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DISSERTATIO ASTRONOMICA,

DE

PARALLAXI ANNUA PLANETARUM PRIMARIORUM AC COMETARUM,

QUAM,

CONS. AMPL. FAC. PHILOS. ABOËNS.

PRÆSIDE

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PRO GRADU

PUBLICE EXAMINANDAM SISTIT

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OSTROBOTNIENSIS,

IN *AUDITORIO MAJORI*

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HORIS ANTE MERIDIEM SOLITIS.

ABOÆ, TYPIS FRENCKELLIANIS.

§. I.

Ejusdem objecti, pro diversitate stationum, ex quibus conspicitur, diversum fore locum opticum, res est notissima. Hæc locorum opticorum differentia *Parallaxin*, latissimo sensu sumtam, constituit. Quum vero diversæ istæ stationes ad certum aliquod punctum, tanquam centrum, referri communiter soleant, Parallaxeos nomine speciatim insignitur angulus, directis ex hoc centro & ex quavis spectatoris statione versus idem objectum lineis interceptus. Doctrinæ Parallaxium in nulla scientia amplior quam in Astronomia est usus. Hac etenim unica scala inaccessibiles alias siderum regiones quasi ascendere, verasque stellarum & distantias & magnitudines mensurare licet. Duplicis vero generis præcipue in Astronomicis considerandæ occurrunt Parallaxes, aliæ diurnæ, aliæ annuæ. Illæ in diversitate consistunt locorum, quibus idem corpus cœleste ex centro terræ & hujus superficie eodem tempore apparet. Ejusmodi Parallaxis in Luna maxime sensibilis est, quippe integrum haud raro gradum excedens; in sole vero & Planetis reliquis admodum exigua, paucisque scrupulis secundis finita. Parallaxis autem annua vel orbitæ sideris cujusvis est distantia locorum opticorum

ejus-

ejusdem, ex terra & ſole ſeu ſyſtematis noſtri planetarii centro ſpectati, vel brevius: differentia inter locum ejus geocentricum atque heliocentricum. Hujus conſideratio in Planetarum inprimis primariorum ac Cometarum Theoria maximi eſt momenti, quippe cujus ope apparentes horum corporum irregularitates ex annuo telluris circa Solem motu ortæ explicantur, eorumque phænomena calculo facile ſubjiciuntur. Duplex præcipue in hoc capite Aſtronomiæ occurrit problema ſolvendum, unum ſcilicet, quo ex dato loco Planetæ vel Cometæ geocentrico quæritur locus ejus heliocentricus, alterum, quo viciſſim dato loco heliocentrico reſpondens inveſtigatur geocentricus, cognitis in utroque caſu poſitione Nodi & inclinatione orbitæ nec non longitudine Solis, hujusque a tellure diſtantia. Horum problematum ut frequens eſt in Aſtronomia uſus, ita varias eorundem ſolutiones apud Aſtronomos paſſim videre licet, quas vero quum ſigillatim recenſere valde prolixum foret, aptius judicavimus novam eadem ſolvendi methodum adferre, quam in prælectionibus Aſtronomicis propoſuerat Cel. Præſes. Quæ videlicet quum concinna admodum nobis videretur atque directa, eandem Speciminis Academici loco jam paucis exponere, exemplisque nonnullis illuſtrare decrevimus.

§. II.

In Sphæra cæleſti ſit ♈ ♌ *B* Ecliptica, in qua ♈ initium Arietis ſeu punctum, a quo verſus *B* nume-

 ren-

rentur longitudines, referatque circulus ejusdem sphæræ maximus ☊ AC planum orbitæ Planetæ vel Cometæ (*), cujus nodus ascendens sit ☊, adeoque ♈☊ longitudo nodi, quam in sequentibus nominemus N, nec non angulus A☊B orbitæ inclinatio, quæ dicatur i. Si jam existente Sole in ☉, adeoque ejus longitudine = ♈☉, quam dicamus S, fuerit P locus Planetæ ex centro Terræ spectati; descripto per ☉ & P, circulo maximo $\Theta PP'$, circulum ☊AP' secante in P', erit hoc punctum P' locus Planetæ heliocentricus ad idem tempus. Patet videlicet planum circuli $\Theta PP'$ per centra Solis, Terræ & Planetæ transire, adeoque locum hujus utrumque geocentricum atque heliocentricum, in eodem hoc circulo positum esse. Locus vero heliocentricus semper manet in plano orbitæ. Ergo hic locus erit in communi intersectione horum planorum, adeoque in puncto P'. Vicissim hinc sequitur, ductum ex loco planetæ heliocentrico P' per locum Solis Θ circulum maximum transire per locum ejusdem geocentricum. Porro manifestum est in Triangulo a rectis, centra Solis, Terræ & Planetæ conjungentibus comprehenso, angulos ad centra Terræ & Planetæ constitutos mensurari arcubus ΘP & PP' respective, adeoque Sinus horum arcuum esse

(*) Ubi in sequentibus brevitatis studio Planetas solum nominamus, Cometas simul subintelligimus; generatim scilicet agimus de corporibus, quorum centrum motus est Sol.

esse in ratione laterum iisdem angulis oppositorum, hoc est: ut distantia Planetæ a Sole ad distantiam inter Solem & terram. Si igitur hæ distantiæ dicantur r & c respective: erit $r : c :: Sin\ \Theta P : Sin\ PP'$. Si denique ex P agatur arcus PL perpendicularis ad eclipticam, erit ΥL longitudo Planetæ geocentrica, quam dicamus L, & PL ejusdem latitudo geocentrica, quae nominetur λ. His præmissis jam patet, utrumque nostrum problema (§. I.) reductum esse ad resolutionem triangulorum sphæricorum ΘPL & $\Omega P'\Theta$. Primo videlicet, quando loco geocentrico respondens quæritur heliocentricus, data supponuntur L, λ, S, c, N & i, atque investigandus est arcus $\Omega P'$ (qui *Argumentum Latitudinis* nominari solet), nec non distantia Planetæ a Sole r. Hoc in casu in Triangulo rectangulo $PL\Theta$ ex datis lateribus $PL = \lambda$ & $L\Theta = L - S$, inveniuntur hypothenusa $P\Theta$ atque angulus $P\Theta L$. Porro in $\Delta P'\Omega\Theta$ ex invento hoc ang. $P\Theta L$ atque dato ang. $P'\Omega\Theta = i$, nec non his intercepto latere $\Theta\Omega = S - N$, investigantur reliqua latera $\Theta P'$ & argumentum latitudinis $\Omega P'$. Cognitis itaque $P\Theta$ & $P'\Theta$, adeoque etiam horum differentia PP'; ope analogiæ superius allatæ $r : c :: Sin\ \Theta P : Sin\ PP'$ innotescit r. Si vero loco ipsius $\Omega P'$ desideretur Planetæ anomalia vera, quæ nominetur z, data præterea erit Aphelii A a nodo distantia ΩA, quæ si dicatur a, erit $z = \Omega P' - a$.

Pari ratione in problemate inverso ex datis S, c,

N, i,

N, i, r & argumento latitudinis ☊P' investigantur L & λ seu longitudo atque latitudo Planetæ geocentrica. Scilicet in Triangulo ☊$P'\Theta$ ex datis lateribus ☊P' nec non Θ☊ $= S - N$, atque angulo intercepto P'☊$\Theta = i$, quæritur latus tertium $P'\Theta$ atque ang. $P\Theta L$. Dato vero latere $P'\Theta$, eruuntur $P'P$ & $P\Theta$ ope analogiæ nuper memoratæ. Hac namque componendo atque dividendo sumta, obtinetur: $r + c : r - c :: Sin\, \Theta P + Sin\, PP' : Sin\, \Theta P - Sin\, PP' :: Tang\, \frac{1}{2}\, \Theta P : Tg\, \frac{1}{2}\, (\Theta P - PP')$, unde innotescit arcuum horum semidifferentia $\frac{1}{2}\, (\Theta P - PP')$, qua ipsi $\frac{1}{2}\, \Theta P'$ addita invenitur ΘP. Ex hac vero hypothenusa ΘP & ang. $P\Theta L$ in Triangulo rectangulo $P\Theta L$ inveniuntur latera ΘL & LP adeoque longitudo atque latitudo planetæ geocentrica quæsita.

Hæ solutiones utut admodum simplices videntur, in applicatione tamen calculum exigunt justo prolixiorem. Quamobrem jam dispiciendum erit, annon ex iisdem principiis pro utroque casu concinniores erui queant regulæ; qua in re ut distinctius agatur, utrumque Problema sigillatim tractemus.

§. III.

PROBLEMA. *Datis ad tempus quodvis Planetæ cujusdam vel Cometæ geocentrica longitudine* ☊$L = L$ *atque latitudine* $PL = \lambda$, *nec non longitudine Solis* ♈$\Theta = S$ *hujusque a terra distantia* $= c$; *cognitisque præterea longitudine Nodi* ♈☊ $= N$, *inclinatione orbitæ* A☊B

$= i$

$= i$ *atque Aphelii* A *a nodo distantia* $A\Omega = a$: *invenire ad idem tempus locum ejusdem heliocentricum* P', *scilicet Anomaliam veram* $AP = z$, *atque distantiam a Sole* $= r$.

Demisso ex P' arcu circuli maximi ad Eclipticam perpendiculari $P'L'$, est $\Upsilon L'$ longitudo atque $P'L'$ latitudo planetæ heliocentrica, quæ dicantur H & h respective. Quum jam in Triangulis rectangulis $P'\Theta L'$ & $P\Theta L$ ob angulum ad Θ communem, sit $Sin\ L'\Theta : Sin\ L\Theta :: Tg\ P'L' : Tg\ PL$, seu $Sin\ (H - S) = \frac{Sin\ (L - S)\ Tg\ h}{Tg\ \lambda} = Sin\ (L - S)\ Cotg\ \lambda\ Tg\ h$, (posito Sinu toto $= 1$); atque in Triangulo rectangulo $P'\Omega L'$ sit $Tg\ h = Tg\ i\ Sin\ (H - N)$: sequitur fore $Sin\ (H - N) : Sin\ (H - S) :: 1 : Tg\ i\ Cotg\ \lambda\ Sin\ (L - S)$. Hinc ut facilius inveniatur longitudo heliocentrica H, quærendus primo est angulus ω talis, ut sit $Tg\ \omega = Tg\ i\ Cotg\ \lambda\ Sin\ (L - S)$ (I). Quo facto habetur $Sin\ (H - N) : Sin\ (H - S) :: 1 : Tg\ \omega$ adeoque (comp. & div.) $Sin\ (H - N) + Sin\ (H - S) : Sin\ (H - N) - Sin\ (H - S) :: 1 + tg\ \omega : 1 - tg\ \omega$, seu $Tg\ (H - \frac{1}{2}.\overline{S + N}) : Tg\ \frac{1}{2}\ (S - N) :: Tg\ (45^\circ + \omega) : 1$. Quamobrem erit $Tg\ (H - \frac{1}{2}.\overline{S + N}) = Tg\ \frac{1}{2}\ (S - N)\ Tg\ (45^\circ + \omega)$ (II). Porro ope longitudinis heliocetricæ H invenitur argumentum latitudinis $\Omega P = a + z$; est namque $Cotg\ (a + z) = Cos\ i\ Cotg\ (H - N)$ (III), unde ob datum a, anomalia vera z simul innotescit. Ad inveniendam denique planetæ a Sole distantiam r, producantur arcus PL & $P'L'$, donec sibi mutuo occurrant in E atque jungantur E, $\odot$. Hoc facto, quum ob angulos ad L, L' rectos, sit E polus eclipticæ, adeoque $L'E$, LE, $\odot E$ quadrantes circulorum maximorum: erunt arcus $\odot L$, LL' respective mensuræ angulorum $\odot EP$, PEP', atque EP' complementum ipsius $P'L'$ seu latitudinis heliocentricæ h, nec non $Sin\, E\odot = 1$. In triangulis vero $\odot EP$, PEP' est $Sin\, \odot P.\, Sin\, EPP' = Sin\, E\odot.\, Sin\, \odot EP = Sin\,(L - S)$, & $Sin\, PP'.\, Sin\, EPP' = Sin\, PEP'.\, Sin\, EP' = Sin\,(H - L)\, Cos\, h$, adeoque $Sin\, \odot P : Sin\, PP' :: r : c$ (§. II) $:: Sin\,(L - S) : Sin\,(H - L)\, Cos\, h$. Sed ob $\Delta\, \Omega P'L'$ rectangulum est $Cos\, h = \frac{Cos\,(a + z)}{Cos\,(H - N)}$

Ergo $r = \frac{c\, Sin\,(L - S)\, Cos\,(H - N)}{Sin\,(H - L)\, Cos\,(a + z)}$ (IV). Solutio igitur præsentis problematis sequentibus absolvitur formulis:

I. $Tang\, \omega = Tg\, i\, Cotg\, \lambda\, Sin\,(L - S)$.

II. $Tg\,(H - \tfrac{1}{2}.\overline{S + N}) = Tg\, \tfrac{1}{2}\,(S - N)\, Tg\,(45^\circ + \omega)$.

III. $Cotg\,(a + z) = Cos\, i\, Cotg\,(H - N)$.

IV. $r = \frac{c\, Sin\,(L - S)\, Cos\,(H - N)}{Sin\,(H - L)\, Cos\,(a + z)}$.

Scholion I. In quibus quadrantibus suorum circulorum posita sint puncta L' & P' pro diverso situ ipsorum $\odot$ & P, ex signis, quæ in formulis allatis pro casu quovis speciali locum obtinent, ut & ex inspectione ipsius figuræ facilius dignoscitur, quam regulis quibusbusdam prolixioribus, quas igitur adferre supervacaneum censemus.

Scholion II. Si desideretur locus heliocentricus planetæ ad eclipticam reductus, mox ex formula II. colligitur longitudo heliocentrica H. Latitudo vero heliocentrica h computari potest secundum alterutram harum formularum: $Tg\,h = \frac{Tg\,\lambda\,(Sin\,H - S)}{Sin\,(L - S)}$, vel $Cos\,h = \frac{Cos\,(a + z)}{Cos\,(H - N)}$, vel $Sin\,h = Sin\,i\;Sin\,(a + z)$.

Scholion III. Si ex iis, quæ in hoc Problemate data supponuntur, invenire simul oporteat planetæ a terra veram distantiam, quæ dicatur t: eodem ac in §. II. ratiocinio demonstrabitur, fore $t : c :: Sin\,\Theta P' : Sin\,PP'$. Sed ex comparatione Triangulorum $\Theta EP'$, PEP' deducitur: $Sin\,\Theta P' : Sin\,PP' :: Sin\,\Theta EP' : Sin\,PEP'$. $Sin\,EP :: Sin\,(H - S) : Sin\,(H - L)\,Cos\,\lambda$; adeoque $t = \frac{c\,Sin\,(H - S)}{Cos\,\lambda\,Sin\,(H - L)}$.

Scholion IV. In casu conjunctionis vel oppositionis, seu existente $L = S$ vel $L = 180° + S$, quum simul sit $H = S$ vel $H = 180° + S$; patet fore I:o $Cotg\,(a + z) = \pm\,Cos\,i\;Cotg\,(S - N)$, II:o $Tg\,h = Tg\,i\;Sin\,(S - N)$, III:o $r = \frac{c\,Sin\,\lambda}{Sin\,(h - \lambda)}$, nec non IV:o $t = \frac{c\,Sin\,h}{Sin\,(h - \lambda)}$.

Exempl. Cometæ, qui Anno 1770 apparuit, secundum observationes D:ni MESSIER, mens. Junii ejusdem

Anni $29^d\ 11^h\ 59'\ 26''$ Temp. med. ad merid. Parif. fuit longitudo geocentrica $L = 9^s\ 9^\circ\ 42'\ 45''$ atque latitudo geocentrica $\lambda = 37^\circ\ 57'\ 32''$ Bor. cfr. *Mem. de Acad. Roï. des Sc. de Paris A. 1776 p. 639, 640.* Ifto tempore fuit longitudo Solis $S = 3^s\ 8^\circ\ 6'\ 25''$ atque diftantia $c = 1{,}01677$, pofità diftantia telluris a Sole media $= 1$. Affumtis jam ejusdem Cometæ elementis loco cit. allatis, fcil. $N = 4^s\ 12^\circ$, $i = 1^\circ\ 33'\ 40''$ atque $a = 44^\circ\ 17'\ 3''$, anomalia ejus vera atque diftantia a Sole fequente ratione inveftigantur:

$i = 1^\circ\ 33'\ 40''$	$Log\ Tg\ i = \bar{2}.4354187$
$\lambda = 37^\circ\ 57'\ 32''$	$L\ Cotg\ \lambda = 0.1078326$
$L = 9^s\ 9^\circ\ 42'\ 45''$	$L\ Sin\ (L-S) = \bar{2}.4474459$
$S = 3^s\ 8^\circ\ 6'\ 25''$	$L\ Tg\ \omega = \bar{4}.9906972$
$L-S = 6^s\ 1^\circ\ 36'\ 20''$	$L\ Tg\ (45^\circ + \omega) = \bar{1}.9991494$
$\omega = -\ 3'\ 22''$	$L\ Tg\ \frac{1}{2}(S-N) = \bar{1}.4838874$
$45^\circ + \omega = 44^\circ\ 56'\ 38''$	$L\ Tg\ (H - \frac{1}{2}(S+N)) = \bar{1}.4830368$
$\frac{1}{2} S = 1^s\ 19^\circ\ 3'\ 12'',5$	$L\ Cos\ i = 1.9998388$
$\frac{1}{2} N = 2^s\ 6^\circ$	$L\ Cotg\ (H-N) = 0.1732752$
$\frac{1}{2}(S-N) = -\ 16^\circ\ 56'\ 47'',5$	$L\ Cotg\ (a+z) = 0.1731140$
$\frac{1}{2}(S+N) = 3^s\ 25^\circ\ 3'\ 12'',5$	$L\ c = 0.0072220$
$H - \frac{1}{2}(S+N) = 5^s\ 13^\circ\ 5'\ 5'',2$	$L\ Sin\ (L-S) = \bar{2}.4474459$
$H = 9^s\ 8^\circ\ 8'\ 17'',7$	$L\ Cos\ (H-N) = \bar{1}.9192792$
$N = 4^s\ 12^\circ$	$-\ L\ Sin\ (H-L) = 1.5611060$
$H-N = 4^s\ 26^\circ\ 8'\ 17'',7$	$-\ L\ Cos\ (a+z) = 0.0807708$
$H-L = -\ 1^\circ\ 34'\ 27'',3$	$L\ r = 0.0158239.$
$H-S = 6^s\ 0^\circ\ 1'\ 52'',7$	

$a + z = 146^\circ\, 7'\, 42'',3$	$L\, c = 0.0072220$
$a = 44^\circ\, 17'\, 3''$	$L\, Sin\, (H - S) = 4.7374952$
$z = 101^\circ\, 50'\, 39'',3$	$-L\, Cos\, \lambda = 0.1032245$
$r = 1.03710$	$-L\, Sin\, (H - L) = 1.5611060$
$t = 0.02565$	$Log\, t = \bar{2}.4090477$

Quum inventa cometæ a tellure distantia t admodum sit exigua, quippe ipsius c partem quadragesimam vix excedens, manifestum est deductas ex immediatis observationibus longitudinem atque latitudinem non esse vere geocentricas, sed notabili quadam parallaxi diurna affectas; quapropter in hoc & similibus casibus correctione quadam opus est. Hæc vero ita institui potest, ut ex inventa per hanc primam approximationem distantia t investigetur effectus parallaxis diurnæ & hinc verus locus geocentricus; quo facto repetito calculo secundum formulas superiores exactus obtinetur locus heliocentricus.

§. IV.

Problema II. *Invenire ad tempus quodvis geocentricam Planetæ cujusdam vel Cometæ longitudinem* $\Upsilon L = L$, *atque latitudinem itidem geocentricam* $PL = \lambda$: *datis pro eodem tempore distantia ejus a Sole* $= r$ *atque anomalia vera* $AP' = z$, *nec non longitudine Solis* $\Upsilon \Theta = S$ *hujusque a Tellure distantia* $= c$, *cognitisque præterea longitudine Nodi* $\Upsilon \Omega = N$, *inclinatione orbitæ* $A \Omega B = i$ *atque Aphelii A a nodo distantia* $A \Omega = a$.

Data anomalia vera $AP' = z$ atque distantia Aphelii a nodo $A☊ = a$, datur argumentum latitudinis $P'☊ = a + z$. Hinc quum in $\Delta\, P'☊L'$ rectangulo ad L' sit $Tg\, ☊L' = Cos\, P'☊L'\, Tg\, P'\Omega$ seu $Tg\,(H - N) = Cos\, i\, Tg\,(a + z)$ (*I.*) innotescit longitudo heliocentrica H. Posito jam $\frac{c\, Cos\,(H - N)}{r\, Cos\,(a + z)} = Tg\,\varphi$, (*II*), erit (§. III. *form. IV*) $Sin\,(L - S) : Sin\,(H - L) :: 1 : Tg\,\varphi$, adeoque (comp. & div.) $Sin\,(L - S) + Sin\,(H - L) : Sin\,(L - S) - Sin\,(H - L) :: 1 + Tg\,\varphi : 1 - Tg\,\varphi$, seu $Tg\,\tfrac{1}{2}\,(H - S) : Tg\,(L - \tfrac{1}{2}.\overline{H + S}) :: 1 : Tg\,(45^\circ - \varphi)$. Unde invenitur L per formulam: $Tg\,(L - \tfrac{1}{2}\overline{H + S}) = Tg\,\tfrac{1}{2}\,(H - S)\, Tg\,(45^\circ - \varphi)$ (*III*). Porro quum ob ang. $P\Theta L$ communem in $\Delta\Delta\, P\Theta L,\, P'\Theta L'$ sit $Tg\, h = \frac{Tg\,\lambda\, Sin\,(H - S)}{Sin\,(L - S)}$, atque $\Delta\, P'\Omega L'$ præbeat $Tg\, h = Tg\, i\, Sin\,(H - N)$; his ipsius $Tg\, h$ valoribus æquatis obtinetur $Tg\,\lambda = \frac{Tg\, i\, Sin\,(H - N)\, Sin\,(L - S)}{Sin\,(H - S)}$ (*IV*), unde datur λ. Problema igitur nostrum ope quatuor harum formularum solvitur:

$$I.\ Tg\,(H - N) = Cos\, i\, Tg\,(a + z).$$

$$II.\ Tg\,\varphi = \frac{c\, Cos\,(H - N)}{r\, Cos\,(a + z)}.$$

$$III.\ Tg\,(L - \tfrac{1}{2}.\overline{H + S}) = Tg\,\tfrac{1}{2}(H - S)\, Tg\,(45^\circ - \varphi).$$

$$IV.\ Tg\,\lambda = \frac{Tg\, i\, Sin\,(H - N)\, Sin\,(L - S)}{Sin\,(H - S)}.$$

Schol. I. Quæ in *Schol. I. Probl. præc.* de situ punctorum L' & P' monuimus, hic pariter de ipsis L & P observanda sunt.

Schol.

Schol. II. Inventis H, L & λ, computari potest distantia Planetæ a terra per formulam *Schol. III. Probl. præc.* allatam.

Schol. III. Si loco ipsorum a & z data fuerint H & h, mox invenietur $Tg\,\varphi = \frac{c}{r\,Cos\,h}$, unde porro per form. III investigatur longitudo geocentrica. Latitudo vero geocentrica hoc in casu facilius computatur per Formulam: $Tg\,\lambda = \frac{Tg\,h\,Sin\,L - S}{Sin\,(H-S)}$.

Exempl. Secundum Tabulas Astronomicas a R. Acad. Scient. Berolin. 1776 editas, fuit A. 1786 m. Maji $3^d\,19^h$ Temp. med. ad Merid. Berolin. Planetæ Mercurii anomalia vera $z = 10^s\,29^\circ\,53'\,27''$ ejusque distantia a Sole $r = 0.45102$, Solis vero longitudo $S = 1^s\,13^\circ\,52'\,31''$ atq; hujus distantia a terra $c = 1.00934$. adeoque $Log.\,\frac{c}{r} = 0.3498445$. Datis præterea $a = 6^s\,27^\circ\,59'\,46''$, $N = 1^s\,15^\circ\,59'\,16''$ nec non $i = 7^\circ$; quæritur ad idem tempus hujus planetæ locus geocentricus.

$a =$	$6^s\,27^\circ\,59'\,46''$	$Log\,Tg\,(a+z) =$	$\bar{2}.5669851.$
$z =$	10. 29. 53. 27.	$L\,Cos\,i$	$\bar{1}.9967507.$
$a+z =$	5. 27. 53. 13.	$L\,Tg\,(H-N) =$	$\bar{2}.5637358.$
$i =$	7. 0. 0.	$L\,Cos\,(H-N) =$	$\bar{1}.9997090.$
$H-N =$	5. 27. 54. 9. 7.	$L\,\frac{c}{r}$	0.3498445.
$N =$	1. 15. 59. 16.		
$H =$	7. 13. 53. 25. 7.	$-\,L\,Cos\,(a+z) =$	0.0002951.
$\varphi =$	65. 55. 23.	$L\,Tg\,\varphi$	0.3498486.

$45^\circ - \varphi = -20^\circ\, 55'\, 23''$	$L\,Tg\,(45 - \varphi) = \bar{1}.5824315.$
$\frac{1}{2} H = 3.\,21.\,56.\,42,8.$	$L\,Tg\,\frac{1}{2}(H - S) = 3.8775334.$
$\frac{1}{2} S = 0.\,21.\,56.\,15,5.$	$L\,Tg\,(L - \frac{1}{2}(H + S)) = 3.4599649.$
$\frac{1}{2}(H - S) = 3.\,0.\,0.\,27.\,3.$	$L\,Tg\,i = \bar{1}.0891438.$
$\frac{1}{2}(H + S) = 4.\,13.\,52.\,58.\,3.$	$L\,Sin\,(H - N) = \bar{2}.5634448.$
$L - \frac{1}{2}(H + S) = 8.\,29.\,58.\,48.\,3.$	$L\,Sin\,(L - S) = 4.3324685.$
$L = 1.\,13.\,51.\,46,6.$	$-\,L\,Sin\,(H - S) = 3.5764378.$
$S = 1.\,13.\,52.\,31.$	$L\,Tg\,\lambda = \bar{3}.5614949.$
$L - S = -\,-\,44,4.$	
$H - S = 6.\,0.\,0.\,54,7.$	
$\lambda = 12.\,31,\,5.$	

Ex iſtis igitur datis obtinetur ad dictum tempus Mercurii geocentrica longitudo $= 1^s\, 13^\circ\, 51'\, 46'', 6$ atque latitudo borealis $= 12'\, 31'', 5$.

§. V.

Generaliter quidem per *Form. III.* §. III. ex cognito angulo ſic dicto Commutationis $H - N$ invenitur Argumentum latitudinis $a + z$, pariter ac viciſſim ex hoc dato ope *Form. I.* §. IV. ille innoteſcit. Quando vero, ſicut in planetis primariis ac quibusdam cometis, admodum exigua eſt orbitæ inclinatio i, commodius erit, immo (cum adhibitis vulgaribus Tabulis Logarithmicis calculus ſubducitur) exactius, ope ſerierum approximando reductionem hanc inſtituere. Ejusmodi ſeries eruuntur facile ex formulis, quas exhibuit Dn. De la Gran-

GRANGE (Recueil de Tables Aſtron. Berl. 1776. Vol. III p. 226. 227). Poſito ſcil. $Tg\,\frac{1}{2}\,i = p$, ex his obtinetur: $H - N = a + z - p^2\, Sin\, 2\,(a + z) + \frac{1}{2}\, p^4\, Sin\, 4\,(a + z) - \frac{1}{3}\, p^6\, Sin\, 6\,(a + z) + \frac{1}{4}\, p^8\, Sin\, 8\,(a + z) -$ &c. (I.)

Unde porro per reverſionem ſequitur fore:

$a + z = H - N + p^2\, Sin\, 2\,(H - N) + \frac{1}{2}\, p^4\, Sin\, 4\,(H - N) + \frac{1}{3}\, p^6\, Sin\, 6\,(H - N) + \frac{1}{4}\, p^8\, Sin\, 8\,(H - N) +$ &c. (II.)

Ope harum ſerierum differentia inter $a + z$ & $H - N$ obtinetur in partibus radii; quæ vero ut minutis ſecundis exprimatur, ulterius multiplicetur per $206264{,}8 = R$, numerum videlicet ſcrupulorum ſecundorum, quos continet arcus radio æqualis, cujus numeri Logarithmus eſt $= 5{,}3144251$. Sic ut in exemplo §. III. ex cognito $H - N = 4^s\, 26^\circ\, 8'\, 17''{,}7$ inveniatur $a + z$, calculus ita inſtituitur:

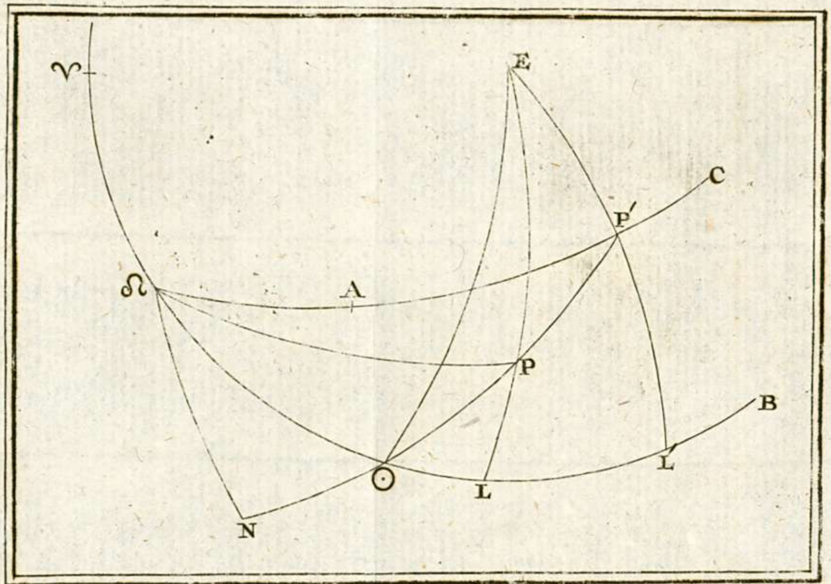
$\frac{1}{2}\, i = 46'\, 50''$	$L\, R$ ——— 5. 3144251
$2\,(H - N) = 9^s\, 22^\circ\, 16'\, 35''$	$2\, L\, p$ ——— 4. 2686162
$p^2\, Sin\, 2\,(H - N) = -\, 35''{,}43$	$L\, Sin\, 2\,(H - N) = 1.9663135$
	$L\, p^2\, Sin\, 2\,(H\text{-}N) = 1.5493548$

Eodem modo terminus ſequens $\frac{1}{2}\, p^4\, Sin\, 4\,(H - N)$ invenietur $= -\, 0''{,}0025$, qui vero ut & reliqui omnes ob ſuam parvitatem negligi tuto poſſunt; quamobrem in hoc exemplo erit $a + z = H - N - 35'{,}43 = 4^s\, 26^\circ\, 7'\, 42''{,}27$.

§. VI.

Ex iis, quæ (§. II.) attulimus, principiis alia adhuc utriusque noſtri problematis ſolutio deduci poteſt. Ducto nempe per ☊, P arcu circuli maximi ☊P & ex ☊ demiſſo arcu ☊N perpendiculari ad P'⊙, quærantur primum anguli N☊⊙, ⊙☊P & P☊P'. In caſu quidem priori, ſive dato loco geocentrico, datur in △ P⊙L utraque cathetus, unde invenitur $Cotg\, P$⊙$L = Sin\,(L - S)\, Cotg\, \lambda$. Eſt autem in △ ☊⊙$N$, $Cotg$ ☊⊙$N = Cotg\, P$⊙$L = Tg$ ⊙☊$N\, Cos\,(S - N)$. Ergo erit Tg ⊙☊N ———

Sin



C.L.S fc:

$\frac{Sin(L-S)\,Cotg\,\lambda}{Cos(S-N)}$. Cognito hinc ang. $\odot\Omega N$, datoque præterea ang $P'\Omega\odot = i$, datur etiam ang. $P'\Omega N$. Quamobrem quum sit $Cos\,\odot\Omega N : Cos\,P'\Omega N :: Cotg\,\odot\Omega : Cotg\,\Omega P'$, dabitur quoque argumentum latitudinis $a + z$ & hinc anomalia vera z. Ad inveniendam vero planetæ a Sole distantiam r, ex datis in $\Delta\, P\Omega L$ cathetis ΩL & PL investigetur ang. $P\Omega L$: est namque $Cotg\,P\Omega L = Sin(L-N)\,Cotg\,\lambda$, unde simul obtinetur ang. $P'\Omega P$; quo facto quum in $\Delta\Delta\,\Omega P\odot$, $\Omega PP'$ sit $Sin\,P\odot : Sin\,PP' :: Sin\,\odot\Omega.\,Sin\,P\Omega\odot : Sin\,\Omega P'.\,Sin\,P'\Omega P ::$ (§. II.) $r : c$, invenitur $r = \frac{c\,Sin(S-N)\,Sin\,P\Omega\odot}{Sin(a+z)\,Sin\,P'\Omega P}$.

In problemate inverso, quo ex dato loco heliocentrico quæritur geocentricus, fiat primo $\frac{c\,Sin(S-N)}{r\,Sin(a+z)} = Tg\,\psi$. Hinc erit (*dem.*) $Sin\,P\Omega\odot : Sin\,P'\Omega P :: 1 : Tg\,\psi$ adeoque (comp. & div.) $Tg\,\tfrac{1}{2}i : Tg\,\tfrac{1}{2}(P\Omega\odot - P'\Omega P) :: 1 : tg(45° - \psi)$, seu $Tg\,\tfrac{1}{2}(P\Omega\odot - P'\Omega P) = Tg\,\tfrac{1}{2}i\,Tg(45° - \psi) = Tg\,\xi$. Datur igitur ang. $P\Omega\odot = \tfrac{1}{2}i + \xi$ & $P'\Omega P = \tfrac{1}{2}i - \xi$. Porro ob $Cos\,N\Omega\odot : Cos\,N\Omega P' :: Cotg(S-N) : Cotg(a+z)$, est (comp. & div.) $Cotg(N\Omega\odot + \tfrac{1}{2}i) : Tg\,\tfrac{1}{2}i :: Sin(a+z+S-N) : Sin(a+z-S+N)$, unde innotescit ang $N\Omega\odot = \eta$. Cognitis vero his angulis, quum in $\Delta\Delta\,P\Omega N$, $\odot\Omega N$ sit $Cos\,\eta : Cos(\eta + \xi + \tfrac{1}{2}i) :: Cotg(S-N) : Cotg\,\Omega P$, nec non in $\Delta\,\Omega PL$, $Cotg\,\Omega P = Cos(\xi + \tfrac{1}{2}i)\,Cotg(L-N)$, invenitur $Tg(L-N) = \frac{Tg(S-N)\,Cos(\xi + \tfrac{1}{2}i)\,Cos\,\eta}{Cos(\eta + \xi + \tfrac{1}{2}i)}$, atque $Tg\,\lambda = Sin(\lambda - N)\,Tg(\xi + \tfrac{1}{2}i)$. Hæ autem formulæ pro utroque problemate solvendo prioribus (§ §. III. IV.) minus sunt concinnæ, nec in orbitis, quarum minor est inclinatio, idem calculi compendium (§. V.) admittunt; quamobrem his ulterius illustrandis supersedemus.