DISSERTATIO ACADEMICA,

SPECIMEN CONTINENS *METHODI RESIDUA, EX DIVISIONE NUMERORUM ORIUNDA, INVESTIGANDI.*

QUAM,

CONS. AMPL. FACULT. PHILOS. ABOËNS.

PRÆSIDE

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ABOÆ, Typis FRENCKELLIANIS.

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Prodiit nuperrime, iisdem ac nostrum hoc qualecunque opusculum, auspiciis, Diss. Gradualis Cl. FREDR. ELFGREN *Theoremata* exhibens *generalia inveniendis residuis, ex divisione numerorum oriundis, inservientia*. Quæ quidem Theoremata adeo late patent, ut universo quæstionum generi, quod spectant, solvendo ex asse sufficiant. At multum tamen abest, ut inchoatam in hac tractatione disquisitionum telam pertextam jure censere possis. Desunt peculiaria Theoriæ ad casus speciales adplicandæ præcepta, ipsam praxin adjuvantia: quæ nisi accesserint, ipsius *Methodi* (ex principiis in laudata Dissertatione traditis derivandæ) *residua, ex divisione numerorum oriunda, investigandi*, ratio minus liquet, & paullo impeditior, tironibus præsertim, manet. Quare spernendum minime nobis visum est, commendatum ab ipso cit. Dissertationis Auctore, Methodi generaliter tantum a se præceptæ uberius exponendæ innocuum consilium. In quo exsequendo ita versari nobis conveniet, ut inceptam in superiori tractatione §:rum

 seriem

ſeriem continuemus, receptaque haud inconcinna numeros exhibendi ſchemata retineamus.

§. 11.

Exhibeatur numerus quicunque N per formulam (*I*) ſupra (§. 3) traditam, & fiat in formula (*II*) (§. 4 exhibita) $r = o$. Quo pacto transibit hæc formula $N(m, r)$ in $N(m, o) = (a_{m-1}, a_{m-2}, \ldots a_o)$. Unde definito ſimul exponente *m*, erit $N(1,o) = a_o$, $N(2,o) = (a_1, a_o)$, $N(3,o) = (a_2, a_1, a_o)$ & ſic porro. Obſervandum vero eſt, valores numericos harum formularum non minus quam generalis illius $N(m, r)$ ad eam a nobis in ſequentibus reſtringi hypotheſin, quæ vulgaris computandi Syſtematis baſin conſtituit, nimirum quod ſit $(1, o) = 10$. Cujus rei ratio ex exemplis ſequentibus facile perſpici poterit.

N	$N(m, o)$, ſumto *m* =				
	1.	2.	3.	4.	5.
6431597.	7.	97.	597.	1597.	31597.
9254928.	8.	28.	928.	4928.	54928.
538460.	0.	60.	460.	8460.	38460.
7968950.	0.	50.	950.	8950.	68950.
3195275.	5.	75.	275.	5275.	95275.

§. 12.

Sit $D = 2^{\alpha} . 5^{\beta}$, ſumto indice *m* ita, ut neuter exponentium α & β eum exceſſerit, numerus 10^m

non

non potest non esse dividuus ipsius D. Unde (§ 8) constat, fore $N_{(m,o)}$ h. e. $(a_{m-1}, a_{m-2} \ldots a_o)$ ejus naturæ, ut per D divisus idem præbeat residuum ac ipse numerus N.

Per se patet minimum obtineri valorem ipsius $N_{(m,o)}$, si ponatur m = majori exponentium α, β.

Sumtis *e. gr.* hisce valoribus ipsius D: $8 = 2^3$, $40 = 2^3.5$, $200 = 2^3.5^2$, $125 = 5^3$, $250 = 2.5^3$, $500 = 2^2.5^3$, nec non $1000 = 2^3.5^3$, quia sunt hi singuli factores numeri 10^3; sufficiet in formula $N_{(m,o)}$ posuisse $m = 3$. Dabit igitur, pro hisce divisoribus, numerus N eadem residua respective ac $N_{(3,o)} = (a_2, a_1, a_o)$. Quare retentis valoribus numeri N in §. præced. adhibitis, ex valoribus ipsius $N_{(3,o)}$ erui possunt hæc residua, qualia in Tabella sequenti occurrunt.

$N_{(3,o)}$	Residua pro Divisoribus:						
	8	40.	200.	125.	250.	500.	1000.
597	5.	37.	197.	97.	97.	97.	597.
928	0.	8.	128.	53.	178.	428.	928.
460	4.	20.	60.	85.	210.	460.	460.
950	6.	30.	150.	75.	200.	450.	950.
275	3.	35.	75.	25.	25.	275.	275.

Pro divisoribus $4 = 2^2$, $20 = 2^2.5$, $25 = 5^2$, $50 = 2.5^2$, atque $100 = 2^2.5^2$, residua commodius obtinentur dividendo (per hosce numeros) valores formulæ $N_{(2,0)} = (a_1, a_0)$, qui pro assumtis valoribus ipsius N sunt hi: 97, 28, 60, 50, 75. Residua pro divisoribus 2, 5, 10, oriunda vel primo adspectu ipsius termini ultimi a_0 patent. Ex cujus consideratione trita hæc fluunt criteria dividuos numerorum 2, 5 & 10 dignoscendi: *binarium* eos metiri numeros, quorum terminus ultimus est numerus par; *quinarium* non nisi eos, qui in 0 vel 5 desinunt; omnes denique *denarii* dividuos desinere in 0.

§. 13.

Definitis sic limitibus, intra quos usus formulæ, $N_{(m,0)}$ restringitur, tractationis nostræ series postulat, ut, quæ sit vis & quis usus formulæ (II), si fiat r vel $= 1$ vel $= -1$, jam plenius exponamus. Quibus positis abit formula hæc

pro $r = 1$, in $N_{(m,r)}$	pro $r = -1$, in $N_{(m,-r)}$
$= (a_n, a_{n-1}, \ldots a_{sm})$	$= \pm (a_n, a_{n-1}, \ldots a_{sm})$
$+ (a_{sm-1}, a_{sm-2}, \ldots a_{(s-1)m})$	$\mp (a_{sm-1}, a_{sm-2}, \ldots a_{(s-1)m})$
$+ \ldots\ldots$	$\pm \ldots\ldots$
$+ (a_{2m-1}, a_{2m-2}, \ldots a_m)$	$- (a_{2m-1}, a_{2m-2}, \ldots a_m)$
$+ (a_{m-1}, a_{m-2}, \ldots a_0)$ (III).	$+ (a_{m-1}, a_{m-2}, \ldots a_0)$ (IV).

In formula (IV), in qua signa + & − alternatim semet excipiunt, adhibenda sunt signa superiora, si fuerit s numerus par: sin minus, inferiora.

Sit

Sit *e. gr.* $N = 52391164761859$.

Quod, si 1:o sumatur $m = 1$; habebitur ope formulæ (III)

$N_{(1,1)} = 5+2+3+9+1+1+6+4+7+6+1+8+5+9 = 67 = N'$, $N'_{(1,1)} = 6+7 = 13 = N''$, $N''_{(1,1)} = 1+3 = 4 = N'''$;

nec non ope formulæ (IV).

$N_{(1,-1)} = 5+2-3+9-1+1-6+4-7+6-1+8-5+9 = 11 = N'$,

atque $N'_{(1,-1)} = -1+1 = 0 = N''$.

Ponatur 2:o $m = 2$. Quo pacto erit, ex formula (III),

$N_{(2,1)} = 52+39+11+64+76+18+59 = 319 = N'$,

$N'_{(2,1)} = 3+19 = 22 = N''$, nec non ex formula (IV), $N_{(2,-1)} = \left.\begin{matrix} 52+11+76+59 \\ -39-64-18 \end{matrix}\right\} \begin{matrix} = 198 \\ -121 = 77 = N'. \end{matrix}$

Si denique 3:o fiat in iisdem formulis $m = 3$; emerget ex illa

$N_{(3,1)} = 52+391+164+761+859 = 2227 = N'$.

$N'_{(3,1)} = 2+227 = 229 = N''$, atque ex hac

$N_{(3,-1)} = \left.\begin{matrix} 52+164+859 \\ -391-761 \end{matrix}\right\} \begin{matrix} = 1075 \\ -1152 = -77 = N'. \end{matrix}$

§. 14.

Quod si sit D factor numeri $10^m - 1$, & a numero quolibet N formetur juxta formulam (III) series: N', N'', *&c.*; erit (§. 8.) terminus quivis hujus seriei ejus indolis, ut divisus per numerum D idem exhibeat residuum ac ipse numerus N.

Idem

Idem quoque valet de terminis seriei per formulam (IV) producendæ, quoties fuerit divisor D factor numeri 10^m+1.

Resumto valore ipsius N, in §. præced. obvio, quæratur: quænam peracta divisione hujus numeri per $D=3, 7, 9, 11, 13, 33$, prodeant residua? Quia vero sunt numeri 3 & 9 factores formulæ 10^m-1, facto $m=1$, numerus 33 factor numeri $99=10^2-1$, nec non 37 factor numeri $999=10^3-1$; habebitur pro divisoribus 3 & 9 (dividendo terminum quemcunque seriei: $67, 13, 4$), residuum $R=1$ & 4 respective; pro divisore 33, (ob $N_{(2,1)}=22$) $R=22$, atque pro divisore 37 (dividendo utrumvis terminorum: $2227, 229$) $R=7$. Præterea est $10^3+1=1001=7.11.13$ dividuus numerorum $7, 11, 13$. Quare obtineri possunt residua pro his divisoribus oriunda, dividendo $N_{(3,-1)}=-77$. Qui numerus cum sit ex formis hisce: $7q, 11q, 13q+1$; præbebit quoque ipse numerus N, peracta divisione per $7, 11, 13$, hæc residua: $0, 0, 1$, respective.

§. 15.

Si fuerit D factor numeri $10^\alpha-1$; erit D quoque factor numeri $10^{\alpha m}-1$, manente m numero integro arbitrario. Est enim generaliter $A^{\alpha n}-1=(A^\alpha-1)(A^{\alpha(m-1)}+A^{\alpha(m-2)}+\ldots+A^\alpha+1)$.

Sumto

Sumto e. gr. 1:o $\alpha=1$, abit $10^\alpha-1$ in $10-1=9$ (cujus numeri factores sunt 3 & 9), atque $10^{\alpha n}-1$ in 10^n-1. Erunt ergo quoque $10^2-1=99$, $10^3-1=999$, $10^4-1=9999$ &c. dividui numerorum 3 & 9.

Facto 2:o $\alpha=3$, abit $10^\alpha-1$ in numerum $10^3-1=27\cdot37$ nec non $10^{\alpha m}-1$ in $10^{3m}-1$. Quia vero est numerus 10^3-1 dividuus numerorum $27, 37, 111, 333, 999$; sequitur hinc reliquos quoque numeros sub forma $10^{3m}-1$ comprehensos, ut: 10^6-1, 10^9-1, $10^{12}-1$, &c. dividuos esse eorundem numerorum.

§. 16.

Si fuerit D factor numeri $10^\alpha+1$, erunt omnes numeri, sub formis: $10^{2\alpha m}-1$, $10^{\alpha(2m+1)}+1$ comprehensi, dividui ejusdem D.

Quo hoc probemus, fiat $10^\alpha=A$. Unde $10^\alpha+1=A+1$, $10^{2\alpha m}-1=A^{2m}-1$, atque $10^{\alpha(2m+1)}+1=A^{2m+1}+1$. Quum vero ex elementis Algebræ notissimum sit, formulas: $A^{2m}-1$, $A^{2m+1}+1$ continere factorem $A+1$; veritas asserti in dubium vocari nequit.

Ex. I. Exinde, quod sit 11 factor formæ $10^\alpha+1$, posito $\alpha=1$, concluditur esse numeros tam

formæ

formæ $10^{2m}-1$ (ut: 10^2-1, 10^4-1, 10^6-1, &c.) quam formæ $10^{2m+1}+1$ (ut: 10^3+1, 10^5+1, 10^7+1, &c.) dividuos numeri 11.

Ex. 2. Vidimus supra (§. 14) esse 10^3+1 dividuum numerorum 7 & 13. Quare, ob $\alpha=3$ erit $10^{2\alpha m}-1=10^{6m}-1$ atque $10^{\alpha(2m+1)}+1=10^{3(2m+1)}+1$ $=10^{6m+3}+1$. Unde conficitur, esse terminos seriei utriusque 10^6-1, $10^{12}-1$, $10^{18}-1$ *&c*; 10^9+1, $10^{15}+1$, $10^{21}+1$, *&c.* dividuos numerorum 7 & 13.

§. 17.

Quo vero generaliter definiri possit, quosnam divisores formula 10^m-1 admittat, sequens e re est ut adponatur

Lemma. Sit $D=d^\alpha$ potestas numeri primi d, $\beta=d^{\alpha-1}(d-1)$, nec non A numerus ad ipsum D primus; erit $A^\beta-1$ dividuus ipsius D.

Lemmatis hujusce demonstrationem qui desideraverit, abunde sibi satisfactum reperiet ab Ill. Eulero in tractatu: *Theoremata Arithmetica nova methodo demonstrata*, Nov. Comm. Acad. Sc. Petrop. Tomo VIII, pagg. 74 seqq. inserto. Vide *Theoremata 3 & 11 cit. tractatus.*

§. 18.

§. 18.

THEOREMA. Sint $d, d_1, d_2, .. d_s$ numeri primi factores videlicet numeri $D = d^{\alpha} . d_1^{\alpha 1} . d_2^{\alpha 2} . d_s^{\alpha_s}$, $\beta = d^{\alpha - 1} (d - 1)$, $\beta_1 = d^{\alpha_1 - 1}(d_1 - 1)$, $\beta_2 = d_2^{\alpha 2 - 1} (d_2 - 1), .. \beta_s = d_s^{\alpha_s - 1} (d_s - 1)$, nec non δ dividuus minimus numerorum: $\beta, \beta_2, \beta_1, .. \beta_s$; erit, manentibus numeris A & D inter se primis, $A^{\delta} - 1$ dividuus ipsius D.

Vi Lemmatis praeced. constat, esse $A^{\beta} - 1, A^{\beta_1} - 1, A^{\beta_2} - 1, .. A^{\beta_s} - 1$ dividuos numerorum d^{α}, $d_1^{\alpha 1}$, $d_2^{\alpha 2}, .. d_s^{\alpha_s}$ respective. Existente vero δ dividuo communi exponentium $\beta, \beta_1, \beta_2, .. \beta_s$, erit (Cfr. §. 15) $A^{\delta} - 1$ dividuus communis formularum $A^{\beta} - 1$, $A^{\beta 1} - 1$, $A^{\beta 2} - 1, .. A^{\beta s} - 1$. Quare erit quoque $A^{\delta} - 1$ dividuus communis numerorum $d^{\alpha}, d_1^{\alpha_1}, d_2^{\alpha 2}, .. d_s^{\alpha_s}$. Unde, quia sunt hi numeri ad se invicem primi, concluditur esse $A^{\delta} - 1$ dividuum producti ex his compositi D.

Coroll. Posito $A = 10$, formula $A^{\delta} - 1$ abit in $10^{\delta} - 1$. Erit ergo $10^{\delta} - 1$ dividuus numeri cujuscunque D, ad denarium primi. Omnes vero numeri, qui communem cum denario factorem habent, desinunt vel in *o*, vel in *2*, vel in 4, vel in *5*, vel in *6*, vel denique in *8*. Quos si exceperis; poterit, pro quolibet reliquorum $= D$ (h. e. pro singulis desinentibus in: *1*, *3*, *7*, *9*), exponens *m* in formula $10^m - 1$ ita determinari, ut obtineatur dividuus ipsius *D*.

Sumto *e. gr.* $D = 63 = 3^2 . 7$, est $\beta = 3^1 . 2 = 6$, $\beta_1 = 7^0 . 6 = 6$, ideoque $\delta = 6$. Quare erit $A^6 - 1$ dividuus numeri *63*, quoties fuerit *A* ad numeros *3* & 7 primus. Sic facto $A = 2, 5, 10$; transit formula $A^6 - 1$ in numeros: $2^6 - 1 = 63$, $5^6 - 1 = 15624 = 63 . 248$, $10^6 - 1 = 999999 = 63 . 15873$.

Schol. Manente numero *A* indeterminato, exponens δ est minimus, qui, substitutus pro *m* in formula $A^m - 1$, producat dividuum numeri *D*. At tamen pro specialibus numeri *A* valoribus sæpe evenit, ut potestas quædam $A^m < A^\delta$ reddat $A^m - 1$ dividuum numeri *D*. In iis vero casibus, fore minimum valorem exponentis *m*, qui adhiberi possit, partem quandam aliquotam ipsius δ, demonstravit EULERUS *Tract. cit. Theor.* 10. Sic si fuerit $A = 2$ & $D = 31$, minimus valor ipsius *m*, qui efficiat $2^m - 1$ divisibilem per *31*, est $5 = \frac{30}{6} = \frac{\delta}{6}$. Facto $A = 10$ & $D = 11$, evadit $A^2 - 1 = 10^2 - 1$ dividuus numeri 11. Est vero δ in

hoc

hoc casu $= 11^0 . 10 = 10$. Qui numerus est quintuplus exponentis 2.

§. 19.

Sit D factor numeri $10^{\mu}+1$, sumto exponente μ ita, ut sit $10^{\mu}+1$ minimus numerus formæ $10^{m}+1$, quem metiatur D; erit $10^{2\mu}-1$ minimus numerus formæ $10^{m}-1$, quem metitur idem D.

Quia est $10^{2\mu}-1 = (10^{\mu}-1)(10^{\mu}+1)$, erit hic numerus dividuus ipsius D. Si neges exponentem 2μ esse minimum, quo adhibito pro m in formula $10^{m}-1$ huic conditioni satisfiat, sit minimus ille, quem quæris, $\mu' < 2\mu$. Hoc pacto erit (cfr. §. præced. *Schol.*) μ' pars aliquota exponentis 2μ, h. e. vel $= \mu$ vel factor quidam ipsius μ. Cum vero $(10^{\mu}-1) = (10^{\mu}+1)-2$, & $10^{\mu}+1$, pro quovis valore exponentis μ, sint numeri inter se primi; nequit esse $10^{\mu}-1$ dividuus numeri D. Sin urgeas esse μ' factorem ipsius μ, sit necesse est $\mu = \mu' q$. Quare, si esset $10^{\mu'}-1$ dividuus numeri D, ejusdem quoque indolis (§. 15) foret $10^{\mu}-1$. Quod ipsum fieri haud posse nuperrime demonstratum est.

Coroll. 1. Sit dividuus minimus ipsius D, ad formam $10^{m}-1$ pertinens $= 10^{\mu}-1$, atque μ numerus impar; formula $10^{m}+1$ ita determinari nequit, ut prodeat dividuus numeri D.

Coroll. 2. Sit $D = d^{\alpha}$ potestas numeri primi, nec non dividuus minimus numeri D, sub forma $10^{m}-1$ comprehensus $= 10^{2\mu}-1$; metitur in hoc casu divisor D numerum $10^{\mu}+1$, nullumque minorem ejusdem formæ.

§. 20.

Qua ratione, pro divisore D desinente in: 1, 3, 7, 9, determinari possit dividuus minimus vel formæ $10^{m}-1$ vel formæ $10^{m}+1$, ex præceptis (§. §. 18, 19) jam traditis omnino patet. Numerorum, qui restant, desinentium nimirum in 0, 2, 4, 6, 8, duo distinguenda sunt genera; *unum* formæ $2^{\alpha}.5^{\beta}$, *alterum* eos complexum numerus, quos præter 2, 5 alii quoque numeri primi metiantur. Numeri *illius* generis metiuntur potestatem aliquam denarii, ut supra (§. 12) ostensum est. Qui vero sub *hoc* comprehenduntur, nullos habent dividuos formæ $10^{m}\pm r$, nisi qui sumendo $r > 1$ prodeant. Quippe horum dividui sunt numeri ex formis hisce: $10^{m}\pm 2$, $10^{m}\pm 4$, $10^{m}\pm 5$, $10^{m}\pm 6$, $10^{m}\pm 8$.. sive generatim $\{10^{m}\pm 2^{\alpha}.5^{\beta}.q$ (existentibus α, β, q numeris integris).

Sic *e. gr* est $10^{2}-2 = 2.7^{2}$ dividuus numerorum 14 & 98; nec non $10^{2}+5 = 3.5.7$ dividuus numerorum 15 & 35.

§. 21.

Coronidem opellæ imposituri Tabellam adposuimus, omnes exhibentem valores formulæ $10^m - r$, pro exponente m sumto intra terminos: 1 & 5, nec non numero r intra terminos: $+5$ & -5, inclusive.

	m					
r	1	2	3	4	5	.. α
0.	2.5	$2^2.5^2$	$2^3.5^3$	$2^4.5^4$	$2^5.5^5$	.. $2^\alpha.5^\alpha$
$+1.$	$3^2.$	$3^2.11.$	33.37	$3^2.11.101$	$3^2.11111$	.. $10^\alpha - 1$
$-1.$	11	101	$7.11.13$	10001	11.9091	.. $10^\alpha + 1$
$+2.$	2^3	2.7^2	2.499	2.4999	2.49999	.. $10^\alpha - 2$
$-2.$	$2^2.3$	$2.3.17$	$2.3.167$	$2.3.1667$	$2.3.16667$	.. $10^\alpha + 2$
$+3.$	7	97	997	9997	99997	.. $10^\alpha - 3$
-3	13	103	17.59	10003	100003	.. $10^\alpha + 3$
$+4.$	2.3	$2^5.3$	$2^2.3.83$	$2^2.3.7^2.17$	$2^2.3.8333$	.. $10^\alpha - 4$
$-4.$	$2.7.$	$2^3.13$	$2^2.251$	$2^2.2501$	$2^2.25001$	.. $10^\alpha + 4$
$+5.$	5	5.19	5.199	5.19999	5.19999	.. $10^\alpha - 5$
$-5.$	3.5	$3.5.7.$	$3.5.67$	$3.5.23.29$	$3.5.6667$	.. $10^\alpha + 5$