DISSERTATIO PHYSICO-MATHEMATICA,

PHÆNOMENA LUMINIS, VIRIBUS ATTRACTIVIS & REPULSIVIS CORPORUM SUBJACERE & EX HIS DERIVARI POSSE,

STATUENS;

CUJUS PARTEM SECUNDAM,

CONSENTIENTE AMPLISS. ORDINE PHILOSOPH. IN IMPERIALI ACADEMIA ABOËNSI,

PRÆSIDE

Mag. JOH. FREDR. AHLSTEDT,

Mathem. Professore, Publ. & Ordin.

PRO GRADU

PUBLICE VENTILANDAM SISTIT

JACOBUS NICOLAUS CUMENIUS,

Stipend. Publ. Satacundensis.

In Auditorio Juridico die 3 Junii 1815.

h. a. m. solitis.

ABOÆ, Typis FRENCKELLIANIS.

6.

Liquet ergo esse $x = \pm \sqrt{\frac{b}{a\,tg\,\varphi^2}} \left(\sqrt{(a \pm y)(b \pm y)} \right.$ $\left. + (a - b) \text{ Log } \left(\sqrt{a \pm y} + \sqrt{b \pm y} \right) \right) + Const.$ Evanescet vero x una cum y, unde $Const = \mp \sqrt{\frac{b}{a\,tg\,\varphi^2}} \left(\sqrt{ab} \right.$ $\left. + (a - b) \text{ Log } \left(\sqrt{a} + \sqrt{b} \right) \right)$, & Integrale demum completum

$$x = \pm \sqrt{\frac{b}{a\,tg\,\varphi^2}} \left(\sqrt{(a \pm y)(b \pm y)} - \sqrt{ab} + (a - b) \text{ Log } \left(\frac{\sqrt{a \pm y} + \sqrt{b \pm y}}{\sqrt{a} + \sqrt{b}} \right) \right). \quad (G).$$

Examinemus jam alterum illum casum æquationis (A), in quo P est negativa. Facili vero patebit negotio, hunc casum, exceptis valoribus constantium a & b, priori esse plane similem. Habetur enim

$$dx = \sqrt{\frac{ce \mp f^2}{cetg\,\varphi^2 \pm f^2}} \cdot dy \sqrt{\frac{ce^2 : (ce \mp f^2) \pm y}{ce^2 tg\varphi^2 : (cetg\varphi^2 \pm f^2) \pm y}}, (A_{''}),$$

unde positis $\frac{ce^2}{ce \mp f^2} = a_,$ & $\frac{ce^2\,tg\,\varphi^2}{cetg\,\varphi^2 \pm f^2} = b_,$ emergit

$$x = \pm \sqrt{\frac{b_,}{a_,\,tg\,\varphi^2}} \left(\sqrt{(a_, \pm y)(b_, \pm y)} - \sqrt{a_, b_,} + (a_, - b_,) \text{ Log } \left(\frac{\sqrt{a_, \pm y} + \sqrt{b_, \pm y}}{\sqrt{a_,} + \sqrt{b_,}} \right) \right). \quad (H.)$$

 Pro-

Progrediamur ad integrandam æquationem (B), ſive

$$dx = \frac{dy}{\sqrt{\frac{C \mp \int P\,dy}{D,} - 1}} \quad (B).$$

Conſtans D, ex æquatione $\frac{ds}{dx} = \sqrt{\frac{C \mp \int P\,dy}{D,}}$, poſito $y = o$, oritur $= c\,Coſ\varphi^2$. Pro ſigno ſuperiori integralis $C \mp \int P\,dy$ obtinetur: $C - \int P\,dy = c \mp \frac{f^2}{e} \pm \frac{f^2}{e \pm y}$,

unde $dx = \frac{dy\,\sqrt{c}\,.\,Coſ\varphi}{\sqrt{c\,Sin\varphi^2 \mp \frac{f^2}{e} \pm \frac{f^2}{e \pm y}}}$, quæ hanc induere poteſt formam: $dx = \frac{\sqrt{ce}\,.\,Coſ\varphi}{\sqrt{ce\,Sin\varphi^2 \mp f^2}}\,.\,dy\sqrt{\frac{e \pm y}{\frac{ce^2\,Sin\varphi^2}{ce\,Sin\varphi^2 \mp f^2} \pm y}}$.

Facto $\frac{ce^2\,Sin\varphi^2}{ce\,Sin\varphi^2 \mp f^2} = b_{\prime\prime}$, evadit $\sqrt{\frac{ce\,Coſ\varphi^2}{ce\,Sin\varphi^2 \mp f^2}}$ $= \sqrt{\frac{b_{\prime\prime}}{etg\,\varphi^2}}$, & $dx = \sqrt{\frac{b_{\prime\prime}}{etg\,\varphi^2}}\,.\,dy\sqrt{\frac{e \pm y}{b_{\prime\prime} \pm y}}$, quæ formula a ſuperioribus non niſi in valoribus conſtantium differt. Oritur ergo

$x = \pm$

$$x = \pm \sqrt{\frac{b_{{\prime\prime}}}{etg\,\phi^2}} \cdot \left(\sqrt{(e \pm y)(b_{{\prime\prime}} \pm y)} - \sqrt{b_{{\prime\prime}}e} + (e - b_{{\prime\prime}}).\right.$$

$$\left.\text{Log.}\left(\frac{\sqrt{e \pm y} + \sqrt{b_{{\prime\prime}} \pm y}}{\sqrt{e} + \sqrt{b_{{\prime\prime}}}}\right)\right). \quad (I).$$

Pro ſigno tandem inferiori Integralis $C \mp \int P dy$ obtinebitur, facto $\frac{ce^2 \, Sin\,\phi^2}{ce\,Sin\,\phi^2 \pm f^2} = b_{{\prime\prime\prime}}$:

$$x = \pm \sqrt{\frac{b_{{\prime\prime\prime}}}{etg\,\phi^2}} \left(\sqrt{(e \pm y)(b_{{\prime\prime\prime}} \pm y)} - \sqrt{b_{{\prime\prime\prime}}e} + (e - b_{{\prime\prime\prime}}).\right.$$

$$\left.\text{Log}\left(\frac{\sqrt{e \pm y} + \sqrt{b_{{\prime\prime\prime}} \pm y}}{\sqrt{e} + \sqrt{b_{{\prime\prime\prime}}}}\right)\right). \quad (K).$$

Suppoſuimus ſupra, valores quantitatum conſtantium a, $a_{\prime}$, b, $b_{\prime}$, $b_{{\prime\prime}}$, $b_{{\prime\prime\prime}}$ esſe poſitivos, ſub qua conditione Integralia inventa G, H, I & K functiones quoque præbent reales. Quod ſi vero una harum quantitatum in quavis æquatione fuerit negativa, manente altera poſitiva, imaginaria evaderent hæc Integralia.

Huic caſui reſpondent:

1:o Si in æquatione A, fuerit $f^2 > cetg\,\phi^2$, ſive b negativa, unde $dx = \sqrt{\frac{-b}{atg\,\phi^2}} \cdot dy \sqrt{\frac{a+y}{y-b}}$, quæ in hanc abit:

$$dx = \sqrt{\frac{b}{atg\,\phi^2}} \cdot dy \sqrt{\frac{a+y}{b-y}}.$$

Ponatur, quo ab irrationalitate liberetur hæc formula, $\sqrt{\frac{a+y}{b-y}} = z$, unde eliciuntur

$$y = \frac{bz^2 - a}{1+z^2},\ \sqrt{a+y} = \frac{z\sqrt{a+b}}{\sqrt{1+z^2}},\ \sqrt{b-y} = \frac{\sqrt{a+b}}{\sqrt{1+z^2}}\ \&$$

$dy = \frac{2(a+b)zdz}{(1+z^2)^2}$. Hinc Integrale reale

$$\int dy \sqrt{\frac{a+y}{b-y}} = \int \frac{2(a+b)z^2 dz}{(1+z^2)^2} = \int \frac{(a+b)dz}{1+z^2}$$

$$-\int \frac{(a+b)(1-z^2)dz}{(1+z^2)^2} = Const. - \frac{(a+b)z}{1+z^2} + (a+b).$$

$$Arc\ Tg\ z = Const - \sqrt{(a+y)(b-y)} + (a+b).$$

$Arc\ Tg \sqrt{\frac{a+y}{b-y}}$; & infertis Conftantibus debitis:

$$x = \sqrt{\frac{b}{atg\varphi^2}}\left(\sqrt{ab} - \sqrt{(a+y)(b-y)} + (a+b)\right.$$

$$\left. Arc\,Tg \sqrt{\frac{a+y}{b-y}} - (a+b)\ Arc\ Tg \sqrt{\frac{a}{b}}\right).\ (L).$$

2:o Si fuerit $f^2 > ce$, five a negativa, unde

$$dx = \sqrt{\frac{b}{-atg\varphi^2}}.\ dy\sqrt{\frac{-a-y}{b-y}} = \sqrt{\frac{b}{atg\varphi^2}}.\ dy\sqrt{\frac{a+y}{b-y}}.$$

Hujus vero Integrale, formulæ (L) plane eft fimile. (M).

3:o

3:o Si in æquatione (A_{II}) ponatur b_I negativa; five $f^2 > cetg\,\varphi^2$, unde $dx = \sqrt{\frac{-b_I}{a_I tg\,\varphi^2}} \cdot dy \sqrt{\frac{a_I - y}{-b_I - y}}$
$= \sqrt{\frac{b_I}{a_I tg\,\varphi^2}} \cdot dy \sqrt{\frac{a_I - y}{b_I + y}}$; Cujus integrale eft

$$x = \sqrt{\frac{b_I}{a_I tg\,\varphi^2}} \left(\sqrt{(a_I - y)(b_I + y)} - \sqrt{a_I b_I} - (a_I + b_I) \, ArcTg \sqrt{\frac{a_I - y}{b_I + y}} + (a_I + b_I) \, ArcTg \sqrt{\frac{a_I}{b_I}} \right). \quad (N).$$

4:o Si in eadem æquatione (A_{II}) a fuerit negativa five $f^2 > ce$. Quo facto oritur $\sqrt{\frac{b_I}{-a_I tg\varphi^2}} \cdot dy \sqrt{\frac{-a_I + y}{b_I + y}}$
$= \sqrt{-\frac{b_I}{a_I\, tg\varphi^2}} \cdot dy \sqrt{\frac{a_I - y}{b_I + y}}$, hujusque Integrale non disfimile Integrali (N).

Tria poftremo enafcuntur Integralia mere Algebraica, fcilicet duo e formula (A), pofito $cetg\,\varphi^2 = f^2$ & $ce = f^2$, tertium vero e formula (B), pofito $ce Sin\,\varphi^2 = f^2$, quibus in cafibus f^2 fignum competit negativum. Innotefcit ex (A_I): $dx = dy \sqrt{\frac{(ce \pm f^2)(e \pm y) \mp ef^2}{(cetg\,\varphi^2 \mp f^2)(e \pm y) \pm ef^2}}$,

quæ

quæ, asſumta $cetg\,\varphi^2 = f^2$, abit in

$$dx = dy\sqrt{\frac{ce^2 + (ce + f^2)y}{ef^2}},$$

ſive inſerto pro f^2 valore $cetg\,\varphi^2$, in

$dx = \frac{dy}{Sin\,\varphi\sqrt{e}}\sqrt{e\,Cof\,\varphi^2 + y}$, cujus Integrale eſt

$$x = \frac{2}{3\,Sin\,\varphi.\sqrt{e}}\left((e\,Cof\,\varphi^2 + y)^{\frac{3}{2}} - e\sqrt{e}.\,Cof\,\varphi^3\right).\ (O).$$

Poſito vero $ce = f^2$, invenitur

$$dx = dy\sqrt{\frac{e\,Cof\,\varphi^2}{e\,Sin\,\varphi^2 - y}},\ \&$$

$$x = 2\sqrt{e}\,Cof\,\varphi\,(\sqrt{e}\,Sin\,\varphi - \sqrt{e\,Sin\,\varphi^2 - y}).\ (P).$$

Ex æquatione tandem (B), ſive

$$dx = dy\sqrt{\frac{ce\,Cof\,\varphi^2\,(e \pm y)}{(ce\,Sin\,\varphi^2 \pm f^2)(e \pm y) \mp ef^2}},$$

poſito $ce\,Sin\,\varphi^2 = f^2$ habebitur

$$dx = \frac{Cotg\,\varphi}{\sqrt{e}}.\,dy\sqrt{e - y},\ \&$$

$$x = \frac{2\,Cotg\,\varphi}{3\sqrt{e}}\left(e\sqrt{e} - (e - y)^{\frac{3}{2}}\right).\ (Q).$$

Æquatio (R): $dy = dx\sqrt{\frac{(cetg\,\varphi^2 \mp f^2)(e \pm y) \pm ef^2}{(ce \pm f^2)(e \pm y) \mp ef^2}}$,

po-

posito $dy = o$, maximum præbet valorem ipsius y. Erit enim $(cetg\,\phi^2 \mp f^2)(e \pm y) \pm ef^2 = o$, unde emergit

$$y = \mp \frac{ce^2\,tg\,\phi^2}{cetg\,\phi^2 \mp f^2}.$$

Pari modo ex æquatione

$$(S):\ dy = dx\sqrt{\frac{(ce\,Sin\,\phi^2 \pm f^2)(e \pm y) \mp ef^2}{ce\,Cos\,\phi^2\,(e \pm y)}},$$

posito $dy = o$, oritur valor maximus ipsius $y = \mp \frac{ce^2\,Sin\,\phi^2}{ce\,Sin\,\phi^2 \pm f^2}$, quibus pro y substitutis valoribus, proveniunt: in casu priori, *Subtangens* $\left(= \frac{ydx}{dy}\right) = \infty$, *Tangens* $\left(= \frac{yds}{dy}\right) = \infty$, *Subnormalis* $\left(= \frac{ydy}{dx}\right) = o$ & *Normalis* $\left(= \frac{yds}{dx}\right) = \pm \frac{ce^2\,tg\,\phi^2}{cetg\,\phi^2 \mp f^2}$; in posteriori autem *Subtangens* $= \infty$, *Tangens* $= \infty$, *Subnormalis* $= o$ & *Normalis* $= \mp \frac{ce^2\,Sin\,\phi^2}{ce\,Sin\,\phi^2 \mp f^2}$. Innotescit ergo, esse directionem lineæ curvæ in puncto D Plano FK parallelam, valoremque lineæ CD esse $\mp \frac{ef^2}{cetg\,\phi^2 \mp f^2}$ & $\mp \frac{ef^2}{ce\,Sin\,\phi^2 \mp f^2}$, respective.

Valores maximi ipsius x, posito $dx = o$, inveniuntur ex æquatione (R), facto $y = \mp \frac{ce^2}{ce \pm f^2}$

&

& ex æquatione (S), poſito $y = \mp e$. In caſu priori oriuntur: *Subtangens* $= o$, *Tangens* $= \pm \frac{ce^2}{ce \pm f^2}$, *Subnormalis* $= \infty$ & *Normalis* $= \infty$, atque in poſteriori: *Subtangens* $= o$, *Tangens* $= \mp e$, *Subnormalis* $= \infty$ & *Normalis* $= \infty$. Patet igitur directionem lineæ curvæ in puncto *D*, esſe in planum *FK* perpendicularem.

Lineæ deinde curvæ, quæ viam Luminis definiunt, ex æquationibus: (pag. 5 & 6.)

$$ds = dy \sqrt{\frac{D}{D - C \mp \int P dy}} \; \& \; ds = dy \sqrt{\frac{C \mp \int P dy}{C - D, \mp \int P dy}},$$

inveniendæ, easdem involvunt functiones, ac ipſæ æquationes coordinatarum, in valoribus tantummodo conſtantium diverſas; quare his ſuperſedemus.

Quæ in præcedentibus, asſumta Lege Newtoniana, eruimus, ſatis ſuperque probant, viam Luminis, corpus pellucidum penetrantis, experientiæ haud esſe conſentaneam. Radius enim Luminis pro quocunque angulo incidentiæ, in ſuperficie plani, ubi vi infinita afficitur, directionem Catheti obtinet, ſive perpendiculariter in planum incidit, unde neque varia refrangibilitas, neque Colores diverſi oriri posſunt; quod vero quam maxime a veritate aberrat. Hunc errorem neque diverſis gradibus caloris, (cujus in phænomena lucis vim minime negare posſumus), neque ſiccitati, humiditative aëris esſe tribuendum, jure cenſentes, ipſi Legi Newtonianæ adſcribendum esſe putamus.

In-