DISSERTATIO ACADEMICA
de invenienda
Parallaxi Altitudinis,
Ex datis Parallaxi Sideris Horizontali
& vera ejus a Zenith distantia,

Quam
Conf. Ampl. Facult. Philof. Aboënf.
PRÆSIDE
Mag. JOHANNE HENRICO LINDQUIST,
Math. Prof. R. & Ord. R. Acad. Scient. Stockh. & R. Societ. Scient. Upfal. Membro,

Publice defendere conabitur
THOMAS TIMOTHEUS KRIANDER,
SATACUNDA-FENNO,

In Auditorio Majori die XV Junii MDCCXCVI,
Horis ante meridiem folitis.

ABOÆ, TYPIS FRENCKELLIANIS.

VIRO

Plurimum Reverendo atque Præclarissimo

D:no THOMÆ KRIANDER,

Ecclesiarum, quæ Deo in Sastmola & Sijkais colliguntur, Pastori meritissimo,

Parenti Indulgentissimo.

Ob paterna in me multifariam collata beneficia, eaque innumera hoc Tibi, Optime Parens, specimen Academicum, in tesseram meæ in te pietatis, dedicatum lubentissimus volui, debui; quod ut soles, Parens Indulgentissime, affectu animoque excipias paterno, ardentissimus cernuusque rogo & obsecro; ad cineres usque permansurus

Parentis Indulgentissimi

Filius obsequentissimus
THOM. TIM. KRIANDER.

§. I.

Frequentissimus est in calculis Astronomicis usus Problematis, quo ex data corporis cœlestis parallaxi horizontali, quæritur parallaxis ejus ad datam quamvis altitudinem vel ad hujus complementum seu distantiam sideris a zenith. Duo vero hic occurrunt casus, prout scilicet distantia hæc a vertice aut apparens est aut vera. In illo casu, quo ex loco apparente colligendus est locus stellæ verus, nihil occurrit difficultatis, nec admodum prolixa opus est computatione. Quoties autem ad datam veram sideris a Zenith distantiam investiganda est ejusdem parallaxis, quamvis nec nisi simplicissimis Trigonometriæ elementis hujus casus solutio superstruatur, prolixior tamen & magis impeditus fit calculus. Quamobrem quum sæpius occurrat hujus problematis applicatio, in eodem solvendo concinnitati, quantum fieri potest, atque compendiis calculi, non neglecto rigore Mathematico, studendum erit. Supervacaneum igitur non judicavimus nostra etiam qualiacunque circa hanc rem tentamina publicæ luci committere.

§. II.

Licet a sphærica parum aberret figura telluris, hujus tamen aberrationis in parallaxibus præsertim Lunæ accuratius definiendis, ratio habenda erit. In antecessum igitur, pro quovis loco observationis determinanda est primo differentia inter Zenith ejus apparens & verum, hoc est: inter duo puncta cœli, versus quæ diriguntur rectæ, quarum una cum directione gravitatis in loco dato congruit, altera ex centro Terræ per eundem locum transit; deinde cognita ex Ephemeridibus vel Tabulis Astronomicis pro tempore observationis parallaxi Sideris horizontali æquatorea, invenienda erit parallaxis ejusdem horizontalis pro loco spectatoris. Ad has reductiones inveniendas posita figura terræ ellipsoidica, (quæ quidem a veritate minime abludere videtur), sit *QAP* (Fig. 1.) quadrans meridiani elliptici, in quo *P* est polus, *Q* punctum æquatoris, *A* locus observationis, *C* centrum terræ, atque ductis *CQ*, *CA* & *CP*, fiat ad Ellipseos punctum *A* normalis *AN*, ipsi *CQ* occurrens in *N*. Describatur porro centro *C* radio *CQ* circulus *QDB*, cui ex *A* in *CQ* demissa perpendicularis *AE* occurrat in *D*, & jungantur *D*, *C*. Si jam sumta semidiametro æquatoris $CQ = 1$, sit semiamiaxis terræ $CP = n$, Latitudo Loci A seu ang. $ANQ = \lambda$, ang. $DCQ = M$ & ang. $ACQ = N$, erit $Tg\ M = n\ Tg\ \lambda$, $Tg\ N = n\ Tg\ M = n^2\ Tg\ \lambda$, & $CA = \frac{Cos\ M}{Cos\ N} = \frac{n\ Sin\ M}{Sin\ N}$ (cfr. *Diss. qua resolvuntur nonnulla Problemata, posita figura Terræ ellipsoidica*, Præside Cel. M. J. WALLENIO a TH. MATTHEISZEN edit. Aboæ 1767. §. 3). Hinc invento ang. N ad datam Latitudinem A datur distantia inter Zenith loci apparens X & verum Z, quorum videlicet illud in producta recta NA, hoc in CA positum erit; est namque ang. $XAZ = CAN = \lambda - N$. Si porro existente parallaxi Sideris horizontali æquatorea $= P$, sit in loco A parallaxis ejus horizontalis $= \pi$; ob $Sin\ P$: $Sin\ \pi$: : $CQ : CA$: : $I : \frac{Cos\ M}{Cos\ N}$: : $I : \frac{n\ Sin\ M}{Sin\ N}$, erit $Sin\ \pi = \frac{Cos\ M\ Sin\ P}{Cos\ N} = \frac{n\ Sin\ M\ Sin\ P}{Sin\ N}$. Cognitis hac ratione ad datum locum Zenith vero Z & parallaxi horizontali $= \pi$, reliquus parallaxium calculus idem pro figura sphæroidica ac pro terra perfecte sphærica obtinet.

§. III.

Indirecta vulgo adhibetur methodus inveniendi parallaxin altitudinis ex datis parallaxi horizontali atque vera sideris a Zenith distantia, calculum scilicet instituendo secundum regulam, cujus ope ex data distantia apparente eadem parallaxis eruitur. Quamobrem hujus brevem expositionem præmittere haud abs re erit. Sit itaque (Fig. 2.) A locus Observatoris, C centrum terræ & L locus stellæ, atque ducantur rectæ AL, CL & CA, quarum hæc CA producta versus Zenith verum Z dirigetur. Porro in plano ACL centro C per L describatur Circulus, cui ex A ad CA ducta perpendicularis AH occurrat in H & jungantur H, C. Hac constructione facta, angulus AHC erit Sideris L parallaxis horizontalis, quæ dicatur π; ang. ALC ipsa altitudinis parallaxis $= p$; ang. ZAL distantia Sideris a Zenith apparens $= x$; & ang. ACL ejusdem distantia vera $= z$. Jam vero per Elem. Trigon. in $\Delta\,\Delta$ AHC, ALC, posito Sinu toto $= 1$, est $1 : Sin\,\pi$ (: : $1 : Sin\,AHC$: : $CH : CA$: : $LC : CA$: : $Sin\,LAC : Sin\,ALC$) : : $Sin\,x : Sin\,p$, unde sequitur: $Sin\,p = Sin\,\pi\;Sin\,x$. Hujus formulæ ope ad datam quamvis stellæ a Zenith distantiam apparentem $= x$, data parallaxi horizontali $= \pi$, exacte invenitur ejus parallaxis altitudinis $= p$. Quumque anguli π & p sint admodum exigui, (pro Luna scilicet est semper $\pi < 1^\circ\,2'$; parallaxes vero Solis & planetarum primariorum aliqnot solummodo scrupulos secundos efficiunt); erit quam proxime $p = \pi\;Sin\,x$ (*). Eadem hæc formula indirectæ

(*) Ut pateat, quantus committi poterit error, loco formulæ: $Sin\,p = Sin\,\pi\;Sin\,x$, adhibendo $p = \pi\;Sin\,x$, rectæ problematis nostri solutioni inservit. Quum videlicet ob $x - z = p$ vel $x = z + p$, sit *Sin* x fere $=$ *Sin* z, ponendo π *Sin* $z = p'$ & π *Sin* $(z + p')$ $= p''$, erit quam proxime $p'' = p$. (*cfr. Leçons elem. d'Astron. par M.* DE LA CAILLE §. 651. Edit. Paris, A. 1761.) Si major desideretur exactitudo, calculus pluries repetendus erit, ponendo scilicet ulterius π *Sin* $(z + p'') = p'''$ & sic porro, donec obtineantur duo va-

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ponatur exacte esse: $p = \pi$ *Sin* $x - u$. Ex hac æquatione cum æqu. *Sin* $p =$ *Sin* π *Sin* x collata, invenitur $u = (\pi -$ *Sin* $\pi)$ *Sin* $x - p +$ *Sin* p; unde posito *Sin* $x = m$, *Sin* $\pi = s$ (adeoque *Sin* $p = ms$) & radio circuli in scrupulis secundis expresso $= 206264''806 = R$, atque evolvendo angulos π & p per series secundum dignitates ipsorum Sinuum progredientes, erit generatim error iste $u = Rms^3 \left[\frac{1}{2.3}(1 - m^2) + \frac{1.3}{2.4.5}(1 - m^4) s^2 + \frac{1.3.5}{2.4.6.7}(1 - m^6) s^4 + \&c.\right]$. Si igitur assumatur $\pi = 1° 2'$, obtinetur $u = 0.''20166 m - 0.''20163 m^3 - 0.00003 m^5 - \&c.$ adeoque pro $x = 35° 16'$ in hoc casu erit $u = 0''0776$. Quumque Lunæ parallaxis horiz. semper sit $< 62'$ atque pro hoc valore ipsius π error u maximus fere pro $x = 35° 16'$ obtineat; patet ex adhibita formula: $p = \pi$ *Sin* x, loco ipsius *Sin* $p =$ *Sin* π *Sin* x, errorem nunquam oriri $> 0''0776$. In computandis vero parallaxibus planetarum primariorum, pro quibus omnibus est $\pi < 35''$, error istius formulæ semper est $< 0.''000000065$ adeoque omnino negligendus.

valores approximati ipsius p, parum aut nihil a se invicem differentes. In computandis quidem parallaxibus planetarum primariorum, ob π semper $< 35''$, hæc methodus indirecta haud incommode adhiberi potest; pro Luna vero calculus per eam sæpe nimis prolixus foret.

§. IV.

Ex iis quæ §. præc. attulimus, haud difficile est directam eruere methodum problema nostrum solvendi. Iisdem enim retentis denominationibus, quum (dem.) sit $1 : Sin\,\pi :: Sin\,x : Sin\,p$, adeoque ob $x = p + z$, $1 : Sin\,\pi :: Sin(p+z) : Sin\,p$; sequitur hanc analogiam mixtim sumendo fore $1 + Sin\,\pi : 1 - Sin\,\pi :: Sin\,(p+z) + Sin\,p : Sin\,(p+z) - Sin\,p$. Jam vero denotantibus a & b angulos quosvis, (Elem. Trigon.) est generatim: $Sin\,a + Sin\,b : Sin\,a - Sin\,b :: tg\tfrac{1}{2}(a+b) : tg\tfrac{1}{2}(a-b)$. Quare sumendo primum $a = p + z$ & $b = p$, adeoque $\tfrac{1}{2}(a+b) = p + \tfrac{1}{2}z$ & $\tfrac{1}{2}(a-b) = \tfrac{1}{2}z$, erit $Sin\,(p+z) + Sin\,p : Sin\,(p-z) - Sin\,p :: tg\,(p+\tfrac{1}{2}z) : tg\tfrac{1}{2}z$. Si autem ponatur $a = 90^\circ$, & $b = \pi$, adeoque $Sin\,a = 1$, $\tfrac{1}{2}(a+b) = 45^\circ + \tfrac{1}{2}\pi$, & $\tfrac{1}{2}(a-b) = 45^\circ - \tfrac{1}{2}\pi$; ob $tg\,(45^\circ - \tfrac{1}{2}\pi) = Cotg\,(45^\circ + \tfrac{1}{2}\pi) = \frac{1}{tg\,(45^\circ + \tfrac{1}{2}\pi)}$ erit $1 + Sin\,\pi : 1 - Sin\,\pi :: tg\,(45^\circ + \tfrac{1}{2}\pi)^2 : 1$ (*). His

(*) Quod sit $1 + Sin\,\pi : 1 - Sin\,\pi :: tg\,(45^\circ + \tfrac{1}{2}\pi)^2 : 1$, etiam sic demonstratur. Sit ADB (Fig. 3.) circulus

His igitur substitutionibus in superiori analogia factis, obtinetur $tg\,(p + \frac{1}{2}z) : tg\,\frac{1}{2}z :: tg\,(45 + \frac{1}{2}\pi)^2 : 1$, adeoque

$$tg\,(p + \tfrac{1}{2}z) = tg\,(45^\circ + \tfrac{1}{2}\pi)^2\, tg\,\tfrac{1}{2}z;$$

cujus formulæ ope, ex datis parallaxi Sideris horizontali $= \pi$, & vera ejus a vertice distantia $= z$, invenitur parallaxis altitudinis $= p$.

Exempl. Si existente parallaxi Lunæ horizontali $= 59' 30'' = \pi$, ad veram ejus a zenith distantiam $= 85^\circ 46' 50'' = z$, invenienda sit parallaxis altitudinis $= p$, calculus ita instituendus erit:

$45^\circ + \frac{1}{2}\pi = 45^\circ\ 29'\ 45''$	$2 Log Tg (45^\circ + \frac{1}{2}\pi) = 0{,}0150341$
$\frac{1}{2}z = 42.\ 53.\ 25$	$Log Tg \frac{1}{2}z = 1.9679882$
$p + \frac{1}{2}z = 43.\ 52.\ 49.\ 3.$	$Log tg (p + \frac{1}{2}z) = 1.9830223$
$p = 0.\ 59.\ 24.\ 3.$	

§. V.

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radio $CA = 1$ descriptus, cujus diametro AB ex centro C perpendicularis ducatur radius CD, ad quem in C constituatur ang. $DCE = \pi$, secetque CE circulum in puncto E, ex quo ducantur rectæ EA, EB & ipsi AB perpendicularis EF. His factis erit $CF = Sin\,\pi$, $BF = 1 + Sin\,\pi$, $AF = 1 - Sin\,\pi$, $\angle BCE = 90^\circ + \pi$, $\angle BAE = \frac{1}{2} BCE = 45^\circ + \frac{1}{2}\pi$, & $\angle BEA = 90^\circ$: quare $\triangle BEF \sim \triangle BAE \sim \triangle EAF$, $BE^2 = AB.BF$, & $AE^2 = AB.AF$, adeoque $1 + Sin\,\pi : 1 - Sin\,\pi$ (:: $BF : AF$ :: $AB.BF : AB.AF$: $BE^2 : AE^2$ :: $tg\,BAE^2 : 1$) :: $tg\,(45^\circ + \frac{1}{2}\pi)^2 : 1$. Q. E. D.

§. V.

Regula (§. 4.) tradita hoc quidem nomine se maxime commendat, quod unica absolvatur æquatione, adeoque applicatu commodissima videatur. Sed tamen neque hæc suis caret difficultatibus. Calculo enim secundum istam regulam Canonum Trigonometricorum etiam optimorum auxilio instituendo, summa plerumque adhibenda est diligentia in logarithmis interpolandis, ne valor ipsius p, ex differentia angulorum $p + \frac{1}{2}z$ & $\frac{1}{2}z$ inveniendus egregie fallat. Sic si in exemplo allato, neglecta interpolatione loco $45° + \frac{1}{2}\pi = 45° 29' 45''$ & $\frac{1}{2}z = 42° 53' 25''$ assumeretur $45° + \frac{1}{2}\pi = 45° 29' 40''$ & $\frac{1}{2}z = 42° 53' 20''$, foret $p = 59' 5''$, unde error fere $= 20''$. Præterea si valor ipsius p usque ad partes decimales scrupuli secundi exactus desideretur, hac adhibita methodo saltim pro $z > 80$, vulgares canones, qui logarithmos non nisi 7 figuris decimalibus expressos exhibent, non sufficerent. Quamobrem pro hujusmodi casibus alia tentanda erit via. Resumatur igitur analogia (§. 4.) $1 : Sin\,\pi :: Sin\,(p + z) : Sin\,p$; & pro $Sin\,(p + z)$ substituatur (Elem. Trig.) $Sin\,p\ Cos\,z + Cos\,p\ Sin\,z$. Hinc facta debita reductione obtinetur $Tg\,p\,(1 - Sin\,\pi\ Cos\,z) = Sin\,\pi\ Sin\,z$; quæ æquatio, posito $Sin\,\pi = n$ & $tg\,\frac{1}{2}z = t$, ob $Sin\,z = \frac{2t}{1 + t^2}$ & $Cos\,z = \frac{1 - t^2}{1 + t^2}$, transformatur in sequentem:

$Tg\,p\left(\frac{1-n(1-tt)}{1+tt}\right)=\frac{2nt}{1+tt}$, vel $Tg\,p\left(1+\frac{(1+n)t^2}{1-n}\right)$ $=\frac{2nt}{1-n}$. Sumto ulterius $\frac{1+n}{1-n}=m^2$, erit $Tg\,p$ $(1+m^2t^2)=\frac{2nt}{1-n}$ adeoque $Tg\,p=\frac{2nt}{(1-n)(1+m^2t^2)}$ $=\frac{n}{m(1-n)}\cdot\frac{2mt}{1+m^2t^2}$.

Est autem $m(1-n)=\sqrt{1-n^2}$, $\frac{n}{\sqrt{1-n^2}}=Tg\,\pi$, $m=\sqrt{\frac{1+n}{1-n}}=\sqrt{\frac{1+Sin\,\pi}{1-Sin\,\pi}}=Tg\,(45^\circ+\frac{1}{2}\pi)$. (§. 3. not †), & assumto angulo v tali ut sit $mt\,(=$ $Tg\,\overline{45^\circ+\frac{1}{2}\pi}\;Tg\,\frac{1}{2}z)=Tg\,\frac{1}{2}v$, erit $\frac{2mt}{1+m^2t^2}$ $=Sin\,v$, adeoque $Tg\,p=Tg\,\pi\;Sin\,v$. Hinc obtinetur methodus, ex data parallaxi Sideris horizontali $=\pi$, ad datam veram ejus a zenith distantiam $=z$ inveniendi altitudinis parallaxin $=p$, ope harum formularum:

I.) $Tg\,\frac{1}{2}v=Tg\,(45^\circ+\frac{1}{2}\pi)\;Tg\,\frac{1}{2}z$; &

II.) $Tg\,p=Tg\,\pi\;Sin\,v$.

Et hac quidem methodo ope canonis trigonometrici, tangentes artificiales pro singulis scrupulis secundis saltem a 0° ad 1° 2' exhibentis (*), parallaxis altitudinis Lunæ etiam ad partes usque 1000:mas scrupuli secundi exacta commode computari posset, si de cetero Theoria Lunæ adeo perfecta foret, ut tanta exactitudo locum obtineret.

Illustrationis caussa idem exemplum (§. 4.), quo ex datis $\pi = 59' 30''$ & $z = 85° 46' 50''$ quæritur p, secundum hanc methodum computabimus:

$45° + \frac{1}{2}\pi = 45° 29' 45.''$	$Log. Tg (45° + \frac{1}{2}\pi) = 0,0075170$
$\frac{1}{2} z = 42. 53. 25.$	$Log\ Tg\ \frac{1}{2} z = \bar{1}.9679882$
$\frac{1}{2} v = 43. 23. 6.$	$Log\ Tg\ \frac{1}{2} v = 1.9755052$
$v = 86. 46. 12.$	$Log\ Sin\ v = \bar{1}.9993095$
$p = 0. 59. 24, 33.$	$Log\ Tg\ \pi = \bar{2}.2382865$
	$Log\ Tg\ p = 2.2375960$

§. VI.

Quum sit semper $\pi < 1° 2'$, erit quam proxime $Tg\,p : Tg\,\pi :: p : \pi$, unde loco formulæ II (§. 5.) assumi poterit: $p = \pi\ Sin\ v$. Videndum igitur erit, quanta exactitudo per hanc approximationem attingatur. Sit exacte $p = \pi\ Sin\ v + \omega$; hanc æqu. subtrahendo ab æqu. $Tg\,p = Tg\,\pi\ Sin\ v$, obtinetur $Tg\,p - p = (Tg\,\pi - \pi)\ Sin\ v - \omega$ adeoque $\omega = (Tg\,\pi - \pi)\ Sin\ v - Tg\,p + p$. Unde evolvendo angulos π & p per series secundum dignitates Tangentium progredientes & compendii caussa ponendo $Tg\,\pi = q$, $Sin\ v = y$, (adeoque $Tg\,p = q\,y$) & $206264,''806 = R$, inve-

(*) Qualis ex. gr est *Recueil de Tables Logarithmiques* &c. *par* I. C. Schulze. *Berol.* 1778.

invenitur error quaesitus $\omega = Rq^3 y [\frac{1}{3}(1-y^2) - \frac{1}{5}q^2 (1-y^4) + \frac{1}{7}q^4 (1-y^6) - \&c.]$

Quamobrem adhibendo formulam $p = \pi$ *Sin* v, pro Luna erit hic error $< 0.'' 15$ & pro planetis primariis $< 0.'' 00000013$. De cetero ex hac serie semper sufficit primus terminus, adeo ut generatim assumi poterit $\omega = \frac{1}{3} R q^3 y (1-y^2)$; quoniam existente $\pi = 1°2'$ in quovis casu fit $R q^5 y [\frac{1}{5}(1-y^4) - \frac{1}{7}q^2 (1-y^6) + \&c.] < 0.'' 00004$.

Hinc etiam vulgarium tabularum Logarithmicarum auxilio pro Luna ex datis π & z, usque ad 1000:am scrupuli secundi partem exacta inveniri potest parallaxis p secundum has formulas:

I.) $Tg \frac{1}{2} v = (Tg\, 45° + \pi)\, Tg \frac{1}{2} z$; II.) $\varphi = \pi\, Sin\, v$;

III.) $\omega = \dfrac{\varphi\, \pi^2\, Cos\, v^2}{3 R^2}$ & IV.) $p = \varphi + \omega$.

Sic si exemplum (§. §. 4. 5.) allatum secundum hanc methodum computetur, invento per form. I. pariter ac in §. praec. $v = 86° 46' 12,''$ calculus ita continuatur:

$\pi = 59' 30'' = 3570''$	$Log\ \pi = 3.5526682$
$\varphi = 3564.'' 329$	$L.\ Sin\ v = 1.9993095$
$\omega = 0.'' 001$	$L.\ \varphi = 3.5519777$
$p = 3564.'' 330$	$2\ L.\ \pi = 7.1053364$
$= 59' 24.'' 33$	$2\ L.\ Cos\ v = 3.5016996$
	$-\ Log\ 3 R^2 = 12.8940284$
	$L.\ \omega = 3.0530421$

§. VII.

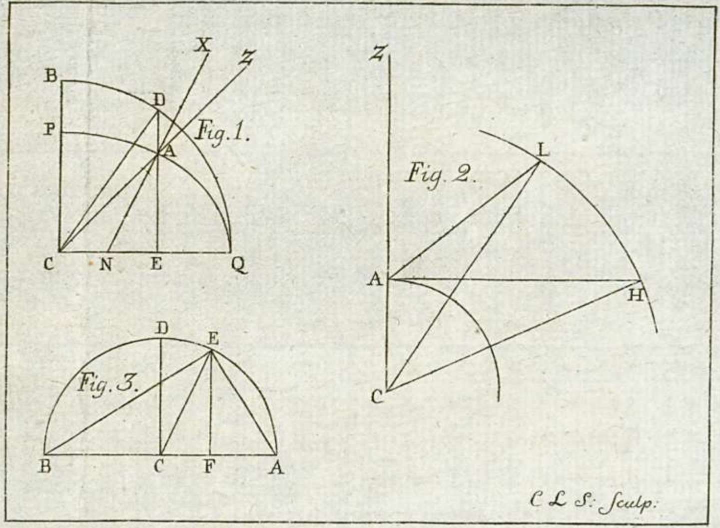
Fig. 1.
X
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Fig. 2.
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§. VII.

Brevitatis studio et ad similitudinem eorum, quae de parallaxibus in hypothesi figurae terrae perfecte sphaericae traduntur, nomen parallaxeos horizontalis retinuimus ad designandum angulum istum π, cujus Sinus est ad Sinum totum, ut semidiameter telluris sub loco observationis ad distantiam sideris a centro terrae, quem angulum ex data parallaxi horizontali sub aequatore (§. 2.) determinare docuimus. Si vero secundum etymologiam vocis, parallaxin horizontalem dicamus illam $= \pi'$, quae stellae (manente hujus a terra distantia) in ipso horizonte, seu plano tellurem in loco observationis contingente, constitutae competit; prius determinanda erit relatio angulorum π & π'. Ex allatis (§. 2.) manifestum est, pro figura terrae ellipsoidica, invento (Fig. 1.) angulo $CAN = XAZ = \lambda - N$, fore $Sin\,\pi' : Sin\,\pi :: Cos\,(\lambda - N) : 1$; unde dato uno angulorum π & π' obtinetur alter. Hujus autem anguli π' in calculis parallaxium rarior est usus.