

DISSERTATIO ASTRONOMICA,

DE

*INVENIENDA ELEVATIONE POLI
EX OBSERVATIS DUARUM STELLARUM ALTITU-
DINIBUS ÆQUALIBUS.*

Q V A M

Conf. Ampl. Fac. Philos. Reg. Acad. Aboëns,

PRÆSIDE

Mag. ANDR. JOH. METHER

Mathem. Prof. Reg. & Ord.

PRO GRADU

Publice examinandam sifit

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Angermannus.

In Auditorio Majori die **XX** Junii MDCCCV,

H. a. m. s.

ABOË, Typis FRENCKELLIANIS.

MAGNÆ
IN SACRAM REG. MAJESTATEM
FIDEI VIRO,
DIOECSEOS HERNOSANDENSIS EPISCOPO,
VENERANDI IBIDEM CONSIST. ECCLES. PRÆSIDI,
GYMNASII SCHOLARUMQUE EPHORO,
REG. ORD. DE STELLA POLARI,
ACADEMIÆQUE LITTERAR. HUMANIOR. HIST. &
ANTIQUIT. MEMBRO,
SVECANÆQUE OCTODECIMVIRO,
REVERENDISSIMO DOMINO S.S.THEOL.DOCATORI,
CAROLO G. NORDIN,
MÆCENATI SUMMO,

*Quem & sanguinis vincula, & beneficia insignia sibi
reddidere amatisimum, & meritorum in Regem, Patriam,
Ecclesiam, Litteras numerus bonis omnibus deveneran-
dum, pagellas hasce sacratas voluit, debuit*

REVERENDISSIMI NOMINIS

cultor humillimus
RETR. ANDR. HELLZÉN.



§. I.

Inter Problemata quibus solvendis operam in pri-
mis Astronomi impenderunt, determinatio Ele-
vationis Poli seu Latitudinis loci, maximi fere mo-
menti est censenda. Vario idcirco etjam modo illam
determinare conati sunt Scientiae Sideralis Doctores,
plerumque autem ex observatis diversis Solis alti-
tudinibus. Sunt vero hæ ipsæ altitudines ejus in-
dolis, ut prolixum haud raro requirant calculum,
quo exactæ satis evadant. Densitas etenim aëris
major vel minor et reliquæ ejus vicissitudines, va-
riabilem efficiunt refractionem radiorum lumenis, e-
oque ipso altitudines observatas erroneas reddunt;
unde etiam ipsa Elevatio Poli, altitudinem observa-
tarum ope inventa, persæpe incerta est. Illud præ-
terea in observandis altitudinibus obvenit incommo-
dum, quod instrumentum huic usui inserviens, præ-
ter quam quod haud parvo vendatur pretio, ple-
rumque correctione egeat, quo sit, ut perpauci ex-
actas observationes altitudinem instituere possint, et
ut prolixior ad ipsam Latitudinem inveniendam e-
vadat calculus. His perspectis difficultatibus, aliam
pro determinanda Elevatione Poli excogitarunt A-

A

stro-

stronomi rationem, quæ incommodis jam allatis non est obnoxia. Ex observatis videlicet duarum Stellarum altitudinibus æqualibus, quamvis quoad veram magnitudinem incognitis, Latitudo Loci investigari potest, et methodum qua in hoc Problemate solvendo uti debet, nobis tradidit Cl. BOHNENBERGER in *Auleitung zur Geographischen ort bestimmung.* Götting 1795. § 162, cujus pleniorum explicationem in sequentibus tradere nobis proposuimus, L. B. censoræ jam submittentes quæ ad materiam tractandam spectant.

§ 2.

Sit P polus & Z zenith, ductoqve arcu circuli maximi ZP , erit PZ complementum Latitudinis quæsitæ. Sint porro A & B Stellaræ quædam, quarum altitudines æquales observantur, erunt, ducetis ZA & ZB , utpote complementa altitudinum observatarum, inter se quoque æquales. Iunctis deinde A & P , B & P arcubus circulorum maximorum, designant hī ipsi arcus complementa declinationum Stellarum observatarum, quorum semi summa, si dicatur D atque semi differentia δ , habebitur $AP = D + \delta$ & $BP = D - \delta$. Bisecto porro $>APB$ arcu PE , si fuerit $>APE = m$ & $>ZPE = n$, erit $>ZPA = n - m$ & $>ZPB = n + m$. Dicto insuper compleemento Elevationis Poli quæsitæ z & posito sinu toto $= 1$, habebitur in $\triangle ZAP$, $\cos AZ = \cos ZP \cos AP + \sin ZP \sin AP \cos ZPA$ & pa-

ri-

riter in $\triangle ZPB$, $Cof BZ = Cof ZP Cof BP + Sin ZP Sin BP Cof ZPB$ seu $Cof AZ = Cof z Cof(D + \delta) + Sin z Sin(D + \delta) Cof(n - m)$ & $Cof BZ = Cof z Cof(D - \delta) + Sin z Sin(D - \delta) Cof(n + m)$, unde ob $Cof AZ = Cof BZ$ eruitur $Cof z Cof(D + \delta) + Sin z Sin(D + \delta) Cof(n - m) = Cof z Cof(D - \delta) + Sin z Sin(D - \delta) Cof(n + m)$, atque divisa æquatione per $Sin z$, habebitur terminis rite dispositis $Cotg z (Cof(D - \delta) - Cof(D + \delta)) = Sin(D + \delta) Cof(n - m) - Sin(D - \delta) Cof(n + m)$. Evolutis vero terminis, prodit $2 Cotg z Sin D Sin \delta = 2 Cof D Sin \delta Cof n Cof m + 2 Sin D Cof \delta Sin n Sin m$ atque divisa æquatione per $2 Sin D Sin \delta$ erit $Cotg z = Cotg D Cof n Cof m + Cotg \delta Sin n Sin m$.

Hæc vero formula, quo faciliorem admittat Logarithmorum usum, transformetur, statuendo

$$\frac{Cotg \delta \tg m \tg n}{Cotg D} = \tg \varphi^2, \text{ si } n < 90^\circ \text{ & habebitur,}$$

ob $1 + \tg \varphi^2 = \frac{1}{Cof \varphi^2}$, $Cotg z =$

$$\frac{Cotg D Cof n Cof m}{Cof \varphi^2}; \text{ in casu vero quo } n > 90^\circ \text{ erit}$$

$$Cotg z = Cotg \delta Sin n Sin m - Cotg D Cof m Cof n$$

& facto $\frac{Cotg D Cotg n Cotg m}{Cotg \delta} = Sin \psi^2$, eruitur ob

$$1 - Sin \psi^2 = Cof \psi^2, Cotg z = Cotg \delta Sin n Sin m Cof \psi^2.$$

EXEMPL. I. Si die XX Januarii anni currentis obseruata fuerit Stella β Ursæ majoris $4^h 47' 56''$ post meridiem ad orientem coeli partem in altitudine apparente $51^\circ 40'$, cujus pro isto die declinatio borealis fit $52^\circ 33' 35''$; præterlapso tempore $28' 20''$ visa est Stella θ Ursæ majoris ad eandem altitudinem & eandem coeli plagam, cujus declinatio borealis pro eodem die fit $57^\circ 25' 22''$. Elevatio Poli pro loco observationis his datis sic computabitur.

$$\begin{aligned}
 D + \delta &= 37^\circ 26' 25'' \quad \text{Log Cotg } \delta = 1,3719828. \\
 D - \delta &= 32^\circ 34' 38'' \quad \text{Log tg } m = 2,7916387. \\
 D &= 35^\circ 0' 31'' 5 \quad \text{Log tg } n = 0,5882113. \\
 \delta &= 2^\circ 25' 53'', 5 \quad \underline{\text{Log Cotg } D = 1,8453679.} \\
 n + m &= 79^\circ 4' 10'' \quad \text{Log tg } \phi^2 = 0,5972007. \\
 n - m &= 71^\circ 59' 10'' \quad \underline{- \text{Log Cof } \phi^2 = 0,6950874.} \\
 n &= 75^\circ 31' 40'' \quad \text{Log Cotg } D = 0,1546321. \\
 m &= 3^\circ 32' 30'' \quad \text{Log Cof } n = 1,39777847. \\
 z &= 29^\circ 32' 19'' 3 \quad \text{Log Cof } m = 1,9991698. \\
 &\quad \text{Log Cotg } Z = 0,2466740.
 \end{aligned}$$

Invento jam $z = 29^\circ 32' 19'' 3$, complementum ipsius seu $60^\circ 27' 40'', 7$ exhibet Elevationem Poli quæsitam.

EXEMPL. II. Si eodem die & anno obseruatæ fuerint eædem Stellæ in altitudine apparente $29^\circ 50'$, horis post meridiem $6^h 43' 43''$ & $7^h 12' 3''$ respecti-
ve, pro invenienda Latitudine loci calculus sic in-
stituitur:

$$D + \delta$$

$D + \delta = 37^\circ 26' 25''$	$\log \operatorname{Cotg} D = 0,1546321.$
$D - \delta = 32^\circ 34' 38''$	$\log \operatorname{Cotg} m = 1,2083613.$
$D = 35^\circ 0' 31'', 5$	$\log \operatorname{Cotg} n = 1,4117887.$
$\delta = 2^\circ 25' 53'', 5$	$-\log \operatorname{Cotg} \delta = 2,6280173.$
$n + m = 108^\circ 0' 50''$	$\log \sin \psi^2 = 1,4027994.$
$n - m = 100^\circ 55' 50''$	$\log \operatorname{Cof} \psi^2 = 1,8734294.$
$n = 104^\circ 28' 30''$	$\log \operatorname{Cotg} \delta = 1,3719828.$
$m = 3^\circ 32' 30''$	$\log \sin m = 2,7908085.$
$z = 43^\circ 32' 6'', 5$	$\log \sin n = 1,9859959.$
	$\log \operatorname{Cotg} z = 0,0222166.$

Adeoque Latitudo Loci invenitur $46^\circ 27' 53'', 5$

Schol. 1. Inventa jam Latitudine Loci, altitudines veræ Stellarum observatarum determinari possunt. In triangulis videlicet ZPA & ZPB , datis duobus lateribus & angulo intercepto, dabitur latus tertium. Habebitur enim in $\triangle ZPA$, $\operatorname{tg} \frac{1}{2}(AZP - ZAP) =$

$$ZAP = \frac{\sin \frac{1}{2}(AP - ZP)}{\sin \frac{1}{2}(AP + ZP)} \operatorname{Cotg} \frac{1}{2} ZPA \quad \&$$

$$\sin \frac{1}{2} AZ = \frac{\sin \frac{1}{2}(AP - ZP)}{\sin \frac{1}{2}(AZP - ZAP)} \operatorname{Cof} \frac{1}{2} ZPA$$

Si in exemplo 1 desiderentur altitudines Stellarum veræ, valor ipsius AZ , sequenti calculo eruitur

$$AP - ZP = 3^\circ 57' 2'', 85 \quad \log \sin \frac{1}{2}(AP - ZP) = 2,8382172$$

$$\log \operatorname{Cotg} \frac{1}{2} ZPA = 0,1388497$$

$$AP + ZP = 33^\circ 29' 22'', 15 \quad -\log \sin \frac{1}{2}(AP + ZP) = 0,2582310$$

$$\frac{1}{2} ZPA = 35^\circ 59' 35'' \quad \log \operatorname{tg} (\frac{1}{2} ZPA) = 1,2352979$$

$$\frac{1}{2}ZA = 19^\circ 12' 36'', 51$$

$$ZA = 38^\circ 25' 13'', 02 \quad - \log \sin(\underline{AZP} - \underline{ZAP}) = 0,7710274$$

2

$$\log \sin(\underline{AP} - \underline{ZP}) = \overline{2,8382172}$$

2

$$\log \operatorname{Cof} \frac{1}{2} ZPA = \overline{1,9079959}$$

$$\log \sin \frac{1}{2} AZ = \overline{1,5172405}$$

Unde itaque altitudo vera seu complementum ipsius $AZ = 90^\circ - 38^\circ 25' 13'' 02 = 51^\circ 34' 46'', 98.$

Schol. 2. Si ad diversas meridiani partes observatae fuerint Stellæ, ita scilicet ut Stella A ad occidentalem coeli plagam, altera vero B ad orientalem visa fuerit, eodem plane modo quo jam ostendimus, Elevatio Poli quæsita determinatur. Observandum solummodo est, loco anguli $n - m$, adhibendum esse $m - n$, quoniam retenta eadem denominatione, qua supra usi sumus, angulus AP hoc in casu æqualis sit summæ angulorum ZPA & ZPE .

§. 3.

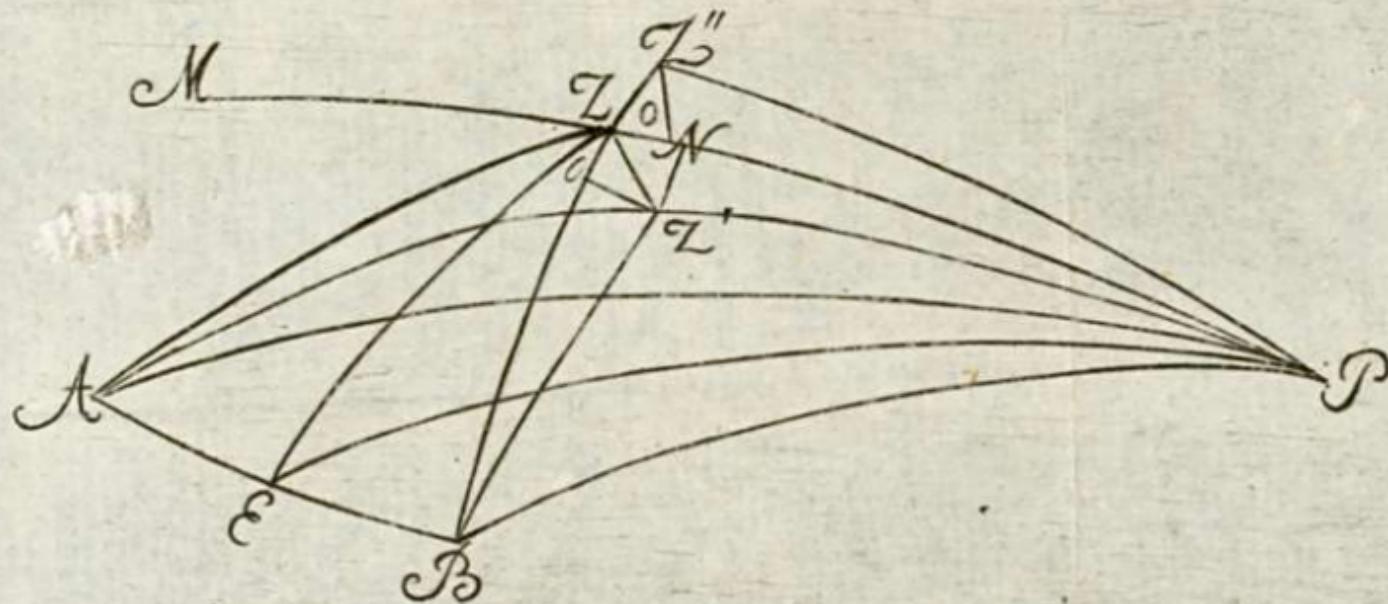
In solutione Problematis nostri altitudines exakte quidem æquales assumsimus; si vero error quidam in his observandis commisus fuerit, dispiciendum erit, quæ hinc ipsius Elevationis Poli proveniat variatio. Positis itaque declinationibus Stellarum invariatis, manente quoque angulo ZPB eodem, sit ex.

ex. gr. BZ revera paullo major AZ , exacte autem $BO = AZ$; descriptis Polis A, B & P arcubus ZZ' , $Z'O$ & NZ' , ductoque arcu $Z'P$, erit ZN variatio Latitudinis quæsita. Ad hanc vero inveniendam sequens jam instituendus est calculus: in triangulo admodum exiguo $Z'OZ$ erit $ZZ':ZO::1:\cos Z'ZO::1:$ $\sin AZB$, ob $>AZB + BZ' = 90^\circ$, & in $\triangle ZNZ'$, $ZZ':ZN::1:\cos Z'ZN::1:\sin MZA::1:\sin AZP$; est enim $>MZA + AZZ' + ZZN = 180^\circ$ & $AZZ' = 90^\circ$. Hinc vero eruitur $ZO = ZZ' \sin AZB$, & $ZN = ZZ' \sin AZP$, adeoque $ZO:ZN::ZZ' \sin AZB : ZZ' \sin AZP :: \sin AZB : \sin AZP$, & $ZN = \frac{ZO \sin AZP}{\sin AZB}$ unde itaque dato ZO , dabitur ZN . Iunctis etenim A & B , in $\triangle APB$ ex datis AP & BP cum angulo intercepto APB , habebitur $\tan \frac{1}{2}(PBA - PAB) = \frac{\sin \frac{1}{2}(AP - PB) \cot \frac{1}{2}APB}{\sin \frac{1}{2}(AP + PB)}$
& $\sin \frac{1}{2}AB = \frac{\sin \frac{1}{2}(AP - PB) \cos \frac{1}{2}APB}{\sin \frac{1}{2}(PBA - PAB)}$. Inven-
to jam $\frac{1}{2}AB$, & cognitis insuper (§. 2 Schol. 1.) ZA & ZB erit, ducto ZE , in $\triangle ZEB$ rectangulo, $\sin EZB = \frac{\sin \frac{1}{2}AB}{\sin ZB}$ unde itaque innoteſcit $AZB = 2EZB$.

In triangulo deinde AZP vel ZPB cognitis duobus lateribus AZ & AP vel BZ & BP cum angulo ZPA vel ZPB , inveniuntur secundum, regulas vulgares Trigonometricas $>AZP$ vel BZP .

§. 4.

Angulum horarum in antecedentibus invariatum supposuimus; posito vero hoc quoque erroneo, examinandum erit, quantus exfurgat error Elevacionis Poli, dato errore anguli horarii, existentibus tamen altitudinibus atque declinationibus stellarum observatarum invariatis. Si itaque auctus fuerit $> ZPB$ vel diminutus angulo $Z'PZ$, perspicuum est, locum ipsius Zenith transferri, ita videlicet, ut Zenith hoc in casu non maneat in Z , sed in alio quodam puncto jam determinando. Iunctis igitur A & B , erit AB portio circuli ad horizontem paralleli, Almicantarat nominati, ob æquales altitudines Stellarum A & B . Bisecto deinde AB , ductoque EZ , ad angulos rectos hic arcus insistat ipsi AB necesse est, adeoque etjam Horizonti perpendicularis erit, unde sequitur, ut sit ZE pars circuli verticalis. Erit autem Zenith semper in circulo verticali, adeoque situs ipsius est in arcu EZ : est vero etjam locus ipsius Zenith in meridiano, ergo in intersectione horum arcuum Z'' . Descripto deinde Polo P arcu $Z''o$, erit Zo variatio Latitudinis quæsita. Ad hanc inveniendam, si inferatur $Z''PZ : \sin ZZ''P$
 $:: Z''Z : \sin ZP$, habebitur $ZZ'' = \frac{Z''PZ \sin ZP}{\sin ZZ''P}$. Quum autem sit $ZZ''P = ZZ''o + oZ''P = ZZ''o + 90^\circ$, & $ZZ''o + Z''Zo = 90^\circ$, erit $ZZ''P = 180^\circ - Z''Zo = EZP$, ade-



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adeoque $ZZ'' = \frac{Z''PZ \ Sin ZP}{Sin EZP}$. Est autem in $\Delta ZoZ''$ res
Et angulo $ZZ'' : Zo :: 1 : Cof Z''Zo$ seu $\frac{Z''PZ \ Sin ZP}{Sin EZP} :$
 $Zo :: 1 : Cof EZP$ unde itaque $Zo = Z''PZ \ Sin ZP$
 $Cotg \frac{1}{2}(AZP + BZP)$ ob $EZP = \frac{1}{2}(AZP - BZP)$
 $+ BZP = \frac{AZP + BZP}{2}$. Investigatis denique valo-
ribus angulorum AZP & BZP , eodem quo in §. an-
tecedente ostendimus modo, innotescit variatio Ele-
vationis Poli, dato errore anguli horarii.