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Dissertatio Academica
De
Invenienda

Longitudine Loci

Ex observata Distantia Lunæ a Stella quadam,

Cujus
Particulam Posteriorem
Cons. Ampl. Fac. Philos. Aboëns.
Præside
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Pro Gradu
Publicæ censuræ submittit
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In Auditorio Majori die XXIX Aprilis MDCCXCV,
H. a. m. c.

ABOÆ,
TYPIS FRENCKELLIANIS.

§. VII.

Singulæ, quas (§. §. V. VI.) exhibuimus, methodi ex data distantia apparente Lunæ a stella quadam inveniendi veram, geometrice quidem spectatæ exactæ omnino sunt; in applicatione vero logarithmorum ope instituenda aliquo plerumque incommodo laborant. Arcus videlicet vel anguli in his calculis occurrentes quum rarissime in numeris (ut ajunt) rotundis expressi habeantur, necesse est ut logarithmi plurimi ex Tabulis Trigonometricis per partes proportionales investigandi sint, quæ quidem interpolatio in his methodis eam ob caussam exactior requiritur, quod ut ex ipsis formulis allatis patet correctio distantiæ per easdem invenienda ex ultimis logarithmorum figuris tota fere dependeat. Iis igitur, qui rigidiorem hanc interpolationem evitare malint, inservituri methodum pro eodem problemate solvendo adferemus, quæ quidem approximatione quadam absolvitur, exactitudinem tamen omnino sufficificientem præſtat, nec logarithmos omni numero abſolutos poſtulat. Hæc vero methodus ſequente nititur lemmate.

§. VIII.

LEMMA. *Si tria latera Trianguli Sphærici ſint* a, b, c, *& angulus lateri* c *oppoſitus dicatur* z; *erit* $Cos\, c = Cos\frac{1}{2} z^2\, Cos(a-b) + Sin\frac{1}{2} z^2\, Cos(a+b)$.

Veritas hujus propoſitionis facile evincitur ex vulgari illa formula trigonometrica: $Cos\, c = Sin\, a\, Sin\, b\, Cos\, z + Cos\, a\, Cos\, b$. Hinc ſcilicet ob $Cos\frac{1}{2} z^2 - Sin\frac{1}{2} z^2 = Cos\, z$ & $Cos\frac{1}{2} z^2 + Sin\frac{1}{2} z^2 = 1$, ſequitur eſſe $Cos\, c = Sin\, a\, Sin\, b\, (Cos\frac{1}{2} z^2 - Sin\frac{1}{2} z^2) + Cos\, a\, Cos\, b\, (Cos\frac{1}{2} z^2 + Sin\frac{1}{2} z^2) = Cos\frac{1}{2} z^2 (Cos\, a\, Cos\, b + Sin\, a\, Sin\, b) + Sin\frac{1}{2} z^2\, (Cos\, a\, Cos\, b - Sin\, a\, Sin\, b)$; unde porro, ob $Cos\, a\, Cos\, b \pm Sin\, a\, Sin\, b = Cos\, (a \mp b)$, erit $Cos\, c = Cos\frac{1}{2} z^2\, Cos\, (a-b) + Sin\frac{1}{2} z^2\, Cos\, (a+b)$. Q. E. D.

§. IX.

Si igitur obſervata fuerit Lunæ a ſtella quadam diſtantia $= c$, exiſtente altitudine apparente Lunæe $= 90° - a$, & ſtellæ $= 90° - b$; manifeſtum eſt, in Triangulo Sphærico inter Zenith obſervatoris & loca apparentia lunæ atque ſtellæ conſtituto, tria latera fore a, b & c. Hujus autem Trianguli ſi angulus ad verticem ponatur $= z$, erit per lemma præc.

1.) $Cos\, c = Cos\frac{1}{2} z^2\, Cos\, (a-b) + Sin\frac{1}{2} z^2\, Cos\, (a+b)$.

Sit pro eadem obſervatione altitudini lunæ $= 90° - a$ competens parallaxis refractione imminuta $= f$, & altitudini ſtellæ $= 90° - b$ reſpondens refractio parallaxi (ſi opus fuerit) imminuta $= g$, adeoque altitudo vera lunæ $= 90° - a + f$ & ſtellæ $= 90° - b - g$. His datis & poſita lunæ a ſtella diſtantia vera $= c - x$, erunt $a - f$, $b + g$ & $c - x$ latera Trianguli Sphærici inter Zenith & loca vera utriusque Sideris conſtituti, cujus angulus ad verticem $= z$; unde, factis compendii cauſſa $f + g = m$ & $f - g = n$, vi ejusdem lemmatis erit

II.) $Cos\,(c - x) = Cos^{\frac{1}{2}}\, z^2\, Cos\,(a - b - m) + Sin^{\frac{1}{2}}\, z^2\, Cos\,(a + b - n)$.

Subtrahendo æquationem I a II & reducendo reſiduum juxta formulam Trigonometricam: $Cos\,A - Cos\,B = 2\,sin\,\frac{1}{2}\,(B + A)\,Sin\,\frac{1}{2}\,(B - A)$, *obtinetur*

III.) $2\,Sin\,\frac{1}{2}\,x\,Sin\,(c - \frac{1}{2}x) = 2\,Sin\,\frac{1}{2}\,m\,Cos^{\frac{1}{2}}\,z^2\,Sin\,(a - b - \frac{1}{2}\,m)$.

$+ 2\,Sin\,\frac{1}{2}\,n\,Sin\,\frac{1}{2}\,z^2\,Sin\,(a + b - \frac{1}{2}\,n)$.

Jam vero patet $2\,Sin\,\frac{1}{2}\,x$, $2\,Sin\,\frac{1}{2}\,m$ & $2\,Sin\,\frac{1}{2}\,n$ eſſe in ratione ipſorum x, m, & n, quum anguli hi ſint admodum evigui, atque *Sinum* $(c - x)$ parum differre a $Sin\,c$; unde erit ſatis exacte

IV.) $x\,Sin\,c = m\,Cos\,\frac{1}{2}\,z^2\,Sin\,(a - b - \frac{1}{2}\,m) + n\,Sin\,\frac{1}{2}\,z^2\,Sin\,(a + b - \frac{1}{2}\,n)$. Si igitur fiat $\frac{1}{2}\,(c + a + b) = p$, $a - b - \frac{1}{2}\,m = h$, & $a + b - \frac{1}{2}\,n = k$; ob

Cos

$Cos\ \frac{1}{2}\ z^2 = \frac{Sin\ p\ Sin\ (p-c)}{Sin\ a\ Sin\ b}$, nec non $Sin\ \frac{1}{2}\ z^2 = \frac{Sin\ (p-a)\ Sin\ (p-b)}{Sin\ a\ Sin\ b}$ (Elem. Trigon. Sphær.), æquatio IV transformatur in

$$V.)\ x = \frac{m\ Sin\ p\ Sin\ (p-c)\ Sin\ h + n\ Sin\ (p-a)\ Sin\ (p-b)\ Sin\ k}{Sin\ c\ Sin\ a\ Sin\ b}$$

Hinc ad inveniendum x ope logarithmorum, sequentes commodissimæ prodeunt formulæ:

$$Sin\ c\ Sin\ a\ Sin\ b = \frac{1}{r};$$

$$rm\ Sin\ p\ Sin\ (p-c)\ Sin\ h = u;$$

$$\&\ rn\ Sin\ (p-a)\ Sin\ (p-b)\ Sin\ k = v.$$

His scilicet computatis, habetur $x = u + v$.

Illustrationis caussa idem secundum hanc methodum computabimus exemplum, quod supra (§ V. VI) adhibuimus, a D:no De la Lande (*Astron. Tom. IV. p. 757, Edit. II*, vel *Tom. III. p. 664 Edit. III.*) mutuatum.

$c =$	102°	30′	0″	$L\ Sin\ c = \bar{1}.9895815$
$a =$	62.	30.	0	$L\ Sin\ a = \bar{1}.9479289$
$b =$	74.	35.	0	$L\ Sin\ b = \bar{1}.9840852$
$f =$		48.	47.	$L\ \frac{1}{r} = \bar{1}.9215956$
$g =$		3.	17.	$L\ r = 0.0784044$
$m =$		52.	4 = 3124″	$L\ m = 3.4947110$
$n =$		45.	30 = 2730″	

$a-b=$	$-12.$	$5.$	0
$\frac{1}{2}m=$		$26.$	2
$b=$	$-12.$	$31.$	2
$a+b=$	$137.$	$5.$	0
$\frac{1}{2}n=$		$22.$	$45.$
$k=$	$136.$	$42.$	$15.$
$2p=$	$239.$	$35.$	$0.$
$p=$	$119.$	$47.$	$30.$
$p-c=$	$17.$	$17.$	$30.$
$p-a=$	$57.$	$17.$	$30.$
$p-b=$	$45.$	$12.$	$30.$

$L\,Sin\,p=$	$\bar{1}.9384385$
$L\,Sin\,(p-c)=$	$\bar{1}.4731014$
$L\,Sin\,b=$	$\bar{1}.3359261$
$L\,u=$	2.3205814
$L\,r=$	0.0784044
$L\,n=$	3.4361626
$L\,Sin\,(p-a)=$	$\bar{1}.9250191$
$L\,Sin\,(p-b)=$	$\bar{1}.8510584$
$L\,Sin\,k=$	$\bar{1}.8361756$
$L\,v=$	3.1268201

$$v = 1339''.1$$
$$u = -209.2$$
$$x = 1129.9 = 18'.49'\,9$$
$$c = 102^\circ.30,0'.0.$$
$$c-x = 102^\circ\,11'\,10''\,1$$

Qui valor per hanc approximationem inventus a vero ne ſcrupulum quidem ſecundum aberrat. Quod in hoc exemplo *u* negativus fiat, ex valero negativo ipſius *Sin h* intelligitur.

§. X.

Ut ex obſervata diſtantia Lunæ a ſtella quadam inveniatur longitudo loci, requiritur ut datum etiam

ſit

ſit tempus verum hujus obſervationis. Notato igitur tempore horologii, quo diſtantia iſta obſervata fuit, mox vel ſaltim intra paucas horas ante vel poſt obſervationem menſurandæ ſunt altitudines vel ejusdem ſtellæ vel alius cujusdam ad hunc ſcopum maxime idoneæ, ex quibus tempus verum computari poterit; in quo quidem negotio ſufficit horologium portatile, cujus motus intra breve hoc temporis inter vallum, quo ad has obſervationes opus eſt, ſenſibilem inæquabilitatem non prodat. Sit itaque ſub meridiano loci iſtius, qui dicatur M, tempus verum $= t$, quo obſervata fuit diſtantia Stellæ alicujus a luna, unde per methodos allatas inventa ſit diſtantia earum vera $= \gamma$. Ad inveniendam hinc longitudinem loci M, ex ephemeridibus ad datum meridianum A computatis pro eodem die ſumantur ejusdem ſtellæ a luna diſtantiæ duæ ad γ proxime accedentes, quæ ſint C pro tempore T, & D pro tempore $T + n$; pro intervallo ſcilicet trium horarum, quale adhibetur in *Nautical Almanac* & *Connoiſſance de Tems*, generatim ponimus n. Ex his datis obtinetur ad meridianum A tempus verum $= T + x$ reſpondens tempori t ſub meridiano M, inferendo $D - C : \gamma - C :: n : x$. Differentia vero temporum $T + x$ & t eſt ipſa differentia meridianorum A & M, unde ex data longitudine illius colligitur longitudo hujus. Plura quæ circa hanc rationem longitudines inveniendi in praxi obſervanda ſunt præcepta attentione digna videre licet in *Explanation of the Tables requiſite to be uſed with the*

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Nautical Ephemeris & in *Nouveau Traité de Navigation par* M. BOUGUER, *revu par* M. DE LA CAILLE *L. IV. Chap. VIII.*

§. XI.

Methodus igitur hæc longitudines computandi duabus diversis absolvitur operationibus, una scilicet, qua distantia corrigitur (§. §VII---IX.). altera qua ex distantia hac correcta ipsa longitudo investigatur (§.X). Possunt vero hæ binæ operationes commode in unam contrahi, quoties differentia meridianorum conjectura seu æstimatione tam prope cognita est, ut error saltim tempus dimidiæ horæ non excedat; quod quidem semper fere obtinet, plerumque autem de paucorum tantum scrupulorum temporis correctione quæstio est. Sit itaque meridianorum *M* sub quo observatio instituitur, & A, ad quem Ephemerides supputatæ sunt, differentia in tempus conversa, secundum æstimationem $= m$, exacte vero $= m + x$, adeo ut sit tempcs $x < 30'$. Tempore vero $= t$ sub meridiano *M* mensurata sit stellæ alicujus a luna distantia $= c$, existente altitudine apparente lunæ $= 90° - a$ & stellæ istius $= 90° - b$. Hinc ex data parallaxi & refractione inveniatur (§. 2) altitudo vera Lunæ $= 90° - a + f$ & stellæ $= 90° - b - g$. Porro quum ob differentiam meridianorum $= m + x$, momento hujus observationis sub meridiano *A* respondeat tempus verum $= t - m - x$, posito scilicet meridiano *M* orientalio.

taliori ipso A, (in casu vero contrario tempus $m+x$ negative sumendum erit); quæratur ex ephemeridibus ad tempus $t-m$ stellæ observatæ a luna distantia $=\gamma$, hujusque distantiæ incrementum $=\delta$ pro tempore $=n$; adeo ut sit ad tempus $t-m-x$ distantia horum siderum vera $=\gamma-\frac{\delta x}{n}$. His igitur positis erunt in duobus triangulis, quorum unum inter Zenith Observatoris & loca apparentia lunæ atque stellæ, alterum inter zenith & loca vera horum siderum constituitur, latera illius c, a, b, atque hujus $\gamma-\frac{\delta x}{n}$, $a-f$, $b+g$; unde ob angulum ad verticem communem qui dicatur z, invenietur x. Hoc vero pluribus methodis fieri potest, quarum unicam afferre sufficiat lemmati nostro (§. VIII) superstructam. Hinc videlicet erit:

$$Cos\, c = Cos\, \tfrac{1}{2} z^2\, Cos\,(a-b) + Sin\, \tfrac{1}{2} z^2\, Cos\,(a+b)\text{; \&}$$

$$Cos\,(\gamma-\frac{\delta x}{n}) = Cos\, \tfrac{1}{2} z^2\, Cos\,(a-b-f-g) + Sin\, \tfrac{1}{2} z^2\, Cos\,(a+b-f-g);$$

Quas æquationes eodem modo ac in §. IX. reducendo, positis brevitatis caussa $\frac{n\,(c-\gamma)}{\delta}=e$, $\frac{1}{2}(c+\gamma)=\lambda$, $f+g=s$, $f-g=q$, $a-b-\frac{1}{2}s=h$, $a+b-\frac{1}{2}q=k$, & $\frac{1}{2}(a+b+c)=p$; ob quantitates s, q & x admodum exiguas obtinetur $(e+x)\,Sin\,(\lambda-\frac{\delta x}{2n}) = s\,Cos^2 z^2$

 Sin h

$Sin\, h + g Sin \frac{1}{2} z^2 Sin\, k$ unde quum sit $Cos \frac{1}{2} z^2 = \frac{Sin\, p\, Sin\,(p-c)}{Sin\, a\, Sin\, b}$, $Sin \frac{1}{2} z^2 = \frac{Sin\,(p-a)\, Sin\,(p-b)}{Sin\, a\, Sin\, b}$ & quam proxime $Sin\left(\lambda - \frac{\delta x}{2n}\right) = Sin\,\lambda$, valor ipsius x commode computari potest secundum has formulas:

I.) $\frac{1}{r} = \frac{\delta\, Sin\, a\, Sin\, b\, Sin\, \lambda.}{n}$;

II) $u = rs\, Sin\, p\, Sin\,(p-c)\, Sin\, h$;

III.) $v = rq\, Sin\,(p-a)\, Sin\,(p-b)\, Sin\, k$;

IV.) $e = \frac{n\,(c-\gamma)}{\delta}$;

V.) $x = u + v - e$

Quoties error longitudinis æstimatæ major est, per primam approximationem valor ipsius x satis exactus non obtinetur, quo in casu repetito opus est calculo. Hæc vero repetitio multum negotii non facessit, quum præter m & quæ hinc dependent γ & λ, reliqua omnia data in hoc calculo invariata maneant.

Illustrationis caussa secundum hanc methodum sequens computare lubet exemplum a D:no MASKELYNE in *Explanation of the Tables requisite* &c. Edit. 2. p. 45-47 propositum:

$m =$

$m = -$ 7h 0′ 0″		$n =$ 3h 0.′ 00″ = 10800″	
$t =$ 23h 55′ 10″		$\delta =$ 1° 40′ 5″ = 6005″	
$c =$ 59° 25′ 34″		$f =$ 0°. 51′. 33.	
$a =$ 62. 57. 30		$g =$ 0°. 0′ 30.″	
$b =$ 30. 41. 8		$\gamma =$ 58. 42. 34	

His datis calculus ita se habet:

$t - m =$	6h 55′ 10″	$L\,\delta = 3.\,7785130$
$2\,p =$	153° 11′ 12″	$L\,n = 4.\,0334238$
$p =$	76. 35. 36.	$L\,\frac{\delta}{n} = \bar{1}.\,7450892$
$p - c =$	17. 10. 2.	$L\,Sin\,a = \bar{1}.\,9497198$
$p - a =$	13. 38. 6	$L\,Sin\,b = \bar{1}.\,7093345$
$p - b =$	45. 47. 28	$L\,Sin\,\lambda = \bar{1}.\,9333738$
$s =$	0. 52. 3 = 3123″	$L\,\frac{1}{r} = \bar{1}.\,3375173$
$q =$	0. 51. 3 = 3063″	$L\,r = 0.\,6624827$
$a - b =$	32. 9. 22.	$L\,s = 3.\,4945720$
$\frac{1}{2}\,s =$	0. 26. 1.	$L\,Sin\,p = \bar{1}.\,9880008$
		$L\,Sin\,(p - c) = \bar{1}.\,4700597$
		$L\,Sin\,h = \bar{1}.\,7208253$
		$L\,u = 3\,3359405$
$h =$	31° 43′ 21″	$L\,r = 0.\,6624827$
$a + b =$	23. 45. 38	$L\,q = 3.\,4861470$
$\frac{1}{2}\,q =$	0. 25. 31	$L\,Sin\,(p - a) = \bar{1}.\,3724256$

 k =

$k = 93.\ 20.\ 7$	$L\ Sin\ (p-b) = \bar{1}.\ 8553995$
$\lambda = 59.\ 4.\ 4$	$L\ Sin\ k = \bar{1}.\ 9992638$
$c-\gamma = 0.\ 43.\ 0. = 2580''$	$L\ v = 3.\ 3757186$
$u = 2167.''\ 4$	$L\ (c-\gamma) = 3.\ 4116197$
$v = 2375.\ 3$	$\frac{L\ \delta}{n} = \bar{1}.\ 7450892$
$4542.\ 7$	
$e = 4640.\ 2$	$L\ e = 3.\ 6665305$
$x = -97''.\ 5 = -1'\ 37.''\ 5$	
$m = -7^h\ 0'\ 0''$	
$m + x = -7^h\ 1'\ 37.''\ 5$	

Dn. Maskelyne per suam methodum (l. c.) invenit hanc differentiam meridianorum $= - 7^h\ 1'\ 38.''$

§. XII.

Supposuimus in antecedentibus Observatorem instructum Ephemeridibus Astronomicis, in quibus stellarum a luna distantiæ computatæ inveniuntur, quales sunt *Connoissance des Tems*, *Nautical Almanac*, & quædam aliæ. Plures vero dantur Ephemerides, in quibus hæ distantiæ desiderantur, licet de cetero pro quovis die motum lunæ curatius definitum exhibeant. Dispiciendum igitur erit, quomodo etiam harum ope, ex observatis stellarum a luna distantiis longitudines geographicæ sine prolixo admodum calculo determinari queant. Notandum vero est, quod admodum

par-

parvæ ſemper ſint variationes latitudinis lunæ, adeo ut etiamſi in æſtimanda longitudine loci aliquot graduum error committeretur, pro tempore tamen obſervationis latitudo lunæ a vera non multum aberrans obtineatur, & præterea ob latitudines ſemper exiguas tam lunæ quam ſtellarum ad hujusmodi obſervationes adhibendarum, hic error latitudinis lunæ, longitudinem inveniendam parum afficiat. Hinc ſequentem detegimus rationem hoc problema ſolvendi. Sit ſub meridiano loci *M* tempore vero $= t$ obſervata ſtellæ cujusdam a luna diſtantia, ex qua & obſervatis ſimul altitudinibus horum ſiderum, per regulas traditas colligatur eorundem diſtantia vera $= \gamma$; ſitque pro hoc tempore latitudo ſtellæ iſtius $= L$ & latitudo lunæ quam proxime $= \lambda$. Ex his datis primo invenienda eſt differentia inter longitudines lunæ & ſtellæ, quæ differentia dicatur y. Sunt autem in triangulo ſphærico inter polum Eclipticæ & loca vera lunæ atque ſtellæ conſtituto, tria latera γ, $90° - L$, & $90 - \lambda$, nec non angulus ad polum $= y$, quamobrem erit

$$Sin\, \tfrac{1}{2}\, y^2 = \frac{Sin\, \tfrac{1}{2}(\gamma - L + \lambda)\; Sin\, \tfrac{1}{2}(\gamma + L - \lambda)}{Cof\, L\; Cof\, \lambda}, \text{ vel}$$

$$Cof\, \tfrac{1}{2}\, y^2 = \frac{Cof\, \tfrac{1}{2}(\gamma + L + \lambda)\; Cof\, \tfrac{1}{2}(\gamma - L - \lambda)}{Cof\, L\; Cof\, \lambda}.$$

Unde invento y, dataque præterea longitudine ſtellæ invenitur longitudo lunæ pro tempore obſervatiovationis. Cognita vero hac longitudine, interpolatione debita ex ephemeridibus ad datum meridianum *A* invenietur tempus verum $= T$, quo luna hanc longitudinem habuit. Cfr. *Dissert. De interpolatione pro inveniendo loco lunæ ex ephemeridibus*, Aboæ 1789. §. 7. Horum temporum differentia $t - T$ est ipsa differentia meridianorum *M* & *A*, unde data longitudine ipsius *A* innotescit longitudo loci *M*.