VATT-KESKUSTELUALOITTEITA
VATT-DISCUSSION PAPERS

285

# LABOUR TAXATION, TAX PROGRESSION AND JOB MATCHING – COMPARING ALTERNATIVE MODELS OF WAGE SETTING

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Valtion taloudellinen tutkimuskeskus
Government Institute for Economic Research
Helsinki 2002

ISBN 951-561-426-0

ISSN 0788-5016

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Oy Nord Print Ab

Helsinki, November 2002

SINKO PEKKA: LABOUR TAXATION, TAX PROGRESSION AND JOB MATCHING – COMPARING ALTERNATIVE MODELS OF WAGE SETTING. Helsinki, VATT, Valtion taloudellinen tutkimuskeskus, Government Institute for Economic Research, 2002, (C, ISSN 0788-5016, No 285). ISBN 951-561-426-0.

**Abstract:** In this paper we consider the effects of labour taxation on wages, unemployment and efficiency in a job matching framework. We derive labour market equilibrium with taxes in a model of endogenous job creation and job destruction under three alternative hypothesis of wage formation: Nash bargain, monopoly union and efficiency wages. We find that labour taxes harm employment irrespective of the wage formation mechanism. However, employment turns out to be much less sensitive to taxation in the models involving wage bargaining. Our results also suggest that increased progression in labour taxation may improve employment with low or even non-existent efficiency cost if wages are set in a bargaining framework.

**Key words: Labour taxation, wage setting, job matching**

**Tiivistelmä:** Tutkimuksessa tarkastellaan työhön kohdistuvien verojen vaikutusta palkkoihin, työllisyyteen ja tehokkuuteen etsintäteoriaan perustuvassa mallissa, jossa sekä työpaikkojen syntyminen että päättyminen ovat endogeenisia. Palkanmuodostuksen osalta tarkastellaan kolmea vaihtoehtoista tapausta: Nash neuvottelua, ”monopoliliittoa” ja tehokkuuspalkkoja. Tulosten mukaan verot kasvattavat tasapainotyöttömyyttä kaikissa palkanmuodostusvaihtoehdoissa. Työllisyys reagoi kuitenkin vähemmän verotukseen kahdessa ensin mainitussa tapauksessa eli ns. ammattiliittomalleissa. Näissä malleissa myös veroprogressi-on lisääminen parantaa työllisyyttä vain vähäisillä tehokkuustappioilla.

**Asiasanat: Työn verotus, palkanmuodostus, matching-mallit**

Earlier version of this paper has been presented at the EALE 2002 conference in Paris, France and the XIX Summer Seminar in Jyväskylä, Finland. I would like to thank the participants of these meetings as well as Kari Hämäläinen and Heikki Räisänen for their helpful comments and suggestions.

# Contents

# 1 Introduction

High and persistent European unemployment has given rise to numerous national and international policy advice programs directed to reduce the structural causes of joblessness. Most of the programs include suggestions to mitigate the tax burden on labour, in particular for low-paid wage earners. The European Union Employment Guidelines, for instance, name a target of reducing fiscal pressure on labour as well as non-wage labour costs. When appropriate, the measures should be targeted to relatively unskilled and low-paid labour (EU, 2000). Similar targets has been adopted by national governments as a recent evaluation study shows for the case of Finland (Ministry of Labour, 2002).

By suggesting to target tax cuts on the lower end of the wage distribution, the policy guidelines would not only reduce total tax burden, but also make the taxation of labour income more progressive.[1] One of the most simple ways of increasing the overall progression is to increase the lower limit for taxable income in an otherwise close to proportional tax schedule. Wagstaff & van Doorslaer (2001) demonstrate that tax allowances indeed constitute a remarkable source of progressivity in the personal income taxation among the OECD countries. In most of the advice programs, the recommendation for cutting taxes especially among low-paid workers is based on their potentially higher labour supply and demand elasticities (e.g. CEPR, 1995), whereas much less attention has been paid to the potential employment effects of the implied increase in progressivity of the labour taxation. Yet, there is considerable amount of both theoretical and empirical evidence that a pure increase in tax progression may bring about wage moderation and improved employment (e.g. Lockwood and Manning, 1993, Holmlund and Kolm, 1995, Pissarides, 1998.)

The aim of this paper is to contribute to this discussion by considering the effects of labour taxation and increased progression in an equilibrium model of labour market with endogenous job creation and job destruction. As for the tax instruments, we cover proportional and progressive tax on labour income as well as a proportional payroll tax. To emphasise the role of wage setting, we consider three alternative hypothesis of wage determination: Nash bargain, monopoly union and efficiency wages. This setting facilitates an interesting comparison between the widely used models of non-competitive wage determination in a matching framework. The inclusion of alternative wage setting mechanisms can be justified by the lack of a definitive model for the European labour market as suggested by Pissarides (1998). For instance, a recent study by Rocheteau (2001) argues that both bargain and efficiency wage mechanisms may coexist: which one is binding, depends on the tightness of the labour market.

We find that, not surprisingly, labour taxes generally harm employment irrespective of the wage formation mechanism.[2] However, the magnitudes of the employment

[1] By progression we mean the property that average tax rate increases with income. For alternative interpretations, see e.g. Atkinson & Stiglitz (1987).

[2] Altenburg & Straub (2002) show that proportinal labour taxes can actually improve employment in a combined effciency wage-union bargaining framework. Their result stems from decomposing the effective labour input into (endogenous) effort and employment.

effects vary between alternative models of wage setting. In particular, employment turns out to be much less sensitive to taxation in the models involving wage bargaining. Our results also suggests that increased progression of labour taxes may improve employment with low or even non-existent efficiency cost if wages are set in a bargaining framework.

Our approach bears resemblance to that of Pissarides (1998), who considers the effects of labour taxes and tax structure in alternative standard models of the labour market. Different from theirs, however, our analysis is cast in a matching framework with endogenous job destruction. On the other hand, we do not explicitly consider the role of unemployment benefits and their indexation, but implicitly confine ourselves to the case where unemployment benefits are fixed in real terms. Mortensen & Pissarides (1999) provide analysis of tax policy effects in a similar matching framework with Nash bargaining over wages. They also discuss alternative wage formation models, but do not consider their implications to the effects of tax policies. Effects of labour taxes and tax progression in the union models with no explicit modeling of search behaviour has been studied by Herzoug (1984) and Koskela & Vilmunen (1996). Pisauro (1991) provides an analysis of the effects of labour taxes in the basic efficiency wage framework.

The structure of the paper is as follows. Section 2 introduces the model and alternative processes of wage determination. Section 3 presents simulations of tax policy effects. Concluding remarks are presented in Section 4. Some of the technical details are presented in the appendices A-C.

# 2 The Model

The framework of our analysis is that of equilibrium unemployment with endogenous job destruction presented in Mortensen and Pissarides (1999) and Pissarides (2000). In what follows, we first introduce taxation to the basic model with Nash bargaining on wages. We then proceed to consider two alternative hypothesis of wage determination: the monopoly union and the efficiency wage models.

## 2.1 Individuals

Since jobs are only created at the highest productivity level ($x = 1$) the expected present value of unemployment search $U$ is given by

$$rU = b - a + \lambda \left( W\left( 1\right) - U\right) \tag{1}$$

where $b$ is the value of leisure, $a$ is net cost of search per period and $W$ is the expected present value of a work offer. Furthermore, $r$ is the discount rate of interest and $\lambda$ is the endogenous probability of encountering a vacancy. For a worker employed in a job with productivity $x$ we have

$$rW(x) = (1-t)\,w(x) - g + \delta\left[\int_0^1 \max\left(W(z), U\right) dF(z) - W(x)\right] \tag{2}$$

where $w$ is the wage rate and $\delta$ is the exogenous probability to encounter a productivity shock. $g$ and $t$ are income tax parameters such that the income tax paid by an employed worker per period is

$$T = g + tw(x)$$

Notice that with $g = 0$ the tax schedule is purely proportional. With $g < 0$, tax schedule is progressive and with $g > 0$, regressive. For later purposes, notice that (2) can be written as

$$W(x) = \frac{(1-t)\,w(x) - g + \delta\left(UF(R) + S_W\right)}{r+\delta} \tag{3}$$

where $R$ is the endogenous reservation productivity to be determined below and $S_W = \int_R^1 W(z)\,F(z)$ is the average value of a filled job for a worker .

## 2.2 Firms

Since jobs are created at the highest productivity level, value of a vacancy $V$ is given by

$$rV = -c + \eta\left(J(1) - V\right) \tag{4}$$

where $c$ is the cost of a vacancy per period, $\eta$ is the probability of encountering an unemployed worker [3]and $J(1)$ is the value of a filled vacancy. The value of a filled vacancy with productivity $x$ is given by

$$rJ(x) = x - (1+s)\,w(x) + \delta\left[\int_0^1 \max\left(J(z), V\right) dF(z) - J(x)\right] \tag{5}$$

where $s$ is the proportional payroll tax levied on employer. Again, we find it useful to rewrite (5) in the form

$$J(x) = \frac{x - (1+s)\,w(x) + \delta\left(VF(R) + S_J\right)}{r+\delta} \tag{6}$$

where $S_J = \int_R^1 J(z)\,F(z)$ is the average value of a filled job for the firm .

## 2.3 Matching

We assume that the unemployed workers and vacancies are matched through are constant returns to scale matching function. Then, for an unemployed worker, the probability of receiving a work offer is given by

[3] $\eta$ is necessarily linked to $\lambda$ which is made explicit below.

$$\lambda = m(\theta) \tag{7}$$

where $m$ is the matching function and $\theta$ is the ratio of vacancies to unemployment referred to as the *labour market tightness*. Consequently, for a firm with a vacancy, the probability of encountering an unemployed worker is given by

$$\eta = \theta^{-1} m(\theta) \tag{8}$$

In some cases we specify a Cobb-Douglas type matching technology with

$$m(\theta) = \theta^{\rho} \tag{9}$$

where $0 < \rho < 1$. With this specification and the notion that expected duration of a job vacancy is the inverse of (8), we notice that the elasticity of the expected duration of a vacancy with respect to the number of vacancies is $1 - \rho$.

## 2.4 Nash bargain on wages

The standard assumption in this type of models is that the rents associated with a job match are shared between workers and firms in a Nash bargain over wages. In our notation, the match specific wage rate is given by

$$w(x) = \arg\max (W(x) - U)^{\beta} (J(x) - V)^{1-\beta} \tag{10}$$

where $0 < \beta < 1$ is an exogenous parameter reflecting relative "bargaining power" of the two parties. In the symmetric case $\beta$ equals one half. It can be shown (e.g. Pissarides, 2000) that if $\beta$ is equal to the elasticity of the expected duration of a vacancy with respect to the number of vacancies, the equilibrium produced by the model with no taxes is "socially efficient". This result holds generally for homogenous of degree one matching functions and is referred to as the Hosios condition (Hosios, 1990).[4]

The first order condition related to (10) (see Appendix A) can be written in the form

$$\frac{W(x) - U}{J(x) - V + W(x) - U} = \frac{\beta(1-t)}{(1+s) - \beta(s+t)} \tag{11}$$

which conveniently shows the effect of taxes on the worker's relative share of the surplus from a job. The left hand side of (11) relates the workers surplus, $W(x) - U$, to the total surplus from a match, $J(x) - V + W(x) - U$. Differentiating the right hand side shows that both of the proportional tax rates, $s$ and $t$, reduce worker's relative share of the surplus. This is because taxes that are proportional to wages induce a common incentive to wage moderation for the worker and the firm as noticed by Pissarides (2000). The per head tax $g$ does not have this property and therefore only affects the total surplus and not the relative shares.

[4] In the present set up, satisfaction of the Hosios condition would require $\beta = 1 - \rho$. There is, however, no logical reason for this to hold automatically.

For later purposes, we substitute (3) and (6) in (11) and solve for $w(x)$ (see Appendix A for details) to get

$$w(x) = \beta \frac{(x - rV)}{1+s} + (1-\beta) \frac{rU + g}{1-t} \tag{12}$$

which is the wage equation in the flow form. Equation (12) shows that the resulting wage rate is a weighted average of the fall back position of the worker, $rU$ and the productivity of a match net of the flow cost of holding a vacancy $x - rV$, both of which are corrected for the relevant taxes.

## 2.5 Equilibrium with taxes

The labour market equilibrium is defined by imposing the conditions for free entry and mutual acceptance of job destruction. Free entry for firms is assumed to bring the value of a vacancy to zero

$$V = 0 \tag{13}$$

With (13) holding, the mutual acceptance of job destruction implies[5]

$$W(R) - U = J(R) = 0 \tag{14}$$

Utilising (13) we may also rewrite (12) as

$$w(x) = \frac{\beta x}{1+s} + \frac{1-\beta}{1-t}(rU + g) \tag{15}$$

Substituting this into (6), under (13) and rearranging then gives

$$(r+\delta) J(x) = (1-\beta) x - \frac{1+s}{1-t}(1-\beta)(rU+g) + \delta S_J \tag{16}$$

To get a more convenient expression for the value of a filled job, we notice that $J(x)$ is linear in $x$ and develop the Taylor series around $J(R) = 0$ to get

$$J(x) = \frac{1-\beta}{r+\delta}(x - R) \tag{17}$$

Applying (16) at the level of reservation productivity ($x = R$) then gives

$$rU + g = \frac{(1-t) R}{(1+s)} + \frac{(1-t)\delta}{(1+s)(1-\beta)} \int_R^1 J(z)\, dF(z) \tag{18}$$

and further substituting (17) for $J(z)$ yields

$$rU + g = \frac{(1-t) R}{(1+s)} + \frac{(1-t)\delta}{(1+s)(r+\delta)} \int_R^1 (z - R)\, dF(z) \tag{19}$$

[5] Alternatively, (14) follows from (11) and (13) and the condition that total surplus from a match is zero at the reservation productivity i.e. $J(R) - V + W(R) - U = 0$.

To express the left hand side more conveniently, apply the wage equation (11) to $x = 1$ with (13) binding to get

$$W(1) = U + \frac{(1-t)\beta}{(1+s)(1-\beta)} J(1) \tag{20}$$

Then notice that with (13) binding, (4) implies

$$J(1) = c\eta(\theta)^{-1} \tag{21}$$

which shows that for given labour market tightness, the value of a new match is increasing in the recruiting cost, $c$.

Next, substituting (20) and (21) into (1) yields

$$rU = b - a + \frac{(1-t)\beta c\theta}{(1-\beta)(1+s)} \tag{22}$$

where we utilised for $\lambda = m(\theta)$ and $\eta = \theta^{-1}m(\theta)$ as defined in (7) and (8).

Substituting (22) into (19) now gives

$$\frac{(1+s)}{(1-t)}(b-a+g) + \frac{\beta c\theta}{(1-\beta)} = R + \frac{\delta}{(r+\delta)} \int_R^1 (z-R)\, dF(z) \tag{23}$$

which is the *job destruction condition in the presence of taxation* and constitutes an upward sloping curve in the $(R, \theta)$ -space. Notice that substituting (22) into (15) yields yet another useful form of the wage equation

$$w(x) = (1-\beta)\frac{b-a+g}{1-t} + \beta\frac{x+c\theta}{1+s} \tag{24}$$

which shows the dependency of wages (with given labour market tightness) on value of leisure ($b$) search cost ($a$) and the cost of holding a vacancy ($c$). Wages depend positively on the labour market tightness, because the expected cost for the firm to find another match increases. This is reflected by the term $c\theta$ on the right hand side of (24). Similarly, wages depend positively on vacancy cost $c$ for given labour market tightness.

To derive another independent equation in the two unknowns, apply (17) to $x = 1$ and substitute (21) for the value of a job to get

$$c = \frac{1-\beta}{r+\delta}\eta(\theta)(1-R) \tag{25}$$

which is the *job creation condition* and constitutes a downward sloping curve in $R, \theta$ -space. The labour market equilibrium in the presence of taxation is the tuple $\theta, R$ defined by (23) and (25). Noteworthy, the tax instruments only enter the job destruction condition (23), whereas the job creation condition (25) is unaffected by the taxes. Intuitively, with given labour market tightness, the reservation productivity is sufficient to transmit the effects of the tax changes to job creation decision based on the free entry condition (13).

Looking at the job destruction condition (23), it is immediately clear, that the proportional tax on income and the payroll tax have identical effects on the equilibrium if rates are chosen such that

$$(1+s) = (1-t)^{-1} \tag{26}$$

By (24) the labour cost $(1+s)\,w$ is also identical for the tax rates such as defined by (26). The net wage $w$ is lower in proportion to $(1+s)$ if payroll tax is used, which guarantees that revenues from the two taxes are equal. Thus, the so called tax wedge argument applies and what is true for the proportional income tax also holds for a proportional payroll tax in the model.[6] In what follows, we therefore mainly concentrate on the former.

Following the approach of Pissarides (2000) it is straightforward to show that the equilibrium with taxes is socially efficient if the matching elasticity parameter $\rho$ is chosen to satisfy the Hosios condition $\beta = 1-\rho$ discussed above, and tax parameters satisfy the following condition

$$g = \frac{-\,(b-a)\,(s+t)}{1+s} \tag{27}$$

which follows from imposing $\frac{(1+s)}{(1-t)}\,(b-a+g) = b-a$ and makes (23) equal to the case with no taxes. What (27) effectively states is that if the value of leisure net of search cost ( $b-a$) is subsidised at the rate equal to the effective tax rate on wages, the overall tax system is neutral with respect to job creation and destruction. However, if $\beta \neq 1-\rho$ the no tax equilibrium is inefficient and "full efficiency" cannot be restored with the available tax instruments .[7]

Effects of exogenous changes in the tax rates can be presented diagrammatically in the $\theta, R$ space: An increase in either of the tax parameters shift the job destruction schedule to the left, whereas the job creation schedule remains stable. Consequently, labour market tightness reduces and reservation productivity increases (Figure 1).

## 2.6 Monopoly union wage setting

In above, we employed the standard assumption of search models according to which wages are an outcome of a Nash bargain between firms and workers. When setting the wage, firms and workers take the action of other agents, and therefore the aggregate variables as given. Also, the parameter reflecting the workers share of the outcome, $\beta$, was exogenously fixed. In this section, we extend the framework to allow

[6] With the tax wedge we refer to the situatiation where effects of the tax are independent of the nominal incididence and the wedge $\frac{(1+s)}{(1-t)}$ is a sufficient statistic to describe the taxation (see e.g. Layard et al, 1991).

[7] If $\beta \neq 1-\rho$, an approriate combination of the tax instruments can, however, improve the efficiency over the no tax equilibrium as we will show below in one of the simulations.

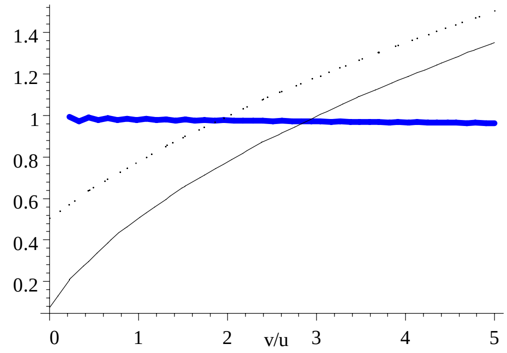


Figure 1: Effect of higher labour taxes in the model with Nash bargain on wages: the upward sloping job destruction schedule shifts up and to the left with a consequent drop in the labour market tighness (v/u) and an increase in the reservation productivity (vertical axis).

for endogenous determination of $\beta$. While there may be several alternatives, we rely on the formulation suggested by Mortensen and Pissarides (1999) which assumes that the worker's share is determined by a "monopoly union" so as to maximize its objective.[8]

An important aspect of this model is the choice of union's objective. Pissarides (2000) shows that if union maximises the expected utility of the unemployed, it will set the worker's share equal to the elasticity of the expected duration of a vacancy with respect to the number of vacancies. In this case, the worker's share would satisfy the Hosios condition for social efficiency. We consider, instead, the hypothesis suggested by Mortensen and Pissarides (1999) that the union maximises the utility of a median member who is currently employed in a job with some given level of productivity. With the notation of the previous section, using (13), (11) and (17), the value function of the median worker can be written as

$$W(x_m) = U + \frac{\beta(x_m - R)(1-t)}{(r+\delta)(1+s)} \tag{28}$$

where $x_m > R$ is the productivity of the job held by the median worker. Differentiating (28) with respect to $\beta$ and setting equal to zero yields the first order condition for the union optimum

$$\frac{\partial W(x_m)}{\partial \beta} = \frac{\partial U}{\partial \beta} + \frac{(1-t)}{(r+\delta)(1+s)}\left((x_m - R) - \beta\frac{\partial R}{\partial \beta}\right) = 0 \tag{29}$$

To develop (29) further, we need to derive suitable expressions for $U$ and $R$, respectively. Starting with $U$, we assume that the productivity shock is evenly distributed

[8] It is noteworthy that this formulation is somewhat different from the standard definition of monopoly union in the static models (e.g. Booth, 1995). Rather than just altering the bargaining power of the union, this formulation adds some features of higher degree of "centralisation" to the model (e.g. Calmfors & Driffill 1988).

in the interval $]0,1[$ (see Appendix B) and apply (19) to get

$$rU = \frac{(1-t)}{(1+s)}\frac{2Rr+\delta R^2+\delta}{2(r+\delta)} - g \tag{30}$$

Differentiating (30) with respect to $\beta$ and substituting into (29) we can rewrite the first order condition as

$$\frac{(1-t)(x_m - R)}{(r+\delta)(1+s)} + \frac{(1-t)(r+R\delta-\beta r)}{r(r+\delta)(1+s)}\frac{\partial R}{\partial \beta} = 0 \tag{31}$$

As for the reservation productivity $R$, we solve (23) for $\theta$ and substitute this into (25) to get

$$\frac{(1-\beta)^{\rho}(1-R)}{r+\delta}\left(\frac{R(2r+R\delta)+\delta-2(1+s)(1-t)^{-1}(r+\delta)(b-a+g)}{2\beta(r+\delta)c}\right)^{\rho-1} = c \tag{32}$$

which is $\Omega(R,\beta) - c = 0$ and implicitly defines $R$ as a function of $\beta$. Evoking the implicit rule we have

$$\frac{\partial R}{\partial \beta} = \frac{(1-\beta-\rho)(1-R)}{\beta(1-\beta)\left(1+2(2Rr+R^2\delta+\Theta)^{-1}(1-\rho)(1-R)(R\delta+r)\right)} \tag{33}$$

where $\Theta \equiv \delta - 2(1+s)(1-t)^{-1}(r+\delta)(b-a+g)$. Notice that setting the right hand side equal to zero implies $\beta = 1-\rho$ which is the Hosios condition in our set up. To determine the "optimal worker's share" we solve (31) for $\partial R/\partial\beta$ to get

$$\frac{\partial R}{\partial \beta} = \frac{-x_m + R}{r+R\delta-\beta r}r \tag{34}$$

and set that equal to the right hand side of (33) to get

$$\frac{(1-\beta-\rho)(1-R)}{(\beta(1-\beta))}\left(1+\frac{(1-\rho)(1-R)(R\delta+r)}{Rr+\frac{1}{2}\delta(R^2+1)-\frac{1+s}{1-t}(r+\delta)(b-a+g)}\right)^{-1}$$
$$= -\frac{x_m - R}{r(1-\beta)+R\delta}r \tag{35}$$

which defines a locus in $(\beta, R)$-space that satisfies the monopoly union condition. It is noteworthy that all three tax rates enter the equation that determines $\beta$. Since the right hand side of (35) is negative, we can infer that $\beta$ must exceed $(1-\rho)$ in the monopoly union optimum. Also notice that by (34) $\frac{\partial R}{\partial \beta}$ is negative in the equilibrium. These findings are in line with the more general results reported by Mortensen and Pissarides (1998) according to which the reservation productivity achieves its maximum at the social optimum.

The labour market equilibrium with monopoly union wage determination is the three tuple $\beta, R, \theta$ defined by (23), (25) and (35). The wage rate is determined by (24). Notice that the sharing rule of (11) still holds in the monopoly union framework. Rearranging (11) a bit, we have

$$\frac{W(x) - U}{J(x) - V + W(x) - U} = \frac{\beta(1-t)}{(1+s) - \beta(s+t)} \tag{36}$$

which defines the worker's relative share of the surplus. The difference to (11) is that $\beta$ is no longer constant in the equilibrium. By (35), value of $\beta$ will depend on the tax parameters as well as on the reservation productivity. As for the tax effects on the worker's share of proportional tax rates $s$ and $t$, in addition to the negative direct effect they now have an indirect effect through $\beta$ which may either mitigate or reinforce the direct effect. Our numerical simulations show that the indirect effect is negative and thus reinforces the drop in the worker's share due to higher proportional taxes. As for the per head tax $g$, there is still no direct effect, but now a change in $g$ will affect worker's share indirectly through $\beta$. So we can conclude that the "neutrality" result of the per head tax of the previous section does not hold in the monopoly union framework. In the simulations we will show that the effect of $g$ on worker's relative share is negative with plausible parameter values. Therefore, the effects of proportional and per head income taxes are closer to each other in the monopoly union model.

## 2.7 Efficiency wage setting

As another alternative mechanism of wage determination consider the "efficiency wage" model by Shapiro and Stiglitz (1984). In this framework, those who are employed can either exert effort or "shirk".[9] For the workers, exerting effort is costly, so they need some incentives to do so. Because the productivity of those supplying effort is higher, it is in the interest of the firms that their employees exert effort. If a firm finds someone shirking, she or he will be dismissed and will end up searching for a new job. However, monitoring is imperfect and there is only a positive probability of detecting a less-productive worker. The solution to the problem is that firms end up paying a wage rate that makes the workers indifferent between exerting effort and "shirking". This wage level is referred to as the "efficiency wage". For the employed exerting effort the valuation of a job can be written as

$$rW_e(x) = (1-t)w - g + \delta(UF(R) + S_e(x)) - \delta W_e(x) \tag{37}$$

For the employed not exerting effort we have

$$\begin{aligned} rW_s(x) &= (1-t)w - g + e + \delta(UF(R) + S_s(x)) - \delta W_s(x) \\ &\quad + q(U - W_s(x)) \end{aligned} \tag{38}$$

[9] The dichotomy in the level of effort is a simplified assumption that can be relaxed for a continuous level of effort. For risk neutral workers, the results will not be affected as shown by Pisauro (1991).

where $e$ is the value of the extra leisure from exerting no effort and $q$ is the monitoring frequency. Notice that the wage rate $w$ is independent of job productivity. The valuation of a job for a worker only depends on whether she or he exerts effort or not. So we have

$$W_i(x) = W_i, \forall x, i = e, s \tag{39}$$

which implies

$$S_i = \int_R^1 W_i dF(x) = W_i(1 - F(R)) \tag{40}$$

Imposing indifference between exerting effort and "shirking"

$$W_e = W_s = W \tag{41}$$

into (37) and (38) yields the *no-shirking condition*

$$e = q(W - U) \tag{42}$$

Notice that (37) can be rewritten as

$$rW = (1 - t)w - g + \delta F(R)(U - W) \tag{43}$$

For the unemployed recall (1) and apply (39) to get

$$rU = b + \lambda(W - U) \tag{44}$$

The efficiency wage can be solved from (43), (42) and (44) to get

$$w = \frac{b + g}{1 - t} + \frac{e(r + \lambda(\theta) + \delta F(R))}{q(1 - t)} \tag{45}$$

According to (45), wages depend positively on both labour market tightness ($\theta$) and reservation productivity ($R$). Intuitively, labour market tightness puts an upward pressure on wages because finding another job becomes easier for a potential shirker. Wages increase in reservation productivity because jobs became more insecure with higher $R$. Furthermore, for given labour market tightness and reservation productivity, wages decrease in the monitoring frequency ($q$) and increase in the value of extra leisure to a shirker ($e$) as well as proportional and per head income taxes, $t$ and $g$. However, slightly rearranging (45) shows that after tax wages $(1 - t)w - g$ are independent of taxes, for given labour market tightness and reservation productivity. Whether after tax wages increase or decrease after a tax hike depend on the indirect effects through $\theta$ and $R$. It turns out in the simulations that there are two offsetting effects that cancel out each other. After tax wages thus remain unchanged if taxes are increased.

To determine the labour market equilibrium in the efficiency wage case, we note that the reservation productivity is now determined by (with (13) binding)

$$J(R) = 0 \tag{46}$$

Substituting wage equation (45) into (6) under (13) and rearranging gives

$$(r+\delta)\,J(x) = x - (1+s)\left(\frac{b+g}{1-t} + \frac{e\,(r+\lambda(\theta)+\delta F(R))}{q\,(1-t)}\right) + \delta S_J \qquad (47)$$

which corresponds to (16) in the basic model with Nash bargain over wages. Again, developing a Taylor series around $J(R) = 0$ yields

$$J(x) = \frac{1}{r+\delta}(x-R) \qquad (48)$$

Applying (47) to $x = R$ (and (46)) and further substituting (48) for $J(z)$ gives

$$\frac{(1+s)}{1-t}\left(b+g+\frac{e}{q}(r+\lambda(\theta)+\delta F(R))\right) = \frac{\delta}{r+\delta}\int_R^1 (z-R)\,dF(z) + R \qquad (49)$$

which is the *job destruction condition in the efficiency wage model* and constitutes an upward sloping schedule in the $R, \theta$ -space by appropriate choice of the exogenous parameters.[10]

To derive another independent equation in the two unknowns, first notice that with (13) binding, (4) still implies

$$J(1) = \frac{c}{\eta(\theta)} \qquad (50)$$

which is (21). Next, apply (48) to $x = 1$ and substitute (21) for the value of a job to get

$$c = \frac{(1-R)}{r+\delta}\eta(\theta) \qquad (51)$$

which is the *job creation condition in the efficiency wage model* and constitutes a downward sloping curve in $R, \theta$ -space. Similar to the basic model with Nash bargain, tax parameters only enter the job destruction condition (49), causing a shift up and to the left. Since the job creation schedule remains stable, a tax hike unambiguously causes an increase in the reservation productivity and a decline in the labour market tightness. It is easy to see that the equivalence of proportional income tax and the payroll tax still holds under the condition (26). In addition, in the present set up, the proportional income tax is equivalent to the per head tax as long as rates are chosen such that $g = tw$. This can be verified by imposing $t = 0$ and $g = t_0 w$ into (45) and (49): the resulting equilibrium will be identical to the case $t = t_0$ and $g = 0$.

Notice that in equilibrium, combining (42), (13) and (17) we can derive the following expression for the worker's share of the surplus

$$\frac{W-U}{J(x)-V+W-U} = \left(1+\frac{q\,(x-R)}{e\,(r+\delta)}\right)^{-1} \qquad (52)$$

[10] Mortensen and Pissarides (1999) have shown that generally the job destruction schedule may be non-monotonic and therefore multiple equilibria is possible.

Expression (52) reveals that tax rates (both proportional and per head) affect workers relative share only indirectly through the reservation productivity $R$, which increases with all of the three tax rates. Since by (52) the workers share is increasing in $R$, we can conclude that taxes increase workers relative share of the surplus in the efficiency wage specification. This is a clear contrast to the models with wage bargaining presented in the previous sections.

# 3 Simulations

In the previous section we derived the equilibrium of the labour market under alternative assumptions concerning the process of wage determination. We already noticed that the effects of tax parameters differ depending on the prevailing mechanism of wage setting. In this section we put the formulas in work to find more specific quantitative responses to tax policy. For that purpose we specify functional forms for the matching function and for the dispersion of the productivity shock. We then choose some plausible numerical values for the exogenous parameters of the model.

Before turning to the simulations, notice that the dynamics of unemployment in the model is described by

$$\frac{du}{dt} = \delta F(R)(1-u) - \lambda(\theta)u \tag{53}$$

where the first term in the right hand side is the flow into unemployment and the second term is the flow out of unemployment. Substituting (9) for $\lambda$, the steady state unemployment can be derived as

$$u = \frac{\delta F(R)}{\delta F(R) + \theta^{\rho}} \tag{54}$$

which is the Beveridge curve of the economy with endogenous job destruction (Pissarides, 2000). Let us also define the *unemployment incidence* as $I \equiv \delta F(R)$ and the *expected duration of unemployment spells* as $D \equiv \theta^{-\rho}$ that we utilise in reporting the simulations results.

To develop a measure of overall welfare, notice that the steady state aggregate income net of search and recruiting costs $y$ can be defined as

$$y = \left(F(R) + \int_R^1 x dF(x)\right)(1-u) + (b - a - c\theta)u \tag{55}$$

where the first term in the right hand side defines the total product in steady state with $F(R)$ representing the fraction of matches of type $x = 1$ (see Appendix C for details). Notice that the value of leisure is taken as exogenous income and the incomes are defined gross of taxes.

To facilitate simulations, we specify a distribution for the productivity shock. For simplicity, we employ the assumption already utilised above in the monopoly union set up that $x$ is uniformly distributed in the interval $[0, 1]$. With this assumption and after substituting (9) into (23) and (25), the aggregate income, the job destruction condition and the job creation condition for the Nash bargain model can be rewritten as follows (see Appendix B for details):

$$y = \left(1 - \frac{1}{2}(1-R)^2\right)(1-u) + (b - a - c\theta)\, u \tag{56}$$

$$\frac{(1+s)}{(1-t)}(b - a + g) + \frac{\beta c\theta}{(1-\beta)} = R + \frac{\delta (1-R)^2}{2(r+\delta)} \tag{57}$$

$$c = \frac{1-\beta}{r+\delta}\theta^{\rho-1}(1-R) \tag{58}$$

The endogenous worker's share parameter of the monopoly union model is then determined by (35). As for the efficiency wage model, the job destruction condition (49) , job creation condition (51) and the wage equation (45) simplify to

$$\frac{1+s}{1-t}\left(b + g + \frac{e}{q}(r + \theta^\rho + \delta(R-\gamma))\right) = \frac{\delta(1-R)^2}{2(r+\delta)} + R \tag{59}$$

$$c = \frac{1-R}{r+\delta}\theta^{\rho-1} \tag{60}$$

$$w = (1-t)^{-1}\left(b + g + \frac{e(r + \theta^\rho + \delta R)}{q}\right) \tag{61}$$

At this stage, we could choose the exogenous parameters of the model to reflect the stylised facts of some particular economy (see e.g. Millard & Mortensen, 1997, Holm et al, 1999). Instead, we use "rule of thumb" values that roughly correspond to the ones used in earlier studies on policy impact (e.g. Mortensen and Pissarides, 1999, Pissarides 1998). The matching function elasticity parameter $\rho$ is chosen so as to satisfy the Hosios condition in the case of symmetric Nash bargain. Parameter values are adjusted so as to produce a reasonable low unemployment rate (roughly 1.5 per cent) at the no tax benchmark of the Nash bargain model. The extra parameters of the monopoly union model (median worker productivity) and efficiency wage model (value of extra leisure and monitoring frequency) are then chosen so that the unemployment rate with no taxes is close to that of the Nash bargain model. The parameter values used in the simulations are presented in Table 1.

| $a$ | $b$ | $c$ | $r$ | $s$ | $\rho$ | $\delta$ | $x_m$ | $e$ | $q$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.2 | 0.6 | 0.2 | 0.01 | 0.0 | 0.5 | 0.03 | 0.98 | 0.1 | 0.5 |

**Table 1**: The parameter values used in the model simulations. $a$ is the search cost per period, $b$ is the value of leisure gross of search cost, $c$ is the per period cost of holding a vacancy, $r$ is the (quarterly) rate of discount, $s$ is the payroll tax, $\rho$ is the matching elasticity parameter, $\delta$ is frequency of a productivity shock, $x_m$is the productivity of the median worker, $e$ is the value of extra leisure for a shirker and $q$ is the monitoring frequency of the firm.

## 3.1 Effects of labour taxes

Let us first consider the effect of an exogenous increase in the proportional tax rate on labour income, $t$. The results of a simulation where $t$ was increased from zero to 0.1 are presented in Table 1. The effect on equilibrium job creation and job destruction rates are, as expected, of equal sign irrespective of the wage setting mechanism. Job creation rate is reduced due to introduction of the tax leading to longer unemployment spells ($\Delta D > 0$). Reservation productivity increases causing more job destruction at given level of employment. This is demonstrated in the higher unemployment incidence ($\Delta I > 0$)

The main difference between the models is the magnitude of the employment effects. In the efficiency wage models the effects are much larger than in the models with wage bargaining. This reflects the fact demonstrated in section 2.7 that labour taxes increase the worker's share of the surplus in the efficiency wage model. This is then reflected in the much stronger response of wages to a tax hike. In terms of the equilibrium condition, while the job creation condition of efficiency wage model (51) is identical to the Nash bargain model, the job destruction schedule (49) is much flatter than its counter part in the bargaining framework.

| | $t$ | $g$ | $\Delta D$ | $\Delta I$ | $\Delta u$ | $\Delta y$ |
|---|---|---|---|---|---|---|
| Efficiency wage | 0.1 | 0.0 | 33.2 | 0.4 | 33.5 | 0.08 |
| Monopoly union | 0.1 | 0.0 | 3.6 | 0.1 | 3.5 | $-0.004$ |
| Nash bargain $\beta = 0.5$ | 0.1 | 0.0 | 4.0 | 0.1 | 4.1 | $-0.002$ |

**Table 2**: Results from the simulations where proportional income taxation was introduced at rate $t = 0.1$. $D$ is the average duration of unemployment spells, $I$ is unemployment incidence, $u$ is unemployment rate and $y$ aggregate income measuring overall efficiency. The symbol $\Delta$ refers to percentage change.

In the monopoly union and Nash bargaining models, the wage and employment responses are much more modest due to the wage moderation effect inherent in wage bargaining already discussed above in section 2.4. Higher proportional tax rate reduces worker's share of the surplus supporting a moderate wage response. Somewhat surprisingly though, the monopoly union does not try to counteract, but rather fortifies wage moderation by reducing $\beta$. Thus, the negative employment effect is smallest in the monopoly union model.

The effects of tax changes on the overall efficiency $y$ mainly reflect the efficiency of the selected benchmark in the models. In the efficiency wage model, unemployment is below the efficient level in the no tax equilibrium and efficiency slightly improves after a tax hike. In the monopoly union model, the opposite is true. In the Nash bargain model the no tax equilibrium is efficient and cannot be improved with tax policy as discussed above in section 2.5. Therefore, efficiency necessarily drops when taxes are increased.

The effects on key endogenous variables of an increase in the per head tax $g$ is presented in Table 3. The results are relatively similar to those of the proportional tax. In fact, for the efficiency wage model, the effects of the two taxes would be identical, if the rates would be chosen such that $g = tw$ and with our choice of parameters, the equilibrium wage is close to unity. The main difference to the proportional tax is that now the wage and employment response in the bargaining models is not so much apart from the efficiency wage model. As noticed in section 2.4 and 2.6 the fixed component of income tax does not reduce the worker's share of the surplus. It does so, however, indirectly in the monopoly union model through lower $\beta$. These facts help to explain why the per head tax has relatively strong effect (in comparison to proportional tax) on unemployment in Nash bargaining model and somewhat weaker effect in the monopoly union model.

| | $t$ | $g$ | $\Delta D$ | $\Delta I$ | $\Delta u$ | $\Delta y$ |
|---|---|---|---|---|---|---|
| Efficiency wage | 0.0 | 0.1 | 34.1 | 0.3 | 34.3 | 0.08 |
| Monopoly union | 0.0 | 0.1 | 8.5 | 0.25 | 8.6 | −0.01 |
| Nash bargain $\beta = 0.5$ | 0.0 | 0.1 | 9.8 | 0.25 | 9.9 | −0.009 |

**Table 3**: Results from the simulations where per head income taxation was introduced at rate $g = 0.1$. $D$ is the average duration of unemployment spells, $I$ is unemployment incidence, $u$ is unemployment rate and $y$ aggregate income measuring overall efficiency. The symbol $\Delta$ refers to percentage change.

## 3.2 Revenue neutral progressive taxation

To compare the response of the alternative wage determination models to a progressive labour taxation, we simulated a simultaneous introduction of proportional wage tax ($t = 0.1$) and a per head tax allowance or subsidy ($g < 0$). The size of the sudsidy was adjusted to just exhaust the revenue raised by the proportional tax. Thus, the net revenue raised by the tax system was zero. With this set up we are able to focus on the effects of pure progression of the tax system. For simplicity, our starting point is the no tax equilibrium of each model, but the results should be applicable to cases with pre-existing (proportional) taxes as well.

| | $t$ | $g$ | $\Delta D$ | $\Delta I$ | $\Delta u$ | $\Delta y$ |
|---|---|---|---|---|---|---|
| Efficiency wage | 0.1 | −0.0985 | – | – | – | – |
| Monopoly union | 0.1 | −0.0986 | −4.5 | −0.15 | −4.59 | – |
| Nash bargain $\beta = 0.5$ | 0.1 | −0.0986 | −5.13 | −0.15 | −5.18 | −0.004 |
| Nash bargain $\beta = 0.55$ | 0.1 | −0.0987 | −5.14 | −0.15 | −5.19 | 0.008 |

**Table 4**: Results from the simulations where progressive income taxation was introduced in a revenue neutral manner. $D$ is the average duration of unemployment spells, $I$ is unemployment incidence, $u$ is unemployment rate and $y$ aggregate income measuring overall efficiency. The symbol $\Delta$ refers to percentage change.

The results from the balanced budget simulations are presented in Table 4. Efficiency wage model results are close to those expected from a competitive labour markets. The two taxes are essentially equivalent as long as the rates are chosen such that $g = tw$, which indeed results from the balanced budget constraint. Consequently, higher progression, involving no change in the total tax burden, has no effect on the labour market equilibrium.

In the Nash bargaining model and monopoly union models, a revenue neutral introduction of progressive taxation evokes wage moderation and reduces unemployment. The mechanisms behind the result are essentially the same as discussed above with reference to the isolated increases in the two tax instruments with exception that $g$ now works in the opposite direction and dominates the net effect. As for the monopoly union case, the adjustment of $\beta$ (upwards) again mitigates the (this time negative) wage response and leaves the improvement of employment somewhat smaller than in the Nash bargain model.

As for the effect on aggregate output, it is noteworthy that despite improved employment, efficiency drops in the Nash bargain model (with $\beta = 0.5$). This is reflection of the fact discussed above in section 2.4 that if the "no tax" equilibrium satisfies the Hosios condition, the efficiency cannot be improved. This is exactly the case in our model when $\beta = 0.5$ (and $\rho = 0.5$ as reported in Table 1). Then introduction of any distortionary policy reduces efficiency. To put it differently, the rate of unemployment in the no tax equilibrium is exactly right for the efficient functioning of the labour market. This does not necessarily hold in a "second best" situation where $\beta \neq 1 - \rho$ as demonstrated by the entries in the last row of Table 4. The final row shows the results for Nash bargaining model where $\beta$ was fixed to value 0.55. The employment effects are close to those of the $\beta = 0.5$ case, but efficiency improves, because unemployment is too high in the initial equilibrium. In the monopoly union case, efficiency remains unchanged, but this result is sensitive to the magnitude of the tax changes.

# 4 Concluding Remarks

We have considered the effects of progressive labour taxation in an equilibrium model of labour market with endogenous job creation and job destruction. To emphasise the role of wage setting, we applied three alternative hypothesis of wage determination: Nash bargain, monopoly union and efficiency wage. Having derived the equilibrium with taxes under alternative wage determination processes, we discussed the key features of the resulting three distinct models.

We find that, among others, the effects of tax parameters on the worker's share of the match surplus differ between the models. This helps to understand the different response to tax policy changes demonstrated in the numerical simulations. Simulating the models, we find that while both proportional and per head labour taxes harm employment in all three models, the magnitude of these effects are much smaller in the models with wage bargaining. The difference is particularly large for the proportional tax, which triggers a wage moderation effect in the bargaining models.

Finally, we combine the two tax instruments in a revenue neutral manner to facilitate a pure increase in the tax progression. We find that increased progression of labour taxes may improve employment with low or even non-existent efficiency cost if wages are set in a bargaining framework. Throughout the simulations, the employment effects of taxation are smaller in the monopoly union model than in the simple Nash bargain model.

Despite the somewhat different framework, our results are broadly in line with those of Pissarides (1998), who suggests that a revenue neutral increase in the tax progression comes close to a "free lunch". From a policy point of view, these findings might provide another justification for the tax reforms that aim to mitigate the tax burden of the low-income workers by introducing tax exemptions and increasing the lower limit for taxable income. Also, to the extent that formulation of the "monopoly union" case can be interpreted as representing more centralised wage bargaining, the results may help to explain the recent empirical findings suggesting that employment effects of labour taxes tend to be smaller in the corporatist economies (Daveri & Tabellini, 2000, Kiander et al, 2000).

As for the future research, it would be interesting to try to fit the models with more realistic parameter values. As this may prove relatively difficult, the sensitivity of the results to changes in the parameter values should be more rigorously considered. A characteristic feature of the models employed in this study is that the policy effects to employment come mainly through implied changes in job creation and therefore unemployment duration. The effects through job destruction seem to be of minor importance. Though this feature may not be all that unrealistic, it might need to be reconsidered in the future work.

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# A Derivation of the Nash wage rule with taxes

When wage setting is assumed to be based on a Nash bargain between workers and firms, the wage rate is determined by

$$w(x) = \arg\max (W(x) - U)^{\beta} (J(x) - V)^{1-\beta} \tag{62}$$

where $\beta$ is an exogenous parameter. Workers and firms are assumed to be "small" in the sense that they do not consider the effects of their action on aggregate variables, in particular on the value of unemployment search and on the value of a vacancy. To derive the first order condition of the problem, differentiate the right hand side of (62) with respect to $w(x)$ and set equal to zero to get

$$\beta (J(x) - V) \Omega_W + (1-\beta)(W(x) - U) \Omega_J = 0 \tag{63}$$

where $\Omega_W = \frac{\partial(W(x)-U)}{\partial w(x)}$ and $\Omega_J = \frac{\partial(J(x)-V)}{\partial w(x)}$ and both $W(x) - U$ and $J(x) - V$ are assumed strictly positive. Utilising the formulas (1), (3), (4) and (6) we can derive expressions for the two derivatives as follows

$$\Omega_W = \frac{1-t}{r+\delta} \tag{64}$$

$$\Omega_J = -\frac{1+s}{r+\delta} \tag{65}$$

Substituting these into ( 63) and some manipulation then yields

$$W(x) - U = \left(1 + \frac{(1-\beta)(1+s)}{\beta(1-t)}\right)^{-1} (J(x) - V + W(x) - U) \tag{66}$$

which is (11). Notice that the derivatives with respect to the proportional tax rates of the first term on the right hand side of (66) are given by

$$\frac{\partial}{\partial t}\left(\frac{\beta(1-t)}{(1+s) - \beta(s+t)}\right) = -\frac{\beta(1-\beta)(1+s)}{(-1-s+\beta s+\beta t)^2} < 0 \tag{67}$$

and

$$\frac{\partial}{\partial s}\left(\frac{\beta(1-t)}{(1+s) - \beta(s+t)}\right) = -\frac{\beta(1-t)(1-\beta)}{(-1-s+\beta s+\beta t)^2} < 0 \tag{68}$$

Substitution of (3) and (6) into the wage rule (11) gives

$$\begin{aligned} & (1-\beta)\left(\frac{(1-t)w(x) - g + \delta(UF(R) + S_W)}{r+\delta} - U\right)(1+s) \\ = \; & \beta\left(\frac{x - (1+s)w + \delta(VF(R) + S_J)}{r+\delta} - V\right)(1-t) \end{aligned} \tag{69}$$

which can be solved explicitly for the wage rate to get

$$w(x)(1+s) = \beta x + (1-\beta)\frac{1+s}{1-t}(rU+g) - \beta r V \tag{70}$$

which is (12).

# B Distribution for the Productivity Shock

Assume productivity shock is uniformly distributed in the interval $[0,1]$. Then $f(z) = 1$ and we have

$$\int_R^1 (z-R)\,dF(z) = \int_R^1 (z-R)\,dz = \frac{1}{2}(1-R)^2$$

$$F(R) = \int_0^R dz = R$$

Also notice that

$$\int_R^1 z dF(z) = \int_R^1 z dz = \frac{1-R^2}{2}$$

# C Tax Revenues at the Steady State

Derive the steady state shares of different type of matches. The stock of matches with productivity $x = 1$, $n_1$ evolves according to

$$\frac{dn_1}{dt} = m(\theta)u - \alpha_1\delta(1-u)(1-F(R)) - \alpha_1\delta(1-u)F(R) \tag{71}$$

where $\alpha_1$ is the share of matches with productivity $x = 1$ of all matches. The three terms on the right hand side represent creation of new jobs, revaluation of jobs with $x = 1$ and destruction of jobs with $x = 1$, respectively. Setting the right hand side equal to zero and solving for $\alpha_1$ yields

$$\alpha_1 = \frac{m(\theta)u}{\delta(1-u)} = F(R) \tag{72}$$

where the last equality follows from the right hand side of (53) being equal to zero in steady state. Consider the revenue from the labour income tax at the steady state. For the proportional part, the revenue is given by

$$T_t = \left(w(1)F(R) + \int_R^1 w(z)dF(z)\right)(1-u)t \tag{73}$$

For the constant part the revenue is

$$T_g = (1-u)g \tag{74}$$

Substituting (24) for $w(x)$ and utilising $f(x) = 1$ and $F(R) = R$ we get

$$\begin{aligned} \int_R^1 w(x)dF(x) &= \int_R^1 \left((1-\beta)\frac{b-a+g}{1-t} + \beta\frac{x+c\theta}{1+s}\right) dF(x) \\ &= (1-R)\left(\frac{1-\beta}{1-t}(b-a+g) + \frac{\beta}{1+s}\left(c\theta + \frac{1+R}{2}\right)\right) \end{aligned} \quad (75)$$

Substituting this into (73) and applying (24) for $w(1)$ then yields

$$T_t = \left(\frac{1-\beta}{1-t}(b-a+g) + \frac{\beta}{1+s}\left(c\theta + 1 - \frac{1}{2}(1-R)^2\right)\right)(1-u)t \quad (76)$$

Repeating similar procedure for the payroll tax yields

$$T_s = \left(\frac{1-\beta}{1-t}(b-a+g) + \frac{\beta}{1+s}\left(c\theta + 1 - \frac{1}{2}(1-R)^2\right)\right)(1-u)s \quad (77)$$

Combining (74), (76) and (77), the total revenue can be expressed as follows

$$\begin{aligned} T &= T_t + T_g + T_s \\ &= \left(\left(\frac{(1-\beta)}{1-t}(b-a+g) + \frac{\beta}{1+s}\left(c\theta + 1 - \frac{1}{2}(1-R)^2\right)\right)(t+s) + g\right) \\ &\quad \times (1-u) \end{aligned} \quad (78)$$

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