

# Is mathematical logic needed in electrical engineering?

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**Abstract:** Electrical engineering is based on deep know-how over the science and mathematical thinking. Of course, as all engineering science, electrical engineering is focused to those problems active in that particular field. When thinking of control and high level automation systems a wide understanding over mathematical logic is needed. In this article an overview of power electronic systems are given where logic thinking is a vital part of the solving problems. Descriptions named here should understand as examples of those fields of engineering where mathematical logic skills are implemented. Also a example of learning tools used in this are is presented.

**Keywords:** power electronic, control systems, mathematical logic, switch

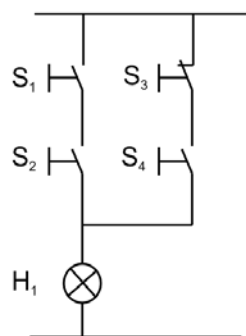
## 1. POWER ELECTRONICS AND LOGIC

The learning materials used in learning engineering should be based on companies' real-life problems, which require mathematical logic skills to be solved. Thus, these materials can easily also be used in the training of professionals in work life. This, together with the engineering students who will graduate with better mathematical logic skills and be the future work force in enterprises, will contribute to the higher performance of the European enterprises.

A mathematical thinking is a vital skill needed in electrical engineering. In this paper there are three examples of real life cases in which a logical thinking is needed and combined to electrical engineering know-how. These example cases are switching on light, a motor start up system and a pulse width modulated systems. Examples are presented from very basic and simple problem to a very challenging vector modulated system.

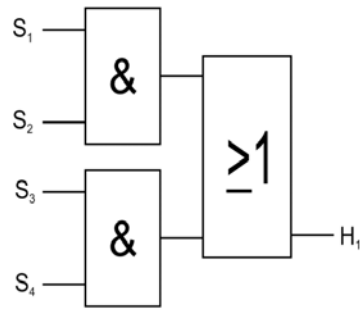
## 2. CASE STUDY: SWITCHING ON LIGHT

To switch on and off equipment in certain application may depend on several conditions that have to be filled. In this example a simple lamp connection diagram is presented.



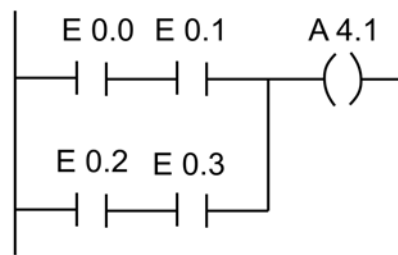
**Fig. 1** A lamp control system shown as electrical line drawing.

Figure 1 should be clear to all electricians. The actual control of this simple lamp connection can be done by using switches as shown in figure. If the system is more complex or the lamp is a part of wider system a programmable logic (PLC) can be used to control it. The same control function can be shown as a functional chart, see figure 2.



**Fig. 2** Functional chart of the lamp control circuit.

Functional chart can be transferred to a PLC program chart as following.



**Fig. 3** PLC program chart of the lamp control circuit.

Also the same control can be described by using the PLC program language.

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U      E 0.0
U      E 0.1
O
U      E 0.2
U      E 0.3
=      A 4.1

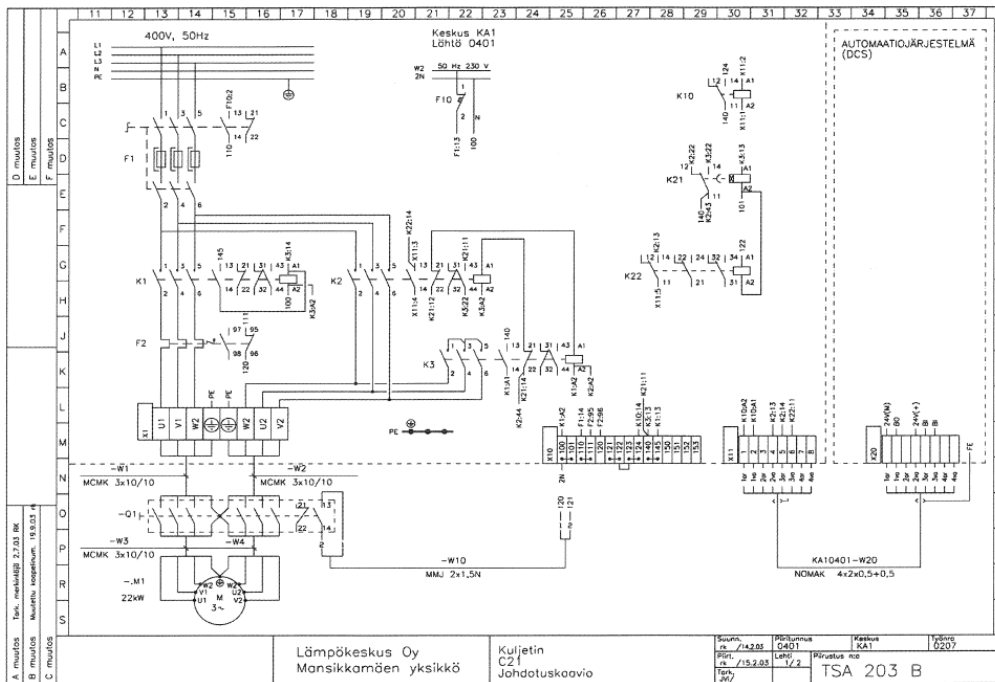
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In this syntax U is AND function, O is OR function and = transfers the status to output address A 4.1.

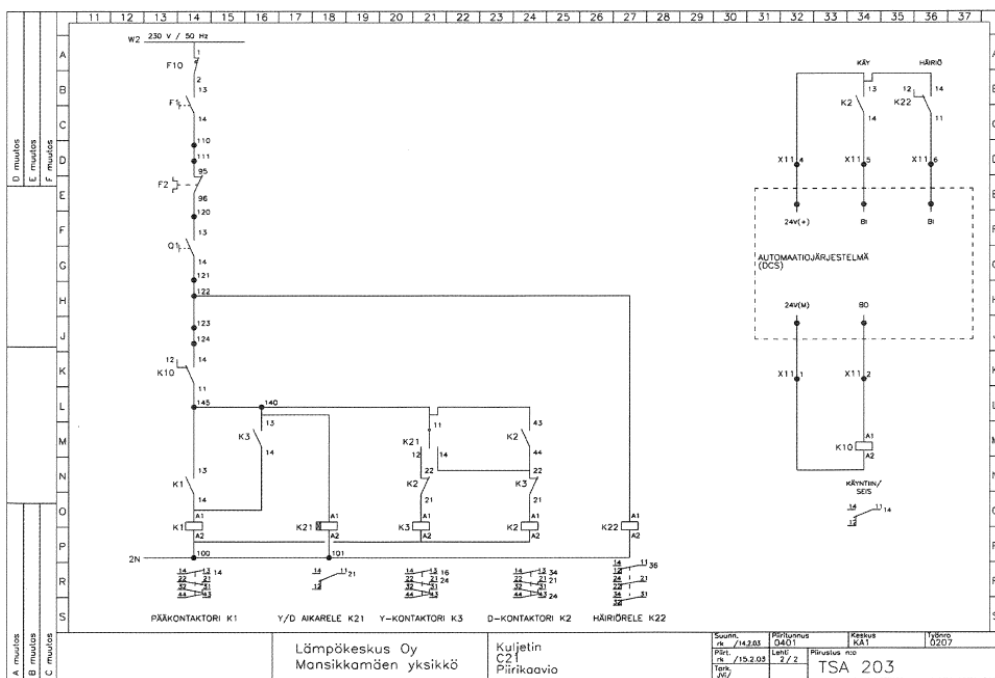
This example shows that a logical way of thinking is attached to electrical systems. From engineering point of view one has to have capability to understand the nature of electrical engineer, rules and design criterions to generate suitable design and component selection and above all understanding of the use of the system. This last part leads easily to discussion how the implementation is supposed to operate and control. An understanding over mathematical logic and how it can be implemented is certainly needed.

### 3. CASE STUDY: A MOTOR START UP SYSTEM

An electrical motor connected directly to feeding line is widely used in industrial installations for generating rotational movement for fans, pumps, conveyors, extruders and other equipments. In those installations the control and protection units are located at electrical cabinet while the actual motor is connected to the process. At figure 4 and figure 5 there is a typical three phase induction motor line diagram and control circuit. The control circuit has been executed by using electromechanical relays.



**Fig. 4** An AC motor line diagram used for conveyor use at heating station.



**Fig. 5** Control diagram for controlling and protecting the actual AC motor.

In this application the motor is connected to the electrical network by using so called YD starting procedure. From engineering point of view one has to understand how the actual application is supposed to operate and combine that information to electrical design. A logical type of thinking is necessary which is easy to notice from following description of the starting procedure.

- Motor is started by closing the contactor K1 and K3. This will be done by closing contactor K10 which enables the control voltage to point 140. The K3 will produce a star

point to the motor end winding. Also contactor K21 will be switch on and a time delay starts to run but it won't change the status at the beginning.

- At the same time K2 has to be locked from the net in order to avoid full scale three phase shortcircuit. This will be done by cross coupling the control circuit. When K3 is activated it will block the feeding from contactor K2 by opening it's auxiliary connections installed in series with K2.
- After a suitable time has based the K21 contactor will change it's status and the auxiliary connection will be transferred from point 12 to 14.
- This will cut of feeding from K3 which will open the star connection. After that K2 will be activated and the delta connection will come into force. While K2 is active it will block the K3 out by cross coupling.

As a result we can have a safe operating starting procedure done by using mechanical relays. By using controllable logic the same control can be done by programming following logical function to suitable PLC logic. Problem described above gives an idea of those applications where once again mathematical logic thinking is needed. To understand this and far more complex systems a deep know-how of the technology is needed together with all the mathematical skills aiding the understanding of the problem.

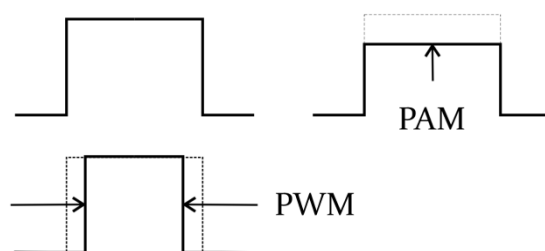
### 3. CASE STUDY: A PULSE WIDTH MODULATED SYSTEMS

In power electronic engineering we are interested to control the power fed to consuming equipment. Normally this has been done by using sinusoidal feeding systems in power grids and distribution networks. When entering to actual equipment level we have to do something in order to achieve power controllability. This can be done for example by using an ACDC conversion to generate a DC signal from an AC signal like voltage and then controlling this DC signal.

#### 3.1. PWM system

A pulse width modulation (PWM) based systems are widely used to produce from a discontinuous signal a continuous usable signal for example to power feeding purposes. PWM technology is a basic energy processing technique applied in power converter systems.

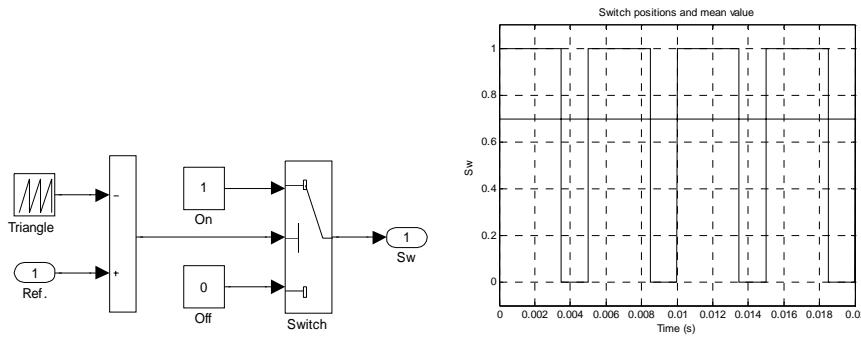
In general a power capacity of a pulse can be controlled by two separate methods. The power of the pulse related either to the height of the pulse (PAM) or to the width of the pulse (PWM), see figure 6.



**Fig. 6** A pulse which typically is a voltage pulse in electrical systems

From these two power control methods the PWM is more popular and used widely in practical applications. There is a maximum type of pulse that can be used for power feeding. This pulse is then 'sliced' to smaller parts by cutting it in order to control the power level. This feature is used in switched mode DC converters to generate from one level DC voltage another level DC

voltage. The control of this type of system is based on comparing DC signal to a ramp type AC signal.



**Fig. 7** Switch control based on comparing triangle shape signal to control value.

From this we can generate a quite simple logical function to describe the operation like following.

$$\begin{aligned} \text{IF (CONT} > \text{TRI)} &\rightarrow S_w = 1 \\ \text{IF (CONT} \leq \text{TRI)} &\rightarrow S_w = 0 \end{aligned}$$

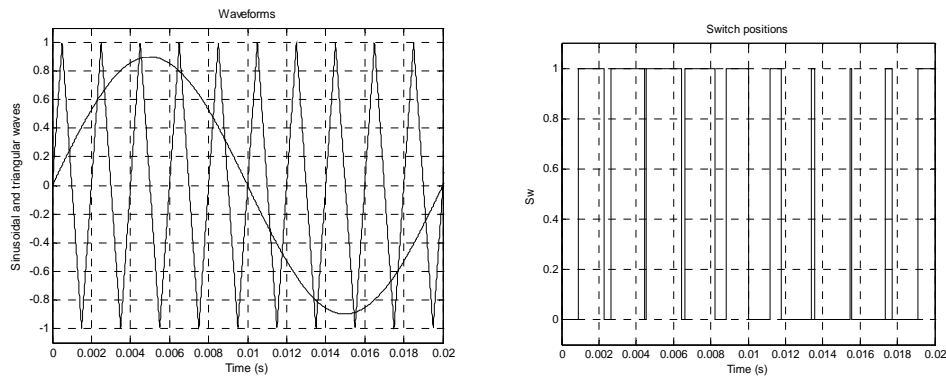
The actual effective power and mean value of a signal can be calculated by following equations.

$$U = \sqrt{\frac{1}{T} \int_0^T u^2(t) dt}, \quad U_o = \frac{1}{T} \int_0^T u(t) dt \quad (1)$$

This equation gives an idea of the effective of the voltage supply but don't tell anything how the signal behaves. For example in electrical drive and net applications we need to generate a sinusoidal voltage in certain frequency in order to control the rotational movement of the shaft or the status of the net. So we have to calculate the first harmonic component of the signal by using Fourier formulas.

$$\begin{aligned} f(t) &= a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t) \\ a_n &= \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \cos n\omega_0 t dt \\ b_n &= \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \sin n\omega_0 t dt \end{aligned} \quad (2)$$

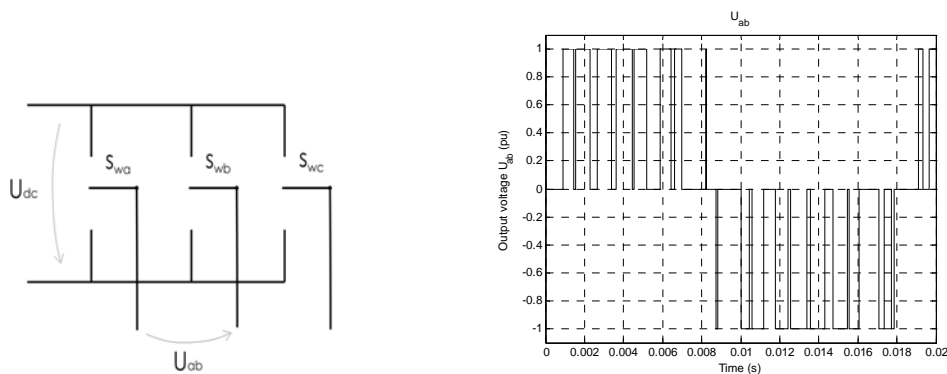
Also the rest of the Fourier components are needed for evaluating the noise of the signal. So we have a problem: how to control a pulse type of feeding in order to produce as sinusoidal voltage as possible. For that a comparison method is used where triangular signal is compared to a sinusoidal one. When ever the sinusoidal control signal has higher value a switch is connected to positive line and otherwise to negative line as was the case with DC converters.



**Fig. 8** A comparison method to produce signal for controlling a switch and switch positions.

This type of comparison is easy to execute by using analogy electronic. Industry and research centers use simulation tools in their research and development work to analyze this type of systems. Those same simulation tools are also very illustrative from studying and learning point of view. In this case a Matlab/SIMULINK environment has been used.

Let's us study a DCAC circuit which will produce a three phase AC voltage from a DC voltage, see figure 9. Output of the figure 8 type of control is far a way from sinusoidal signal but one can find the dominant frequency component. Now the purpose is to feed this type of voltage signal to an electrical system like motor or an electrical net. In practice the electrical circuit will determine together with the voltage fed to the system the resulting current which in turn is supposed to be as sinusoidal as possible.



**Fig. 9** Output voltage of a three switch DCAC circuit.

With a simple PWM system a good enough current shape can be generated. However this control method has some drawbacks like slow step response and low accuracy for many applications. These features can be improved by using sensors like speed and angle measurement systems in motor drives. Those type equipments means in practice extra costs and a failure risk for future. What can be done?

## 2.2. PWM system modeled by using vector calculation

A sinusoidal signal, like current below, can be modeled also by using a rotating vector, equation 3.

$$\underline{i} = \hat{i} \cdot e^{j\omega t} \quad (3)$$

In this equation both  $\hat{i}$  and  $\upsilon$  can be and in our case are time dependent.

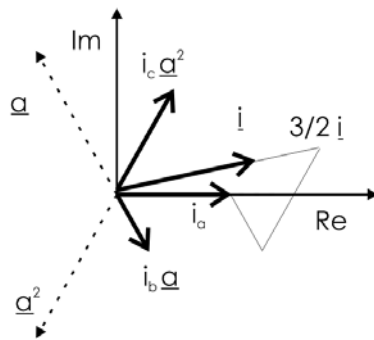
The current vector is modeled in complex system where the real axle is the positive reference direction. Equation three can be generalized by taking a unit vector into use.

$$\underline{a} = e^{j0} \quad (4)$$

Now if we can also model the motor or net behavior by accurate enough circuit model we should be able to calculate the feeding logic by using rotating vector calculation. This can be done for example by using so called dq-model to estimate the status of the motor in every time step. We are entering to the vector modulation world. How can we produce suitable switching signal by using vector calculation method? A switching function can be stated with marking  $s_w$ . The output current vector in three phase system in general can be written on complex space vector as following.

$$\underline{i} = \frac{2}{3}(i_a + a \cdot i_b + a^2 \cdot i_c) \quad (5)$$

This expression can be shown as a vector diagram, see figure 10.

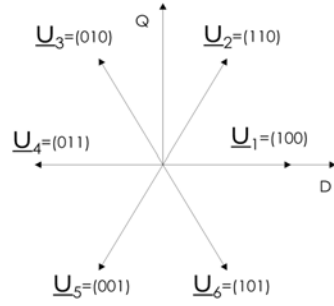


**Fig. 10** A vector diagram based on

In case of switching type of solution the output voltage is depending of the status of the individual switches. Now the task is to form the voltage vector in right manner. In machines it will generate a current which has a smooth form due to the damping phenomena affected by the inductances in the machines. By using different type of voltage switching the shape of the current can be affected.

$$\begin{aligned} \underline{u}_{ab} &= (S_{wa} - S_{wb}) \cdot u_{dc} \\ \underline{u}_{bc} &= (S_{wb} - S_{wc}) \cdot u_{dc} \\ \underline{u}_{ca} &= (S_{wc} - S_{wa}) \cdot u_{dc} \end{aligned} \quad (6)$$

Output vectors can be shown as switch options where 1 indicates that the switch is connected to + feeding line and 0 indicates connection to – feeding line.

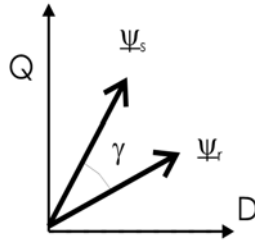


**Fig. 11** Switch options available at one level DC voltage inverter.

So called induction motor torque can be calculated for example by following equation.

$$t_e = \frac{3}{2} p \frac{L_m}{L_s \cdot L_r} |\underline{\psi}_r| \cdot |\underline{\psi}_s| \sin \gamma \quad (7)$$

As a vector diagram following equation can be shown as following figure.



**Fig. 12** Rotor and stator flux vector positions. The torque will increase when the angle between these two vectors increases. Flux in turn has an practical maximum value that cannot be exceeded.

Now the main task is to control the switched so that the flux vectors will rotate smoothly. If stator resistance is neglected the voltage vector can be expressed by following equation.

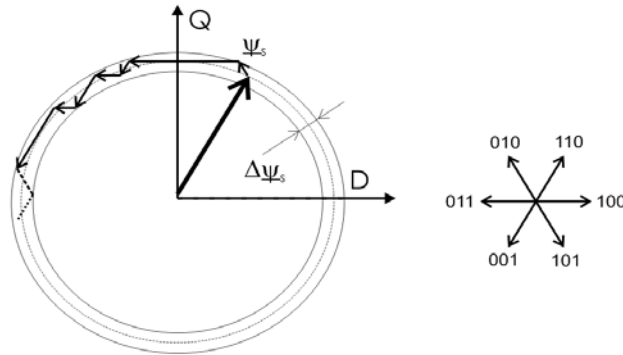
$$\underline{u}_s = \frac{d\underline{\psi}_s}{dt} \quad (8)$$

If the time is short enough the flux vector will follow the direction of the voltage vector and the speed of the change is related to the length of the voltage vector.

$$\Delta\underline{\psi}_s = \underline{u}_s \cdot \Delta t \quad (9)$$

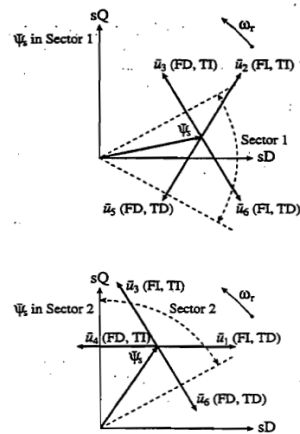
This will result to following vector diagram and switching options. The task is to generate a circular shape of flux vector end line. To produce circular shape we have only flux directions shown in figure 11.





**Fig. 13** Stator flux and the relation to voltage vectors. The shape of the end line of the rotating flux vector is not circular shape but close enough.

To arrange the actual flux and as a result torque control we need to find the optimum switching combinations. In figure 14 the FI indicates flux increase (1), FD flux decrease (0), TI torque increase (1), TD torque decrease (0).



**Fig. 14** Selection of the switch positions in different sectors.

From this we can generate logical functions to operate the machine. At the first sector the logical functions are:

$$\begin{aligned} \text{IF } (F = 1) \text{ AND } (T = 1) &\rightarrow \underline{u}_2 \\ \text{IF } (F = 0) \text{ AND } (T = 1) &\rightarrow \underline{u}_3 \\ \text{IF } (F = 1) \text{ AND } (T = 0) &\rightarrow \underline{u}_4 \\ \text{IF } (F = 0) \text{ AND } (T = 0) &\rightarrow \underline{u}_5 \end{aligned}$$

At the second sector the same logical functions are:

$$\begin{aligned} \text{IF } (F = 1) \text{ AND } (T = 1) &\rightarrow \underline{u}_3 \\ \text{IF } (F = 0) \text{ AND } (T = 1) &\rightarrow \underline{u}_4 \\ \text{IF } (F = 1) \text{ AND } (T = 0) &\rightarrow \underline{u}_1 \\ \text{IF } (F = 0) \text{ AND } (T = 0) &\rightarrow \underline{u}_2 \end{aligned}$$

Same logical functions can be described in one table, shown in figure 15.

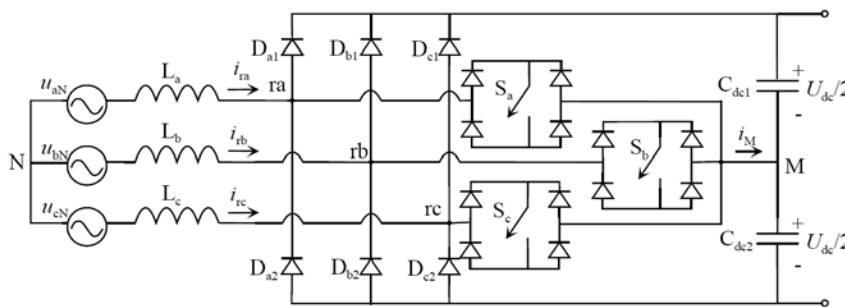
$d\psi$	$dr_s$	$\alpha(1)$ sector 1	$\alpha(2)$ sector 2	$\alpha(3)$ sector 3	$\alpha(4)$ sector 4	$\alpha(5)$ sector 5	$\alpha(6)$ sector 6
1	1	$\vec{u}_2$	$\vec{u}_3$	$\vec{u}_4$	$\vec{u}_5$	$\vec{u}_6$	$\vec{u}_1$
	-1	$\vec{u}_7$	$\vec{u}_8$	$\vec{u}_7$	$\vec{u}_8$	$\vec{u}_7$	$\vec{u}_8$
0	1	$\vec{u}_3$	$\vec{u}_4$	$\vec{u}_5$	$\vec{u}_6$	$\vec{u}_1$	$\vec{u}_2$
	-1	$\vec{u}_8$	$\vec{u}_7$	$\vec{u}_8$	$\vec{u}_7$	$\vec{u}_8$	$\vec{u}_7$

Active switching vectors:  $\vec{u}_1(100)$ ;  $\vec{u}_2(110)$ ;  $\vec{u}_3(010)$ ;  $\vec{u}_4(011)$ ;  $\vec{u}_5(001)$ ;  $\vec{u}_6(101)$   
Zero-switching vectors:  $\vec{u}_7(111)$ ;  $\vec{u}_8(000)$ .

**Fig. 15** Table of the optimal switch vector selection.

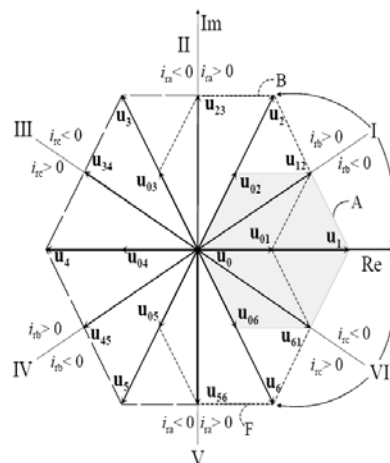
As an end it means that the voltage vector has to be controlled by using a suitable logic table. Once again mathematical thinking is widely needed to program a controller to follow this table and to control the output of the DCAC switch converter.

On the other hand a one level switch inverter is quite simple to program and to implement into use. In case of a compensation unit a three level inverter can be used. One of many topologies is so called VIENNA I inverter unit where only three controllable power components are needed.



**Fig. 16** Three level VIENNA I inverter used for compensating over waves from electrical net.

In this case we have a higher number of voltage vectors from which to choose the right switch combination.

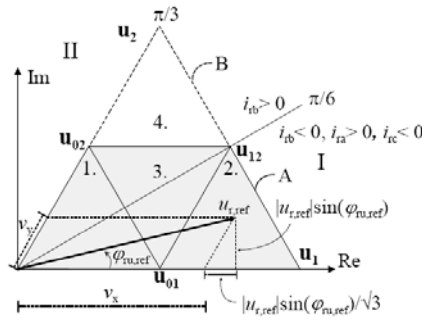


**Fig. 17** Voltage vectors from which to choose in case of VIENNA I inverter.

Control vector can be listed as following figure.

$\mathbf{u}$	swv	sw	$\text{SIGN}(i_{ra}, i_{rb}, i_{rc})^{**}$	$i_{M}$
$\mathbf{u}_0$	(0 0 0)	(1 1 1)	x x x	0
$\mathbf{u}_{01-}$	(1 0 0)	(0 1 1)	+ - -	$-i_{ra} (< 0)$
$\mathbf{u}_{01+}$	(0 -1 -1)	(1 0 0)	+ - -	$i_{ra} (> 0)$
$\mathbf{u}_1$	(1 -1 -1)	(0 0 0)	+ - -	0
$\mathbf{u}_{12}$	(1 0 -1)	(0 1 0)	+ x -	$i_{rb}$
$\mathbf{u}_{02-}$	(1 1 0)	(0 0 1)	+ + -	$i_{rc} (< 0)$
$\mathbf{u}_{02+}$	(0 0 -1)	(1 1 0)	+ + -	$-i_{rc} (> 0)$
$\mathbf{u}_2$	(1 1 -1)	(0 0 0)	+ + -	0
$\mathbf{u}_{23}$	(0 1 -1)	(1 0 0)	x + -	$i_{ra}$
$\mathbf{u}_{03-}$	(0 1 0)	(1 0 1)	- + -	$-i_{rb} (< 0)$
$\mathbf{u}_{03+}$	(-1 0 -1)	(0 1 0)	- + -	$i_{rb} (> 0)$
$\mathbf{u}_3$	(-1 1 -1)	(0 0 0)	- + -	0
$\mathbf{u}_{34}$	(-1 1 0)	(0 0 1)	- + x	$i_{rc}$
$\mathbf{u}_{04-}$	(0 1 1)	(1 0 0)	- + +	$i_{ra} (< 0)$
$\mathbf{u}_{04+}$	(-1 0 0)	(0 1 1)	- + +	$-i_{ra} (> 0)$
$\mathbf{u}_4$	(-1 1 1)	(0 0 0)	- + +	0
$\mathbf{u}_{45}$	(-1 0 1)	(0 1 0)	- x +	$i_{rb}$

**Fig. 18** Control vector selection table.



**Fig. 19** Options that are available for switching while the reference vector is located at subtriangle two.

Control vector at subtriangle two can be determined by using following equation and logical functions.

$$\begin{aligned} \alpha_x &= v_x / (U_{dc} / 3) \\ \alpha_y &= v_y / (U_{dc} / 3) \end{aligned} \quad (10)$$

IF ( $\alpha_x \geq 1$ )  $\rightarrow$  2. subsector

IF ( $\alpha_y \geq 1$ )  $\rightarrow$  4. subsector

IF (( $\alpha_x < 1$ ) AND ( $\alpha_y < 1$ ) AND ( $\alpha_x + \alpha_y < 1$ ))  $\rightarrow$  1. subsector

IF (( $\alpha_x < 1$ ) AND ( $\alpha_y < 1$ ) AND ( $\alpha_x + \alpha_y \geq 1$ ))  $\rightarrow$  3. subsector

In practice we have reached an application where the mathematical skills of the engineer are really on trial. System complexity comes from the deep know-how needed to understand power electronic devices. At the end the control of the real process is based on mathematical modeling of the problem and to understanding of mathematical logic.

#### 4. CONCLUSIONS

Mathematical logic develops the kind of thinking that is needed by the future planners of technical devices in order to have end-user friendly, logical and safe technical applications. From simple to complex applications the mathematical skills of the engineer are really on trial. System complexity comes from the deep know-how needed to understand power electronic

devices. At the end most of the electrical equipments and systems the control of the real process is based on mathematical modeling of the problem and to understanding of mathematical logic as described in this article.

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