# Dissertatio Astronomica,
Methodum Sistens
Inveniendi
# *Tempus Verum*
Ex Observatis Æqualibus Diversarum
Stellarum Altitudinibus.

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Quam
*Cons. Ampl. Fac. Philos. Aboëns.*
Præside
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Math. Profess. Reg. & Ord. nec non Reg. Acad. Scient. Svec. Membro
*Pro Gradu*
Publicæ Censuræ offert
*Adamus Johannes Tammelander*,
Tavastensis.
In *Auditorio Majori* die XXVI. Novembr.
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Horis ante meridiem consvetis.

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*Aboæ*, Typis Viduæ R. Acad. Typogr. *J. C. Frenckell*.

Kongl. Maj:ts

Tro-Tjenare, Lieutenanten

Wålborne

# Herr *Carl Gustaf Wünsch*;

Min Högtärade Herr Morfar!

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Min Högtärade Herr Morfaders

Ödmjuklydigste Dotterson

*Adam Joh. Tammelander.*

Kongl. Maj:ts

Tro-Tjenare, Cornetten

Wälädle

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Samt

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*Adam Joh. Tammelander.*

## §. I.

*Tempus Verum*, quod, aliis *Apparens* dictum, motu Solis diurno mensuratur, variis modis ab Astronomis investigari solet. Usitatissima vero est & simplicitate sua semet præ ceteris commendans Methodus, quæ dicitur *Altitudinum correspondentium*. Observatis scilicet æqualibus unius ejusdemque sideris ante & post culminationem altitudinibus, medium inter utramque observationem tempus, ipsum culminationis momentum exhibet; nisi interea immutata fuerit stellæ declinatio, quo in casu aliqua correctione opus est, quæ *correctio meridiei* vel *æquatio Altitudinum correspondentium* dicitur. Hæc vero methodus licet ad determinandum absque prolixo calculo tempus verum sit admodum idonea, suis tamen non caret difficultatibus. Major inprimis quæ inter hujusmodi observationes requiritur, temporis mora Methodum hanc incommodam reddit. Quum enim stellarum, quo ad meridianum sunt propiores, eo minor sit altitudinis variatio, necesse est, ut plurium, (quatuor minimum vel sex) horarum intervallo distent ipsæ observationes, quo debita exactitudine altitudines correspondentes notari queant. Tanto vero temporis spatio, præterquam quod deficiens cœli serenitas laborem haud raro frustretur, sæpissime etjam refractionum Astronomicarum variatio, differentiam altitudinum haud contemnendam parit. Ex vario namque calore ceterisque caussis quum continuis fere mutationibus obnoxia sit aëris densitas & quæ hinc pendet radiorum luminis in atmosphæra terrestri refractio, interlapsis pluribus horis hujus inæqualitas ita increscere facile potest, ut error inde satis notabilis in altitudinibus correspondentibus oriatur. Ut igitur huic incommodo quantum fieri posset medelam adferrent, & minori temporis dispendio institutis observationibus propositum obtinerent, alias Astronomi tempus verum inveniendi rationes excogitarunt. Inter has attentione omnino digna nobis visa est methodus Dni Köhler, Astronomi *Dresdensis* sollertissimi, ex correspondentibus diversarum stellarum altitudinibus horam noctis inveniendi, cujus expositionem in Ephemeridibus Berolinensibus seu *Astronomisches Jahrbuch für das Jahr 1784 pagg. 153--155.* videre licet, ubi rationem suæ inventionis ita reddit: "Die Unbequemlichkeit bey correspondirenden Sternhöhen 5 bis 6 "Stunden auf die nach ihrem Durchgang durch den "Meridian zu nehmenden Sternen, warten zu müssen, "hat mich schon långst auf einen Einfall gebracht, "dessen Ausführung ich jedoch nicht für ganz allge- "mein, sondern nur in einigen Fållen für brauchbar "ausgeben kan, weil meistentheils bey denselben die

Auf-

Auflösung eines Dreyecks vorfällt, an den eine Sei-"
te ein Stück eines kleinern Cirkuls ist. - - - Diese"
Methode würde sehr brauchbar seyn, weil die weni-"
ge Zeit, die man zur Berechnung anwendet, nicht"
wie die der Beobachtung von der Witterung abhängt,"
die oft die einseitig genommene Höhen, wenn sie"
sich ändert, vergeblich macht, und hier noch der"
Vortheil eintritt, dass sich in der kurzen Zeit der Be-"
obachtung, die Refraction nicht so viel ändern kann,"
als wenn zwischen den Beobachtungen 5 bis 6 Stun-"
den verfliessen." Quoniam vero Dn. KÖHLER nihil hic datum supponat præter rectas Ascensiones & Declinationes stellarum, quarum altitudines correspondentes observantur, una cum temporis intervallo inter utramque observationem interlapso, problema hoc omnino indeterminatum est, nisi exacte æquales fuerint ipsarum stellarum declinationes. Quod quum rarissime eveniat, ut in omnibus casibus problema hoc solutionem admitteret, ipsam altitudinem observatam in calculum inducendam judicavit. Hoc autem facto, incertitudo refractionis methodum Ejus minus tutam & prolixitas calculi admodum incommodam reddit. His perspectis difficultatibus, de faciliori quadam & magis idonea via ad investigandum tempus verum ex observatis æqualibus diversarum stellarum altitudinibus cogitare cœpimus; benignæ jam C. Lectoris censuræ submittentes, quæ nobis succurrerunt problematum quorundam huc pertinentium solutiones.



§. II.

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PROBLEMA. *Observatis æqualibus duarum Stellarum altitudinibus una cum intervallo temporis inter observationes, datisque latitudine loci atque Stellarum declinationibus & ascensionibus rectis, invenire tempus verum utriusque observationis, veramque Stellarum altitudinem.*

Sit (Fig. 1. 2.) $PZ$ arcus meridiani, $P$ polus, $Z$ zenith, tempore $T$ observata Stella una in $A$, & tempore $T + t$ altera æquialta in $B$, temporibus hisce computatis juxta horologium, in quo toti revolutioni sidereæ seu 360° æquatoris respondeant $24^h - m$ (ita ut si ex. gr. ad motum Solis medium, ut communiter fieri solet, compositum sit horologium, fiat $m = 3'56''$ temporis); dicanturque Stellarum $A$ & $B$ ascensiones rectæ $a$ & $a + \alpha$, earumque declinationes versus polum elevatum numeratæ $D - \delta$ & $D + \delta$ respective, nec non latitudo loci $p$; ductis arcubus circulorum maximorum $ZA$, $ZB$, $PA$ & $PB$, erit in $\Delta\Delta\ PZA$ & $PZB$, $PZ = 90° - p$, $PA = 90° - D + \delta$, & $PB = 90° - D - \delta$ nec non $ZA = ZB$. Quæratur jam primo angulus horarius $= H$, intervallo temporis $t$ respondens, inferendo: $24^h - m : t :: 360° : H$, quo facto invenitur angulus $APB = \alpha - H$, qui compendii caussa dicatur $2\lambda$. Bisecetur porro Ang. $APB$ arcu $PR$, sitque ang. $ZPR = z$ (versus orientem a Zenith sumtus, adeoque si in oppositam partem

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cadat, signo contrario afficiendus); erit $BPZ = \lambda + z$ & $APZ = \pm \lambda \mp z$, ubi valent signa superiora pro (Caf. 1. Fig. 1.) Stellis $A$ & $B$ ad diversas partes meridiani positis, sed signa inferiora, quando (Caf. 2. Fig. 2.) utraque ad eandem partem observatur. Posito igitur Sinu toto $= 1$, erit (Trig. Sphær.) $Cos.\ AZ = Cos.\ p\ Cos\ (D - \delta)\ Cos\ (\pm \lambda \mp z) + Sin\ p\ Sin\ (D - \delta)$ & $Cos\ BZ = Cos\ p\ Cos\ (D + \delta)\ Cos\ (\lambda + z) + Sin\ p\ Sin\ (D + \delta)$, quibus æquationibus inter se collatis & div. per $Cos\ p$, obtinetur $Cos\ (D - \delta)\ Cos\ (\pm \lambda \mp z) - Cos\ (D + \delta)\ Cos\ (\lambda + z) = Tang.\ p\ [Sin\ (D + \delta) - Sin\ (D - \delta)]$. Facta vero terminorum evolutione, & div. per $2\ Cos\ D\ Cos\ \delta\ Sin\ \lambda$, prodit $Sin\ z + Tg\ D\ Tg\ \delta\ Cotg\ \lambda\ Cos\ z = \frac{Tg\ p\ Tg\ \delta}{Sin\ \lambda}$. Ut facilius hinc inveniri queat angulus $z$, statuatur $Tg\ D\ Tg\ \delta\ Cotg\ \lambda = Tg\ \varphi$, quo facto æquatio inventa in hanc transmutatur: $Sin\ z + Tg\ \varphi\ Cos\ z = \frac{Tg\ p\ Tg\ \delta}{Sin\ \lambda}$, & (multipl. per $Cos\ \varphi$) $Sin\ z\ Cos\ \varphi + Cos\ z\ Sin\ \varphi = \frac{Tg\ p\ Tg\ \delta\ Cos\ \varphi}{Sin\ \lambda} = Sin\ (z + \varphi)$. Hinc igitur facile invenitur angulus quæsitus $z$ ope formularum: $Tg\ \varphi = Tg\ D\ Tg\ \delta\ Cotg\ \lambda$, & $Sin\ (z + \varphi) = \frac{Tg\ p\ Tg\ \delta\ Cos\ \varphi}{Sin\ \lambda}$. Invento vero $z$, in alterutro $\Delta\Delta\ APZ$, $BPZ$ ex datis duobus lateribus cum angulo intercepto innotescit latus tertium $AZ$ vel $BZ$ adeoque hujus complementum, scil. altitudo Stellarum vera, cujus differentia ab altitudine observata exhibet refractionem huic altitudini competentem. Nec difficile est ex cognito angulo $z$ colligere tempus Solare verum. Sumtis scilicet ex Tabulis vel Ephemeridibus Astronomicis ad meridiem proxime antecedentem & insequentem ascensionibus rectis Solis, quae sint $S$ & $S + s$ respective, inferatur: Ut $360° + s$ ad $a + \lambda - z - S$ (vel si $S > a + \lambda - z$, ad $360° + a + \lambda - z - S$) ita $24^h$ ad quaesitum tempus verum, a meridie proxime praecedente numeratum & tempori $T$ ad horologium computato respondens.

*Coroll.* Si in Casu 1. fuerit $\delta$ tam parva ut sine sensibili errore poni queat $tg\,\delta = \delta$, nec admodum propinquae ad meridianum observatae sint Stellae $A$ & $B$, erit $z = \delta\left(\frac{Tg\,p}{Sin\,\lambda} - \frac{Tg\,D}{Tg\,\lambda}\right)$, quae formula vulgaris est pro computanda Correctione Meridiei, ex correspondentibus Solis altitudinibus inventi.

*Exempl. 1.* Sub Elevatione Poli $= 60° 27' 10'' = p$ horologio ad motum Solis medium constructo, die 4 Octobris hujus Anni $6^h 22' 10'' = T$ tempore vespertino in altitudine apparente $= 23° 36' 30''$ observatus sit versus occidentem *Arcturus*, cujus isto die est declinatio $20° 19' 12'' = D - \delta$ & Asc. R. $211° 29' 4'' = a$, atque $6^h 40' 35'' = T + t$ seu interjecto tempore $18' 25'' = t$ ad partem coeli orientalem in eadem altituditudine inventa ſit $\gamma$ *Pegaſi*, cujus declinatio eſt $13^\circ\ 59'\ 44'' = D + \delta$ & aſc. R. $0^\circ\ 33'\ 54''$ vel potius $360^\circ\ 33'\ 54'' = a + \alpha$, (add. ſcil. $360^\circ$ ut obtineatur $\alpha < 180^\circ$); exiſtente aſcenſione recta Solis ad tempus meridianum ejusdem diei $= 190^\circ\ 38'\ 6'' = S$ atque hujus variatione diurna $= 54'\ 45'' = s$. His datis pro inveniendo tempore vero calculus ita ſubducitur:

$$24^h - m = 23^h\ 56'\ 4'' : t = 18'\ 25'' :: 360^\circ : H = 4^\circ\ 37'.$$

| | |
|---|---|
| $a + \alpha = 360^\circ\ 33'\ 54''$ | $D + \delta = +\ 13^\circ\ 59'\ 44''$ |
| $a = 211^\circ\ 29.\ 4$ | $D - \delta = +\ 20.\ 19.\ 12$ |
| $\alpha = 149^\circ\ 4.\ 50$ | $2D = +\ 34.\ 18.\ 56$ |
| $H = 4.\ 37.\ 00$ | $2\delta = -\ 6.\ 19.\ 28$ |
| $2\lambda = 144^\circ.27'.50''.$ | $D = +\ 17^\circ\ 9'\ 28''$ |
| $\lambda = 72^\circ.13'.55''$ | $\delta = -\ 3^\circ.9'.44''$ |
| $\text{Log } Tg\ \delta = \bar{2}.7423111.$ | $\text{Log } Tg\ \delta = \bar{2}.7423111.$ |
| $\text{L. } Tg\ D = \bar{1}.4895990.$ | $\text{L. } Tg\ p = 0.2465232$ |
| $\text{L. } Cotg\ \lambda = \overline{\phantom{1}}.5057602.$ | $-\ \text{L } Sin\ \lambda = 0.0212264$ |
| $\text{L } Tg\ \varphi = \bar{3}.7376703.$ | $\text{L } Cof\ \varphi = \bar{1}.9999935$ |
| $\varphi = -\ 18'\ 47''$ | $\text{L } Sin(z + \varphi) = \bar{1}.0100542$ |
| $z + \varphi = -\ 5^\circ\ 52'\ 26''$ | $a = 211^\circ\ 29'\ 4''$ |
| $z = -\ 5^\circ\ 33'\ 39''$ | $\lambda = 72.\ 13.\ 55.$ |
| | $-z = 5.\ 33.\ 39.$ |
| | $a + \lambda - z = 289.\ 16.\ 38$ |
| | $S = 190.\ 38.\ 6$ |
| | $a + \lambda - z - S = 98^\circ\ 38'\ 32''$ |

$360^\circ\ 54'\ 45'' : 98^\circ\ 38'\ 32'' :: 24^h : 6^h\ 33'\ 34''$ tempus verum primæ obſervationis, cujus igitur hoc in caſu a

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tempore juxta horologium computato differentia est $= 11' 24''$. Si porro secundum vulgares Trigonometriæ Sphæricæ regulas solvatur $\Delta APZ$, in quo cognita sunt $AP = 69° 40' 48''$, $PZ = 29° 32' 50''$ & ang. $APZ = 77° 47' 34''$, invenitur $AZ = 66° 25' 44'', 3$ adeoque Stellæ in $A$ vera altitudo $= 23° 34' 15'' 7$.

*Exempl.* 2. Eodem loco eodemque horologio die ejusdem Mensis 10 in altitudine apparente $= 16° 36' 30''$ versus orientem observatæ sint $\beta$ *Orionis* $13^h 26' 8'' = T$ & *Procyon* $13^h 33' 32'' = T + t$, ita ut sit intervallum temporis inter utramque observationem seu $t = 7' 24''$, cui respondet ang. $H = 1° 51' 18'$. Isto die est $S = 196° 8' 11''$ & $s = 55' 23''$; Stellæ prioris Asc. recta $= 76° 4' 21''$ & Decl. $= - 8° 27' 27''$ (signo scil. negativo notanda, quia versus polum depressum ab æquatore hæc stella declinat), atque posterioris Asc. R. $= 112° 1' 49''$ & Decl. $= 5° 46' 22''$; unde sequente ratione tempus verum investigatur:

| | |
|---|---|
| $a + \alpha = 112° 1' 49''$ | $D + \delta = + 5° 46' 22''$ |
| $a = 76. 4. 21.$ | $D - \delta = - 8. 27. 27.$ |
| $\alpha = 35. 57. 28.$ | $2 D = - 2. 41. 5$ |
| $H = 1. 51. 18.$ | $2 \delta = + 14. 13. 49.$ |
| $2 \lambda = 34. 6. 10$ | $D = - 1° 20' 32'', 5$ |
| $\lambda = 17°. 3'. 5''$ | $\delta = + 7° 6' 54, 5$ |

L. Tg

| | |
|---|---|
| $L.\ Tg\ \delta = \bar{1}.\ 0962961.$ | $L\ Tg\ \delta = \bar{1},\ 0962961$ |
| $L.\ Tg\ D = \bar{2}.\ 3698352.$ | $L\ Tg\ p = 0,\ 2465232$ |
| $L.\ Cotg\ \lambda = 0.\ 5132696$ | $-L\ Sin\ \lambda = 0,\ 5327927$ |
| $L.\ Tg\ \varphi = \bar{3}.\ 9794009$ | $L\ Cos\ \varphi = \bar{1},\ 9999803$ |
| $\varphi = -\ 32'\ 47''$ | $L\,Sin(z+\varphi) = \bar{1},8755923$ |
| $z + \varphi = +48^\circ\ 40'\ 12''$ | $a = 76^\circ\ 4'\ 21''$ |
| $z = 49^\circ\ 12'\ 59''$ | $360^\circ + \lambda = 377^\circ\ 3.\ 5.$ |
| $S = 196.\ 8.\ 11.$ | $360^\circ + a + \lambda = 453.\ 7.\ 26$ |
| $z + S = 245.\ 21.\ 10$ | $z + S = 245.\ 21.\ 10$ |
| | $360^\circ + a + \lambda - z - S = 207^\circ\ 46'\ 16''.$ |

$360^\circ\ 55'\ 23'' : 207^\circ\ 46'\ 16'' :: 24^h : 13^h\ 48'\ 58''$ quæsitum tempus verum, tempori $T = 13^h\ 26'\ 8''$ juxta horologium respondens, cujus itaque ab illo differentia est $= 22'\ 50''$.

*Scholion.* Si in casu 1. exacte æquales forent Stellarum observatarum declinationes, adeoque $\delta = 0$, foret etjam $z = 0$, unde simplici regula proportionum inveniretur tempus verum. Hoc vero cum rarissime accidat, necesse est ut præter æqualitatem altitudinum, tempus inter observationes præterlapsum, Stellarumque ascensiones rectas atque declinationes, aliquid adhuc detur, quo determinatum fiat hoc problema. Hanc ob rationem Dn. KÖHLER (§. I.) cognitam supponit altitudinem veram, qua data tempus quidem verum per regulas Trigonometricas ita investigari potest: In

Triangulo *APB* ex datis lateribus *AP*, *BP* & angulo intercepto *APB*, quæritur angulus *PBA* (vel *PAB*) & latus tertium *AB*, quo facto ex cognitis in Δ *ZBA* tribus lateribus invenitur ang. *ZBA* (vel *ZAB*), adeoque etjam angulorum *PBA* & *ZBA* (vel *PAB* & *ZAB*) differentia *ZBP* (vel *ZAP*), unde denique in Δ *ZBP* (vel *ZAP*) ex invento ang. *PBZ* (vel *PAZ*) & lateribus hunc comprehendentibus *PB* (vel *PA*) & *BZ* (vel *AZ*) eruitur ang. *ZPB* (vel *ZPA*), cujus ope eadem ratione qua supra usi sumus, tempus verum innotescit. Præterquam vero quod altitudinem veram ex observata collectam, ut in §. I. monuimus, refractio varia aliquantulum incertam semper reddat, calculum etjam prolixiorem & admodum tædiosum hæc methodus postulat, ut alia ejus incommoda taceamus. Potius igitur inter data in hoc problemate assumendam judicavimus ipsam loci latitudinem, quippe quæ plerumque cognita est & in quovis loco, ubi Astronomicis observationibus vacare licet, primum investigari solet, quam ob caussam etjam in vulgari methodo altitudinum correspondentium Solis, ad inveniendam correctionem meridiei, datam semper elevationem poli supponunt Astronomi. Interim tamen quum pleraque dentur Instrumenta Astronomica, sumendis quidem æqualibus altitudinibus aptissima, quorum autem ope exacta non semper obtinetur ipsarum altitudinum mensura, quæ in directis pro invenienda elevatione poli institutis observationibus supponitur; di-

spi-

spiciendum nobis erit, an non adhibitis etjam hujusmodi instrumentis, observatis scil. in æquali altitudine pluribus Stellis, inveniri queat & tempus verum & loci latitudo, quod ansam nobis dedit solvendi problematis sequentis.

## §. III.

Problema. *Observatis in æquali altitudine tribus Stellis, quarum cognitæ sunt ascensiones rectæ atque declinationes, datisque temporum inter has observationes intervallis: invenire 1:o tempus verum singularum observationum, 2:o latitudinem loci & 3:o veram Stellarum altitudinem.*

In arcu meridiani $PZ$ (Fig. 2.) sit $P$ polus & $Z$ zenith, sitque observata prima Stella in $A$, secunda in $B$, tertia in $C$, & quidem singulæ in æquali altitudine quæ dicatur $x$, ita ut $ZA = ZB = ZC = 90^\circ - x$. Ex datis ascensionum rectarum differentiis atque temporibus inter singulas observationes interlapsis eadem ratione ac in §. præced. innotescunt anguli $APB$ & $APC$, qui dicantur $r$ & $\varrho$, sintque Stellarum $A$, $B$, $C$ declinationes seu arcuum $AP$, $BP$, $CP$ complementa, $\alpha$, $\beta$, $\gamma$ respective, nec non loci latitudo sive ipsius $PZ$ complementum $= y$, atque angulus $ZPA = z$, adeoque $ZPB = z + r$ & $ZPC = z + \varrho$; angulis his a zenith versus orientem seu secundum seriem signorum,

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rum, declinationibus vero ab æquatore verſus polum elevatum ſumtis, ſignoque igitur negativo afficiendis, quoties in partem contrariam cadunt. His poſitis erit (Trig. Sphær.) 1:o $Sin\, x = Cos\,\alpha\; Cos\, y\; Cos\, z + Sin\,\alpha\; Sin\, y$, 2:o $Sin\, x = Cos\,\beta\; Cos\, y\; Cos\,(z + r) + Sin\,\beta\; Sin\, y$, & 3:o $Sin\, x = Cos\,\gamma\; Cos\, y\; Cos\,(z + \varrho) + Sin\,\gamma\; Sin\, y$; quibus $Sin\, x$ valoribus inter ſe collatis & debite reductis, eruitur $Tg\, y = \dfrac{(Cos\,\alpha - Cos\,\beta\; Cos\, r)\; Cos\, z + Cos\,\beta\; Sin\, r\; Sin\, z}{2\, Sin\,\frac{1}{2}(\beta - \alpha)\; Cos\,\frac{1}{2}(\beta + \alpha)}$ (M) &

$$Tg\, y = \frac{(Cos\,\alpha - Cos\,\gamma\; Cos\,\varrho)\; Cos\, z + Cos\,\gamma\; Sin\,\varrho\; Sin\, z}{2\, Sin\,\frac{1}{2}(\gamma - \alpha)\; Cos\,\frac{1}{2}(\gamma + \alpha)}$$

(N). Ut ex his binis æquationibus (M & N) commodius inveniri queant valores ipſorum $z$ & $y$, ſequentes adhibere juvabit ſubſtitutiones:

$$\frac{Cos\,\alpha - Cos\,\beta\; Cos\, r}{2\, Sin\,\frac{1}{2}(\beta - \alpha)\; Cos\,\frac{1}{2}(\beta + \alpha)} = tg\, u\; Sin\,\varphi \;(P);$$

$$\frac{Cos\,\beta\; Sin\, r}{2\, Sin\,\frac{1}{2}(\beta - \alpha)\; Cos\,\frac{1}{2}(\beta + \alpha)} = tg\, u\; Cos\,\varphi \;(Q);$$

$$\frac{Cos\,\alpha - Cos\,\gamma\; Cos\,\varrho}{2\, Sin\,\frac{1}{2}(\gamma - \alpha)\; Cos\,\frac{1}{2}(\gamma + \alpha)} = tg\, v\; Sin\,\psi \;(R), \;\&$$

$$\frac{Cos\,\gamma\; Sin\,\varrho}{2\, Sin\,\frac{1}{2}(\gamma - \alpha)\; Cos\,\frac{1}{2}(\gamma + \alpha)} = tg\, v\; Cos\,\psi \;(S); \text{ adeo}$$

ut æqu. M transmutetur in hanc: $tg\, y = tg\, u\; Sin\,\varphi\; Cos\, z + tg\, u\; Cos\,\varphi\; Sin\, z$, ſeu $tg\, y = tg\, u\; Sin\,(\varphi + z)$ (M'), & æquat. N pariter in $tg\, y = tg\, v\; Sin\,(\psi + z)$ (N')

($N'$) designantibus $u$, $v$, $\varphi$ & $\psi$ novas quantitates incognitas, quarum valores sic investigantur: dividendo æqu. $P$ per $Q$, exterminatur $u$ & fit $tg\,\varphi =$ $\frac{Cos\,\alpha}{Cos\,\beta\,Sin\,r} - Cotg\,r$, adeoque ($tg\,\varphi + cotg\,r$ seu) $\frac{Cos\,(\varphi - r)}{Cos\,\varphi\,Sin\,r} = \frac{Cos\,\alpha}{Cos\,\beta\,Sin\,r}$, & $(Cos\,\varphi - r) : Cos\,\varphi :: Cos\,\alpha : Cos\,\beta$. unde (comp. & div.) $Cos\,(\varphi - r) - Cos\,\varphi : Cos\,(\varphi - r) + Cos\,\varphi :: Cos\,\alpha - Cos\,\beta : Cos\,\alpha + Cos\,\beta$ seu (Elem. Trigon.) $tg\,(\varphi - \frac{1}{2} r) : cotg\,\frac{1}{2} r :: tg\,\frac{1}{2}(\beta + \alpha) : cotg\,\frac{1}{2}(\beta - \alpha)$ adeoque $tg\,(\varphi - \frac{1}{2} r) = cotg\,\frac{1}{2} r.\, tg\,\frac{1}{2}(\beta + \alpha).\, tg\,\frac{1}{2}(\beta - \alpha)$. Invento hinc valore ipsius $\varphi$, ope æquat. $Q$, innotescit $u$. Eodem modo ex æqu. $R$ & $S$ eruitur $tg\,(\psi - \frac{1}{2}\varrho) = Cotg\,\frac{1}{2}\varrho.\, tg\,\frac{1}{2}(\gamma + \alpha).\, tg\,\frac{1}{2}(\gamma - \alpha)$ & $tg\,v =$ $\frac{Cos\,\gamma\,Sin\,\varrho}{2\,Cos\,\psi\,Sin\,\frac{1}{2}(\gamma - \alpha)\,Cos\,\frac{1}{2}(\gamma + \alpha)}$. Cognitis hac ratione ipsis $\varphi$, $u$, $\psi$ & $v$, conferendo inter se æqu. $M'$ & $N'$, quum sit $Sin\,(\varphi + z) : Sin\,(\psi + z) :: tg\,v : tg\,u$, erit (comp. & div.) $Sin\,(\varphi + z) + Sin\,(\psi + z) : Sin\,(\varphi + z) - Sin\,(\psi + z) :: tg\,v + tg\,u : tg\,v - tg\,u$, seu $tg\,(z + \frac{1}{2}\varphi + \frac{1}{2}\psi) : tg\,\frac{1}{2}(\varphi - \psi) :: Sin\,(v + u) : Sin\,(v - u)$, unde innotescit angulus $z$, quo invento ope alterutrius æqu. $M'$ vel $N'$ porro obtinetur elevatio poli $y$. Patet igitur solutionem hujus problematis sequentibus absolvi formulis:

1:o $Tg\,(\varphi - \frac{1}{2} r) = Cotg\,\frac{1}{2} r.\, tg\,\frac{1}{2}(\beta + \alpha)\, tg\,\frac{1}{2}(\beta - \alpha)$.

 2:o

$$2{:}o\ tg\ u = \frac{Cos\ \beta\ Sin\ r}{2\ Cos\ \varphi\ Sin\ \frac{1}{2}(\beta - \alpha)\ Cos\ \frac{1}{2}(\beta + \alpha)}$$

$$3{:}o\ tg\ (\psi - \tfrac{1}{2}\varrho) = Cotg\ \tfrac{1}{2}\varrho\ tg\ \tfrac{1}{2}(\gamma + \alpha)\ tg\ \tfrac{1}{2}(\gamma - \alpha)$$

$$4{:}o\ tg\ v = \frac{Cos\ \gamma\ Sin\ \varrho}{2\ Cos\ \psi\ Sin\ \frac{1}{2}(\gamma - \alpha)\ Cos\ \frac{1}{2}(\gamma + \alpha)}$$

$$5{:}o\ tg\ (z + \tfrac{1}{2}\varphi + \tfrac{1}{2}\psi) = \frac{Sin\ (v + u)\ tg\ \frac{1}{2}(\varphi - \psi)}{Sin\ (v - u)}\ \&$$

$6{:}o\ tg\ y = tg\ u\ Sin\ (\varphi + z)$ vel $tg\ y = tg\ v\ Sin\ (\psi + z)$. Inventis denique $z$ & $y$, facile invenitur altitudo $x$, in alterutro Triangulorum *APZ*, *BPZ* vel *CPZ* ex cognitis duobus lateribus cum angulo intercepto quærendo latus tertium. Dato autem angulo $z$, innotescit tempus verum. (§. II.)

*Scholion 1.* Analysi plane simili eruitur Methodus inveniendi latitudinem loci $= y$ & tempus verum ope quatuor Stellarum, quarum binæ *A* & *B* (Fig. 2.) in eadem altitudine $= x$ & reliquæ *C* & *D* in alia quadam altitudine $= x'$ observentur. Datis scil. rectis Stellarum ascensionibus & temporum intervallis inter singulas observationes, dantur anguli *APB*, *APC* & *CPD*, qui dicantur $r$, $R$ & $\varrho$, sintque Stellarum *A*, *B*, *C*, *D* declinationes $\alpha$, $\beta$, $\gamma$, $\delta$ respective, nec non angulus $ZPA = z$. Ob $ZA = ZB$ in $\Delta\Delta$ *AZP*, *BZP* erit (ut supra) $tg\ y =$

$$\frac{(Cos\ \alpha - Cos\ \beta\ Cos\ r)\ Cos\ z + Cos\ \beta\ Sin\ r\ Sin\ z}{2\ Sin\ \frac{1}{2}(\beta - \alpha)\ Cos\ \frac{1}{2}(\beta + \alpha)}$$

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unde posito $Cotg\ \frac{1}{2} r\ tg\ \frac{1}{2} (\beta + \alpha)\ tg\ \frac{1}{2} (\beta - \alpha) = tg\ (\varphi - \frac{1}{2} r)$ & $tg\ u = \frac{Cos\ \beta\ Sin\ r}{2\ Cos\ \varphi\ Sin\ \frac{1}{2}(\beta - \alpha)\ Cos\ \frac{1}{2}(\beta + \alpha)}$ fit $tg\ y = tg\ u\ Sin\ (z + \varphi)$. Eodem modo ob $CZ = DZ$ in $\Delta\Delta\ CZP,\ DZP$, fit $tg\ y = \frac{(Cos\ \gamma - Cos\ \delta\ Cos\ \varrho)\ Cos\ (z + R) + Cos\ \delta\ Sin\ \varrho\ Sin\ (z + R)}{2\ Sin\ \frac{1}{2}(\delta - \gamma)\ Cos\ \frac{1}{2}(\delta + \gamma)}$ unde posito $Cotg\ \frac{1}{2} \varrho\ tg\ \frac{1}{2} (\delta + \gamma)\ tg\ \frac{1}{2} (\delta - \gamma) = tg\ (\psi - \frac{1}{2} \varrho)$ & $tg\ v = \frac{Cos\ \delta\ Sin\ \varrho}{2\ Cos\ \psi\ Sin\ \frac{1}{2}(\delta - \gamma)\ Cos\ \frac{1}{2}(\delta + \gamma)}$ obtinetur $tg\ y = tg\ v\ Sin\ (z + R + \psi)$. Diversis his ipsius $tg\ y$ valoribus inter se collatis, fit $Sin\ (z + \varphi) : Sin\ (z + R + \psi) :: tg\ v : tg\ u$, adeoque (comp. & div.) $tg\ \overline{z + \frac{1}{2}(R + \psi + \varphi)} : tg\ \frac{1}{2}(R + \psi - \varphi) :: Sin\ (u + v) : Sin\ (u - v)$, unde invenitur ang. $z$ & hinc porro elevatio poli $y$.

*Scholion 2.* Potest etjam tempus verum inveniri ope duarum Stellarum sequenti ratione. Observetur primo in quacunque altitudine $= x$ stella quædam fixa in $C$ (Fig. 1.) & post exactum aliquod tempusculum in alia quacunque altitudine $= x'$ eadem stella observetur in $B$; quo facto instrumentum convertatur versus alteram meridiani partem & investigetur alia quædam fixa $A$ in hac eadem altitudine $= x'$, quæ stella denuo observetur in $D$ quando altitudinem primo observatam $= x$ attigit. Ex datis nimirum temporiporibus inter binas priores observationes, pariter ac inter binas posteriores interlapsis, dantur his proportionales anguli $BPC = 4h$ & $APD = 4h'$ nec non ex differentia inter ascensionem rectam utriusque Stellæ una cum tempore inter secundam & tertiam observationem, datur ang. $APB = 2\lambda$. Si angulus $APB$ bisecetur arcu $PR$, & angulus $ZPR$ dicatur $z$, stellarumque $B$ & $A$ declinationes sint $\beta$ & $\alpha$ respective, nec non latitudo loci $y$: erit $\frac{Sin\, x' - Sin\, x}{Cos\, y} = Cos\, \beta\, (Cos\, \overline{\lambda + z} - Cos\, \overline{\lambda + 4h + z}) = Cos\, \alpha\, (Cos\, \overline{\lambda - z} - Cos\, \overline{\lambda + 4h' - z})$ seu $\frac{Cos\, \frac{1}{2}(x' + x)\, Sin\, \frac{1}{2}(x' - x)}{Cos\, y} = Cos\, \beta\, Sin\, 2h\, Sin\, (\lambda + 2h + z) = Cos\, \alpha\, Sin\, 2h'\, Sin\, (\lambda + 2h' - z)$. Posito igitur $\frac{Cos\, \beta\, Sin\, 2h}{Cos\, \alpha\, Sin\, 2h'} = tg\, \omega$, erit $Sin\, (\lambda + 2h + z) : Sin\, (\lambda + 2h' - z) :: 1 : tg\, \omega$, adeoque (div. & comp.) $tg\, (z + h - h') : tg\, (\lambda + h + h') :: tg\, (45^\circ - \omega) : 1$, unde invenitur $z$.

## §. IV.

Accidit haud raro ut saltim quam proxime cognita sint & tempus verum & loci latitudo, adeo ut non nisi de exigua quadam correctione quæstio sit. His vero positis, præter vulgaria ista compendia, quæ Methodus differentialium subministrat, specialiores dantur

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tur casus, in quibus praxis & solutio praecedentis problematis fit admodum concinna. Exemplum hujus rei nobis praebet Stella Polaris, cujus videlicet quum ob suam cum Polo vicinitatem, minima sit variatio altitudinis, admisso etjam aliquo errore horologii, satis exacte tamen assignari potest tempus, quo haec stella altitudinem elevationi poli aequalem attinget. Hoc igitur tempore si observetur stella ista in $N$ (Fig. 3.), adeo ut, posito Polo in $P$ & Zenith in $Z$, sit $ZN = ZP$, atque in eadem altitudine ulterius observentur Stellae binae aliae $A$ & $B$, quas quidem quantum fieri potest ad primum verticalem proximas, unam versus orientem, alteram versus occidentem sumere praestat: hinc facili calculo eruetur & tempus verum & elevatio poli. Sint enim stellarum $A$ & $B$ a polo distantiae seu declinationum complementa $AP = 2a$ & $BP = 2b$, nec non ang. $APB = 2\lambda$, qui angulus per differentiam ascensionum rectarum & tempus inter observationes harum stellarum datur, bisecetur que ang. $APB$ arcu $PR$, atque dicatur $ZPR$ $h$, (adeo ut $LPZ = \lambda + h$ & $KPZ = \lambda - h$) nec non latitudo loci $p$; demissis a $Z$ in $AP$ & $PB$ perpendiculis $ZL$ & $ZK$, ob $ZA = ZP = ZB$ erit $PL = a$ & $PK = b$. Hinc in Triangulis rectangulis $PZL$ & $PZK$ est $tg\, p = Cos\,(\lambda + h)\; Cotg\, a\; (A')$ & $tg\, p = Cos\,(\lambda - h)\; Cotg\, b\; (B')$, unde $Cos\,(\lambda + h) : Cos\,(\lambda - h) :: tg\, a : tg\, b$ atque (comp. & div.) $tg\, h : Cotg\, \lambda :: Sin\,(b - a) : Sin\,(b + a)$. Cognito hinc valore anguli li $h$ (cujus ope tempus verum datur), ſubſtituto in alterutra æquat. $A'$ vel $B'$ invenitur loci latitudo $p$.

Si vero ſuſpicandum foret, veram Stellarum obſervatarum altitudinem æqualem non fuiſſe elevationi poli, error hic facillime detegi poteſt ſequente ratione. Sit Stellæ polaris a polo diſtantia $PN = 2c$ & angulus $APN$ ex differentia Aſc. R. & tempore inter obſervationes ſtellarum $N$ & $A$ inveniendus $= \mu$, poſito $tg\ c\ tg\ p = Coſ\ \gamma$; ſi æquales fuerint $ZN$ & $ZP$, erit $\lambda + h - \mu = \gamma$. Quoties vero inter $\gamma$ & $\lambda + h - \mu$ differentia quædam notabilis datur, quantitates inventæ $h$ & $p$ correctione aliqua indigent, quæ ita inveniri poteſt. Sit $\gamma - \lambda - h + \mu = \delta$ & inveſtigetur primo differentia inter altitudinem ſtellarum & elevationem poli ſeu $ZP - ZN$ quæ dicatur $2u$. In Triangulo ſcilicet $PZN$ manentibus duobus lateribus $ZP$ & $PN$ eſt variatio anguli intercepti $\delta$ ad variationem lateris tertii $2u$ ut $Coſec.\ PNZ$ ad $Sin\ PN$ (Cotes æſtim. errorum in mixta Math. Theor. 22), ſed ob exiguam differentiam laterum $ZN$ & $ZP$, erit $PNZ = ZPN$ quam proxime: Ergo $2u = \delta\ Sin\ 2c\ Sin\ (\lambda + h - \mu)$. Porro quum ob inæqualitatem laterum $PZ$ & $ZA$ in $\Delta\ PZA$ inæqualia fiant ſegmenta baſeos $PL$ & $LA$, & generatim ſit $Coſ\ PZ : Coſ\ ZA :: Coſ\ PL : Coſ\ LA$, adeoque (comp. & div.) $Cotg\ \frac{1}{2}\ (PZ + ZA) : tg\ \frac{1}{2}\ (PZ - ZA) :: Cotg\ \frac{1}{2}\ PA : tg\ \frac{1}{2}\ (PL - LA)$, ſubſtituendo $tg\ p$ pro $Cotg\ \frac{1}{2}\ (PZ + ZA)$ & pro $tg\ \frac{1}{2}\ (PZ - ZA)$ atque $tg\ \frac{1}{2}\ (PL - LA)$ arcus ipſos exiguos,

guos, bisecto $PA$ in $G$ erit $GL = u$ Cotg $p$ Cotg $a$. Pari modo in $\Delta\, PZB$ bisecto $PB$ in $H$ erit $KH = u$ Cotg $p$ Cotg $b$. Posito jam ang. $ZPR = h - z$, erit in Triangulis rectangulis $LPZ$ & $KPZ$, ang. $ZPL = \lambda + h - z$, $KPZ = \lambda - h + z$, $PL = a + u$ Cotg $p$ Cotg $a$, $PK = b + u$ Cotg $p$ Cotg $b$, atque ob hypothenusam $PZ$ communem Cos $ZPL$ : Cos $ZPK$ : : tg $PK$ : tg $PL$, unde facta substitutione & debita reductione eruitur $z = u$ Cotg $p$ Cotg $a$ Cotg $b$ Sin $2h$. Inventis vero $u$ & $z$ per resolutionem alterutrius $\Delta\, PZL$ vel $PKZ$ facile est invenire latitudinem loci correctam.

## §. V.

Supposuimus in praecedentibus cognitam celeritatem motus horologii, adeo ut si juxta hoc computatum tempus $24^h - m$ respondeat toti revolutioni sidereae, quantitas ista $m$ data sit. Jam dispiciendum erit qua ratione etjam haec celeritas, si incognita fuerit, investigari queat. Et quidem mox patet, data elevatione poli, observatis in aequali altitudine tribus Stellis inveniri posse non tantum ipsam altitudinem stellarum & tempus verum, sed etjam horologii accelerationem. Pari ratione ex observationibus quatuor stellarum aequialtarum praeter easdem incognitas etjam latitudinem loci determinari posse perspicuum est. Methodus vero directa haec solvendi problemata, praeterquam quod non nisi valde prolixis absolvatur calculis, minus idonea est ad hanc accelerationem vel retardationem satis exacte determinandam, quum ob temporis intervallum inter observationes, quod in his problematibus supponitur exiguum, errores observationum vix evitabiles etjamsi minimi valorem quantitatis *m* reddant admodum incertum. Præstat igitur indirecta uti methodo, cujus praxin hoc loco sufficiat exemplis in §. II. allatis illustrasse: adplicatione ejus ad reliquos casus hinc facile invenienda. Tempora vera in his exemplis inventa fuerunt $4^d\ 6^h\ 33'\ 34''$ & $10^d\ 13^h\ 48'\ 58''$ mensis Octobris hujus Anni, quibus respective competunt æquationes temporis $11'\ 30''$ & $13'\ 13''$ adeoque tempora media $4^d\ 6^h\ 22'\ 4''$ & $10^d\ 13^h\ 35'\ 45''$, temporibus secundum horologium computatis $4^d\ 6^h\ 22'\ 10''$ & $10^d\ 13^h\ 26'\ 8''$ correspondentia. Horum differentia $6^d\ 7^h\ 3'\ 58''$ collata cum differentia temporum mediorum $6^d\ 7^h\ 13'\ 41''$ per regulam proportionum dabit tempus horologii $23^h 58' 27'', 32$ pro quovis die solari medio, unde sequitur retardatio horologii diurna $= 1'\ 32'', 68$ quæ dicatur *r*. Hinc invenitur quantitas quæsita *m* inferendo: ut est dies solaris medius ad diem sidereum ($24^h : 23^h\ 56'\ 4''$), ita $24^h - r$ ad $24^h - m$. Erit igitur $m = 5'\ 28'', 43$. Ulterius dispiciendum erit, an satis exacti sint valores *r* & *m* per primam hanc approximationem inventi, & quanta correctione egeant tempora ista vera, quæ in his exemplis supponendo *m* justo minorem scil. $= 3'\ 56''$ computata fuerunt. Hunc in finem calcuculus ita ſubducendus eſt. Pro retardatione horologii diurna $= r$ ſit generatim temporis veri correctio invenienda $= \daleth$, & his correſpondentes angulorum $H$, $\lambda$ & $z$ (§. II.) variationes $H'$, $\lambda'$ & $z'$ reſpective. Quum jam ſit $24^h - m : t :: 360^\circ : H + H'$ & $t : 23^h\, 56'\, 4'' :: H : 360^\circ$ nec non $24^h - r : 24^h - m :: 23^h\, 56'\, 4'' : 24^h$, his proportionibus compoſitis eruitur $24^h - r : 24^h :: H : H + H'$ adeoque $24^h - r : r :: H : H'$, unde $H' = \dfrac{rH}{24^h - r} = -2\lambda$. Porro quoniam admodum exigui ſunt anguli $\lambda'$ & $z'$, & utcunque varientur ang. $\lambda$ & $z$, conſtans ſemper eſt $Coſ\,(D - \delta)\, Coſ\,(\pm\lambda \mp z) - Coſ\,(D + \delta)\, Coſ\,(\lambda + z)$ ſcil. $= tg\, p\, [Sin\,(D + \delta) - Sin\,(D - \delta)]$ (§. II.), fiet $(\lambda' + z')\, Coſ\,(D + \delta)\, Sin\,(\lambda + z) = (\lambda' - z')\, Coſ\,(D - \delta)\, Sin\,(\lambda - z)$, unde facta debita reductione obtinetur $\lambda' - z' : 2\lambda' :: Sin\,(\lambda + z)\, Coſ\,(\lambda + \varphi) : Sin\, 2\lambda\, Coſ\,(z + \varphi)$, adeoque ob $2\lambda' = -\dfrac{rH}{24^h - r}$ (dem.), $\lambda' - z' = \dfrac{-rH}{24^h - r} \cdot \dfrac{Sin\,(\lambda + z)\, Coſ\,(\lambda + \varphi)}{Sin\, 2\lambda\, Coſ\,(z + \varphi)}$.

Eſt autem $360^\circ + s : a + \lambda - z - S :: 24^h$ ad tempus verum (§. II.); Ergo $360^\circ + s : \lambda' - z' :: 24^h : \daleth$. Subſtituto itaque pro $\lambda' - z'$ valore invento erit $\daleth = - r \cdot \dfrac{24^h}{24^h - r} \cdot \dfrac{H}{360^\circ + s} \cdot \dfrac{Sin\,(\lambda + z)\, Coſ\,(\lambda + \varphi)}{Sin\, 2\lambda\, Coſ\,(z + \varphi)}$

Facta jam adplicatione hujus formulæ ad exempla §.

 II.

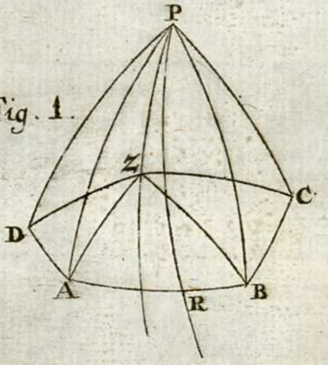

Fig. 1.

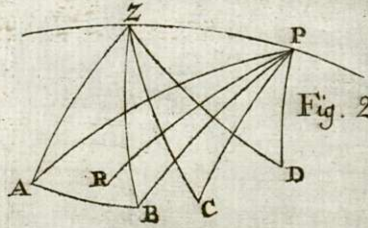

Fig. 2.

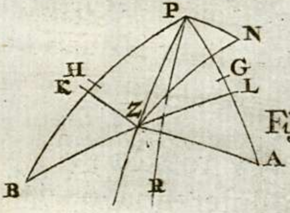

Fig. 3.

C.L.S. sc.

II. invenitur correctio temporis veri in *exemplo* 1 = − 0″. 525 & in *ex.* 2 = − 1″, 240. Quumque harum differentia non ſit niſi 0″, 715, patet valores ipſorum *r* & *m* per primam approximationem detectos, ſatis accuratos eſſe, quippe quorum error vix decimam ſcrupuli ſecundi partem ſuperat. Nec facile obvios fore caſus autumamus, in quibus approximationem ulterius repetere opus ſit.