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Dissertatio Academica

De

Invenienda

Longitudine Loci

Ex observata Distantia Lunæ a Stella quadam,

Cujus

Particulam Priorem

Cons. Ampl. Fac. Philos. Aboëns.

Præside

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ABOÆ, Typis *Frenckellianis.*

§. I.

Quamdiu horologia maritima ſatis exacta vulgo non proſtant, nulla ad longitudines locorum determinandas in re præſertim nautica melior tutiorque datur methodus, quam quæ obſervatis Lunæ a ſtellis diſtantiis innititur. Omnium videlicet corporum cœleſtium velociſſimus eſt motus Lunæ, unde ejus a Sole & reliquis ſtellis diſtantia exiguo temporis intervallo notabiliter variat. Quumque theoria Lunæ, quamvis nondum omni numero abſoluta, eum tamen perfectionis gradum jam attigerit, ut ex Tabulis vel Ephemeridibus Aſtronomicis pro dato quovis tempore locus ejus, adeoque etiam diſtantia a ſtella quacunque cognita, ſatis exacte aſſignari queat; viciſſim pro meridiano, ad quem Tabulæ iſtæ vel Ephemerides computatæ ſunt, inveniri poterit tempus, quo Luna datam diſtantiam ab aliqua ſtella obtineat. Obſervato igitur tempore, quo in loco quocunque alio eadem ſit horum aſtrorum mutua diſtantia, differentia horum temporum exhibet differentiam meridianorum utriusque loci, unde ex cognita longitudine geographica unius innotescit longitudo alterius. Ad hunc calculum sublevandum Cel. Anglorum Astronomus MASKELYNE inde ab anno 1767 in *The Nautical Almanac* pro quolibet die & tertia quaque hora ad tempus meridiani Grenovicensis computatas exhibuit distantias geocentricas Lunae a praecipuis stellis, cum quibus hic planeta simul observari poterit; unde vicissim facillima interpolatione datae cuivis distantiae respondens invenietur tempus. Hinc igitur nullo fere negotio obtineretur longitudo loci cujusvis a dicto meridiano, si pro dato quocunque tempore in isto loco determinari posset distantia vera Lunae ab aliqua ejusmodi stella. Quum vero distantiae observatae duplici errore, scilicet vi refractionis & parallaxeos afficiantur, ad hos errores corrigendos ulteriori opus est calculo, cujus autem prolixitas nautas ab applicatione hujus methodi ad longitudines inveniendas maxime deterrere solet. Quamobrem multo studio in id incubuerunt Astronomi, ut concinniores detegerent methodos, quibus hi refractionis atque parallaxeos effectus computari, adeoque distantiae Lunae a Stellis verae ex observatis deduci possent. Has autem singulas percensere, neque instituti ratio neque proposita brevitas nobis jam permittit. Symbolam aliquantam his solum addituri, nostra qualiacunque hac in re tentamina benignae C. Lectoris censurae submittimus.

§. II.

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Quum effectus refractionis atque parallaxeos pro distantia Lunæ a stella quadam computandus sit, necesse est ut præter distantiam apparentem horum siderum simul observetur apparens utriusque altitudo supra horizontem; ex hac enim magnitudinem tam refractionis quam parallaxeos dependere notissimum est. Ad refractionem quod attinet, ejus quidem lex generalis nondum exacte cognita est, quippe quæ ex ratione, qua pro diversa altitudine minuitur densitas Atmosphæræ, deducenda est. Lex ista, qua statuitur refractio cotangenti altitudinis apparentis proportionalis, non nisi pro altitudinibus 20 gradus excedentibus admitti potest, pro stellis vero supra horizontem minus elevatis, a vero notabiliter aberrat. Hypothesis Bradleji, qua refractionem proportionalem cotangenti altitudinis apparentis ipsa refractione triplicata auctæ assumit, experientiæ magis consentanea videtur. Verum etiamsi veritas hujus regulæ omni dubio careret, ejus tamen applicatio calculum nimis implicatum redderet. Quamobrem in praxi commodissimum erit ad Tabulas Refractionum recurrere, quales v. g. extant in *Tables Astronomiques par M.* De la Lande *Tab. CLIX (Edit. 2)*, *Recueil de Tables Astron. de l' Acad. R. de Berlin Vol. III. p. 228. 229* & alibi passim. Quumque hæ Tabulæ ad mediam aliquam aëris temperiem constructæ sint, pro alia quavis constitutione aëris, ope Barometri & Thermometri examinanda, haud negligendæ sunt correctiones, quales videre licet apud De la Lande l. cit. *Tab. CLX* & in *Tabulis Berolinensibus Vol. III. p. 230 &c.* Hæc quidem correctio refractionis vulgo, præsertim a nautis, omitti solet; quum vero eandem aliquando decimam partem totius refractionis excedere citatæ tabulæ doceant, errorem minime contemnendum ex hac omissione oriri posse apparet. Quare usum Thermometri atque Barometri itinerarii etiam hoc respectu in re navali commendandum judicamus.

Parallaxis horizontalis Lunæ pro quolibet die ex Ephemeridibus assignari solet, qualis sub æquatore obtinet, & licet ob figuram terræ a sphærica parum aberrantem, exigua quidem sit variatio parallaxeos horizontalis pro diversis Latitudinibus, hanc tamen variationem, quippe facillimo calculo inveniendam, negligere opus non est. Si scilicet sit pro dato tempore parallaxis horizontalis æquatorea $= P$ &, quæ eodem tempore sub latitudine $= L$ obtinet, parallaxis horizontalis dicatur π; posito Sinu Toto $= 1$ erit satis exacte $\pi = P\,(1 - \frac{1}{200} Sin\,L^2)$. Ex data vero parallaxi horizontali $= \pi$ cuivis altitudini apparenti $= 90° - a$ vel distantiæ a zenith $= a$ competens invenitur parallaxis $= \pi\, Sin\, a$. Quoties ad scrupulos secundos attendere opus non est, parallaxis altitudinis Lunæ commode depromi etiam potest ex tabula, quam præbet Dn. Maskelyne in *Tables requisite to be used*

 by

by the Nautical Ephemeris, Tab. XII. In eodem hoc libro Tabula VIII reductionem altitudinis Lunæ apparentis ad veram vi refractionis & parallaxeos simul exhibet, cujus igitur insignis est usus, quoties refractionem mediam adhibere licet. Cum vero refractio ob diversam aëris densitatem, ut supra monuimus, corrigenda sit, utramque tam refractionem quam parallaxin seorsim secundum regulas allatas computare præstat. Eædem denique formulæ, quas pro parallaxi Lunæ dedimus, etiam pro invenienda parallaxi altitudinis Solis vel planetæ cujusvis adhibendæ sunt, quoties ex observata horum a Luna distantia longitudo loci colligenda est.

§. III.

Inventa magnitudine refractionis & parallaxeos, ex observata altitudine facile obtinetur altitudo vera. Quum videlicet vi refractionis elevatiores & vi parallaxeos depressiores appareant stellæ, de cetero autem in utroque casu eundem circulum verticalem occupent; manifestum est, altitudinem apparentem refractione minuendam & parallaxi augendam esse, quo altitudo vera inveniatur. Vel si loco altitudinum adhibeantur earundem complementa seu distantiæ stellarum a zenith, quod quidem in calculis Astronomicis plerumque commodius est, vicissim addendo refractionem & subtrahendo parallaxin, distantia a vertice apparens ad veram reducitur. Si igitur mensurata sit

Lunæ a stella quadam distantia apparens $= c$, simulque observata sit apparens a zenith distantia Lunæ $= a$ & stellæ ejusdem $= b$, ex quibus secundum regulas allatas colligitur vera a zenith distantia Lunæ $= \alpha$ & stellæ $= \beta$; eo præcipue reducitur Problema nostrum, ut ex datis a, b, α, β & c inveniatur hujus stellæ a Luna distantia vera $= \gamma$. Quocirca in antecessum observasse convenit, quod cum parallaxis Lunæ multum excedat ejus refractionem, in stellis vero fixis nulla & in sole planetisque admodum exigua parallaxis deprehendatur, semper erit $\alpha < a$ & $\beta > b$.

Quæ mox primo intuitu sese offert problematis propositi solutio, ex vulgaribus Trigonometriæ Sphæricæ regulis depromitur. Quum enim circulus verticalis cujusvis stellæ vi refractionis & parallaxeos non varietur, manifestum est Triangulum, cujus latera sunt arcus a, b & c angulum ad verticem communem habere cum Triangulo arcubus α, β & γ comprehenso. Hic igitur angulus communis, qui dicatur z, in priori horum triangulorum ex datis tribus lateribus a, b, c inveniri potest secundum alterutram harum formularum:

$$Sin\,\tfrac{1}{2}\,z^2 = \frac{Sin\,\frac{1}{2}(c+a-b)\;Sin\,\frac{1}{2}(c-a+b)}{Sin\,a\;Sin\,b};$$

$$Cos\,\tfrac{1}{2}\,z^2 = \frac{Sin\,\frac{1}{2}(a+b+c)\;Sin\,\frac{1}{2}(a+b-c)}{Sin\,a\;Sin\,b};$$

$$Tg\,\tfrac{1}{2}\,z^2 = \frac{Sin\,\frac{1}{2}(c+a-b)\;Sin\,\frac{1}{2}(c-a+b)}{Sin\,\frac{1}{2}(a+b+c)\;Sin\,\frac{1}{2}(a+b-c)}.$$

Ex hoc invento angulo z lateribusque eum intercipientibus α & β, in altero triangulo invenitur tertium latus quæsitum γ. Divisio etenim hujus Trianguli in duo Triangula rectangula secundum methodum vulgarem sequentes offert formulas:

$$Tg\,x = tg\,\alpha\,Cos\,z,\ \&\ Cos\,\gamma = \frac{Cos\,\alpha\,Cos\,(\beta - x)}{Cos\,x},$$

vel $$Tg\,y = tg\,\beta\,Cos\,z\ \&\ Cos\,\gamma = \frac{Cos\,\beta\,Cos\,(\alpha - y)}{Cos\,y}.$$

Hæc vero methodus præterquam quod prolixiorem supponat calculum, eo etiam laborat incommodo, quod in casu quovis sigillatim discernendum sit, utrum arcus x (vel y) & γ quadrante minores sumantur vel majores, quod quidem nautis in calculo trigonometrico minus exercitatis magno est impedimento, & errori locum facile præbet. Quamobrem alias breviores minusque impeditas tentare oportet vias. Primo igitur dispicere convenit, annon vulgari illa commodior aliqua obtineatur resolutio casus istius Trigonometrici, quo ex datis duobus lateribus cum angulo intercepto in quovis Triangulo Sphærico reliqua incognita inveniantur; & huic fini inserviens nobis videtur Lemma sequens.

§. IV.

Lemma. *In Triangulo quovis Sphærico si dicantur tria latera a, b, c & anguli his respective oppositi A, B, C, erit*

$1{:}o\ Tg\frac{1}{2}(A-B) = \frac{Sin\frac{1}{2}(a-b)Cotg\frac{1}{2}C}{Sin\frac{1}{2}(a+b)}$; $2{:}o\ Tg\frac{1}{2}(A+B) = \frac{Cos\frac{1}{2}(a-b)Cotg\frac{1}{2}C}{Cos\frac{1}{2}(a+b)}$,

$3{:}o\ Sin\frac{1}{2}c = \frac{Sin\frac{1}{2}(a+b)Sin\frac{1}{2}C}{Cos\frac{1}{2}(A-B)}$, $4{:}o\ Sin\frac{1}{2}c = \frac{Sin\frac{1}{2}(a-b)Cos\frac{1}{2}C}{Sin\frac{1}{2}(A-B)}$;

$5{:}o\ Cos\frac{1}{2}c = \frac{Cos\frac{1}{2}(a+b)Sin\frac{1}{2}C}{Cos\frac{1}{2}(A+B)}$; $6{:}o\ Cos\frac{1}{2}c = \frac{Cos\frac{1}{2}(a-b)Cos\frac{1}{2}C}{Sin\frac{1}{2}(A+B)}$.

Demonstrationem formularum 1 & 2 apud Auctores Trigonometriæ Sphæricæ passim videre licet. Cfr. sis MAUDUIT *Astr. Sphér.* §. 182. KLÜGEL *Analytische Trigon. Cap.* 6. §. 19. *Form.* 158. 159 &c. Reliquarum ex vulgari illa Formula Trigonometrica:

$$Cos\,c = Sin\,a\,Sin\,b\,Cos\,C + Cos\,a\,Cos\,b \quad (\mathfrak{A})$$

sic evincitur. Positis compendii caussa $\frac{1}{2}(a+b) = m$, $\frac{1}{2}(a-b) = n$, $\frac{1}{2}(A-B) = \varphi$ & $\frac{1}{2}(A+B) = \Psi$, quum (Princ. Trigon.) sit $Sin\,a\,Sin\,b = Sin\,m^2 - Sin\,n^2$, $Cos\,a\,Cos\,b = Cos\,m^2 - Sin\,n^2$, $Cos\,c = 1 - 2\,Sin\frac{1}{2}c^2$, & $Cos\,C = 1 - 2\,Sin\frac{1}{2}C^2$; his factis substitutionibus æquatio $\mathfrak{A}$ transformatur in

$1 - 2\,Sin\frac{1}{2}c^2 = (Sin\,m^2 - Sin\,n^2)(1 - 2\,Sin\frac{1}{2}C^2) + Cos\,m^2 - Sin\,n^2$, vel peracta debita reductione:

$$Sin\frac{1}{2}c^2 = Sin\,m^2\,Sin\frac{1}{2}C^2 + Sin\,n^2\,Cos\frac{1}{2}C^2 \quad (\mathfrak{B}).$$

Quumque porro (Form. 1.) sit $Tg\,\varphi = \frac{Sin\,n\,Cotg\frac{1}{2}C}{Sin\,m}$ adeoque $Sin\,n^2\,Cos\frac{1}{2}C^2 = Sin\,m^2\,Sin\frac{1}{2}C^2\,Tg\,\varphi^2$, erit $(\mathfrak{B})$

$$Sin\frac{1}{2}c^2 = Sin\,m^2\,Sin\frac{1}{2}C^2\,(1 + Tg\,\varphi^2),$$

unde pro $1 + Tg\,\varphi^2$ substituendo $\frac{1}{Cos\,\varphi^2}$ & radicem quadratam extrahendo obtinetur formula 3. Pari modo quum sit $Sin\,n^2\,Cos\frac{1}{2}C^2\,Cotg\,\varphi^2 = Sin\,m^2\,Sin\frac{1}{2}C^2$

&

& $1 + Cotg\,\varphi^2 = \frac{1}{Sin\,\varphi^2}$, his substitutionibus in aequ. 𝔅 factis eruitur formula 4. Si vero loco substitutionum superiorum sequentes adhibeantur: $Sin\,a\,Sin\,b = Cos\,n^2 - Cos\,m^2$; $Cos\,a\,Cos\,b = Cos\,m^2 - Sin\,n^2$; $Cos\,c = 2\,Cos\,\tfrac{1}{2}\,c^2 - 1$ & $Cos\,C = 2\,Cos\,\tfrac{1}{2}\,C^2 - 1$; ex aequ. 𝔄 & form. 2 eadem ratione deducuntur formulae 5 & 6.

Scholion 1. Eximius est usus hujus Lemmatis in resolvendo Triangulo quovis Sphaerico ex datis duobus lateribus cum angulo intercepto. Primo enim inveniuntur per form. 1 & 2 reliqui anguli, & dein per alterutram reliquarum formularum latus incognitum. Ubi vero mox desideratur hoc latus tertium, nec reliquos angulos seorsim nosse opus est, commodissime adhibetur aut form. 1 cum 3:ia vel 4:ta, aut form. 2 cum 5:ta vel 6:ta. Sic enim minori negotio deteguntur quantitates incognitae & simul evitatur ambiguitas ista, cujus supra (§. 3.) meminimus.

Schol. 2. Idem compendium etiam adhiberi potest in Trigonometria plana, quando ex datis trianguli rectilinei duobus lateribus a, b, & angulo intercepto C quaeritur tertium latus c. Posito scilicet $\frac{a-b}{a+b}\,Cotg\,\tfrac{1}{2}C = tg\,\varphi$, erit $c = \frac{(a+b)\,Sin\,\frac{1}{2}\,C}{Cos\,\varphi}$ vel $c = \frac{(a-b)\,Cos\,\frac{1}{2}\,C}{Sin\,\varphi}$.

Schol. 3. Vicissim ex datis in Triangulo Sphaerico duobus angulis A, B & latere interjecto c, reliqua latera a, b & tertius angulus C invenientur per sequentes

tes formulas, ope eorum quae in Trigonometria sphaer. de Triangulis Supplementalibus traduntur, ex lemmate nostro facillime deducendas:

$$1{:}o\ Tg\tfrac{1}{2}(a-b)=\frac{Sin\tfrac{1}{2}(A-B)\,tg\tfrac{1}{2}c}{Sin\tfrac{1}{2}(A+B)},\quad 2{:}o\ Tg\tfrac{1}{2}(a+b)=\frac{Cos\tfrac{1}{2}(A-B)\,tg\tfrac{1}{2}c}{Cos\tfrac{1}{2}(A+B)},$$

$$3{:}o\ Cos\tfrac{1}{2}C=\frac{Sin\tfrac{1}{2}(A+B)\,Cos\tfrac{1}{2}c}{Cos\tfrac{1}{2}(a-b)},\quad 4{:}o\ Cos\tfrac{1}{2}C=\frac{Sin\tfrac{1}{2}(A-B)\,Sin\tfrac{1}{2}c}{Sin\tfrac{1}{2}(a-b)},$$

$$5{:}o\ Sin\tfrac{1}{2}C=\frac{Cos\tfrac{1}{2}(A+B)\,Cos\tfrac{1}{2}c}{Cos\tfrac{1}{2}(a+b)},\quad 6{:}o\ Sin\tfrac{1}{2}C=\frac{Cos\tfrac{1}{2}(A-B)\,Sin\tfrac{1}{2}c}{Sin\tfrac{1}{2}(a+b)}.$$

§. V.

Applicatio Lemmatis praec. ad Problema nostrum solvendum ex iis quae §. 3 disseruimus, facilis est. In triangulo scilicet quod distantiis apparentibus comprehenditur, secundum alterutram (§. 3) allatarum formularum ex datis tribus lateribus primo invenitur angulus ad verticem. Quo facto in altero Triangulo distantiarum verarum ex cognito hoc angulo & lateribus eundem intercipientibus invenitur (§. 4. Schol. 1.) latus tertium quaesitum. Quumque pro supputando hoc latere quatuor diversae in Lemmate praec. occurrant formulae, totidem hinc eruuntur methodi problema nostrum solvendi, quas brevitatis studio sequentibus formulis analyticis exhibebimus, in quibus eaedem ac §. 3 obtinent denominationes, adeo ut scilicet sit

Distantia a zenith apparens Lunæ	=	a,
- - - - - - - - - Stellæ	=	b,
- - - - - - - - vera Lunæ	=	α,
- - - - - - - - - - Stellæ	=	β,
Distantia Lunæ a Stella apparens	=	c,
- - - - - - - - - - - vera	=	γ,

Methodus prima: $Sin\,\frac{1}{2}z^2 = \frac{Sin\,\frac{1}{2}(c+a-b)\,Sin\,\frac{1}{2}(c-a+b)}{Sin\,a\;Sin\,b}$;

$Tg\,\varphi = \frac{Sin\,\frac{1}{2}(\alpha-\beta)\,Cotg\,\frac{1}{2}z}{Sin\,\frac{1}{2}(\alpha+\beta)}$; $Sin\,\frac{1}{2}\gamma = \frac{Sin\,\frac{1}{2}(\alpha+\beta)\,Sin\,\frac{1}{2}z}{Cos\,\varphi}$.

Methodus 2:da: $Cos\,\frac{1}{2}z^2 = \frac{Sin\,\frac{1}{2}(a+b+c)\,Sin\,\frac{1}{2}(a+b-c)}{Sin\,a\;Sin\,b}$;

$Tg\,\varphi = \frac{Sin\,\frac{1}{2}(\alpha-\beta)\,Cotg\,\frac{1}{2}z}{Sin\,\frac{1}{2}(\alpha+\beta)}$; $Sin\,\frac{1}{2}\gamma = \frac{Sin\,\frac{1}{2}(\alpha-\beta)\,Cos\,\frac{1}{2}z}{Sin\,\varphi}$.

Methodus 3:tia: $Sin\,\frac{1}{2}z^2 = \frac{Sin\,\frac{1}{2}(c+a-b)\,Sin\,\frac{1}{2}(c-a+b)}{Sin\,a\;Sin\,b}$;

$Tg\,\Psi = \frac{Cos\,\frac{1}{2}(\alpha-\beta)\,Cotg\,\frac{1}{2}z}{Cos\,\frac{1}{2}(\alpha+\beta)}$; $Cos\,\frac{1}{2}\gamma = \frac{Cos\,\frac{1}{2}(\alpha+\beta)\,Sin\,\frac{1}{2}z}{Cos\,\Psi}$.

Methodus 4:ta: $Cos\,\frac{1}{2}z^2 = \frac{Sin\,\frac{1}{2}(a+b+c)\,Sin\,\frac{1}{2}(a+b-c)}{Sin\,a\;Sin\,b}$;

$Tg\,\Psi = \frac{Cos\,\frac{1}{2}(\alpha-\beta)\,Cotg\,\frac{1}{2}z}{Cos\,\frac{1}{2}(\alpha+\beta)}$; $Cos\,\frac{1}{2}\gamma = \frac{Cos\,\frac{1}{2}(\alpha-\beta)\,Cos\,\frac{1}{2}z}{Sin\,\Psi}$.

In his formulis z denotat angulum ad zenith, φ vero semidifferentiam & Ψ semisumman reliquorum angulorum in Triangulo sphærico, quod inter Zenith & loca vera utriusque sideris constituitur. Ut quantitates negativæ in iisdem evitentur, quoties fuerit $\alpha < \beta$, loco $\alpha - \beta$ substituatur $\beta - \alpha$. Primo quidem intuitu hæ methodi æque prolixæ videntur ac vulgaris illa methodus trigonometrica (§. 3), quum in iis singulis pariter ac in hac tres adhibendæ sint formulæ. Re autem attentius considerata patebit, eas præ hac multum præstare & ipsi methodo maxime commendatæ D:ni DE BORDA concinnitate vix cedere. Opus videlicet non est ipsos angulos auxiliares z & φ (vel Ψ) nosse, verum sufficit ipsi *Log Sin* $\frac{1}{2}z$ (vel *L Cos* $\frac{1}{2}z$) respondentem mox sumere *L Cotg* $\frac{1}{2}z$, qui Logarithmi in una eademque linea in Canone Trigonometrico inveniuntur; quod idem de ang. φ (& Ψ) valet. Unde evenit ut totus hic calculus non plures quam novem Logarithmos ex tabulis trigonometricis evolvendos postulet. Illustrationis caussa exemplum adferemus idem, quo ad explicandam methodum D:ni DE BORDA utitur D:nus DE LA LANDE *Astron. Tom. IV. p.* 757. Existente scilicet

$a = 62°\ 30'$. $c = 102°\ 30'$. $\alpha = 61°\ 41'\ 13''$

$b = 74°\ 35'$. $\beta = 74°\ 38'\ 17''$.

secundum methodum nostram primam calculus ita instituitur:

$b - a = 12°\ 5'\ 0''$	$\beta + \alpha = 136°\ 19'\ 30''$.
$\frac{1}{2}(b - a) = 6°\ 2'\ 30''$.	$\beta - \alpha = 12.\ 57.\ 4.$
$\frac{1}{2}c = 51.\ 15.\ 0.$	$\frac{1}{2}(\beta + \alpha) = 68.\ 9.\ 45.$
$\frac{1}{2}(c + a - b) = 45.\ 12.\ 30.$	$\frac{1}{2}(\beta - \alpha) = 6.\ 28.\ 32.$
$\frac{1}{2}(c - a + b) = 57.\ 17.\ 30.$	

$LSin\tfrac{1}{2}(c+a-b) = \bar{1}.8510584$

$LSin\tfrac{1}{2}(c-a+b) = \bar{1}.9250191$

$-LSin\ a = 0.0520711$

$-LSin\ b = 0.0159148$

$2LSin\tfrac{1}{2}z = \bar{1}.8440634$

$LCotg\tfrac{1}{2}z = \bar{1}.8177313 \qquad LSin\tfrac{1}{2}z = \bar{1}.9220317$

$-LSin\tfrac{1}{2}(\beta+\alpha) = 0.0323385 \qquad LSin\tfrac{1}{2}(\beta+\alpha) = 1.9676615$

$LSin\tfrac{1}{2}(\beta-\alpha) = \bar{1}.0522294 \qquad -LCos\varphi = 0.0013803$

$LTg\varphi = \bar{2}.9022992 \qquad LSin\tfrac{1}{2}\gamma = \bar{1}.8910735$

$\tfrac{1}{2}\gamma = 51^\circ\ 5'\ 35''\ 5''' \qquad \gamma = 102^\circ\ 11'\ 11''$

Reliquarum methodorum omnino ſimilis eſt applicatio.

§. VI.

Ex vulgaribus iſtis formulis (§. 3), quarum ope angulus quivis in Triangulo Sphærico ex datis tribus lateribus inveſtigari ſolet, commodæ etiam pro ſolvendo problemate noſtro regulæ haud difficulter eruuntur. Retentis namque iisdem ac §. §. 3. 5 denominationibus, erit

$$\frac{Sin\tfrac{1}{2}(\gamma+\alpha-\beta)\,ſin\tfrac{1}{2}(\gamma-\alpha+\beta)}{Sin\,\alpha\ Sin\,\beta}\left(=ſin\tfrac{1}{2}z^2\right)=\frac{ſin\tfrac{1}{2}(c+a-b)\,ſin\tfrac{1}{2}(c-a+b)}{Sin\,a\ Sin\,b}$$

$$\&\ \frac{ſin\tfrac{1}{2}(\alpha+\beta+\gamma)\,ſin\tfrac{1}{2}(\alpha+\beta-\gamma)}{Sin\,\alpha\ Sin\,\beta}\left(=coſ\tfrac{1}{2}z^2\right)=\frac{ſin\tfrac{1}{2}(a+b+c)\,ſin\tfrac{1}{2}(a+b-c)}{Sin\,a\ Sin\,b};$$

$$\text{ergo}\ ſin\tfrac{1}{2}(\gamma+\alpha-\beta)\,ſin\tfrac{1}{2}(\gamma-\alpha+\beta)=\frac{ſin\tfrac{1}{2}(c+a-b)\,ſin\tfrac{1}{2}(c-a+b)\,ſin\alpha\,ſin\beta}{Sin\,a\ Sin\,b}\ (A),$$

& $\sin\frac{1}{2}(\alpha+\beta+\gamma)\sin\frac{1}{2}(\alpha+\beta-\gamma)=\frac{\sin\frac{1}{2}(a+b+c)\sin\frac{1}{2}(a+b-c)\sin\alpha\sin\beta}{\sin a \sin b}(B)$.

Quumque porro (Princ. Trigon.) sit

$\sin\frac{1}{2}(\gamma+\alpha-\beta)\sin\frac{1}{2}(\gamma-\alpha+\beta)=\sin\frac{1}{2}\gamma^2-\sin\frac{1}{2}(\alpha-\beta)^2$ (C),

vel $\sin\frac{1}{2}(\gamma+\alpha-\beta)\sin\frac{1}{2}(\gamma-\alpha+\beta)=\operatorname{Cos}\frac{1}{2}(\alpha-\beta)^2-\operatorname{Cos}\frac{1}{2}\gamma^2$ (D),

& $\sin\frac{1}{2}(\alpha+\beta+\gamma)\sin\frac{1}{2}(\alpha+\beta-\gamma)=\sin\frac{1}{2}(\alpha+\beta)^2-\sin\frac{1}{2}\gamma^2$ (E),

vel $\sin\frac{1}{2}(\alpha+\beta+\gamma)\sin\frac{1}{2}(\alpha+\beta-\gamma)=\operatorname{Cos}\frac{1}{2}\gamma^2-\operatorname{Cos}\frac{1}{2}(\alpha+\beta)^2$ (F),

facta substitutione C vel D in æqu. A, & E vel F in æqu. B, terminisque debite transpositis, quatuor sequentes obtinentur æquationes:

I. $\sin\frac{1}{2}\gamma^2=\sin\frac{1}{2}(\alpha-\beta)^2+\frac{\sin\frac{1}{2}(c+a-b)\sin\frac{1}{2}(c-a+b)\sin\alpha\sin\beta}{\sin a \sin b}$;

II. $\operatorname{Cos}\frac{1}{2}\gamma^2=\operatorname{Cos}\frac{1}{2}(\alpha-\beta)^2-\frac{\sin\frac{1}{2}(c+a-b)\sin\frac{1}{2}(c-a+b)\sin\alpha\sin\beta}{\sin a \sin b}$;

III. $\sin\frac{1}{2}\gamma^2=\sin\frac{1}{2}(\alpha+\beta)^2-\frac{\sin\frac{1}{2}(a+b+c)\sin\frac{1}{2}(a+b-c)\sin\alpha\sin\beta}{\sin a \sin b}$;

IV. $\operatorname{Cos}\frac{1}{2}\gamma^2=\operatorname{Cos}\frac{1}{2}(\alpha+\beta)^2+\frac{\sin\frac{1}{2}(a+b+c)\sin\frac{1}{2}(a+b-c)\sin\alpha\sin\beta}{\sin a \sin b}$.

In quibus manifestum est, salva æquatione pro $\alpha-\beta$ substitui posse $\beta-\alpha$, quando fuerit $\beta>\alpha$. Singulæ vero hæ æquationes debita substitutione ita transformari possunt, ut ope Logarithmorum calculum satis commodum admittant; unde quatuor aliæ problema nostrum solvendi sequentes oriuntur methodi:

Methodus 5:ta: $Tg M^2=\frac{\sin\frac{1}{2}(c+a-b)\sin\frac{1}{2}(c-a+b)\sin\alpha\sin\beta}{\sin\frac{1}{2}(\alpha-\beta)^2 \sin a \sin b}$;

$$\sin\tfrac{1}{2}\gamma=\frac{\sin\frac{1}{2}(\alpha-\beta)}{\operatorname{Cos} M}.$$

Me-

Methodus 6:ta: $Cos N^2 = \frac{Sin\frac{1}{2}(c+a-b)\, Sin\frac{1}{2}(c-a+b)\, Sin\alpha\, Sin\beta}{Cos\frac{1}{2}(\alpha-\beta)^2\, Sin\, a\, Sin\, b}$,

$Cos\frac{1}{2}\gamma = Cos\frac{1}{2}(\alpha-\beta)\, Sin N.$

Method. 7:ma: $Cos P^2 = \frac{Sin\frac{1}{2}(a+b+c)\, Sin\frac{1}{2}(a+b-c)\, Sin\alpha\, Sin\beta}{Sin\frac{1}{2}(\alpha+\beta)^2\, Sin\, a\, Sin\, b}$,

$Sin\frac{1}{2}\gamma = Sin\frac{1}{2}(\alpha+\beta)\, Sin P.$

Method. 8:va: $Tg Q^2 = \frac{Sin\frac{1}{2}(a+b+c)\, Sin\frac{1}{2}(a+b-c)\, Sin\alpha\, Sin\beta}{Cos\frac{1}{2}(\alpha+\beta)^2\, Sin\, a\, Sin\, b}$,

$Cos\frac{1}{2}\gamma = \frac{Cos\frac{1}{2}(\alpha+\beta)}{Cos Z}.$

Harum methodorum septima est ipsa methodus D:ni DE BORDA, cujus igitur hac ratione multo concinnior habetur analysis quam quæ extat *in Astronomie par M.* DE LA LANDE *Tom. IV. p. 754---756.* Methodos etiam his nostris V---VIII similes, verum alia via inventas adfert Cel. LEXELL in *Act. Acad. Scient. Imp. Petropol. pro A. 1777. P. II. p. 344---346.*

Applicationem harum formularum illustraturi eodem ac in §. 5. utemur exemplo, quod secundum methodum V. ita computatur:

$a = 62°\ 30'\ 0''$	$L\,Sin\frac{1}{2}(c+a-b) = \bar{1}.8510584$
$b = 74.\ 35.\ 0.$	$L\,Sin\frac{1}{2}(c-a+b) = \bar{1}.9250191$
$c = 102.\ 30.\ 0.$	$L\,Sin\,\alpha$ - - - - - $= \bar{1}.9446649$
$\alpha = 61.\ 41.\ 13.$	$L\,Sin\,\beta$ - - - - - $= \bar{1}.9841994$
$\beta = 74.\ 38.\ 17.$	$-L\,Sin\,a$ - - - $= 0.0520711$
$b-a = 12.\ 5.\ 0.$	$-L\,Sin\,b$ - - - $= 0.0159148$
	2) $\bar{1}.7729277$

$\frac{1}{2}(b-a) = 6° \; 2' \; 30.''$ | 2) $\bar{1}.\,7729277$

$\frac{1}{2}c = 51.\,15.\,0.$ | *) $\bar{1}.\,8864639$

$\frac{1}{2}(c+a-b) = 45.\,12\,30.$ | $L\,Sin\,\frac{1}{2}(\beta-\alpha) = \bar{1}.\,0522294$

$\frac{1}{2}(c-a+b) = 57.\,17.\,30.$ | $L\,Tg\,M = 0.\,8342345$

$\beta - \alpha = 12.\,57.\,4.$ | $L\,Cos\,M = \bar{1}.\,1611559$

$\frac{1}{2}(\beta-\alpha) = 6.\,28.\,32.$ | $L\,Sin\,\frac{1}{2}\gamma = \bar{1}.\,8910735$

$\frac{1}{2}\gamma = 51° \, 5' \, 35.'' \, 5;$ $\gamma = 102° \, 11'' \, 11''$

Calculus ejusdem exempli secundum methodum VI plane similis est usque ad Logarithmum, quem asterisco notavimus. Hoc vero invento ita continuatur:

*) $\bar{1}.\,8864639$

$L\,Cos\,\frac{1}{2}(\beta-\alpha) = \bar{1}.\,9972203$

$L\,Cos\,N = \bar{1}.\,8892436$

$L\,Sin\,N = \bar{1}\,8007776$

$L\,Cos\,\frac{1}{2}\gamma = \bar{1}.\,7979979$

$\frac{1}{2}\gamma = 51° \, 5' \, 35.'' \, 5;$ $\gamma = 102° \, 11' \, 11.''$

§. VII.