_1, \delta, r)$ with a scalar p for the homogeneous model, the set $(p_1, p_2, ..., p_q)$ of productivity terms for the model with a discrete distribution of productivity, and Γ for the model with a continuous distribution of productivity. Knowledge of these parameters allows us to compute an estimate for b using (5).[21] Since our sample was drawn from the inflow of unemployment, we observe a spell of unemployment along with the post-unemployment destination for each individual in the data. For those individuals whose unemployment ended in a new job we further observe the wage rate accepted as well as the duration of the subsequent job along with the reason for termination. Since we do not have complete information on all observations, the possibility of censored observations on both unemployment and job duration is explicitly accounted for.

The likelihood contribution from an individual who is unemployed for d periods, accepts then a job with an associated wage w, keeps that jobs for j periods until he gets laid off $(a = 0)$ or finds another job $(a = 1)$ has a general form

$$\ell = \varphi\left(d\right)\left[f(w)\phi\left(j, a\left|w\right.\right)\right]^{1-c_d}, \tag{29}$$

where φ is the density function of unemployment duration, f is the density of the wage offer distribution and ϕ is the density function of job duration and destination conditional on the accepted wage w. The censoring indicator for unemployment

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[21] It can be argued that b is the 'deep' structural parameter of the model rather than r, which follows implicitly from the optimal search strategy of unemployed workers. However, this distinction does not make any difference in practice due to the one-to-one relationship between b and r outlined in (5).

duration c_d takes a value of zero if the unemployment spell is followed by a new job, and a value of one otherwise.

Since job offers arrive at the Poisson rate λ_0 and all offers are acceptable to the unemployed in equilibrium, unemployment duration d is exponentially distributed with intensity parameter λ_0. Thus

$$\varphi(d) = \lambda_0^{1-c_d} e^{-\lambda_0 d}, \tag{30}$$

where the possibility of censored data is taken into account through c_d. As workers search randomly among employers using a uniform sampling scheme, the realisation of the wage w in the accepted job is simply a random draw from the equilibrium wage offer distribution F.

To derive the conditional distribution of job duration j and destination a, we can use the standard competing risks framework for exponential duration models. Recall that layoffs occur at the Poisson rate δ and alternative offers arrive at the Poisson rate λ_1. Since only wage offers exceeding the current wage will be accepted, the actual quit rate is $\lambda_1 (1 - F(w))$, the probability of receiving an offer times the probability that the received offer is acceptable given the current wage w. Conditional on the current wage w, the job duration j has an exponential distribution with intensity parameter $\delta + \lambda_1 (1 - F(w))$. Exit from this job into unemployment occurs with probability $\delta / [\delta + \lambda_1 (1 - F(w))]$ and exit into a higher-paying job with probability $\lambda_1 (1 - F(w)) / [\delta + \lambda_1 (1 - F(w))]$. Putting these together yields

$$\phi(j, a \,|w) = [(1-a)\delta + a\lambda_1 (1 - F(w))]^{1-c_j} \, e^{-(\delta + \lambda_1 [1 - F(w)])j}. \tag{31}$$

Substituting (30) and (31) into (29) and taking logarithm gives the individual contribution to the log-likelihood function

$$\begin{aligned} \log \ell \;\; = \;\; & (1-c_d)(1-c_j) \ln[(1-a)\delta + a\lambda_1 (1 - F(w))] - \lambda_0 d \\ & + (1-c_j)(\ln \lambda_0 + \ln f(w) - (\delta + \lambda_1 [1 - F(w)]) j). \end{aligned} \tag{32}$$

Estimations of different specifications of the Burdett-Mortensen model will all be based on (32), the only difference between the specifications being the functional form assumed for F and f.[22]

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[22] Christensen and Kiefer (1997) show that identification of all structural parameters of the model does not necessarily require information on whether the job spell ends in a quit or layoff, i.e. observations on a are not crucial for identification. The separate identification of λ_1 and δ even without knowledge of a follows from the fact that the conditional job hazard decreases with w. A higher wage does not affect the layoff rate but implies a lower quit rate and the extent of this effect depends on λ_1.

Recall that the shape of the equilibrium wage offer distribution generally does not depend on λ_0. This observation taken together with (32) suggests that λ_0 is identified from the unemployment duration data only. It follows that the estimator of λ_0 is stochastically independent of all other parameters of the model, being robust with respect to different model specifications.

4.2 Identical workers and firms

We begin our empirical analysis with the simplest specification of the model with homogeneous workers and firms. Equilibrium solutions for F and f are given by (11) and (12) respectively. Substituting these expressions into (32) and summing over the observations gives the log-likelihood function in terms of $(\lambda_0, \lambda_1, \delta, p, r)$. The properties of the maximum likelihood estimators are not standard in this case, however. This is due to the dependence of the support of F on the unknown parameters of the model. Kiefer and Neumann (1993) show that one can simplify the estimation problem considerably by reparametrising the model from $(\lambda_0, \lambda_1, \delta, p, r)$ to $(\lambda_0, \lambda_1, \delta, h, r)$ and using order statistics to estimate the bounds of the support of F. To pursue this route, we solve the system (13) and (14) for p and b to obtain

$$\begin{aligned} p &= \frac{\beta}{\beta-\gamma} r + \frac{\gamma}{\gamma-\beta} h, && (33) \\ b &= \frac{\beta-1}{\beta-\gamma} r + \frac{\gamma-1}{\gamma-\beta} h, && (34) \end{aligned}$$

where β and γ are given by (15) and (16) respectively. Using (33) to substitute p out of the expressions for F and f, we can rewrite the log-likelihood function in terms of $(\lambda_0, \lambda_1, \delta, r, h)$.

Following Kiefer and Neuman (1993), we estimate r and h using the sample minimum and maximum respectively.[23] In the second step the frictional parameters are estimated by maximising the likelihood function with respect to $(\lambda_0, \lambda_1, \delta)$ conditional on the estimates of (r, h). Due to the properties of order statistics, the estimates of (r, h) are superconsistent, converging to their true values at a rate faster than $\sqrt{n}$, where n is the sample size. In what follows, the order $\sqrt{n}$ distribution of the estimators of (r, h) depends little on other parameters of the model, suggesting that ignoring variation in the estimates of (r, h) does not affect asymptotic inference about $(\lambda_0, \lambda_1, \delta)$. This allows us to take them as fixed in the maximum likelihood

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[23] Of course, the estimate of r is biased upwards and that of h downwards in finite samples.

Table 2: Estimation results with identical agents

	λ_0	λ_1	δ	p	b
Lower vocational and less					
Men, 16-21	.0285 (.0011)	.0099 (.0011)	.0470 (.0024)	42,697 (3,271)	2,461 (229)
Men, 22-30	.0399 (.0011)	.0130 (.0009)	.0547 (.0018)	58,533 (2,844)	754 (279)
Men, 31-50	.0375 (.0010)	.0117 (.0008)	.0531 (.0018)	70,946 (3,629)	332 (312)
Men, 51-65	.0166 (.0006)	.0138 (.0013)	.0701 (.0029)	78,136 (5,373)	4,532 (205)
Women, 16-21	.0282 (.0012)	.0070 (.0009)	.0486 (.0025)	62,871 (6,164)	1,753 (257)
Women, 22-30	.0292 (.0010)	.0082 (.0007)	.0401 (.0017)	50,235 (3,240)	1,228 (264)
Women, 31-50	.0286 (.0009)	.0051 (.0005)	.0384 (.0015)	65,841 (5,314)	811 (238)
Women, 51-65	.0109 (.0004)	.0054 (.0008)	.0519 (.0024)	78,050 (9,152)	3,988 (114)
Upper vocational					
Men, 16-21	.0385 (.0015)	.0128 (.0013)	.0405 (.0022)	29,884 (1,751)	1,740 (298)
Men, 22-30	.0494 (.0014)	.0118 (.0008)	.0390 (.0014)	46,673 (1,950)	-2,358 (416)
Men, 31-50	.0353 (.0010)	.0080 (.0006)	.0327 (.0012)	71,490 (3,825)	-3,464 (535)
Men, 51-65	.0116 (.0006)	.0071 (.0013)	.0454 (.0029)	94,126 (13,130)	4,687 (342)
Women, 16-21	.0505 (.0018)	.0113 (.0010)	.0458 (.0022)	33,238 (1,982)	635 (291)
Women, 22-30	.0534 (.0015)	.0106 (.0007)	.0357 (.0013)	33,173 (1,426)	-1,366 (346)
Women, 31-50	.0335 (.0010)	.0079 (.0006)	.0324 (.0013)	55,604 (3,089)	-1,489 (419)
Women, 51-65	.0095 (.0009)	.0035 (.0014)	.0506 (.0055)	152,083 (55,257)	3,970 (337)
Lower university					
Men, 16-30	.0562 (.0016)	.0096 (.0007)	.0296 (.0011)	48,443 (2,054)	-7,453 (697)
Men, 31-65	.0495 (.0013)	.0075 (.0005)	.0292 (.0010)	91,523 (4,474)	-14,893 (1,031)
Women, 16-30	.0675 (.0025)	.0119 (.0010)	.0284 (.0014)	28,814 (1,272)	-5,081 (705)
Women, 31-65	.0357 (.0015)	.0080 (.0009)	.0293 (.0017)	66,729 (4,988)	-4,881 (895)
Upper university					
Men, 16-30	.0735 (.0047)	.0180 (.0020)	.0184 (.0018)	34,727 (1,350)	-16,266 (2,945)
Men, 31-65	.0363 (.0016)	.0078 (.0008)	.0218 (.0013)	110,639 (7,135)	-20,876 (2,368)
Women, 16-30	.0770 (.0050)	.0102 (.0015)	.0249 (.0023)	63,666 (5,355)	-26,298 (3,901)
Women, 31-65	.0395 (.0027)	.0123 (.0019)	.0310 (.0029)	64,873 (6,069)	-5,850 (1,781)

Notes: Standard errors are in parentheses.

estimation, even though they are not exogenous. This approach is justified by a general concept of a local cut by Christensen and Kiefer (1994*b*). Given the consistent estimates of $(\lambda_0, \lambda_1, \delta, r, h)$, we can estimate p and b using (33) and (34) and compute their standard errors using the delta method (see Bunzel et al., 2001, for details).

Order statistics estimates of (r, h) for each group can be found from Table 1, where they correspond to the minimum and maximum accepted wage (i.e $\widehat{r} = w_{\min}$ and $\widehat{h} = w_{\max}$). Estimates of other parameters of the model are presented in Table 2. It is found that λ_0 is uniformly higher than λ_1, suggesting that wage offers arrive more frequently when unemployed than when employed. This corresponds to the case where the reservation wage r exceeds the value of non-market time b. Moreover, as δ is uniformly higher than λ_1, jobs are more likely to end in a layoff than in a quit for a better job.

Ignoring workers aged below 22, the layoff rate λ_0 decreases with age, being exceptionally low for less educated workers aged over 50. Moreover, λ_0 increases with education, though not uniformly. In contrast, λ_1 does not exhibit any clear patterns with respect to education nor with respect to age. Less educated women aged over 50 have the lowest chances of finding a job when unemployed, but women with university education tend to receive more offers than their male counterparts when unemployed. Layoff rate δ decreases with education but there is little difference by sex. Workers aged below 22 and those aged over 50 are more likely to be laid off than other workers.

Productivity p increases with age and is often higher for men than women with some exceptions. Young workers with upper vocational education are found to be less productive than their less educated counterparts. Otherwise productivity differentials across education groups do not exhibit very clear insights. The value of non-market time b appears to be positive among workers with the lowest level of education. For more educated groups b is typically negative, being more negative for higher education levels. Negativity of b reflects the need to interpret this parameter not only a function of unemployment benefits but also of search costs and perhaps even of the disutility of unemployment (Bunzel et al., 2001). There is also a tendency for b to be lower for women than men among less educated workers aged below 22 and over 50, while the reverse is true among workers between 22 and 50.

Overall all structural parameters of the model are estimated accurately and their estimates allow for a meaningful economic interpretation. The fit to the wage data is less satisfactory, however. This is illustrated in Figures 1 to 3 where empirical wage distributions and predicted wage offer distributions obtained from the different specifications of the model are shown. The predicted theoretical density for the pure homogeneity model is computed by inserting the parameter estimates into (12). While the empirical (kernel) densities are unimodal and skewed with a long right tail, the equilibrium search model with identical agents restricts the predicted densities to be increasing and convex over the whole support. Such a shape is obviously not supported by the data. The predicted densities are flat over their whole support, leading to a poor fit to the wage data in all worker groups.

Recall that our data do not contain information on working time. With some inaccuracies in the available wage data, this suggests that our wage variables are subjected to measurement error. The existence of measurement error in wages may partly explain the clear discrepancy between the empirical and theoretical wage distributions. The estimates of the structural parameters of the model can be expected to be affected by measurement errors as well. Note that the order statistics estimators of (r, h) are clearly sensitive to measurement error. Moreover, the dependence of (r, h) on other parameters of the model implies that the maximum likelihood estimates of frictional parameters (λ_1, δ) are also affected by measurement error in wage data (Van den Berg and Ridder, 1993). Thus taking the possibility of measurement error in wages explicitly into account may improve the performance of maximum likelihood estimation.

To deal with measurement errors, we assume that the wage observation in the data, say x, is the product of the true unobserved wage w and an error term ε, so that $x = w \cdot \varepsilon$. The proportional measurement error is assumed to be independently and identically distributed across individuals and to be independent of all other variables in the model. Following Christensen and Kiefer (1994*b*) and Bunzel et al. (2001), we assume that ε has a Pearson Type V distribution with unit mean, variance σ^2 and density

$$g_\varepsilon(\varepsilon) = \frac{\left(1+\sigma^{-2}\right)^{2+\sigma^{-2}}}{\Gamma\left(2+\sigma^{-2}\right) \cdot \varepsilon^{3+\sigma^{-2}}} \exp\left(-\frac{1+\sigma^{-2}}{\varepsilon}\right). \tag{35}$$

A consequence of allowing for the measurement error is that we need to add an integral for each wage observation in the individual likelihood contribution. Formally, we replace (29) by

$$\ell = \varphi\left(d\right)\left(\int_{x/h}^{x/r} \phi\left(j, a \left| \frac{x}{\varepsilon}\right.\right) f\left(\frac{x}{\varepsilon}\right) \frac{1}{\varepsilon} g_\varepsilon(\varepsilon) d\varepsilon\right)^{1-c_d}, \tag{36}$$

where φ, ϕ and f are as given in the previous section, and $1/\varepsilon$ is the Jacobian of the transformation between the true and observed wages given the error term. In this case we do not use order statistics for (r, h) but estimate them simultaneously with $(\lambda_0, \lambda_1, \delta, \sigma)$ by maximising the likelihood function based on (36), in which the integral must be evaluated numerically in each iteration.[24] With the estimates of $(\lambda_0, \lambda_1, \delta, r, h)$ in hand, the estimates of (p, b) can be computed using (33) and (34) as before.

Estimation results from the homogeneous model with measurement error in wages are reported in Table 3. The results are missing for four groups of low-educated women as the estimates of r and h converged to the same value in their

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[24] Note that the presence of measurement errors makes the support of the distribution of observed wages independent of the unknown parametes, so the maximum likelihood estimation is standard in this case.

Table 3: Estimation results with identical agents and measurement error in wages

	λ_0	λ_1	δ	p	b	σ
Low voc. & less						
Men, 16-21	.0285 (.0011)	.0236 (.0027)	.0437 (.0023)	9,278 (920)	7,052 (454)	.3012 (.0114)
Men, 22-30	.0399 (.0011)	.0317 (.0024)	.0508 (.0018)	11,915 (848)	8,121 (475)	.3417 (.0108)
Men, 31-50	.0375 (.0010)	.0291 (.0022)	.0495 (.0017)	11,329 (996)	9,995 (545)	.3739 (.0100)
Men, 51-65	.0166 (.0006)	.0339 (.0033)	.0658 (.0028)	14,336 (1,499)	8,855 (517)	.3743 (.0173)
Women, 16-21	–	–	–	–	–	–
Women, 22-30	–	–	–	–	–	–
Women, 31-50	.0286 (.0009)	.0122 (.0013)	.0366 (.0015)	8,013 (1,022)	7,849 (420)	.2700 (.0069)
Women, 51-65	.0109 (.0004)	.0124 (.0018)	.0502 (.0024)	11,457 (1,793)	7,026 (391)	.2883 (.0123)
Upper vocational						
Men, 16-21	.0385 (.0015)	.0300 (.0031)	.0363 (.0021)	10,900 (660)	5,611 (409)	.2294 (.0183)
Men, 22-30	.0494 (.0014)	.0286 (.0020)	.0353 (.0013)	10,682 (635)	8,056 (540)	.3320 (.0091)
Men, 31-50	.0353 (.0010)	.0189 (.0015)	.0301 (.0012)	16,688 (1,268)	8,244 (889)	.3549 (.0170)
Men, 51-65	.0116 (.0006)	.0133 (.0025)	.0439 (.0029)	34,816 (4,476)	8,013 (652)	.2657 (.0327)
Women, 16-21	.0505 (.0018)	.0301 (.0029)	.0412 (.0021)	7,436 (479)	6,464 (364)	.2594 (.0076)
Women, 22-30	–	–	–	–	–	–
Women, 31-50	–	–	–	–	–	–
Women, 51-65	.0095 (.0009)	.0075 (.0031)	.0496 (.0054)	40,043 (13,510)	5,802 (674)	.2217 (.0516)
Lower university						
Men, 16-30	.0562 (.0016)	.0228 (.0016)	.0264 (.0011)	15,660 (612)	4,556 (607)	.2506 (.0151)
Men, 31-65	.0495 (.0013)	.0183 (.0013)	.0265 (.0010)	22,672 (1,127)	4,951 (1,007)	.3550 (.0201)
Women, 16-30	.0675 (.0025)	.0267 (.0024)	.0251 (.0014)	9,190 (570)	7,545 (827)	.3018 (.0103)
Women, 31-65	.0357 (.0015)	.0195 (.0022)	.0265 (.0016)	13,554 (1,640)	9,305 (1,332)	.3832 (.0188)
Upper university						
Men, 16-30	.0735 (.0047)	.0431 (.0047)	.0140 (.0016)	17,131 (528)	2,864 (1,355)	.2071 (.0210)
Men, 31-65	.0363 (.0016)	.0197 (.0020)	.0189 (.0012)	27,896 (1,760)	6,656 (1,901)	.4462 (.0388)
Women, 16-30	.0770 (.0050)	.0294 (.0044)	.0205 (.0022)	12,490 (1,350)	9,911 (2,646)	.3409 (.0206)
Women, 31-65	.0395 (.0027)	.0305 (.0050)	.0271 (.0028)	19,978 (1,861)	6,219 (1,642)	.4045 (.0604)

Notes: Standard errors are in parentheses

cases. Compared to the previous results, λ_1 is now uniformly clearly higher and δ uniformly slightly lower, while λ_0 is of course not affect by the introduction of measurement errors. Among workers with an upper university degree λ_1 now exceeds δ, while the reverse still holds for other groups. Moreover p is uniformly lower and b uniformly higher than previously. The presence of measurement errors in the wage data suggests that a range of wage offers is narrower than previously, so less variation in p and b is required to explain the observations in the data. The estimates of r and h are generally very close to each other, resulting in a small difference between p and b. As an another implication, dispersion in the sequence of wages that the worker can earn over time is predicted to be extremely narrow.

From Figures 1 to 3 we see that the measurement error specification results in a good fit to the wage data.[25] Both tails are captured quite nicely and the mode

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[25] The predicted densities for *observed* wages in the figures are obtained by inserting the param-

point is very close. As allowing for the measurement error in the wages improves the model's fit to the wage data so crucially, one can expect that the estimates of frictional parameters are more appropriate as well. On the other hand, the fact that measurement errors account for such a large part of the observed wage variation can be viewed as a failure of the model as it implies that the theory is unable to explain wage dispersion within the labour market segments. In light of the clear contrast between the theoretical and empirical wage distributions this is not a very surprising finding, however. A large degree of measurement error is needed to account for so divergent shapes of the distributions.

4.3 Discrete productivity dispersion

Next we consider the extended model with a discrete distribution of productivity. Here we do not allow for measurement error in wages, so comparisons with the homogeneous version of the model can be done in a straightforward manner. The individual contribution to the likelihood function is still given by (32), the only difference compared to the homogeneous case without measurement errors being that F and f are now given by (19) and (20) respectively. In addition to the previous problem that the bounds of the support of F depend on unknown parameters, estimation is further complicated by the fact that the likelihood function is not differentiable at the wage cut points $(\overline{w}_1, ..., \overline{w}_{q-1})$. An estimation method which can deal with these complications is developed by Bowlus et al. (1995a).

Once again it is convenient to reparametrise the model in a similar fashion as was done in the homogeneous case. The restriction $\gamma_i = F(\overline{w}_i)$ implies the following relationship between the productivity terms, wage cuts and the fractions of firm types:

$$p_i = \frac{1}{1-\mu_i^2}\overline{w}_i - \frac{\mu_i^2}{1-\mu_i^2}\underline{w}_i, \quad i = 1, 2, ..., q, \tag{37}$$

where

$$\mu_i = \frac{1+\kappa_1\left(1-\gamma_i\right)}{1+\kappa_1\left(1-\gamma_{i-1}\right)} \in (0,1)\,, \quad i = 1, 2, ..., q. \tag{38}$$

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eter estimates into

$$\int_{x/h}^{x/r} f\left(\frac{x}{\varepsilon}\right)\frac{1}{\varepsilon}g_{\varepsilon}(\varepsilon)d\varepsilon.$$

Substituting p_i's out of (19) and (20) using (37) allows us to write the likelihood function in terms of $\left(\lambda_0, \lambda_1, \delta, r, h, \overline{w}_1, ..., \overline{w}_{q-1}, \gamma_1, ..., \gamma_{q-1}\right)$. Since there are kinks at the wage cuts in F, the density f and hence the likelihood function is discontinuous at these points.

Bowlus et al. (1995*a*) show that the maximum likelihood estimates of wage cuts $(\overline{w}_1, ..., \overline{w}_{q-1})$ come from the set of observed wages. As in the homogenous case, we use order statistics to estimate (r, h). Conditional on these estimates, the likelihood function can be maximised using an iterative procedure with two steps in each iteration. In the first step the likelihood function is maximised with respect to $(\overline{w}_1, ..., \overline{w}_{q-1})$ holding $(r, h, \lambda_0, \lambda_1, \delta)$ fixed, using simulated annealing which randomly searches over the possible wage cut combinations according an optimal stopping rule.[26] Given the estimates of $(\overline{w}_1, ..., \overline{w}_{q-1})$, the corresponding discontinuity points in the wage offer distribution $\left(\gamma_1, ..., \gamma_{q-1}\right)$ are estimated by observed frequencies in the wage data. In the second step the likelihood function is maximised with respect to $(\lambda_0, \lambda_1, \delta)$ conditional on $\left(r, h, \overline{w}_1, ..., \overline{w}_{q-1}, \gamma_1, ..., \gamma_{q-1}\right)$. Since this part of the maximisation problem is smooth, standard maximum likelihood algorithms can be applied. These two steps are then iterated until convergence occurs.

In addition to the order statistics estimators for (r, h), the maximum likelihood estimators of the wage cuts in the wage offer distribution $(\overline{w}_1, ..., \overline{w}_{q-1})$ also converge to their true value at a rate faster than $\sqrt{n}$. It follows that they are asymptotically independent of the maximum likelihood estimator of $(\lambda_0, \lambda_1, \delta)$ and the theory of local cuts by Christensen and Kiefer (1994*b*) justifies conditioning on them in the second step of the procedure. The iterative separate maximisations can be shown to lead to a joint maximum of the likelihood function on converge.

There is no formal test for choosing a value of q, the number of firm types. However, the authors of this estimation technique argue that the likelihood ratio test of one value of q against another based on the standard χ^2-criterion can be expected to work reasonably well in practice. This is so even though the exact distribution of the test statistics is not known due to non-regular estimation procedure. Thus, we choose the number of firm types by comparing two times the improvement in the log-likelihood function with each additional firm type to the $\chi^2_{.05}$ critical value.[27]

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[26] For simulated annealing, see Kirkpatrick et al. (1983) and Bowlus et al. (1995*a*).

[27] A Monte Carlo evidence of Bowlus et al. (1995*b*) indicates a tendency towards overfitting the number of heterogeneity types using this criterion. However, they further find that choosing a value of q greater than the true value has only a minor effect on the estimates of (λ_1, δ), while the order statistics estimators of (r, h) and the ML estimator of λ_0 are obviously unaffected by the

Table 4: Estimation results with discrete productivity dispersion

	λ_0	λ_1	δ	q	$\overline{p}$	b
Lower vocational and less						
Men, 16-21	.0285 (.0011)	.0219 (.0023)	.0444 (.0023)	5	21,539	4,266
Men, 22-30	.0399 (.0011)	.0288 (.0019)	.0507 (.0018)	6	28,438	4,049
Men, 31-50	.0375 (.0010)	.0234 (.0016)	.0499 (.0017)	5	36,920	3,761
Men, 51-65	.0166 (.0006)	.0298 (.0026)	.0662 (.0028)	6	37,507	5,793
Women, 16-21	.0282 (.0012)	.0166 (.0020)	.0464 (.0025)	4	27,227	4,069
Women, 22-30	.0292 (.0010)	.0193 (.0017)	.0370 (.0017)	6	22,808	3,971
Women, 31-50	.0286 (.0009)	.0123 (.0012)	.0365 (.0015)	6	29,556	3,465
Women, 51-65	.0109 (.0004)	.0122 (.0017)	.0503 (.0024)	4	36,217	4,715
Upper vocational						
Men, 16-21	.0358 (.0015)	.0254 (.0024)	.0373 (.0021)	4	17,497	3,827
Men, 22-30	.0494 (.0014)	.0254 (.0016)	.0353 (.0013)	4	23,945	2,491
Men, 31-50	.0353 (.0010)	.0181 (.0013)	.0301 (.0012)	5	35,160	2,320
Men, 51-65	.0116 (.0006)	.0133 (.0023)	.0439 (.0029)	3	53,937	5,972
Women, 16-21	.0505 (.0018)	.0276 (.0024)	.0420 (.0021)	5	15,697	3,636
Women, 22-30	.0534 (.0015)	.0230 (.0015)	.0323 (.0013)	6	17,499	2,547
Women, 31-50	.0335 (.0010)	.0174 (.0013)	.0298 (.0013)	4	27,277	2,913
Women, 51-65	.0095 (.0009)	.0072 (.0029)	.0498 (.0054)	2	77,089	4,798
Lower university						
Men, 16-30	.0562 (.0016)	.0213 (.0014)	.0267 (.0011)	5	25,167	275
Men, 31-65	.0495 (.0013)	.0180 (.0012)	.0266 (.0010)	6	41,202	-1,758
Women, 16-30	.0675 (.0025)	.0245 (.0019)	.0250 (.0014)	6	16,807	373
Women, 31-65	.0357 (.0015)	.0205 (.0021)	.0263 (.0016)	6	29,217	2,460
Upper university						
Men, 16-30	.0735 (.0047)	.0410 (.0043)	.0138 (.0016)	5	19,919	-863
Men, 31-65	.0363 (.0015)	.0208 (.0020)	.0188 (.0012)	6	45,053	-584
Women, 16-30	.0770 (.0050)	.0257 (.0033)	.0206 (.0022)	5	26,853	-3,965
Women, 31-65	.0395 (.0027)	.0265 (.0039)	.0275 (.0028)	5	33,265	2,109

Notes: Standard errors are in parentheses. q is the number of firm types and $\overline{p} = \sum_{i=1}^{q} (\gamma_i - \gamma_{i-1}) p_i$ is the average productivity across firms.

Once the other parameters of the model are estimated, unobserved heterogeneity terms $(p_1, ..., p_q)$ can be estimated using (37) and (38),[28] and an estimate of b can be obtained using

$$b = r - \frac{\lambda_0 - \lambda_1}{\lambda_1} \sum_{i=1}^{q} (p_i - \underline{w}_i) \left(1 - \mu_i^2 - \frac{2\delta (1 - \mu_i)}{\delta + \lambda_1 (1 - \gamma_{i-1})} \right), \tag{39}$$

which follows from the substitution of (19) into (5).

Order statistics estimates for $(r, h) \equiv (\underline{w}_1, \overline{w}_q)$ can be found from Table 1 as previously. Other parameter estimates are shown in Tables 4 to 6. Estimates of the frictional parameters (λ_1, δ) are generally very close to the estimates obtained from

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value of q chosen.

[28] It is worth noting that in this model all wage differentials that cannot be explained by differences in the frictional parameters are attributed to productivity differences. Thus the productivity parameters may capture also other sources of wage dispersion than pure productivity differences. (Bowlus, 1997.)

Table 5: Estimation results with discrete productivity dispersion, continued

	p_1	p_2	p_3	p_4	p_5	p_6
Lower vocational and less						
Men, 16-21	13,643	30,392	40,487	44,072	108,892	
Men, 22-30	17,295	17,190	19,007	24,177	41,443	136,864
Men, 31-50	22,671	37,564	47,012	56,809	148,293	
Men, 51-65	20,879	26,750	35,892	40,360	62,652	211,517
Women, 16-21	11,969	33,784	114,231	679,097		
Women, 22-30	12,354	17,426	25,163	42,016	47,827	125,113
Women, 31-50	16,398	17,708	24,134	28,632	45,129	162,682
Women, 51-65	19,360	43,418	123,891	540,357		
Upper vocational						
Men, 16-21	13,375	19,258	26,360	62,415		
Men, 22-30	15,146	21,668	27,620	75,768		
Men, 31-50	22,338	27,545	30,704	41,592	100,363	
Men, 51-65	38,261	72,673	141,549			
Women, 16-21	9,757	15,158	29,653	39,350	87,954	
Women, 22-30	11,946	13,236	16,207	26,410	56,156	56,535
Women, 31-50	16,759	26,769	48,668	195,431		
Women, 51-65	49,328	152,647	721,354			
Lower university						
Men, 16-30	18,213	20,020	24,891	35,263	113,867	
Men, 31-65	25,616	45,985	48,888	83,992	156,640	446,879
Women, 16-30	12,812	13,507	14,208	17,024	22,032	41,729
Women, 31-65	17,605	25,184	30,545	44,957	59,843	127,858
Upper university						
Men, 16-30	16,589	16,960	19,525	21,083	40,111	
Men, 31-65	26,719	28,891	31,640	44,102	58,421	168,181
Women, 16-30	17,355	18,697	25,678	50,459	151,310	
Women, 31-65	22,495	26,773	34,878	40,743	95,614	

Notes: All p_i's are statistically significant at the 5 per cent level.

the homogeneous model with measurement error in wages. Compared to the corresponding estimates from the measurement error specification, there is a tendency for λ_1 to be slightly smaller while δ does not exhibit any systematic differences. Overall the differences in these estimates are so moderate that one can draw basically the same conclusions concerning the frictional parameters from this model and from the homogeneous model with the measurement error.

To get an idea of productivity differences, the average productivity across the firms is computed and shown in Table 4. Conditional on the education level, the average productivity $\overline{p}$ tends to increase with age. Except for workers aged over 50, there is a tendency for $\overline{p}$ to be lower for workers with upper vocational education than for those with lower vocational education. Overall these differences across the worker groups are in line with the findings from the homogeneous model, even though the absolute values of productivity estimates are quite different. Namely, the

Table 6: Estimation results with discrete productivity dispersion, continued

	$\overline{w}_1$	$\overline{w}_2$	$\overline{w}_3$	$\overline{w}_4$	$\overline{w}_5$	γ_1	γ_2	γ_3	γ_4	γ_5
Lower voc. & less										
Men, 16-21	8,700	10,704	11,755	12,511		.7857	.8917	.9297	.9543	
Men, 22-30	9,768	10,008	10,273	11,889	15,874	.6143	.6493	.6807	.8053	.9420
Men, 31-50	12,458	14,803	15,684	16,329		.7477	.8620	.8933	.9110	
Men, 51-65	11,330	11,478	12,988	13,132	18,042	.7283	.7403	.8183	.8247	.9467
Women, 16-21	7,352	9,303	14,857			.8027	.9153	.9927		
Women, 22-30	7,913	8,150	10,445	11,352	12,211	.7080	.7357	.8880	.9173	.9410
Women, 31-50	8,270	8,647	9,378	9,629	12,102	.6623	.7290	.8063	.8270	.9383
Women, 51-65	8,849	11,177	13,265			.7947	.9433	.9823		
Upper vocational										
Men, 16-21	9,379	10,197	11,728			.7967	.8673	.9450		
Men, 22-30	9,563	10,242	14,532			.6310	.6810	.9067		
Men, 31-50	12,429	13,539	14,280	17,680		.6340	.7100	.7527	.8757	
Men, 51-65	16,607	19,303				.7914	.8767			
Women, 16-21	7,118	8,296	10,438	10,646		.7297	.8663	.9517	.9573	
Women, 22-30	8,074	8,349	9,603	11,238	11,580	.6483	.6957	.8377	.9157	.9213
Women, 31-50	10,213	11,963	18,128			.7013	.8103	.9770		
Women, 51-65	13,736	17,585				.8653	.9686			
Lower university										
Men, 16-30	12,405	12,811	13,030	17,481		.7490	.7897	.8030	.9563	
Men, 31-65	15,407	21,470	21,528	25,153	31,672	.7200	.9040	.9057	.9520	.9903
Women, 16-30	8,763	8,951	9,742	10,257	12,828	.5913	.6199	.7297	.7763	.9206
Women, 31-65	10,846	13,638	14,769	15,852	21,243	.6153	.7864	.8373	.8635	.9533
Upper university										
Men, 16-30	12,846	14,301	16,066	17,204		.5553	.7085	.8255	.8872	
Men, 31-65	16,284	17,287	19,185	23,020	29,622	.5637	.6181	.7062	.8022	.9103
Women, 16-30	13,065	13,086	16,651	22,223		.7102	.7122	.8796	.9592	
Women, 31-65	13,888	15,327	17,958	21,768		.6160	.6979	.7914	.9006	

average productivity estimates $\overline{p}$ are approximately only half of the corresponding productivity estimates obtained from the homogeneity model without measurement error but are clearly higher than the estimates from the measurement error specification. Furthermore, it turns out that the value of non-market time b is typically positive, though there are few groups for which b takes a negative value. Differences in b with respect to education and age are similar to those observed in the case of the homogeneous model.

Estimates of productivity parameters, wage cuts and their weights are shown in Tables 5 and 6. Individual productivity values and wage cuts, say p_i and $\overline{w}_i$, exhibit increasing patterns with respect to age among groups with university education while the picture is less clear for less educated workers. Of course, these kind of comparisons are complicated by the fact that the number of firm types q varies across the groups. Given the shape of the empirical wage distribution, it is not very surprising that the bulk of firms is found to be low productivity ones. Firms with the lowest level of productivity represent over half of all firms in each submarket as $\gamma_1 > .5$ holds for all worker groups. Some productivity terms take very high values in some groups of workers, though their relative weights are very low (see the associated γ_i's).

In Figures 1 to 3 the estimated density functions of the model with discrete productivity dispersion are characterised by discontinuous jumps at the estimated wage cuts $(\overline{w}_1, ..., \overline{w}_{q-1})$, between which the densities exhibit locally increasing patterns. It turns out that the model with discrete productivity dispersion is able to capture the shape of the wage distribution quite well but has some difficulties with the both tails of the distribution. In particular the estimated density has generally the left tail which is too fat compared to the observed wage distribution. This failure is not unique to the Finnish data but appears in the previous empirical applications of the same model as well (see Bowlus et al., 1995*a*, Bowlus, 1997, and Bunzel et al., 2001). Additionally, there are difficulties in explaining wage observations at the upper end of the distribution which is often thin, covering wide ranges. It is easy to see that adding more firm types serves as a way of obtaining a more accurate fit to the right tail of the distribution. The estimation procedure aims to attach different firm types to each of these observations, leading to implausibly high productivity values sometimes. However, these high productivity values have only a minor overall effect as their weights are very low (in terms of the associated values of γ_i).

It should be stressed that the trimming procedure applied to the wage data is related to the number of heterogeneity terms needed to match the right hand tail of the wage distribution. Indeed by trimming a higher fraction of wage values from the upper end of the distribution leads to a smaller choice of q, with the highest productivity parameters p_i being in a more reasonable range and the associated values of γ_i being well below one. Changing the upper value of trimming has of course a direct effect on the estimate of h. A brief sensitivity analysis done with the different trimming thresholds suggests that the parameters and conclusions of interest are reasonably robust, however.

4.4 Continuous productivity dispersion

Next we turn our attention to the version of the model with a continuous productivity distribution. The individual likelihood contribution has the same general form as previously defined in (32). With the assumption of continuous productivity distribution Γ, the equilibrium distribution of wage offers is given by $F(w) = \Gamma\left(K^{-1}(p)\right).$

This is a highly nonlinear function of unknown parameters and does not have a closed-form expression in general, suggesting that the standard maximum likelihood estimation could be very cumbersome. The first point to note is that the integrals within the expressions for F and f must be evaluated numerically in each iteration. An additional difficulty follows from the fact that the productivity distribution Γ is not generated by the model but it must be taken as exogenously given. This is not a problem as such but, as argued by Bontemps et al. (2000), the most well-known parametric specifications for Γ are unlikely to generate the wage offer distribution consistent with the shape usually observed in the wage data.

For these reasons Bontemps et al. (2000) propose a flexible estimation procedure which does not restrict Γ to belong in any parametric family. This estimation method consists of three steps. The first step of the procedure is to estimate F and f from the wage data using some nonparametric procedure. In the second step the likelihood function is maximised with respect to $(\lambda_0, \lambda_1, \delta)$ conditional on the nonparametric estimates of F and f. As the final step $p = K^{-1}(w)$ and $\gamma(p)$ are estimated using[29]

$$K^{-1}(w) = w + \frac{1 + \kappa_1 [1 - F(w)]}{2\kappa_1 f(w)}, \tag{40}$$

$$\gamma(p) = \frac{2\kappa_1 f(w)^3}{\kappa_1 f(w)^2 - f'(w)(1 + \kappa_1 [1 - F(w)])}, \tag{41}$$

where F, f and f' are replaced by their nonparametric estimates from the first step and $\kappa_1 = \lambda_1/\delta$ by its maximum likelihood estimate from the second step.[30] To estimate the wage offer density f, we apply the standard Gaussian kernel density estimator and choose the bandwidth by a rule of thumb that minimises the mean integrated square error. Corresponding estimates of F and f' are then obtained by integration and differentiation of the kernel density estimate. Standard errors are finally obtained by bootstrapping the whole estimation procedure outlined above.[31]

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[29] The first equation is simply the first-order condition (21) solved for $p = K^{-1}(w)$. The second equation can be found by differenting the first equation with respect to w and noting that $\left(K^{-1}\right)'(w) = f(w)/\gamma(p)$ by virtue of the relationship $F(w) = \Gamma\left(K^{-1}(w)\right)$.

[30] Obviously, the final step requires the denominator of (41) to be positive. Note that this condition is equivalent to the second-order condition of the firm's problem outlined in (22) or, in other words, that $f(w)(1 + \kappa_1 [1 - F(w)])$ decreases over the support of F.

[31] Our estimation procedure differs slightly from that used by Bontemps et al. (2000) because of the different sampling scheme. Contrary to the inflow sample of unemployment, Bowlus et al. (2000) use a sample from the French Labour Force Survey drawn from the stock of employed and unemployed workers. Consequently, wage observations in their data come from G, not from F as in our data. As such they estimate G (instead of F) using a nonparametric procedure and then recover the associated F using the equilibrium flow relationship. They also replace (40) and (41) by the corresponding equations expressing p and γ as functions of G, g, g' and κ_1, where g is the density of the earnings distribution and g' its derivative.

It is worth emphasising that the application of the kernel density techniques to the wage data does not impose any restrictions on the shape of equilibrium wage distributions. Conditional on the nonparametric estimates of F and f, the likelihood function estimated in the second step relies only on assumptions about the behaviour of individual workers who are taking the wage offer distribution as given. In other words, the assumptions about the wage-setting strategies of firms does affect the estimation of frictional parameters $(\lambda_0, \lambda_1, \delta)$. Only the third step of the estimation procedure exploits the part of the model which describes firm behaviour (that is, the first-order condition of firm's problem). Bontemps et al. (2000) emphasise that the estimates of the frictional parameters can be expected to be consistent under a wide range of assumptions on the demand side of the story. This class of models includes, among others, the specifications of equilibrium search models outlined and estimated in the previous sections.

It should be stressed that the estimation method of Bontemps et al. (2000) is very simple in computational respect. No numerical integration is needed in the second step due to substitution of the kernel estimates in the likelihood function, while the third step does not require even iterations. This a clear advantage compared to the estimation procedure of Bowlus et al. (1995a) for the case of discrete productivity dispersion.

The estimates of the frictional parameters are given in Table 7, whereas the kernel estimates of f are shown in Figures 1 to 3. It appears that λ_1 is generally very close to the estimates obtained from the homogeneous model with measurement error and from the model with discrete productivity dispersion while δ is almost identical. Since the estimates of $(\lambda_0, \lambda_1, \delta)$ in this setting can be expected to be robust with respect to different mechanisms determining the wage distributions, we can conclude that all specifications of the Burdett-Mortensen model allowing for an acceptable fit to the wage data produce appropriate estimates for the frictional parameters. This suggests that to the extent we are concerned with the estimation of frictional parameters it is not so important whether the deviations from the theoretical distribution predicted by the homogeneous model are explained by measurement error or by employer heterogeneity as long as the shape of the wage distribution is captured by the specification. This is essentially the same result as found by Bunzel et al. (2001) from the Danish data.

The average productivity values $\overline{p}$ in the last column are computed by taking the average over workers entering employment based on (40). However, the estimated

Table 7: Estimation results with continuous productivity dispersion

	λ_0	λ_1	δ	$\overline{p}$
Lower vocational and less				
Men, 16-21	.0285 (.0009)	.0234 (.0029)	.0437 (.0034)	20,921
Men, 22-30	.0399 (.0014)	.0291 (.0021)	.0508 (.0026)	28,443
Men, 31-50	.0375 (.0016)	.0254 (.0022)	.0495 (.0025)	34,973
Men, 51-65	.0166 (.0007)	.0298 (.0030)	.0658 (.0040)	36,894
Women, 16-21	.0282 (.0010)	.0179 (.0025)	.0454 (.0032)	25,973
Women, 22-30	.0292 (.0010)	.0194 (.0019)	.0368 (.0022)	22,520
Women, 31-50	.0286 (.0011)	.0118 (.0011)	.0366 (.0019)	30,019
Women, 51-65	.0109 (.0006)	.0117 (.0016)	.0502 (.0034)	38,504
Upper vocational				
Men, 16-21	.0385 (.0014)	.0293 (.0026)	.0362 (.0029)	16,134
Men, 22-30	.0494 (.0017)	.0262 (.0020)	.0352 (.0019)	23,465
Men, 31-50	.0353 (.0012)	.0183 (.0014)	.0301 (.0018)	33,879
Men, 51-65	.0116 (.0006)	.0129 (.0026)	.0440 (.0043)	56,461
Women, 16-21	.0505 (.0023)	.0280 (.0026)	.0412 (.0029)	15,990
Women, 22-30	.0534 (.0019)	.0232 (.0017)	.0322 (.0017)	17,573
Women, 31-50	.0335 (.0013)	.0184 (.0015)	.0296 (.0016)	26,052
Women, 51-65	.0095 (.0012)	.0077 (.0034)	.0495 (.0075)	71,444
Lower university				
Men, 16-30	.0562 (.0018)	.0224 (.0014)	.0264 (.0014)	24,535
Men, 31-65	.0495 (.0018)	.0174 (.0011)	.0266 (.0013)	42,881
Women, 16-30	.0675 (.0026)	.0245 (.0025)	.0251 (.0018)	16,971
Women, 31-65	.0357 (.0015)	.0185 (.0020)	.0265 (.0020)	31,605
Upper university				
Men, 16-30	.0735 (.0067)	.0384 (.0054)	.0139 (.0021)	20,448
Men, 31-65	.0363 (.0018)	.0189 (.0018)	.0190 (.0016)	48,668
Women, 16-30	.0770 (.0051)	.0271 (.0042)	.0204 (.0026)	24,157
Women, 31-65	.0395 (.0031)	.0294 (.0051)	.0272 (.0035)	30,916

Notes: Standard errors are in parentheses. $\overline{p}$ is the average productivity over workers who found a job.

relationship between the wage offer and productivity is not consistent with the theory. The condition that $f(w)\left(1+\kappa_1\left[1-F(w)\right]\right)$ decreases everywhere is violated for small wages in all worker groups. In other words, the model fails to capture a steeply increasing wage density observed at the lower end of the wage distribution. Recall that the model with a discrete distribution of productivity also fails to explain the shape of the left tail of the wage distribution.

The increasing pattern of $f(w)\left(1+\kappa_1\left[1-F(w)\right]\right)$ on small wages implies that the relationship $K(p)$ is downward-sloping for small values of p. This suggests the wages offered by some less productive firms are not optimal as they could increase their profits by reducing their wages. Van den Berg and Van Vuuren (2000) point out that omitted worker heterogeneity can potentially explain this failure of the model. To see this, suppose that workers within a given labour market segment are heterogenous with respect to their value of non-market time (this heterogeneity may result, for example, from differences in unemployment benefits or in the value of leisure). In this case workers will apply different reservation wages when searching from unemployment. Thus a firm which lowers its wage offer may become unattractive for some groups of workers. The firms should take this effect into account when setting wages which may explain the failure of the theoretical model at the lower end of the wage distribution.

5 Conclusion

This papers has provided quite an extensive structural empirical analysis of various specifications of the Burdett-Mortensen model. We begun with the homogeneous version of the model with identical agents. This analysis was followed by the extended versions of the basic model with measurement error in wages and unobserved employer heterogeneity. All model specifications were estimated using maximum likelihood from a sample of Finnish individuals becoming unemployed during 1992. We paid particular attention to how well the different model specifications are able to explain the shape of the wage distribution observed in the Finnish data.

We found that, in the absence of measurement error in wages, the equilibrium search model with identical agents does not fit to the wage data. This failure is due to the prediction of the theory that the equilibrium wage offer distribution has an increasing density everywhere which is at odds with the wage data. Introduction of the measurement error in wages or employer heterogeneity in terms of labour productivity across firms provides a way of making the model more flexible. Indeed these extended versions of the basic model were found to give a much better fit to the wage data. Moreover, the frictional parameters of the model – the layoff rate and the arrival rates of job offers – were found to be fairly robust across the model specifications which fits to the wage data.

In the case of the homogeneous model with measurement error in wages almost all wage dispersion was attributed to measurement error. This indicates that the model without (unobserved) worker or employer heterogeneity cannot explain the observed wage variation. Although the equilibrium models with employer heterogeneity match the overall shape of the wage distributions relatively well, they do have problems in explaining the shape of the left tail of the wage distribution, and in particular the testable theoretical conditions implied by the model with continuous productivity dispersion were rejected in the empirical analysis. Of course, one can expect that incorporating employer heterogeneity and measurement error into the same model provides a way of explaining observations at the lower end of the wage distribution as well. On the other hand, unobserved heterogeneity and measurement errors allowed for in a sufficiently flexible form can be used to explain any discrepancy between the theory and data.

An alternative approach to proceed might involve the reconsideration of the theoretical structure of the model. The demand side of the equilibrium search story especially raises some doubts. The assumption that the only choice the firm has to make is to set its wage optimally is obviously not very realistic. In reality firms face external shocks which lead to the creation and destructions of jobs at the existing firms as wages cannot be fully adjusted in respond to these shocks. Of course, the incorporation of endogenous job creation and destruction would make the equilibrium model of the labour market much more complicated. However, this seems to be important in order to make significant progress in developing a more rigorous basis for empirical equilibrium analysis of the labour market. In my view, the increasing availability of matched worker-firm data sets provides great potential for directing future research to this direction.

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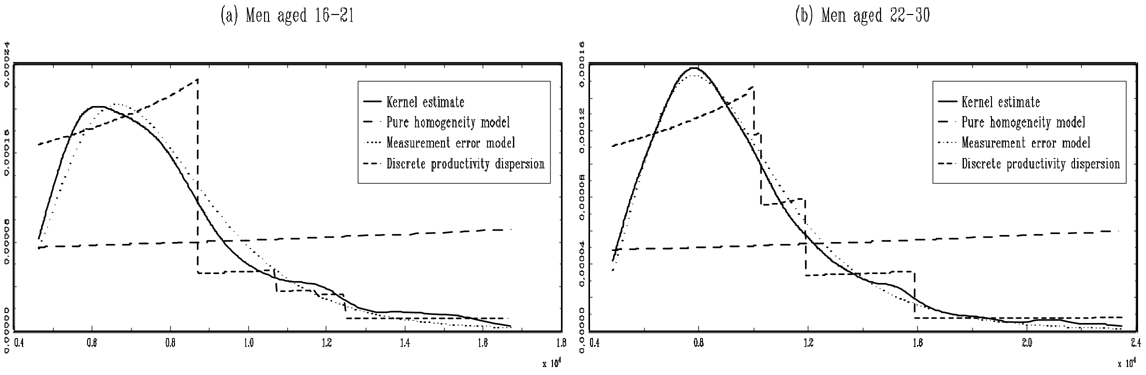
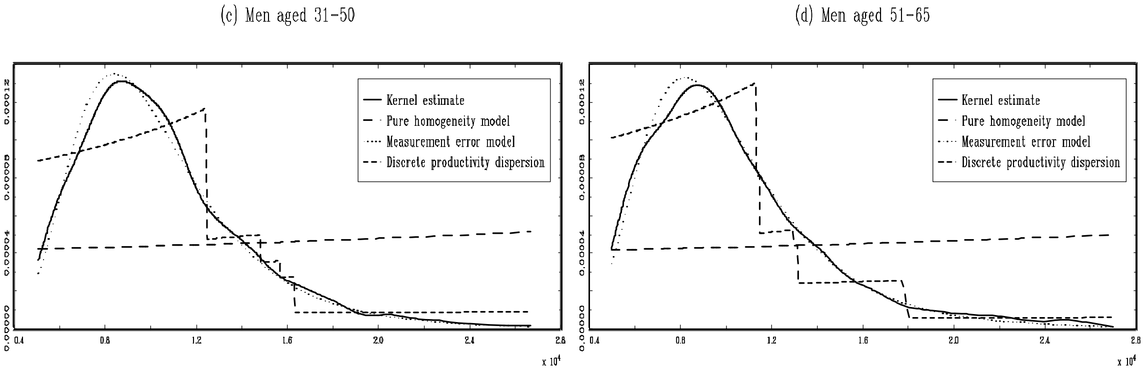
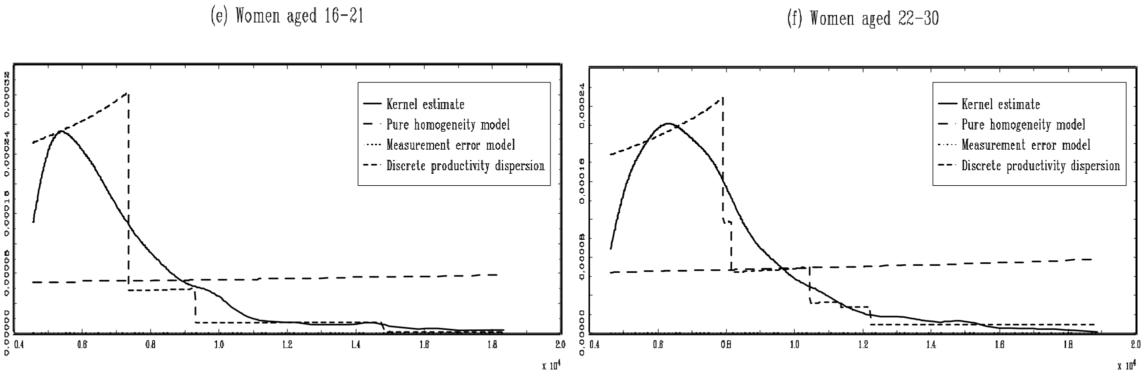
Figure 1: Wage offer densities for workers with lower vocational education or less


(a) Men aged 16-21

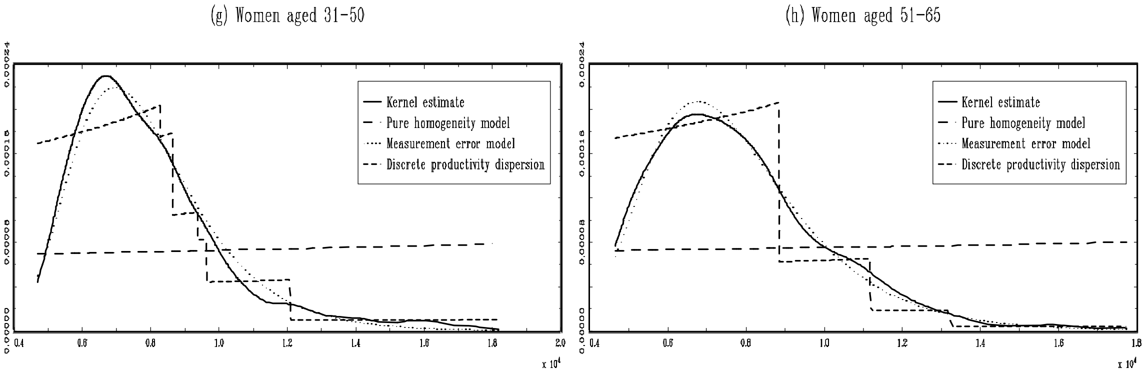
(b) Men aged 22-30


(c) Men aged 31-50

(d) Men aged 51-65

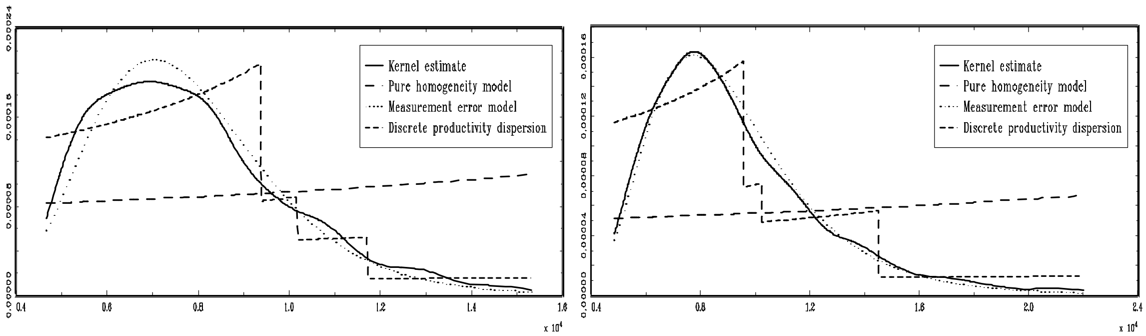

(e) Women aged 16-21

(f) Women aged 22-30


(g) Women aged 31-50

(h) Women aged 51-65

Figure 2: Wage offer densities for workers with upper vocational education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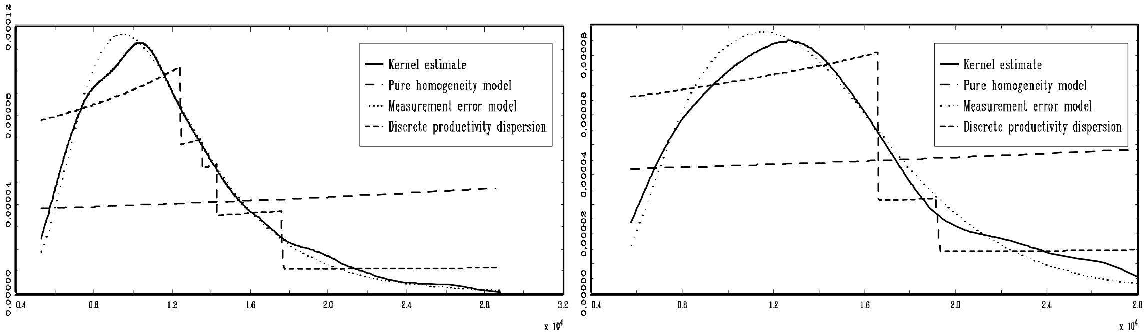
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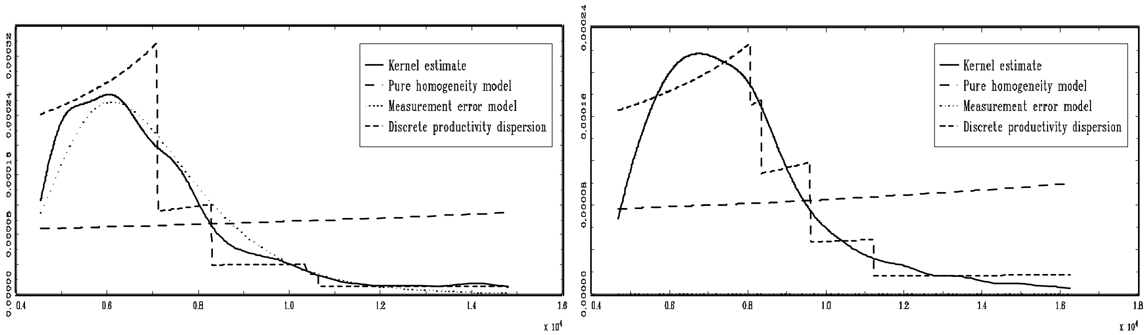
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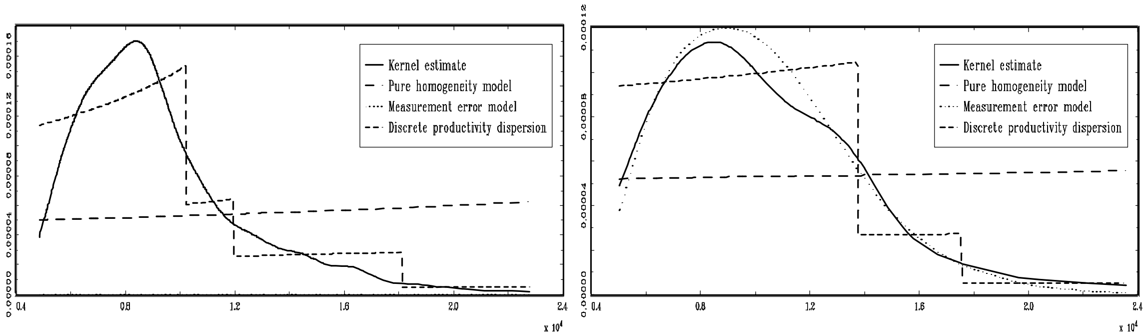
(f) Women aged 22-30

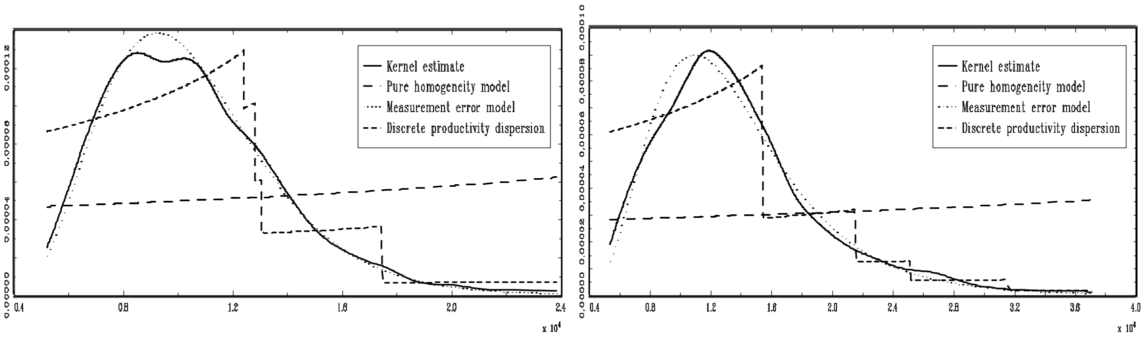

(g) Women aged 31-50

(h) Women aged 51-65

Figure 3: Wage offer densities for workers with university education

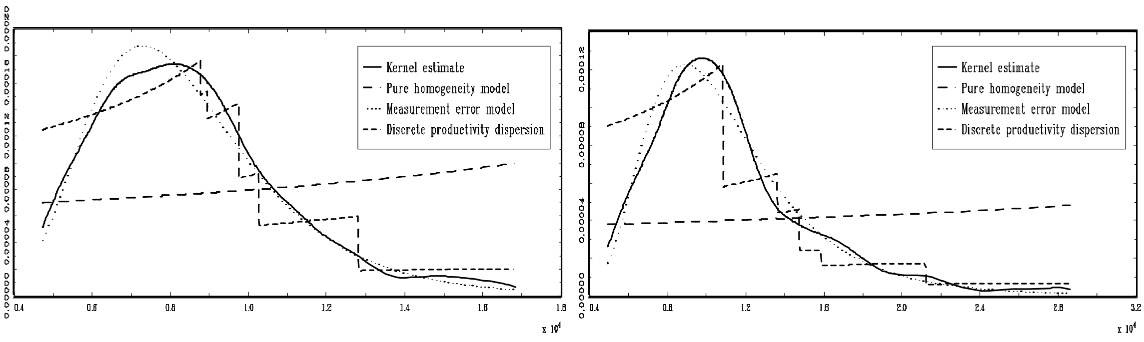
(a) Men aged 16–30 with lower university education

(b) Men aged 31–65 with lower university education



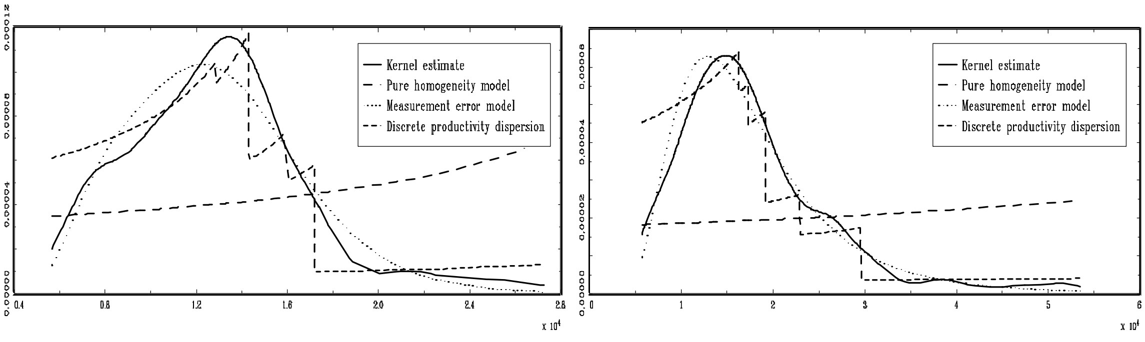
(c) Women aged 16–30 with lower university education

(d) Women aged 31–65 with lower university education



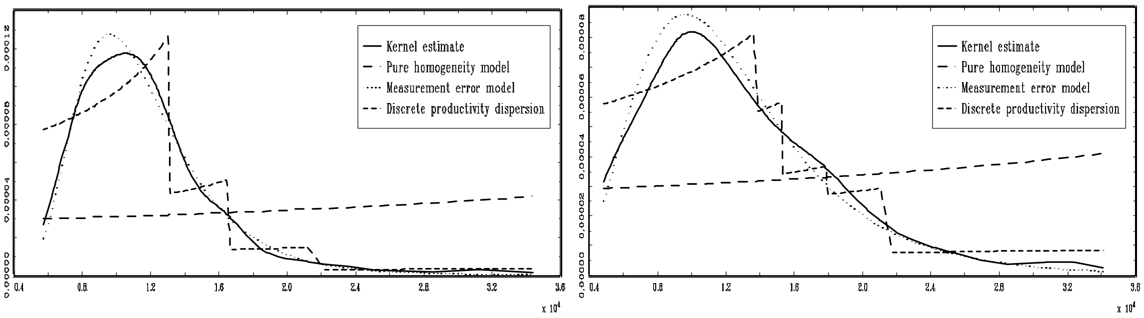
(e) Men aged 16–30 with upper university education

(f) Men aged 31–65 with upper university education



(g) Women aged 16–30 with upper university education

(h) Women aged 31–65 with upper university education



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