

R. Mikkola 075

X

DISSERTATIO ASTRONOMICA,
DE
CREPUSCULIS.

QUAM
CONS. AMPL. FAC. PHIL. IN ACAD. ABOENS.

PRÆSIDE

Mag. ANDR. JOH. METHER;
MATHEM. PROF. P. O.

PRO GRADU

PUBLICÆ CENSURÆ SUBMITTIT

REGINALDUS DE BECKER,
NOB. WIB. STIP. EKESTUBB.

IN AUD. MATHEM. D. XV JUNII MDCCCX.

H. A. M. S.

ABOÆ, TYPIS FRENCKELLIANIS.



V I R O
PERCELEBRI ET PRÆCLARISSIMO
DOMINO MAGISTRO
MAGNO ALOPAEO,
LOGICES ET MATHESEOS IN GYMNASIO BORGOËNSI LECTORI,

Dat, Dicat, Dedicat,

REGINALDUS de BECKER.

Atmosphærām terrestrem, radios Solares refle-
ctendo, creperam illam atque debilem lucem
mane ante ortum solis & vesperi post occasum i-
psius perspicuam, quam crepusculum nominamus,
producere, satis est cognitum. Postquam videlicet
Tellus motu diurno nos e conspectu Solis subduxit,
sublimior aér nobis adhuc a Sole illustratus manet.
Sed magis magisque descendente Sole, minus con-
tinuo illustratur aér. Pariter mane Sol paullatim
Atmosphærām illuminare incipit, coelique faciem
undique minutatim lucidam usque ad ortum ipsius
reddit. Successive itaque crescit & decrescit crepu-
sculum, & a qualitate aëris, ejusque vicissitudinibus
pendet; quamobrem momentum initii atque finis
ipsius difficile admodum est determinatu. Quum
autem ex observationibus liqueat, Stellas minores
oculo inarmato cerni posse eo momento, quo Sol

A

deci-

decimum octavum infra horizontem attigerit gradum, Astronomi assumserunt, limites crepusculorum ita esse determinandos, ut portio arcus diurni, quam Sol a momento ortus vel occasus absolverit, donec ad almicantarat 18° infra horizontem descendat, crepusculum metiatur. Unde patet, duracionem ipsius & a motu Solis in ecliptica & a diversa locorum Latitudine quoque & quidem maxime pendere. Quo enim proprius Solis supra horizontem ortus, vel infra ipsum occasus, ad motum horizonti perpendicularis accederit, eo citius Sol aseendat descendatve, necesse est, unde crepusculum brevius: quo autem obliquior motus Solis orientis vel occidentis versus horizontem fuerit, eo lentores Solis ab horizonte distantia mutationes subibit, unde crepusculum longius evadit. Variante igitur declinatione Solis atque elevatione Poli, variat quoque crepusculum: specimen ideo Academicum edituri, durationem crepusculi nobis ita investigandam proposuimus, ut tradita generali Problematis solutione, specialem ipsius applicationem, assumta Latitudine $60^{\circ} 27' 7''$, pro quovis dimidio declinationis gradu exhibeamus, unde crepusculum pro quovis die facili negotio haberi potest.

§. 2.

Est vero directa solutio Problematis nostri inventu haud difficilis. Sit nempe ZP arcus meridiani,

diani, Z Zenith, P Polus, HR horizon verus, si locus Solis in horizonte fuerit A , jungantur puncta A & Z arcu AZ , ductoque arcu circuli maximi ZB ita, ut æqualis fiat 108° , & productus fecet arcum diurnum DB per A transientem in B , junctisque punctis A & P , B & P , arcibus circulorum maximorum AP & BP habebitur in ΔZAP , posito Sinu toto = 1 (Element. Trigon. Sphær.)

$$\sin \frac{1}{2} ZPA = \sqrt{\frac{\sin \frac{1}{2}(AZ+AP-ZP) \sin \frac{1}{2}(AZ+ZP-AP)}{\sin ZP \sin AP}}$$

& in ΔZPB

$$\sin \frac{1}{2} ZPB = \sqrt{\frac{\sin \frac{1}{2}(BZ+BP-ZP) \sin (BZ+ZP-BP)}{\sin ZP \sin BP}}$$

Ex quibus itaque innescunt anguli ZPA & ZPB , adeoque etiam $APB = ZPB - ZPA$, qui quidem in tempus conversus, dabit crepusculum quæsitum.

Exempl. I. In Latitudine $60^\circ. 27'. 7''$. si pro die 3 Aprilis anni currentis declinatio Solis Borealis in momento occasus fuerit $5^\circ. 2'. 17''$. crepusculum vespertinum pro eodem die sequenti calculo investigatur: $ZP = 29^\circ. 32'. 53''$ Log $\sin \frac{1}{2} AZ + AP - ZP = 1,9799110$

$$AP = PB = 84^\circ. 57'. 43'' \quad \text{Log } \sin \frac{1}{2} AZ + ZP - AP = 1,4731352$$

$$AZ = 90^\circ \quad - \quad \text{Log } \sin \frac{1}{2} ZP = 0,3070180$$

$$BZ = 108^\circ \quad - \quad \text{Log } \sin \frac{1}{2} AP = 0,0016812$$

$$2 \log \sin \frac{1}{2} ZPA = 1,7617454$$

$$\underline{AZ + AP - ZP = 72^\circ.42'.25''}. \quad \text{Log } \frac{1}{2} \sin ZPA = \overline{1,8008727}$$

$$\underline{AZ + ZP - AP = 17^\circ.17'.35''}$$

$$\underline{BZ + BP - ZP = 81^\circ.42'.25''}$$

$$\underline{BZ + \overset{2}{ZP} - BP = 26^\circ.17'.35''}$$

$$\frac{1}{2} ZPA = 49^\circ.28'.24''$$

$$ZPA = 98^\circ.56'.48''$$

$$\frac{1}{2} ZPB = 70^\circ.50'.24'', 94, \quad \text{Log } \sin \underline{BZ + BP - ZP} = \overline{1,9954349}$$

$$ZPB = 141^\circ.40'.49''.88, \quad \text{Log } \sin \underline{BZ + \overset{2}{ZP} - BP} = \overline{1,6463670}$$

$$-\text{Log } \sin \overset{2}{ZP} = \overline{0,3070180}$$

$$-\text{Log } \sin \overset{2}{BP} = \overline{0,0016812}$$

$$2 \text{Log } \sin \frac{1}{2} ZPB = \overline{1,9505011}$$

$$\text{Log } \sin \frac{1}{2} ZPB = \overline{1,9752505}$$

Datis vero iam angulis ZPA & ZPB , iporum quoque dabitur differentia seu angulus $APB = 42^\circ.44'.1''.88$, qui in tempus conversus secundum rationem $360^\circ : 24''$, exhibet crepusculum vespertinum quæstum $2^\circ.50'.56'', 12.$

Exempl. 2. Eodem loco & anno existente declinatione Solis Australi in momento ortus ipsius die 7 Octobris $5^\circ.14'.32''$, crepusculum matutinum sic computatur:

$ZP =$

$$ZP = 29^\circ. 32'. 53'' \quad \text{Log Sin } \underline{AZ + AP - ZP} = \overline{1,9901566}$$

$$AP = PB = 95^\circ. 14'. 32'' \quad \text{Log Sin } \underline{AZ + ZP - AP} = \overline{1,3232965}$$

$$\underline{AZ = 90^\circ} \quad - \text{Log Sin } \underline{ZP} = \overline{0,3070180}$$

$$\underline{BZ = 108^\circ} \quad - \text{Log Sin } \underline{AP} = \overline{0,0018203}$$

$$\underline{AZ + AP - ZP} = 77^\circ. 50'. 49'', 5, \quad 2 \text{Log Sin } \underline{\frac{1}{2} ZPA} = \overline{1,6222914}$$

$$\underline{AZ + ZP - AP} = 12^\circ. 9'. 10'', 5 \quad \text{Log Sin } \underline{\frac{1}{2} ZPA} = \overline{1,8111457}$$

$$\underline{BZ + BP - ZP} = 86^\circ. 50'. 49'', 5$$

$$\underline{BZ + ZP - BP} = 21^\circ. 9'. 10'', 5$$

$$\underline{\frac{1}{2} ZPA} = 40^\circ. 20'. 34'', 1 \quad \text{Log Sin } \underline{BZ + BP - ZP} = \overline{1,9993422}$$

$$\underline{ZPA} = 80^\circ. 41'. 8'', 2 \quad \text{Log Sin } \underline{BZ + ZP - BP} = \overline{1,5573357}$$

$$\underline{\frac{1}{2} ZPB} = 58^\circ. 55'. 57'' \quad - \text{Log Sin } \underline{ZP} = \overline{0,3070180}$$

$$\underline{ZPB} = 117^\circ. 51'. 54'' \quad - \text{Log Sin } \underline{BP} = \overline{0,0018203}$$

$$2 \text{Log Sin } \underline{\frac{1}{2} ZPB} = \overline{1,8655162}$$

$$\text{Log Sin } \underline{\frac{1}{2} ZPR} = \overline{1,9327581}$$

Unde itaque innoteſcit > $APB = 37^\circ. 10'. 45'' 8$, qui eo-dem modo, quo in exemplo antecedenti in tempus conversus dabit crepusculum matutinum 2^h. 28'. 43''.

§. 3.

Quamvis crepusculum secundum methodum
in §. præcedenti allatam investigari possit, ope
tamen formularum Trigonometricarum algebraica
ipsius investigatio multo est concinnior. Manente
igitur constructione figuræ eadem, si declinatio
Solis fuerit d , Latitudo Loci p , arcus $BK = 18^\circ \mp$
 a , $>ZPA = \gamma$ & $>APB = y$, erit $AP = PB = 90^\circ \mp$
 d , $ZP = 90^\circ - p$, $ZA = 90^\circ$, $ZB = 90^\circ + a$ & $>ZPB =$
 $\gamma + y$. In triangulo vero ZPA habebitur (Elem.
Trig. Sphær.) $\cos AZ = \cos ZPA \sin ZP \sin AP \pm$
 $\cos ZP \cos AP$, & in ΔZPB , $\cos BZ = \cos ZPB$
 $\sin ZP \sin BP \pm \cos ZP \cos BP$, seu $\cos \gamma \cos p \cos d \pm$
 $\sin p \sin d = 0$ (I) & $\cos 90^\circ + a = \cos \gamma + y \cos p \cos d \pm$
 $\sin p \sin d$ (II). Subducendo vero æquationem (I)
ab æqu. (II) eruitur $\cos 90^\circ + a = (\cos \gamma + y - \cos \gamma)$
 $\cos p \cos d$ & facta debita reductione $-\sin a = (\cos \gamma -$
 $\cos \gamma) \cos p \cos d$, unde $\cos(\gamma + y) = \cos \gamma \cdot \frac{\sin a}{\cos p \cos d}$ (III).
Quumque dividendo æquat. (I) per $\cos p \cos d$, sit
 $\cos \gamma = \mp \tan p \tan d$, valor ipsius γ est datus & hinc
facillime determinatur $\gamma + y$ & y ope æqu. (III),
qui crepusculum exhibet. Quo autem angulus iste
Logarithmorum ope ex æquat. (III) inveniri queat,
statuatur in casu, quo $\cos \gamma$ positivus fuerit, atque
 $\cos \gamma > \frac{\sin a}{\cos p \cos d}$ vel $\sin d < \frac{\sin a}{\sin p}$, quo in casu $\gamma + y$
 $< 90^\circ$,

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$\angle 90^\circ, \frac{\sin a}{\cos \gamma \cos p \cos d} = \sin \varphi^2$, unde facta substitu-
tione $\cos(\gamma + y) = \cos \gamma \cos \varphi^2$. Si vero fuerit $\cos \gamma =$
 $\frac{\sin a}{\cos p \cos d}$, vel $\sin d = \frac{\sin a}{\sin p}$, erit $\gamma + y = 90^\circ$, e qua æqua-
tione, cognito γ, y facillime determinatur. Quum
autem $\cos \gamma < \frac{\sin a}{\cos p \cos d}$ vel $\sin d > \frac{\sin a}{\sin p}$ & $\gamma + y > 90^\circ$,
ponatur $\frac{\cos \gamma \cos p \cos d}{\sin a} = \sin \varphi^2$, eritque $\cos(\gamma + y) =$
 $\frac{\cos \varphi^2 \sin a}{\cos p \cos d}$. Existente denique $\cos \gamma$ negativo, sup-
ponatur $\frac{\sin a}{\cos \gamma \cos p \cos d} = \operatorname{tg} \varphi^2$, eruiturque facta de-
bita reductione $\cos \gamma + y = -\frac{\cos \gamma}{\cos \varphi^2}$.

Exempl. Si resumantur eadem data, quæ in
exemplo 1. §. præc. attulimus, crepusculum secundum
methodum jam allatam sequenti modo investigatur.
 $d = 5^\circ. 2'. 17''$ $\operatorname{Log} \operatorname{tg} p = 0,2465085$
 $p = 60^\circ. 27. 7''$ $\operatorname{Log} \operatorname{tg} d = 2,9452616$
 $a = 18^\circ$ $\operatorname{Log} \cos \gamma = 1,1917701$
 $y = 98^\circ. 56'. 48''$ $\operatorname{Log} \sin a = 1,4899824$
 $y + y = 141^\circ. 40' 50''$ $- \operatorname{Log} \cos p = 0,3070180$
 $y = 42^\circ. 44'. 2''$ $- \operatorname{Log} \cos d = 0,0016811$
 $- \operatorname{Log}$

$$\begin{aligned}
 -\log \cos \gamma &= 0,8082299 \\
 \log \tan \varphi^2 &= 0,6069114 \\
 \log \cos \gamma &= 1,1917701 \\
 -\log \cos \varphi^2 &= 0,7028566 \\
 \log \cos(\gamma + \varphi) &= 1,8946267
 \end{aligned}$$

Dato itaque γ , crepusculum eodem modo quo supra indicavimus habetur.

Schol. 1. In easu quo Sol 18 gradus infra horizontem non percurrit, vel si ipsius locus exacte 18° fuerit, crepusculum per totam noctem durare, perspicuum est. Accidit vero hoc in Latitudine $60^\circ 27' 7''$, ubi declinatio Solis Borealis $11^\circ. 32'. 53''$ excesferit. In tabula igitur apposita, valores γ & φ continente, crepusculum tantummodo pro declinatione Solis Bor. $11^\circ 30'$, quam circa diem 21 mensis Aprilis obtinet, calculavimus. Nam ab eo tempore crescit & decrescit declinatio Solis Borealis successive, ita tamen ut semper sit major $11^\circ. 30'$, donec circa diem 23 mensis Augusti hanc ipsam iterum habet magnitudinem. Diminuitur vero posthaec declinatio Solis Borealis eodem modo, quo antea crevit, quamobrem durationes crepusculorum cædem sunt.

Declina- tio Solis in So lis a ultra.	y	y	y	y	Declina- tio Solis a ultra.	Austr.							
6°	0°	2°	55°	12°	6h. 0°	0°	2h. 35'	12°	12°	12°	12°	12°	12°
0°	30°	6h. 3°	36°	15°	30° 5h. 56°	28° 2h. 34°	15° 2h. 12°	30° 4h. 27°	55° 2h. 30°	12° 4h. 27°	55° 2h. 30°	12° 4h. 27°	55° 2h. 30°
1°	50°	6h. 10°	35°	2h. 32°	50° 5h. 57°	2h. 32°	15°	13°	30° 4h. 19°	46° 2h. 51°	17° 4h. 25°	52° 2h. 50°	45° 4h. 25°
2°	30°	6h. 14°	47°	40°	50° 5h. 49°	25° 2h. 32°	34°	14°	11° 4h. 11°	26° 2h. 31°	17° 4h. 25°	51° 2h. 50°	45° 4h. 25°
3°	50°	6h. 21°	13°	2h. 42°	50° 5h. 42°	20° 2h. 31°	11°	14°	30° 4h. 11°	14° 30°	15° 4h. 25°	51° 2h. 50°	45° 4h. 25°
4°	30°	6h. 24°	20°	2h. 44°	50° 5h. 31°	40° 2h. 29°	57°	16°	11° 4h. 11°	14° 30°	15° 4h. 25°	52° 2h. 50°	45° 4h. 25°
5°	30°	6h. 28°	24°	2h. 46°	50° 5h. 31°	40° 2h. 29°	57°	16°	11° 4h. 11°	14° 30°	15° 4h. 25°	52° 2h. 50°	45° 4h. 25°
6°	30°	6h. 31°	55°	2h. 48°	50° 5h. 28°	55° 2h. 29°	14°	16°	30° 4h. 11°	14° 30°	15° 4h. 25°	59° 2h. 50°	45° 4h. 25°
7°	30°	6h. 35°	53°	2h. 50°	50° 5h. 24°	29° 2h. 28°	52°	17°	11° 4h. 11°	14° 30°	15° 4h. 25°	59° 2h. 50°	45° 4h. 25°
8°	30°	6h. 39°	57°	2h. 53°	50° 5h. 20°	53° 2h. 28°	54°	17°	10° 4h. 11°	14° 30°	15° 4h. 25°	58° 2h. 50°	45° 4h. 25°
9°	30°	6h. 42°	44°	2h. 55°	50° 5h. 17°	17° 2h. 21°	18°	18°	10° 4h. 11°	14° 30°	15° 4h. 25°	58° 2h. 50°	45° 4h. 25°
10°	30°	6h. 46°	23°	2h. 58°	42°	6°	30° 5h. 13°	37°	2h. 98°	12°	18°	30° 5h. 35°	18° 2h. 40°
11°	30°	6h. 50°	2°	5h. 1°	55°	7°	5h. 9°	58°	2h. 98°	5°	19°	30° 5h. 30°	24° 2h. 42°
12°	30°	6h. 55°	43°	5h. 5°	29°	7°	30° 5h. 6°	17°	2h. 97°	58°	19°	30° 5h. 25°	21° 2h. 43°
13°	30°	6h. 57°	25°	5h. 9°	25°	8°	30° 5h. 5°	21°	2h. 97°	58°	19°	30° 5h. 20°	13° 2h. 45°
14°	30°	6h. 1°	5°	5h. 14°	2°	8°	30° 4h. 58°	55°	2h. 28°	0°	20°	30° 5h. 14°	56° 2h. 47°
15°	30°	7h. 4°	54°	5h. 15°	19°	15°	30° 4h. 4h. 47°	31°	2h. 28°	22°	20°	30° 5h. 9°	31° 2h. 49°
16°	30°	7h. 8°	41°	5h. 25°	23°	9°	30° 4h. 4h. 47°	19°	2h. 28°	15°	21°	30° 5h. 5°	57° 2h. 51°
17°	30°	7h. 12°	29°	3h. 32°	48°	10°	4h. 4h. 47°	31°	2h. 28°	25°	20°	30° 5h. 14°	56° 2h. 47°
18°	30°	7h. 16°	20°	5h. 42°	11°	10°	30° 4h. 4h. 47°	40°	2h. 28°	59°	20°	30° 5h. 9°	45° 2h. 45°
19°	30°	7h. 20°	13°	5h. 55°	19°	11°	4h. 4h. 47°	39°	2h. 28°	57°	20°	30° 5h. 5°	58° 16°
20°	30°	7h. 24°	8°	4h. 29°	42°	11°	30° 4h. 35°	52°	2h. 29°	18°	23°	28° 2h. 40°	5° 3h. 0°

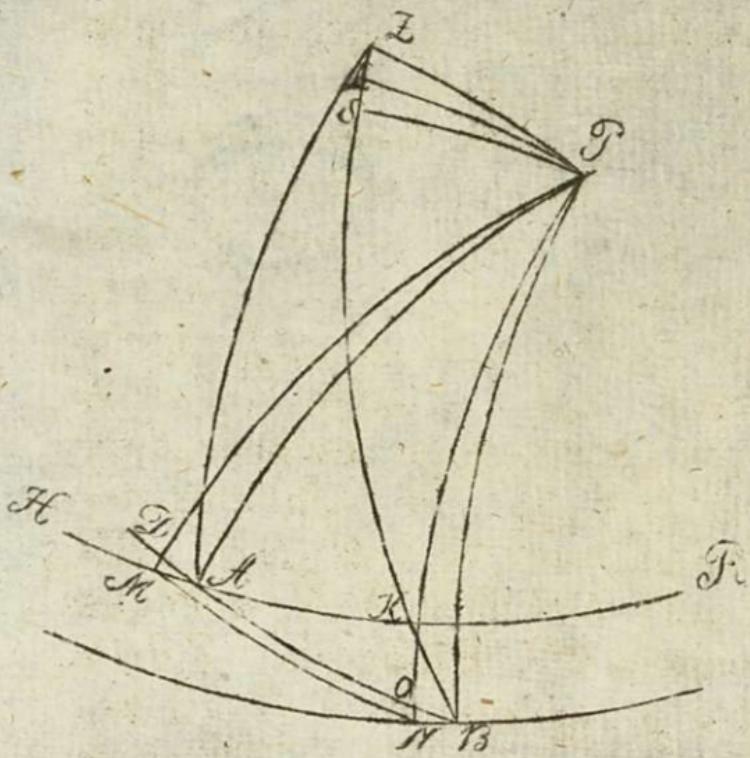
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Schol. 2. Ut vero pateat usus tabulae allatae, quæratur exempli gratia crepusculum vespertinum pro die 9 Mensis Novembris hujus Anni, existente declinatione Solis Australi in momento occasus $16^{\circ} 44' 56''$: In tabula habetur crepusculum pro decl. $16^{\circ} 30'$, $2^{\text{h}} 36' 7''$ & pro decl. $17^{\circ} 2^{\text{h}} 37' 11''$: Inferendo igitur $30'$ (differentia inter declinationes in tabula): $14' 56''$ (differentia inter $16^{\circ} 30'$ & $16^{\circ} 44' 56''$): $3' 4''$ differentia inter crepuscula pro $16^{\circ} 30'$ & 17° : $31'', 9$ (differentia inter crepuscula pro $16^{\circ} 30'$ & $16^{\circ} 44' 56''$), si addatur, quoniam e tabula crepuscula crescere animadvertisimus, $31'', 9$, crepusculo $2^{\text{h}} 36' 7''$ pro $16^{\circ} 30'$, eruitur $2^{\text{h}} 36' 38'' 9$, seu crepusculum vespertinum quæsitus. Si vero decreas fuerit crepusculum, differentia inventa ausepta est.

Schol. 3. Valorem anguli γ , areum semidiurnum exhibentem attulimus, quoniam crepusculum illo incognito investigari nequit, & præterea ortus Solis atque occasus ejus ope facilime determinatur. Si videlicet momento culminationis Solis addatur, exhibet occasum, si autem auferatur, ortum. Necesse tamen est, ut ex Ephemeridibus de promatur declinatio Solis pro occasu vel ortu, & e tabula, eodem plane modo, quo Schol. 2. indicavimus, investigetur angulus γ huic momento respondens, qui denique ope formularum pro invenienda refractione Astronomica configatur.

Quod ad problema de inveniendo crepusculo minimo attinet, sequens illud resolvendi simplicissima nobis videtur methodus. Demonstrandum enim primo est, angulos positionis ZAP & ZBP vel ut a quibusdam nuncupantur parallacticos, in casu, quo crepusculum est minimum, æquales esse, & deinde quærenda est declinatio Solis, existentibus his angularibus æqualibus. Concipiatur igitur arcus MN infinite proximus ipsi AB , atque sit NB portio arcus horizonti parallelī, erit, exsistente $>APB$ minimo, $AB=MN$. Quumque præterea quam proxime æquales sunt arcus $D\sigma$ & MN , atque MN ipsi AB parallelus, sequitur, ut assumi possit $AD=oB$ & $AM=NB$. Äæqualia igitur sunt triangula DMA & BON , adeoque etiam angulus $DAM=$ ang. oBN . Est autem $>ZAH=90^\circ=>PAD$, unde, sublato communi $>ZAD$, habebitur $>PAZ=>DAM$. Pariter angulus $ZBN=90^\circ=>PBA$, quamobrem, si communis $>ZBA$ auferatur, erit $>PBZ=>oBN$, atque hinc sequitur, ut sit $>PAZ=>PBZ$, in casu quo $>APB$ minimus est. Sumatur præterea $SB=90^\circ$, erit $ZS=BK=18^\circ$, atque $\Delta AZP=\Delta PSB$, unde $ZP=PS$ & demisso arcu PL normali ipsi ZB , habebitur $ZL=SL=9^\circ$. In ΔPBL rectangulo erit $Cof PB=Cof LB$ $Cof LP$ & in ΔLPS etiam rectangulo $Cof PS=Cof LP$ $Cof LS$, unde eruitur
 Cof



$$\frac{\cos BP}{\cos LB} = \frac{\cos SP}{\cos LS}, \text{ atque hinc } \cos BP = \frac{\cos SP \cos LB}{\cos SL} = \\ \frac{\cos ZP \sin LS}{\cos LS} = \cos ZP \operatorname{Tg} LS. \text{ Patet autem valorem}$$

ipius BP negativum esse debere, quoniam arcus $LB > 90^\circ$, unde sequitur, ut declinatio Solis Australis sit, ubi crepusculum minimum fuerit. Pro Latitudine igitur $60^\circ. 27. 7''$ habebitur ope formulæ allatæ declinatio Solis Australis in easu crepusculi minimi $7^\circ. 55'. 11''$, unde secundum formulam §. 3. eruitur crepusculum minimum $24^\circ 27'. 57''$, quod etiam a tabula supra allata deducere possumus. Ex Ephemeridibus vero Solem hanc ipsam declinationem habere constat circa diem I. mensis Martii & diem I. mensis Octotris, ita ut quovis anno bis crepusculum sit minimum.

