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# PUBLIC WELFARE SERVICES AND INEQUALITY: INTRODUCTION TO METHODOLOGY AND SOME EXAMPLES WITH THE 1985 FINNISH HOUSEHOLD EXPENDITURE SURVEY DATA

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**ABSTRACT:** In the paper concentration curve analysis is applied to assess the distribution effects of the public provision of welfare services. In contrast with, say estimation of the mean, concentration curve estimation is here argued to be considerably less robust for data imperfections. In the latter case a selection bias-like component creeps in and the choice of the reference distribution has crucial importance for successful management of the bias. Empirical examples using Finnish microdata indicate that public provision of welfare services has an inequality decreasing effect. In addition, marginal reforms which favour public provision and increase commodity taxes either in general or in private counterparts of these services, seem to increase welfare in an inequality averse society. These results are reasonably robust when the analysis is extended considering some important sub-groups of households.

KEY WORDS: Inequality; Welfare Services; Concentration Curve Dominance; Household Expenditure Survey.

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**TIIVISTELMÄ:** Tutkimuksessa analysoidaan yhteiskunnallisten hyvinvointipalvelujen jakautumavaikutuksia keskittymiskäyrien tarkastelun avulla. Tavallisesta keskiarvojen estimoinnista poiketen keskittymiskäyrien estimointi on huomattavasti herkempää aineistopuutteiden suhteen. Keskittymiskäyrien tapauksessa syntyy harhaa, joka on läheistä sukua valikoitumisharhalle, ja lisäksi vertailujakauman perustana olevan muuttujan valinta on olennaista harhan kurissa pitämiseksi. Suomen mikroaineistoon perustuvat esimerkit viittaavat siihen, että yhteiskunnalliset hyvinvointipalvelut vähentävät yhteiskunnan eriarvoisuutta. Lisäksi sellaiset reformit, jotka rahoittavat palvelujen käytön marginaalisen lisäyksen kiristämällä hyödykeverotusta yleensä tai jopa palvelujen yksityisen sektorin tuottamien vastineiden osalta, lisäävät eriarvoisuutta kaihtavan yhteisön hyvinvointia. Nämä tulokset ovat kohtuullisen vakaita, vaikka tarkastelu ulotetaankin joihinkin tärkeisiin kotitalouksien rakenteen perusteella muodostettuihin aliryhmiin.

**ASIASANAT:** Eriarvoisuus; Julkiset hyvinvointipalvelut; Keskittymiskäyrien analysointi; Kotitaloustiedustelu.

**CONTENTS**

## 1. INTRODUCTION

Provision of welfare services is a major component of public consumption. Economic analysis of redistribution has conventionally stressed the role of taxes and transfers as instruments in pursuing equity objectives, neglecting the role of welfare services. In Finland about one-third of total public expenditure was used to provide education, health and social services in 1990. It would be difficult to justify such massive public sector intervention on efficiency grounds without considering the sizable redistribution effect due to public provision of welfare services.[1]

At the turn of the current decade the Finnish economy has been hit by a recession which is unprecedentedly severe. As a result the GNP-share of public expenditure has risen to a very high level and the public sector debt is accumulating rapidly. The hardships in financing the increased public consumption have put forth urgent demand to assess the provision of welfare services both from efficiency and from equity points of view. Preferably the analysis should guide cuts in public expenditure in order to limit free access in services where the consequences are least painful.

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[1] Although redistribution in cash rather than in kind is traditionally advocated by economists, the beneficial redistributive effects of welfare services, eg. in the fields of education and health services, have been recognized in the literature. Consider situations where the commodity is not transferable and individuals choose between commodity bundles offered by the government and by the private sector. Further, the individuals who opt in for the commodity bundle provided by government cannot top up the fixed amount (or quality) they receive by purchases in the private sector. In this case the use of transfers in-kind (public provision) and user fees extends the range of instruments available to the government and is welfare-improving in the presence of distortionary taxes. The channel through which the additional instruments operate is through relaxing the self-selection constraint of the optimal taxation theory. As a result the welfare loss due to pursuing the equity objectives is minimized by spreading over many instruments (see eg. Guesnerie and Roberts, (1984), Blackorby and Donaldson (1988), Boadway and Marchand (1990) and Munro (1992)).

This paper considers the provision of welfare services and their effect on economic inequality using the 1985 Finnish Household Expenditure Survey microdata. Previous Finnish studies with similar databases have established that the utilization of public welfare services in general alleviates economic inequality, eg. Parkkinen (1981), Sullström (1984), Uusitalo (1988) and Tilastokeskus (1988). The previous analysis has been for the most part contended with presenting calculations based on quintile or decile means for the data.[2]

The approach adopted in this paper is to analyse inequality with the use of the more informative Lorenz and concentration curve techniques in connection with derived inequality indicators, generalized Gini and concentration coefficients, see Yitzhaki (1983). These statistical concepts are interwoven to an underlying welfare-based theoretical framework using some recently derived results which neatly combine these two at first sight disconnected approaches. In the paper extensive use is made of the concept of conditional concentration curve dominance at marginal changes of government intervention (MCDS) by Slemrod and Yitzhaki (1987). An additional interesting feature in the study is the selection of the reference distribution which is here scale-corrected household consumption. The scope of possible welfare improving reforms in an inequality averse society is here further extended by considering commodity taxation as a viable finance source of public welfare services.

The decomposition of inequality measures has contributed a great deal towards understanding the determinants of inequality and the relative contribution of various independent factors. These derivations do not however often take account of the data imperfections likely to be encountered in prac-

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[2] The approach used in present study is more similar with Hagfors and Sullström (1989) who studied the concentration of social transfers using Finnish Household Expenditure Surveys 1976 and 1981, and Häkkinen (1991) who studied inequality in health care using micro data from 1987.

tical empirical analysis. A very severe problem concerns the error free measurement of the various subcomponents of consumption (or income if the latter is selected as a basis of comparison). To be more specific, first the use of welfare services is recorded under a relatively short time-period, usually one month in our data. This leads to a higher variance and inequality being over-estimated relative to figures recorded under a longer time period. Further, the concentration curve is based on an expected value conditional on the value of the reference variable not exceeding an upper limit. Therefore, a selection bias component creeps in and leads to inconsistent and biased estimation of the concentration curves. It is thus a more severe problem than in the case of estimating the mean level in the utilization of welfare services. In addition the choice of the reference distribution plays a crucial role here. In the paper it is shown by examples from Finnish data that these problems have relevance when the contribution of public welfare services to economic inequality is assessed.

The positive contribution of this study is to indicate instances where welfare services alleviate economic inequality. In this context it seems natural to consider the use of commodity taxation as a viable finance source. These aspects have been relatively neglected in the Finnish research tradition whereas the areas of direct taxation and public transfers have attracted more interest. In this paper cases are presented which imply that marginal changes in commodity taxation and provision of public welfare services may be welfare improving through their effect on the distribution of utility among the households. In addition similar conclusions are obtainable by changes in the structure of indirect taxation.

On the methological side some independent interest may lie in the empirical application of a Lorenz and concentration curve estimation method utilized in the study. Here a non-parametric estimation method is used which is based on a locally weighted polynomial regression together with end-point restrictions.

The paper is organized as follows. Section 2 introduces the analytical framework. Section 3 presents the data and discusses the effects of data inaccuracy on the empirical estimation and the choice of the reference distribution. Section 4 briefly describes the non-parametric estimation method used in the study. In section 5 the main results are presented, first for the whole data and then for some important sub-groups of households separately. The latter exercise facilitates welfare statements with less dependence on the choice of equivalence scale. As a by-product to our analysis a substantial bias is shown to creep into the estimation if measurement error is dealt with in a relaxed fashion. Section 6 presents the conclusions and points out some avenues for further study.

## 2. ANALYTICAL FRAMEWORK

The main tools of the present paper are the familiar Lorenz and concentration curve analysis and inequality indices based directly on these curves. The **Lorenz curve**, L(p), is defined as the proportion of income which is earned by the least privileged p-fraction of consumers, i.e.

$$L_F(p) = \int_0^{F^{-1}(p)} y\,dF_y / M_y, \quad with\ p \in [0,1]. \tag{1}$$

Above F is the cumulative distribution function of income, y, and $M_y$ is the mean income. The **concentration curve** of expenditure on a commodity, x, is defined similarly as the proportion of aggregate consumption of x which is consumed by the least privileged p-fraction of consumers w.r.t. the distribution of y. More specifically, the integrand in (1) is replaced with x which is defined as a function of y, E(x |y), and $M_y$ replaced with the mean expenditure $M_x$. The concentration curve is convex (concave) to the origin if the income elasticity of the commodity is positive (negative).

Yitzhaki (1983) has suggested a generalization of the familiar **Gini coefficient**. The **extended Gini** is a weighted integral of the area between the forty-five degree line and the Lorenz curve, with the formula

$$G_\nu(y) = 1 - \nu(\nu-1)\int_0^1 (1-p)^{\nu-2} L_F(p)\,dp \tag{2}$$

$$= -\nu\int_0^\infty (1-F)^{\nu-1} dF + \nu\int_0^1 L_F(p)\,d(1-p)^{\nu-1} \tag{3}$$

$$= -\nu\,Cov(y,\ [1-F(y)]^{\nu-1})\,/M_y, \quad with\ \nu > 1. \tag{4}$$

The last formula follows by a change of variables $y = F^{-1}(p)$ and partial integration (Note that $dL_F(p)/dp = y/M_y$ at the

point $y = F^{-1}(p)$). The Gini is a special case of the extended Gini where $\nu = 2$.

The **extended concentration coefficient** of expenditure on a commodity, $C_\nu(x, y)$ is defined similarly

$$C_\nu(x, y) = -\nu\, Cov(x, [1-F(y)]^{\nu-1}) / M_x. \quad (5)$$

By applying the law of iterated expectations one gets the following formula which is useful for theoretical derivations

$$C_\nu(x, y) = -\nu E_y\left[ E(x|y) [1-F(y)]^{\nu-1}\right] - 1. \quad (6)$$

Following Lerman and Yitzhaki (1984) we can decompose the Gini coefficient by income components, $\sum y_k = y$, using the additivity property of the covariance to get

$$G_2 = 2\,Cov(y, F(y)) / M_y = 2\sum Cov(y_k, F(y)) / M_y = \sum G_k R_k S_k, \quad (7)$$

where $G_k$ is the Gini coefficient corresponding to income component k, $S_k$ is the share of component k of total income and $R_k$ is the **Gini correlation** of component k with total income

$$R_k = \frac{Cov(y_k, F(y))}{Cov(y_k, F(y_k))} = \frac{C_2(y_k, y)}{G_2(y_k)}. \quad (8)$$

The formula (7) gives an easily applicable tool of assessing the contribution of different income or consumption expenditure components to inequality.[3]

An alternative approach is to assume cardinal, i.e. comparable individual utility functions and to base the analysis on a well-defined social welfare function. Commonly used functions assume the individually additively separable, symmetric functional or Paretian form, i.e.

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[3] The formula (7) is easily generalized to deal with extended Gini with an extended Gini correlation defined by the means of extended concentration coefficient as in (8).

$$W(F_y) = \int_0^\infty U(y)\, dF_y \ . \qquad (9)$$

Here U is an increasing, concave function of y. The concavity of U implies that the society has aversion against inequality.

A prime example of social welfare functions is the class of functions due to Atkinson (1970) which are based on a constant relative inequality aversion by the society, where the aversion parameter is defined by -yU"/U'.

However, the exact form of the social welfare function is open to debate and therefore is best considered unknown. The following set of results have proved especially useful in stating some general and easily verifiable conditions for Lorenz curves that are equivalent with an improvement in social welfare. These results are based on second-order stochastic dominance criteria and were originally developed in the finance literature to rank portfolios.

**Proposition 1** (Atkinson, 1970) If the income distributions F and G have equal means then the condition[4],
$L_F > L_G$, holds if and only if
the income distribution F is preferable to G with respect to all concave utility functions U.

A more general formulation of the previous result is

**Proposition 2** (Shorrocks, 1983) Define **generalized Lorenz curve** as $GL(p) = M_y L(p)$. For the income distributions F and G the condition,
$GL_F > GL_G$, holds if and only if
the income distribution F is preferable to G with respect to all concave utility functions U.

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[4] The notation $L_F > L_G$ means that the Lorenz curve associated with the distribution F, $L_F(p) \geq L_G(p)$ for all $p \in [0, 1]$ and strict inequality for at least one point.

The following result gives a method of assessing reforms in commodity taxation and provision of public services from the welfare point of view by the use of marginal conditional (second degree) stochastic dominance, MCSD. The dominance criteria are defined by the use of concentration curves.[5]

**Proposition 3** (Slemrod and Yitzhaki, 1987) Assume that the government makes a marginal change in its commodity tax system by subsidizing one commodity and taxing another commodity by an equal marginal amount. The social welfare increases for all concave Paretian social welfare functions if and only if the concentration curve of the first commodity is everywhere above that of the second commodity.

The above condition can be checked by direct comparison of the relevant concentration curves. On the other hand, a necessary condition for the MCSD to hold is that the extended concentration index $C_\nu$ defined by (5) is less for the first commodity with all values of $\nu > 1$.

Yitzhaki (1983) shows that in calculating the extended Gini or concentration coefficients an increase in $\nu$ increases the relative weights attached to the lower end of the income distribution at the expense of those attached to the upper end (see eq. 3).[6] Further, he claims that the extended Gini has most of the properties of the celebrated Atkinson inequality index and, similarly as with the Atkinson index, the analyst may select a range of values for $\nu$ to check for the robustness of inequality ranking of the alternatives to

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[5] The concentration curve analysis has been previously used to decribe the progressivity of taxes, see eg. Lambert (1989) and Hagfors and Sullström (1989).

[6] The aversion to inequality rises as $\nu$ increases. The coefficient implies indifference to inequality at $\nu = 1$, and becomes Rawls' max-min criteria at $\nu = \infty$.

different distributional judgements.[7]

In most cases it is hard or impossible to accurately measure and include all items that are relevant in the formation of a correctly measured consumption or income concept. The following simple result which appears to be new to the literature gives a method of assessing inequality in unobserved total consumption by analyzing an observed sub-component of it.

**Proposition 4** Consider a component of consumption c. Assume that consumption $c(y) = E(c \mid y)$ can be defined as a monotone function of total consumption y, with $c' > 0$. If c is a convex function of y, i.e. $c'' \geq 0$, then for any two distributions of c, F and G the condition,
$GL_F > GL_G$, holds if and only if
the distribution in total consumption that defines F is preferable to the one related with G with respect to all concave utility functions U of y.[8]

**Proof:** By assumption we have Lorenz dominance w.r.t. the distribution of c. Proposition 2 guarantees that we have increased welfare w.r.t. every concave function of c. We prove that this holds also w.r.t. to an arbitrary concave function U of y, by showing that our assumptions guarantee

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[7] Peter Lambert in chapter 5.4 of his book (Lambert, 1989) surveys the connection between generalized Gini coefficients and social welfare. Let the welfare of an individual depend on the income of others. Now individual feeling of deprivation relative to the incomes of the better-off leads to a case where mean income and extended Gini coefficient summarize social welfare.

[8] The above result has an interesting application to the analysis of income tax progression and distribution: Suppose we have a fixed progressive (convex) tax function. This corresponds to c above. If the distribution of paid taxes in a given year dominates one in another year using generalized Lorenz-curves then the same property holds w.r.t. the pre-tax incomes.
The above property supplements the celebrated theorem due to Jakobson (1976) and Kakwani (1977) (see Lambert (1989) Sec's 6 & 7): For a fixed pre-tax income distribution the tax function is progressive if and only if $L_{Y-T} \geq L_Y \geq L_T$.

that U can be expressed as a concave function of c.

Use the inverse function of c to write y = f(c). Write U is a composite function, U(y) = (Uof)(c), where by assumption f' > 0. The second derivative of the function (Uof) is

$$\frac{\partial^2 (Uof)}{\partial c^2} = f'(U'of)\left[\frac{U''of}{U'of}f' + \frac{f''}{f'}\right]. \qquad (10)$$

The above expression is strictly less than zero for every concave U if and only if f'' ≤ 0. Notice that

$$f'(c) = \frac{1}{(c'of)(c)}, \text{ and } \frac{\partial^2 f}{\partial c^2}(c) = -\frac{(c''of)(c)\, f'(c)}{[(c'of)(c)]^2} \qquad (11)$$

Therefore f'' ≤ 0 if and only if the consumption of c is a convex function of y.■

To clarify the conditions of the above result note that one can write (c(0) = 0)

$$c'(y)\,y - c(y) = \int_0^y c''(u)\,u\,du \geq 0, \quad \text{if} \quad c'' \geq 0. \qquad (12)$$

In (12) convexity of c implies that the consumption share of c is everywhere non-decreasing which is equivalent with that its elasticity w.r.t. y is ≥ 1. In terms of concentration curves convexity of c means that the derivative of the concentration curve w.r.t. y is a convex function.

On the other hand, because c and y are connected through a monotone relationship they both imply the same ordering on the data and concentration curves w.r.t. c are the same as those w.r.t. y. To get this result one does not need the convexity of c to hold. Therefore one can apply proposition 3 under weaker observability conditions than propositions 1 and 2.

A simple result that allows one to make assessments w.r.t. a change in inequality in a sub-component with the means of change in inequality in the total is given next.

**Proposition 5** Consider a component of consumption c. Assume that consumption $c(y) = E(c\,|y)$ can be defined as a monotone function of total consumption y, with $c' > 0$. If c is a concave function of y, i.e. $c'' \leq 0$, then for any two distributions of y, F and G, the condition
$GL_F > GL_G$,
implies the corresponding inequality when the induced distributions of c are compared.

**Proof:** By assumption we have Lorenz dominance w.r.t. the distribution of y. Let U be an arbitrary concave function of c. Now the composite function, $v = Uoc$ is a concave function of y. By proposition 2 the condition $GL_F > GL_G$ implies that $E_FU = E_Fv > E_Gv = E_GU$. Since U is an arbitrary concave function, proposition 2 guarantees that we have Lorenz dominance in the marginal distributions of c induced by F and G.■

Taken together propositions 4 and 5 imply that for welfare comparisons to be equivalent when either a sub-component or the total income are used, it is generally necessary that the sub-component is an affine function of the total income. Above the welfare statement is taken to hold w.r.t. an arbitrary Paretian utility function.

To make welfare statements on the basis of consumption of an inferior good the counterparts of propositions 4 and 5 are

**Proposition 6** Consider a component of consumption g. Assume that consumption $g(y) = E(g\,|y)$ can be defined as a monotone function of total consumption y, with $g' < 0$. If g is a concave function of y, i.e. $g'' \leq 0$, then for any two distributions of g, F and G, the condition
$GL_F < GL_G$, holds if and only if
the distribution in total consumption that defines F is preferable to the one related with G with respect to all concave utility functions U of y.

**Proposition 7** Consider a component of consumption g. Assume that consumption $g(y) = E(g|y)$ can be defined as a monotone function of total consumption y, with $g' < 0$. If g is a convex function of y, i.e. $g'' \geq 0$, then for any two distributions of y, F and G, the condition

$GL_F > GL_G$,

implies the opposite inequality when the induced distributions of c are compared.

**Proof:** By substitution of -g for c in the proofs of propositions 4 and 5.■

## 3. THE DATA

The data that are used in the present study originate from the 1985 Finnish Household Expenditure Survey microdata. It consists of 8200 observations with the relevant sample weights allowing for non-response and sampling probabilities. The sample weights are employed in all calculations of this study in order to produce estimates at the population level. Note that the deficiencies in expenditure surveys due to sampling loss affect most both ends of income distribution. This makes the sampled distribution more equal than the actual one. This holds even in the case where weighting may ensure that sample means are unbiased estimates of their population equivalents. An additional point is that the sampling population of household surveys does not include institutionalized members of society, living in hospitals, prisons, etc., which by construction are important users of public welfare services.

In the present study public welfare services consist of health services, education, social services and housing benefits. The subcomponents of health services and education are not commented in any great detail here.[9] Social services consist for the main part of day-care of children and the care of elderly and disabled persons. Here the housing benefits is a less interesting item which covers only the interest subsidies of publicly provided housing loans with below the market rate of interest.[10]

Public welfare services are mainly produced by the local government and are primarily financed by state subsidies and local taxes. In this study their consumption is valued using

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[9] Detailed definitions of the variables and some additional information are found in Tilastokeskus (1987) and (1988 a).

[10] The more important public sector instruments affecting the housing markets such as various tax benefits, rent subsidies or provision of housing services at below the market rent are not covered here.

production cost estimates net of possible user fees. The user fees are taken into account in the corresponding consumption expenditure category, eg. inpatient fees in the health expenditure component of consumption (for a short description of the Finnish system, see eg. Järviö et. al. (1992)).

Table 1 shows the means for some variables utilized in the present study. In the calculations a rather rough equivalence scale - index number that attempts to measure the cost to a household of a change in its composition- has been used to adjust for differing needs in households. The equivalence scale is based on the OECD- recommendations and the quintiles in table 1 are constructed on the basis of the scale adjusted measure of household consumption expenditure.[11]

Consumption expenditure used in the present study is formed in a standard way, covering subgroups 1-85 (see Tilastokeskus, 1988 a) with the exception that the purchase of major durables, i.e. cars and other vechiles, boats, and television sets, video- and music recorders, personal computers, furniture and the like are not included in it.

Public welfare services affect the consumption possibilities in an important way. They are in average roughly 20 per cent of the consumption measure reported in table 1 and about 15 per cent of disposable income in scale corrected units. Further their use is concentrated to the bottom quintile. This observation is in accordance with earlier studies in Finland. In the next section we proceed to a more detailed analysis of their effects using the analytical tools presented in section 2.

In this study an 'extended consumption' measure is also employed. This concept is formed by adding to the above consumption expenditure the use of public welfare services

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[11] In the scale one adult household gets weight equal to one, each additional adult gets weight 0.7 and children get weight 0.5.

and an additional component which consists of gifts to other households. The extended consumption refers to the monetary value of goods and services available for consumption.

Next, some points are made about the effect of data quality to inequality analysis. These points are often not properly confronted in empirical analysis. The stereotypical analyst proceeds rather too often to directly calculating the empirical counterparts of the theoretical formulas with scant reference to various biases involved when dealing with measurement error ridden observational counterparts of the theoretical concepts.

Generally, in assessing inequality one must decide whether to base comparisons on income or consumption. Distribution of income is relevant from the economic welfare point of view only so far as it reflects the relative utility levels of households.

If consumers are rational and forward looking then generally income over a time period is more variable than consumption. In addition marginal utility of consumption is equal to marginal utility due to accumulation of an equivalent marginal unit of wealth instead of consuming it now. Therefore, keeping consumption constant, differences in savings should reflect only differences in income stream but not in wealth and should not count in welfare comparisons. This smoothing does not have to be complete as implied in the case of perfect capital market by the rational expectations - permanent income hypothesis but it may reflect differences in investment possibilities including here possible future relaxation of say liquidity or transaction constraints[12].

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[12] This type of argument should be quite general and apply to a strongly time-separable utility function with or without uncertainty in future prices of assets and consumption goods and under various constraints on liquidity, under transaction costs, etc. There is however a qualification to the above remark which is of some importance in between-cohort comparisons since households in different stages of their life-cycle face different lifetime discounted prices (and liquidity constraints) (for further discussion, see Blundell & Preston, 1991).

On the other hand, consumption based welfare comparisons face some practical difficulties. First consumption should be measured not including investment in durable goods but imputing instead the service flow provided by the ownership of durables.[13] The latter exercise is in many cases difficult or impossible to do due to limitations in the collected data.[14] If the consumption measure that is used is based on non-durable consumption exclusively, then this leads to a systematic bias if the elasticity of durable services w.r.t an appropriate "permanent" income or wealth concept is not equal to one. In particular an elasticity above one implies that inequality is thus underestimated (cf. proposition 4).[15]

Second, non-durable consumption is typically measured in terms of purchases of the goods and services in question on a relatively short period of time. In our case the bulk of consumption data are collected on a two-week time period. In contrast, income data refer to a longer period of time, in our case to one year. Infrequent nature of purchases w.r.t consumption results in negative autocorrelation and high variance in data collected over relatively short time periods.[16]

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[13] Here one should add that durable goods eg. houses and automobiles are also assets and in practice it would be very hard to distinguish between the asset demand and consumption demand. This point, the practical importance of which should, however, not be overstated, is often neglected in theoretical considerations by focusing only on perfectly functioning asset markets together with parallelly operating rental markets.

[14] In our data set an imputed measure of housing expenditure for owner-occupiers is included in the corresponding expenditure component.

[15] Note, however, that if the above relation between the service flow of durables and consumption is a stable one, then concentration of eg. welfare services is not affected by the choice of reference distribution (see, the remark immediately after proposition 4).

[16] Even in the case of a relatively stable consumption category as food, Hall and Mishkin (1982) find using the Panel Study of Income Dynamics that over 80 % of the vari-

This causes the degree of inequality being overestimated. An additional point is that inequality is being overestimated within groups that have more infrequent purchase patterns, e.g. single person households relative to that of some other group, eg. households with several members.

Last, and probably the most important shortcoming of the data is that consumption is exempt from the value of leisure.

In summary, differences in data quality, eg. in the variance of various consumption categories within or between different consumer groups produces a bias that creeps into inequality measurements. Using only cross-section data these effects are difficult to correct for but at least their existence should be acknowledged and preferably one should make some rough statements about the relative magnitude of errors involved.

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ance of year to year change in 'measured' food consumption is attributable to measurement error in comparison to the change attributable to economic variables.

## 4. NONPARAMETRIC ESTIMATION OF CONCENTRATION

The Lorenz and concentration curves presented in this study are estimated by using a nonparametric estimation method. The raw data consists of a sample concentration curve. This curve is based on the empirical distribution function of the data which is formed by using the sampling weights. In effect, the estimation procedure used is a smoothing filter applied on the empirical concentration curve.

The filter is a fourth degree polynomial which is locally fitted to the data using a weighted regression with a normal kernel. The bandwidth is determined locally so that as the sample size, N, goes to infinity the bandwidth goes to zero. Simultaneously the number of observations used in the local fit goes to infinity (in this study by order $N^{1/3}$). In the estimation of the curve $f(x,y) = 0$, the end-point restrictions, $f(0,0) = f(1,1) = 0$ are set to hold in each local fit.

The gini- and concentration coefficients of the data were calculated by two different methods. The first is based on the area under the empirical concentration curve which can be calculated either directly or by the covariance formula (4). The second consists of analytically integrating the locally smoothed curve. The two methods gave closely similar values with no discrepancies at the reported level of accuracy.[17]

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[17] The estimation method is described in greater detail in Suoniemi (1993), where Lorenz-curve estimation is discussed. There the fourth-degree polynomial filter is compared to two other filters which are based on the Kakwani-Podder function and on the elliptic function. The empirical application of the filters are there shown to guarantee a quite accurate estimation of the gini-coefficient when only rough, tabulated information is available, whereas the estimation that is based on the corresponding empirical distribution is considerably less accurate. In the present paper, the number of observations is so large (8200) that the choice of the filter makes no difference at the reported level of accuracy. The programs are written using GAUSS™ (Aptech Systems) programming language.

## 5. EMPIRICAL RESULTS

We start by considering the contribution of public welfare services to overall inequality in consumption. In doing this we define an extended measure of consumption by adding the welfare service use and gifts to other households in consumption expenditure, and decompose the Gini coefficient of the extended consumption w.r.t. its subcomponents following Stark et. al. (1986). Here we are using formula (2.7.). The method employed is in analogy with analyzing the effects of taxation using after tax distribution as the base of comparison. The results for the 1985 data are shown in Table 2. The items in the first four rows represent public intervention and the component 'gifts from other households' represents private transfers between households. The next five rows deal with 'voluntary' consumption type categories, and the last two entries are our rival measures of total consumption.

We find that welfare services seem to increase overall inequality if the above method is used, where utility is evaluated including the service use. This at first sight rather unexpected observation (compare col's 2 & 3 in table 1) is confirmed if we examine the concentration curves of the separate welfare service components in the case where these curves are formed w.r.t. the extended consumption concept. The curves are shown in figure 1 along with the corresponding Lorenz curves and do not seem to have such clearly expressible concavity properties which would be necessary to inequality reduction.

What lies behind this counterintuitive observation? We argue below that a regression type of phenomena is behind this, and the above observational point of view is counter-productive to our original purposes. To be more specific, first the use of welfare services is recorded under a relatively short time-period, usually one month in our data. As argued in the previous section this leads to a higher variance and inequality being over-estimated relative to figures recorded

under a longer time period. The above effect seems to be particularly pronounced in the case of health services where the time period is generally one month, and shows up in table 2, where health services apparently contribute more to inequality relative to its share of consumption than other welfare services[18]. The high dispersion in the utilization of public welfare services is also indicated by the relatively high values of the corresponding Gini coefficients in table 2.

Second, although the mean level of the use of welfare services is unbiasedly estimated in the presence of a measurement error component of the above type, the concentration curve is based on an expected value, conditional on the sum of it and consumption expenditure not exceeding an upper limit. Therefore a selection bias-like component creeps in and leads to a pronouncedly biased estimation of the concentration curves.

The above bias can be partly avoided if we examine instead the concentration curves of welfare services w.r.t. consumption expenditure only. This is so if the measurement errors in welfare services and consumption are independent. In this case the expected value of the former error conditional on the value of consumption is equal to zero and in particular independent of its own variance. These points are illustrated in the appendix using an analytical example based on bi-normally distributed random variables.[19]

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[18] In the case of education, the problem is less severe because the circumstances that affect the access to these services such as the number of school age children stay constant under a relatively long time period.

[19] As a side line one cannot help suspecting whether some of the empirical observations reported by Stark et. al. (1986) are to some degree affected by a similarly contaminated estimation. In the paper the impact of migrants' remittances to village income are compared and the finding is that their contribution to total income inequality is less in the case where the proportion of remittances in final income is large i.e. there is more remittances. One should suspect a large variation coefficient in remittances when there are relatively few of them, i.e. there are many zero observations.

Table 3 represents the corresponding calculations when the data have been sorted according to the values of consumption.[20] Now the point of departure is before the government intervention in the form of provision of public services has taken place. As expected, the above, rather hasty conclusion is reversed and the provision of welfare services significantly reduces inequality.[21] This is in line with earlier Finnish studies where the common tendency has been to base the comparison w.r.t. disposable income and tabulate quintile means as in table 1. One notes that the gini-correlations for both health services and social services are negative. This means that mean expenditure elasticity of these components is negative, i.e. the utilization of these services is on average inversely related to consumption. Social services seem to contribute most to alleviating economic inequality.

Next, we turn to examining the corresponding Lorenz and concentration curves which are shown in figure 2. On the left panels the curves of the various welfare services and private transfers from others are shown together with the Lorenz curve of consumption. The right panels show the curves for consumption and some of its subcomponents together with the variable 'gifts to other households', a consumption-type element. The Lorenz curves for consumption, food and housing consumption go very near each other. Comparison of the concentration curves shows that the concentration of social services lies highest except some short stretches at the very low and high ends of the x-axis. On the other hand, the curve of transportation expenditure is everywhere below the others. This suggests that there exist

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[20] Note that a strict decomposition interpretation as in table 2 is no longer possible because welfare services are no longer included in the total measure.

[21] Naturally there remains problems in estimation as indicated in the above discussion, and these may affect the conclusions of this study. One should lean more on the econometric approach than state-of-art methods currently do. I hope to expand on these points in a future study.

dominance relations which in the light of proposition 3 have interesting welfare implications.

Figure 3 shows the difference between the concentration curves of social services and all other items except transportation expenditure which will be dealt in a later figure.[22] One notes that social services systematically dominate the consumption categories. By proposition 3 this indicates that a reform that marginally increases the utilization of social services and finances it with an equivalent increase in commodity taxation is welfare increasing in an inequality averse society. If non-marginal reforms of this type are considered one should complement the analysis by calculating the change in excess burden of public intervention.

If the corresponding comparison is taken w.r.t. education the dominance continues to hold. In contrast, the concentration curves of public health services and social services cross twice at the low end of the reference distribution, and no conclusions on dominance can be made.

Transportation seems to represent a relative luxury item of consumption if it is compared with other consumption categories (table 3). Figure 4 shows the differences between the other concentration curves and our luxury category of consumption, transportation. One notes that transportation is consistently dominated even by the other consumption categories. Now the additional implication of proposition 3 is that a commodity tax reform that marginally increases the tax on transportation and lowers the tax of the other shown consumption categories should be welfare improving. This holds for non-marginal changes provided that the excess burden of taxation which is a second order term does not dominate the first order effects. Simple application of the Ramsey-rule indicates that this should roughly hold if the own price elasticity of transportation is not markedly

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[22] Comparisons w.r.t. housing benefits and private transfers are in most cases less relevant and will not be commented on.

larger than that of the other categories.

In figure 5 some additional pair-wise comparisons of concentration curves are represented. Starting from the north-west corner one notes that private transfers between households alleviate inequality until the very top end of the reference distribution. Here one should note that social services dominate gifts from other households (fig. 3) and therefore public services apparently seem to be a more effective instrument of reducing inequality than private transfers.[23]

Public health services dominate health expenditure. In contrast the concentration curves of education and public health services show several crossings. Food consumption dominates the other consumption categories but, on the other hand, it is dominated by both education and health services. As a minor point, a comparison of housing benefits and housing expenditure reveals a rather peculiar form which indicates that subsidies due to loans with below the market rate of interest accrue to households with relatively high housing expenditure.

In the above calculations a rather rough equivalence scale has been used to adjust for differing needs in households. Checking for the sensitivity of results to choice of scale is outside the scope of the present study. It should be noted, however, that a recent study by Coulter et. al. (1992) concludes that the choice of the scale may be of crucial importance. Further, they note that generally an equivalence scale defined as a function of household size has not a monotone effect on inequality measures based on scaled income. This means that one cannot simply calculate inequality estimates for two extreme equivalence scales,

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[23] Naturally there exist some inter-generational transfers in the form of voluntary labour services which are not included in our data. Further, with no impression of the relevant counter-factual situation, there are no direct methods to establish how much private transfers (or alternatively consumption demand) there would be without any provision of public welfare services.

here the obvious candidates would be per capita and per household consumption, and conclude that intermediate scales will lead to intermediate inequality estimates.

An alternative, and a more preferable strategy to make the analysis less dependent on the choice of the equivalence scale is to examine whether it is possible to find 'sequential dominance' relations with respect to some homogenous sub-groups of households. In this case one is able to make welfare statements under less demanding conditions if there exists universal agreement whether one group is on average more needy than the other. The method was originally developed by Atkinson and Bourguignon (1989) to compare the effects of taxation to the welfare in sub-groups of households that differ by family size.

Table 5 presents the concentration coefficients for the whole data as well as in some interesting sub-groups which are formed on the basis of household composition. The construction of the groups facilitates life-cycle comparisons in the pattern of public welfare service utililization. The first five of these groups are non-overlapping and the last group 'elderly household' consists of those with the head of household over 65 years old.[24] One notes that the concentration-coefficient for social services is negative in all consumer sub-groups, i.e. the use of this service is on average inversely related to consumption. In the case of other public welfare services the relation is less clear-cut but if one compares these concentration coefficients to those of consumption subcategories, one finds that they are systematically lower than those of the latter. Similarly, as for the pooled data, transportation is a relative luxury item of consumption in all consumer sub-groups shown in table 5, and food is a necessity but not an inferior good. Figures 6 a-f present the Lorenz and concentration curves in

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[24] Although the variables represented remain scaled, the choice of scale has a minor effect on the results since the sub-groups (with the exeption of the residual group "other") are rather homogenous w.r.t. the value of the scale measure.

each consumer sub-group for the same set of variables as in figure 2. The corresponding within-group Gini coefficients are shown in table 4 and indicate the dispersion in the utilization rate of services and consumption.

Similarly as in figure 3, figures 7 a-f show the differences between the concentration curves of social services and all other items within each consumer sub-group. One notes that social services sysstematically dominate the consumption categories. If the corresponding comparison is taken w.r.t. education, the curves cross at the high end in the single adult subgroup, and at the low end in the single provider and in the two adults with children subgroups. The former two crossings may be affected by sample variability due to the relatively low number of observations involved. If public health services are considered, the same effect lies behind the crossing at the high end in the subgroup of a single adult. The lack of domination in the single provider subgroup is more pronounced and this is clearly dependent on the life-cycle pattern in the utilization of the corresponding welfare service. On the other hand, it is interesting to note that the dominance w.r.t. health services seems to hold in the subgroup of elderly households.

Figures 8 a-f show the corresponding comparisons between the concentration curves of our luxury category of consumption, transportation and all other items. One notes that transportation is almost consistently dominated even by the other consumption categories. The dominance is weakest w.r.t. health expenditure. The concentration of health expenditure is rather heavy at the high end of distribution and reflects the high utilization of private versus public health services together with correspondingly weak precision in the estimation of the concentration curve. In the case of elderly households one should naturally dismiss the comparison w.r.t education.

To get a complete picture of possible inequality reduction on the basis of sub-group comparisons one has to consider between-group effects. If we take for granted that differen-

ces in the scale corrected group-means of consumption (the last row of table 6) reflect mostly inadequacies of our scale measure, i.e. the consumer groups have on average reasonably similar utility levels, it suffices to consider the consumption shares given in table 5. Considering social services the sub-groups are ordered w.r.t. the utilization rate in the following way: single provider, couple with children, elderly household, other households, single adult, and couple with no children. The last three groups have only minor differences in the utilization rates. If health services are considered the order from highest to lowest utilization rate is by household type: elderly household, single provider, couple with no children, single adult, other households, and last couple with children. For education the most important finding is that single providers have a larger share of these services than couples with children. The other comparisons across consumer sub-groups are less relevant on the basis of their construction. The above orderings are in rough accordance with popular assessments of the relative needs of the consumer sub-groups w.r.t. the services in question.

Additional pairwise comparisons of concentration curves are shown in figures 9 a-f. Only a few of them are picked up for discussion here. Public health services consistently dominate health expenditure. The relative impact of education and health services in inequality reduction is clearly governed by the life-cycle pattern in utilization. As a curiosity, the provision of housing loans at below the market rate of interest has a desired effect only in the two sub-groups consisting of households with children. Food expenditure dominates health expenditure but is in turn dominated by public health services. The only exception takes place at the low end of distribution in the single provider sub-group where subsidizing of the food would have relatively more impact. Finally, contrary to the common tendency prevailing in the other sub-groups, private transfers increase inequality within the residual group "other".

The final point in this section is based on proposition 7 of section 2. The previous findings indicate that the utilization of welfare services, g, is related to consumption, y, through a decreasing convex function $g(y) = E(g \mid y)$. Assume that the above relation is based on a rational choice between the government commodity bundle and the corresponding private sector product. Now if welfare in the society decreases and access to the government bundle is left unchanged then the generalized Lorenz curve of the utilization of public welfare services dominates the initial position. In particular, adverse economic conditions in the beginning of 1990's would indicate a substantial increase in the use of health and social services.

## 5. CONCLUSION

In the paper concentration curve analysis has been applied to Finnish data to assess the reduction in inequality due to public welfare services. In doing this some general observations were made which are particularly important for successful empirical application of these theory based methods to actual data plagued with incompleteness and measurement errors. The practical relevance of these points is empirically established and this hopefully encourages further discussion about data collection and estimation methods in order to reduce the various sources of bias present in empirical applications.

The main positive findings of this study are the following. First, the concept of marginal conditional stochastic dominance (Slemrod and Yitzhaki, 1987) allows welfare statements to be made using Finnish data on the utilization of public welfare services with quite weak reliance on behavioral assumptions. In particular one finds that public welfare services have a major role in inequality reduction with social services which consists of day-care of children and care of the elderly and disabled persons, being most efficient in this respect. The effects of education and health services reflect the life-cycle pattern of utilization and seem to complement each other over different stages of the life-cycle.

Second, finance of welfare services through indirect taxation seems to increase social welfare under quite weak conditions if marginal changes are considered. Third, the dominance relations detected in the data seem to be reasonably robust if some important sub-groups of households are examined separately. This finding reduces somewhat the need to resort to strong a priori assumptions in selecting the proper equivalence scale. Further, the general feature of public welfare services dominating over the selected consumption categories may hold reasonably well over the life-cycle of households.

The present study has contended to deal with the effects of public welfare services on economic equity.[25] The efficiency aspects have been deliberately left out. One should, however, acknowledge that social welfare services, eg. education and health, may be regarded as merit goods worth subsidising either due to positive external effects or because of paternalistic arguments arising from incomplete information and time or intergenerational inconsistency in individual decision making. On the other hand, from the point of view of allocative and incentive effects, the present provision mechanism may naturally leave a lot to be desired for. Further, if eg. medical care and education are viewed as basic rights or entitlements, there is social concern for the distribution of the use of these services that goes beyond our usual concern for income distribution.

At present, the empirical results offer a snapshot view of the Finnish system in 1985. It would be recommendable to make a more comprehensive time series evaluation based on previous Finnish Household Expenditure Surveys from 1971, 1976, and 1981, and including the 1990 data which has recently become available. In doing this one would consider the role of public welfare services in reducing inequality over time and in relation to the amount of resources used to producing these services. It would be worthwhile to apply the methods of this paper in a wider framework including the effects of public transfers in cash and taxation to get a more extensive view on the effects of public sector in economic inequality and social welfare.[26]

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[25] As a by-product it turns out that private transfers between households are by no means negligible in Finland and seem to reduce economic inequality. In this respect, however, the effect due to the public provision of welfare services dominates. In addition public intervention in the form of providing housing loans with under the market rate of interest does not seem to have favourable inequality reducing effects.

[26] The projected study would extend and complement the previous time-series examination of Finnish income distribution by Uusitalo (1988).

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**APPENDIX**

The following example illustrates the effect of measurement error on concentration curves between two mutually dependent variables. Let $x^*$ and $y^*$ be two jointly normally distributed variables with parameters[27]

$$\begin{bmatrix} x^* \\ y^* \end{bmatrix} \sim N \begin{bmatrix} \mu_x \; ; & \sigma_{xx} & \varrho\sigma_x\sigma_y \\ \mu_y \; ; & \varrho\sigma_x\sigma_y & \sigma_{yy} \end{bmatrix}. \qquad \textbf{(A 1)}$$

A straightforward calculation gives the concentration curve of $x^*$ relative to $y^*$ by application of the general formula $L_{x,z}(p) = pE_z(\, E(x|z) \mid z < F^{-1}{}_z(p)\,)/E(x)$. First

$$E(x^* \mid y^*) = \mu_x + \varrho\frac{\sigma_x}{\sigma_y}(y^* - \mu_y),$$

and

$$F^{-1}_{y^*}(p) = \mu_y + \sigma_y\Phi^{-1}(p).$$

Integration gives

$$L_{x^*,y^*}(p) = p - \varrho\frac{\sigma_x}{\mu_x}\,\phi(\Phi^{-1}(p)). \qquad \textbf{(A 2)}$$

Similarly

$$\begin{bmatrix} x^* \\ x^* + y^* \end{bmatrix} \sim N \begin{bmatrix} \mu_x & ; & \sigma_{xx} & \sigma_{xx} + \varrho\sigma_x\sigma_y \\ \mu_x + \mu_y & ; & \sigma_{xx} + \varrho\sigma_x\sigma_y & \sigma_{xx} + 2\varrho\sigma_x\sigma_y + \sigma_{yy} \end{bmatrix}. \qquad \textbf{(A 3)}$$

$$L_{x^*,x^*+y^*}(p) = p - \frac{1 + \varrho\tau}{\sqrt{1 + 2\varrho\tau + \tau^2}}\frac{\sigma_x}{\mu_x}\,\phi(\Phi^{-1}(p)). \qquad \textbf{(A 4)}$$

where $\tau = \sigma_y/\sigma_x$. Above the function $\Phi$ denotes the cumulative distribution function and $\phi$ is the density of a unit normal, N(0, 1), random variable.

Introduce measurement errors on $x^*$ and $y^*$ to get observable variables $x = x^* + u_x$ and $y = y^* + u_y$. The errors are independent of the true values and jointly normal with parameters

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[27] Note that the variables in question may take negative values. This, however, has no material effect on the argument.

$$\begin{bmatrix} u_x \\ u_y \end{bmatrix} \sim N \begin{bmatrix} 0 \ ; & s_{xx} & rs_x s_y \\ 0 \ ; & rs_x s_y & s_{yy} \end{bmatrix}. \qquad \textbf{(A 5)}$$

The observed variables are jointly normal with parameters

$$\begin{bmatrix} x \\ y \end{bmatrix} \sim N \begin{bmatrix} \mu_x \ ; & \sigma_{xx} + s_{xx} & \varrho\sigma_x\sigma_y + rs_x s_y \\ \mu_y \ ; & \varrho\sigma_x\sigma_y + rs_x s_y & \sigma_{yy} + s_{yy} \end{bmatrix}. \qquad \textbf{(A 6)}$$

By direct application of (A 2) to the parameters given in (A 6) one has

$$L_{x,y}(p) = p - \left[ \frac{1 + (r/\rho)(s_x/\sigma_x)(s_y/\sigma_y)}{\sqrt{1 + s_{yy}/\sigma_{yy}}} \right] \varrho \frac{\sigma_x}{\mu_x} \phi(\Phi^{-1}(p)). \qquad \textbf{(A 7)}$$

The above formula differs from the corresponding formula between the error free variables (A 2) through the term in square brackets. If measurement errors are independent, $r = 0$, this term is positive and less than one. In this case the formula differs from (A 2) only through $s_y/\sigma_y$, i.e. the dependence is through the error in y and not through the error in x.

The above observation is completely general and is not dependent on the normality assumption. The term in square brackets causes the biased estimation of the Lorenz curve of y and results in the inequality in y being overestimated and in the concentration of x being biased towards the diagonal, a 'regression effect'. In particular the estimate of the concentration coefficient is less than the true value, and pronouncedly so, if $r \neq 0$.

On the other hand consider calculating $L_{x,x+y}$. In this case one gets a formula similar with (A 4) if one makes the following substitutions

$$\left[ \frac{1 + (r/\rho)(s_x/\sigma_x)(s_y/\sigma_y)}{1 + s_{xx}/\sigma_{xx}} \right] \varrho\tau \mapsto \varrho\tau,$$

$$\left[ \frac{1 + s_{yy}/\sigma_{yy}}{1 + s_{xx}/\sigma_{xx}} \right] \tau^2 \mapsto \tau^2, \text{ and}$$

$$(1 + s_{xx}/\sigma_{xx})^{0.5}\sigma_x \mapsto \sigma_x.$$

In the case of independent measurement errors this leads to a similar 'regression bias' as in the case of formula (A 7). Now, however, the bias is amplified because the selection is dependent also on the error in x.

In conclusion, if the measurement error in x is severe it is more preferable to base concentration curve analysis on such a total variable where the contribution of the variable x is not included rather than including it. This is even more important if the analysis involves the comparison of concentration curves w.r.t. several x-variables.

**Table 1: Population means[28] of selected variables in 1985.**

| Variable | Population means for Whole Data | First[29] Quintile | Last Quintile |
|---|---|---|---|
| Sample weight | 431 | 425 | 501 |
| Public Services: | 6834 | 7131 | 5886 |
| Education | 3060 | 2838 | 2764 |
| Health services | 2958 | 3479 | 2608 |
| Social services | 670 | 750 | 382 |
| Housing benefits | 145 | 64 | 132 |
| Gifts to | 796 | 449 | 1441 |
| Gifts received | 729 | 541 | 1094 |
| Consumption: | 37745 | 19241 | 64862 |
| Transportation | 5041 | 1597 | 11354 |
| Health expenditure | 1433 | 825 | 2741 |
| Food expenditure | 8052 | 5781 | 10033 |
| Housing expenditure | 7449 | 5085 | 10118 |
| Consumption & Public Services | 45376 | 26821 | 72189 |
| Disposable Income | 45912 | 32833 | 61709 |
| Age of Head | 47.28 | 53.55 | 44.27 |
| No of Adults | 1.82 | 1.81 | 1.74 |
| No of Children | 0.54 | 0.53 | 0.39 |
| No of Retired | 0.29 | 0.56 | 0.14 |
| Equivalent Adults | 1.85 | 1.83 | 1.71 |
| F | 0.50 | 0.10 | 0.90 |
| No of Households | 8200 | 1722 | 1353 |
| representing Households (1000s) | 2045 | 409 | 409 |

[28] Variables referring to various items of monetary variables (FIM), eg. consumption are measured in per equivalent adult terms.

[29] Quintiles are constructed according to the values of nondurable consumption per equivalent adult and refer to the distribution of households.

**Table 2: Decomposition of the 1985 Inequality.**

| Variable | Share S | Gini G | Ginicorr. R | Contribution SGR | %-share |
|---|---|---|---|---|---|
| education | 0.0674 | 0.7825 | 0.4291 | 0.0226 | 0.0942 |
| health serv. | 0.0652 | 0.8242 | 0.5865 | 0.0315 | 0.1312 |
| social serv. | 0.0148 | 0.9194 | 0.2007 | 0.0027 | 0.0114 |
| housing ben. | 0.0032 | 0.9555 | 0.2185 | 0.0007 | 0.0028 |
| gifts from | 0.0161 | 0.8660 | 0.2341 | 0.0033 | 0.0136 |
| gifts to | 0.0175 | 0.8140 | 0.3816 | 0.0055 | 0.0227 |
| transport | 0.1111 | 0.5216 | 0.6613 | 0.0383 | 0.1595 |
| health exp. | 0.0316 | 0.6788 | 0.4253 | 0.0091 | 0.0379 |
| food exp. | 0.1775 | 0.2592 | 0.3163 | 0.0146 | 0.0606 |
| housing exp. | 0.1642 | 0.2642 | 0.4102 | 0.0178 | 0.0740 |
| consumption | 0.8319 | 0.2408 | 0.8853 | 0.1773 | 0.7378 |
| extend. consumpt. | 1.0000 | 0.2403 | 1.0000 | 0.2403 | 1.0000 |

**Table 3: Decomposition of the 1985 Inequality w.r.t. Consumption**

| Variable | Share S | Gini G | Ginicorr. R | Contribution SGR | %-share |
|---|---|---|---|---|---|
| education | 0.0811 | 0.7825 | 0.0129 | 0.0008 | 0.0034 |
| health serv. | 0.0784 | 0.8242 | -0.0704 | -0.0045 | -0.0189 |
| social serv. | 0.0178 | 0.9194 | -0.1202 | -0.0020 | -0.0082 |
| housing ben. | 0.0039 | 0.9555 | 0.0705 | 0.0003 | 0.0011 |
| gifts from | 0.0193 | 0.8660 | 0.1784 | 0.0030 | 0.0124 |
| gifts to | 0.0211 | 0.8140 | 0.3072 | 0.0053 | 0.0219 |
| transport | 0.1335 | 0.5216 | 0.7254 | 0.0505 | 0.2099 |
| health exp. | 0.0380 | 0.6788 | 0.3810 | 0.0098 | 0.0408 |
| food exp. | 0.2133 | 0.2592 | 0.4040 | 0.0223 | 0.0928 |
| housing exp. | 0.1973 | 0.2642 | 0.5117 | 0.0267 | 0.1108 |
| consumption | 1.0000 | 0.2407 | 1.0000 | 0.2407 | 1.0000 |

**Table 4: Gini-coefficients in consumer subgroups 1985**

| | Household type | | | | | | |
|---|---|---|---|---|---|---|---|
| Variable | All | Single adult | Couple no child | Single provider | Couple children | Other | Elderly household |
| education | 0.7825 | 0.9130 | 0.9293 | 0.4233 | 0.4803 | 0.6299 | 0.9548 |
| health serv. | 0.8242 | 0.8854 | 0.8125 | 0.7818 | 0.7344 | 0.7756 | 0.8141 |
| social serv. | 0.9194 | 0.9480 | 0.9606 | 0.7531 | 0.8314 | 0.9073 | 0.8939 |
| housing ben. | 0.9555 | 0.9858 | 0.9709 | 0.9086 | 0.9038 | 0.9341 | 0.9918 |
| gifts from | 0.8660 | 0.8646 | 0.8827 | 0.6073 | 0.8307 | 0.8774 | 0.8839 |
| gifts to | 0.8140 | 0.8226 | 0.7762 | 0.8323 | 0.7901 | 0.7729 | 0.8194 |
| transport | 0.5216 | 0.6052 | 0.5168 | 0.5554 | 0.4286 | 0.4318 | 0.5704 |
| health exp. | 0.6788 | 0.7375 | 0.6295 | 0.6633 | 0.6334 | 0.6038 | 0.6562 |
| food exp. | 0.2592 | 0.3324 | 0.2346 | 0.2435 | 0.1990 | 0.2076 | 0.2724 |
| housing exp. | 0.2642 | 0.2771 | 0.2499 | 0.1752 | 0.2019 | 0.2538 | 0.2779 |
| consumption | 0.2407 | 0.2702 | 0.2487 | 0.1979 | 0.1992 | 0.2196 | 0.2489 |

**Table 5: Concentration-coefficients in consumer subgroups 1985**

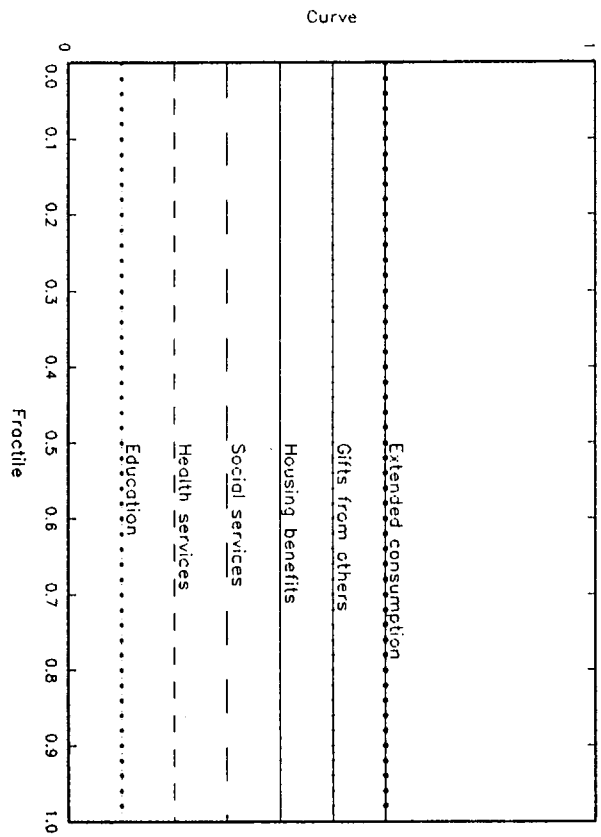
| | Household type | | | | | | |
|---|---|---|---|---|---|---|---|
| Variable | All | Single adult | Couple no child | Single provider | Couple children | Other | Elderly household |
| education | 0.0101 | -0.0947 | 0.1693 | -0.0436 | 0.0208 | 0.0837 | 0.3697 |
| health serv. | -0.0580 | -0.0631 | -0.1210 | -0.1001 | 0.0075 | 0.0168 | -0.0974 |
| social serv. | -0.1105 | -0.2508 | -0.3387 | -0.1051 | -0.0355 | -0.2614 | -0.2460 |
| housing ben. | 0.0674 | 0.1818 | 0.2017 | -0.0078 | -0.0305 | 0.0133 | 0.0908 |
| gifts from | 0.1545 | 0.1297 | 0.1769 | 0.0504 | 0.1437 | 0.3307 | -0.0443 |
| gifts to | 0.2501 | 0.2413 | 0.2673 | 0.2313 | 0.2043 | 0.2481 | 0.3513 |
| transport | 0.3784 | 0.4451 | 0.3927 | 0.3582 | 0.2926 | 0.3238 | 0.4018 |
| health exp. | 0.2586 | 0.2995 | 0.2101 | 0.3051 | 0.2486 | 0.2188 | 0.3300 |
| food exp. | 0.1047 | 0.1243 | 0.0890 | 0.1106 | 0.0938 | 0.1089 | 0.1438 |
| housing exp. | 0.1352 | 0.1476 | 0.1383 | 0.0688 | 0.1036 | 0.1430 | 0.1834 |
| consumption | 0.2407 | 0.2702 | 0.2487 | 0.1979 | 0.1992 | 0.2196 | 0.2489 |

**Table 6: Consumption shares in consumer subgroups 1985**

| | Household type | | | | | | |
|---|---|---|---|---|---|---|---|
| Variable | All | Single adult | Couple no child | Single provider | Couple children | Other | Elderly household |
| education | 0.0811 | 0.0597 | 0.0287 | 0.1969 | 0.1368 | 0.1117 | 0.0042 |
| health serv. | 0.0784 | 0.0831 | 0.0861 | 0.0935 | 0.0646 | 0.0728 | 0.1738 |
| social serv. | 0.0178 | 0.0095 | 0.0048 | 0.0913 | 0.0400 | 0.0105 | 0.0198 |
| housing ben. | 0.0039 | 0.0017 | 0.0031 | 0.0086 | 0.0081 | 0.0030 | 0.0007 |
| gifts from | 0.0193 | 0.0239 | 0.0172 | 0.0731 | 0.0140 | 0.0096 | 0.0209 |
| gifts to | 0.0211 | 0.0284 | 0.0247 | 0.0112 | 0.0110 | 0.0147 | 0.0330 |
| transport | 0.1335 | 0.1276 | 0.1386 | 0.1094 | 0.1272 | 0.1548 | 0.0867 |
| health exp. | 0.0380 | 0.0431 | 0.0425 | 0.0283 | 0.0285 | 0.0345 | 0.0713 |
| food exp. | 0.2133 | 0.1931 | 0.2238 | 0.2091 | 0.2200 | 0.2368 | 0.2745 |
| housing exp. | 0.1973 | 0.2274 | 0.1917 | 0.2123 | 0.1749 | 0.1642 | 0.2645 |
| consumption[1] | 37.7 | 38.5 | 38.5 | 38.0 | 37.0 | 35.9 | 30.7 |

[1] In 1000's FIM.

**Legends for figure 1.**



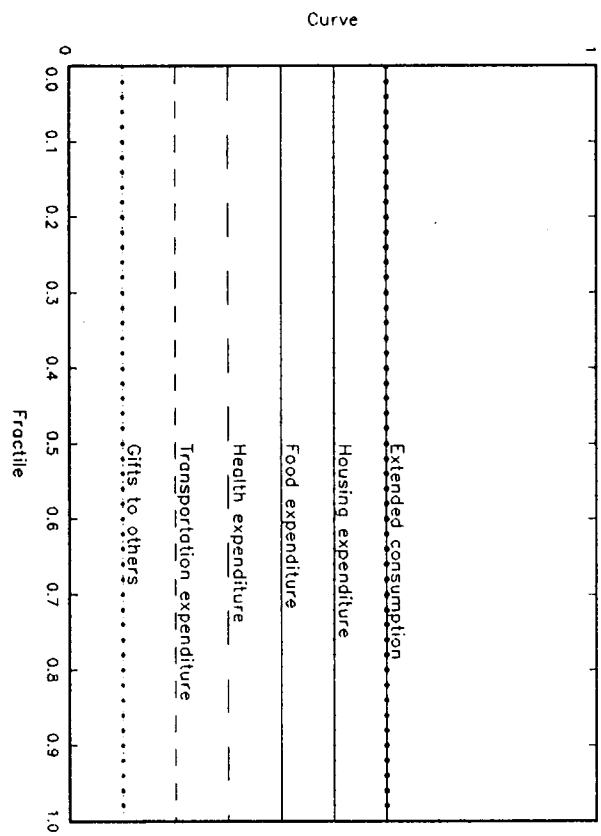
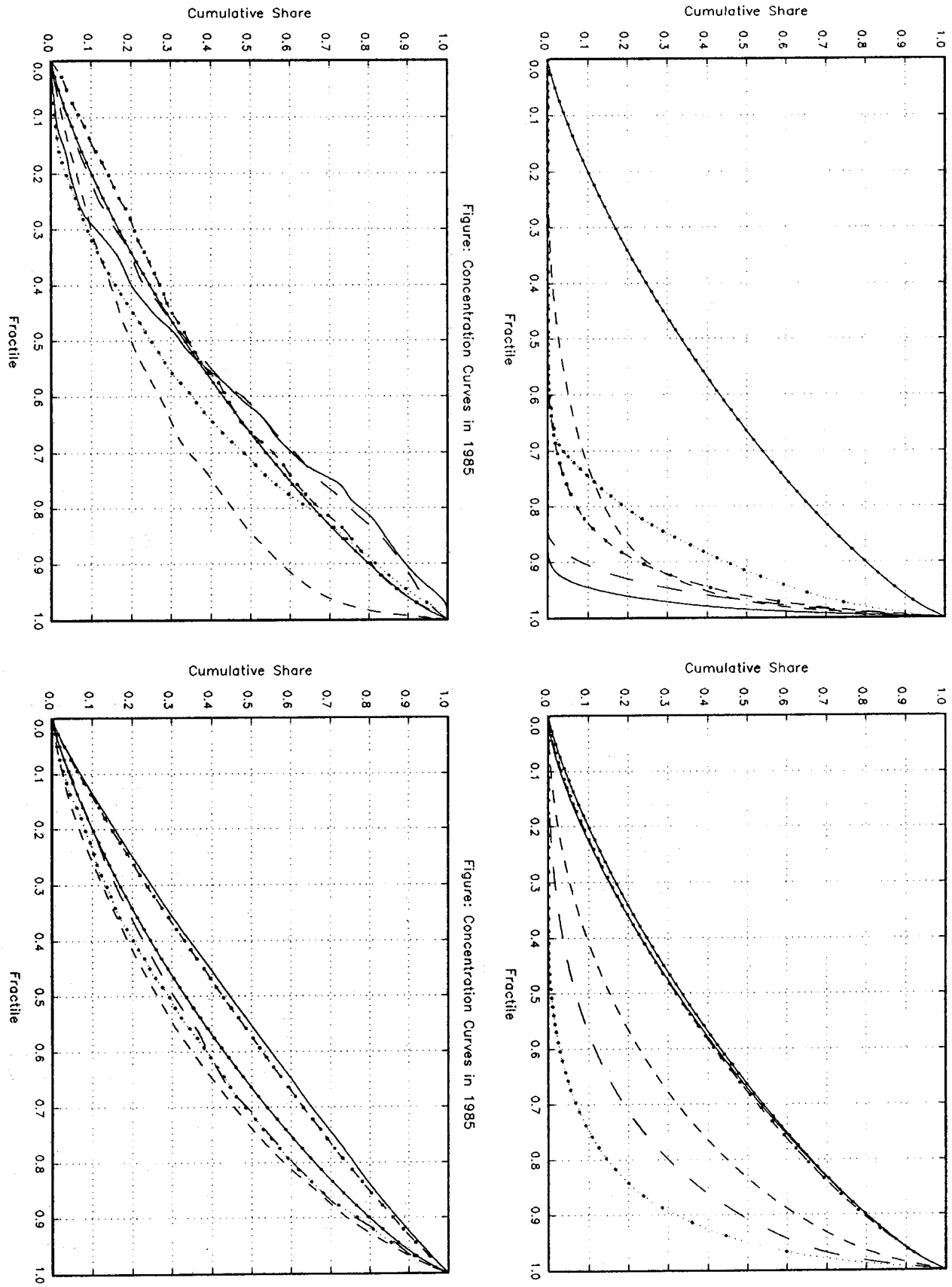
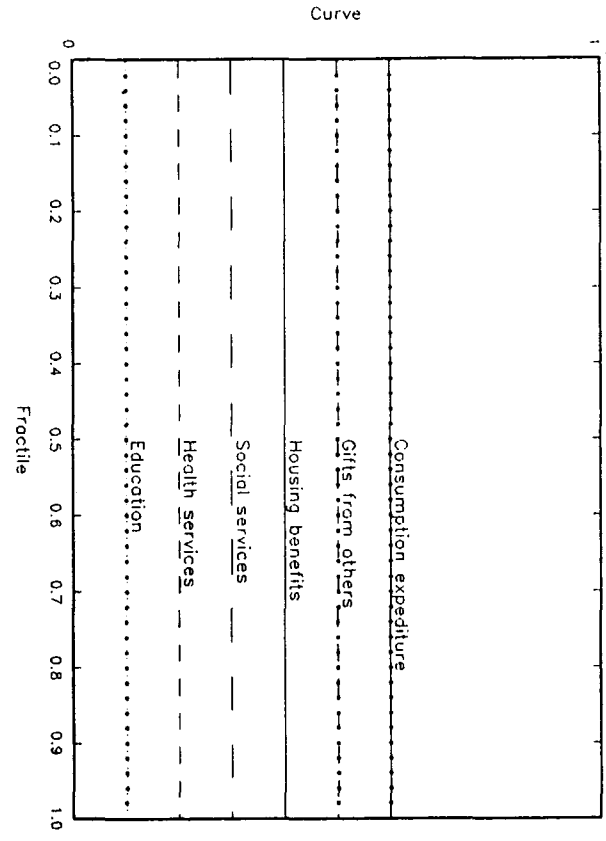



**Figure 1. Lorenz curves and concentration curves w.r.t. extended consumption in 1985; all households**



**Legends for figures 2 and 6 a - 6 f.**



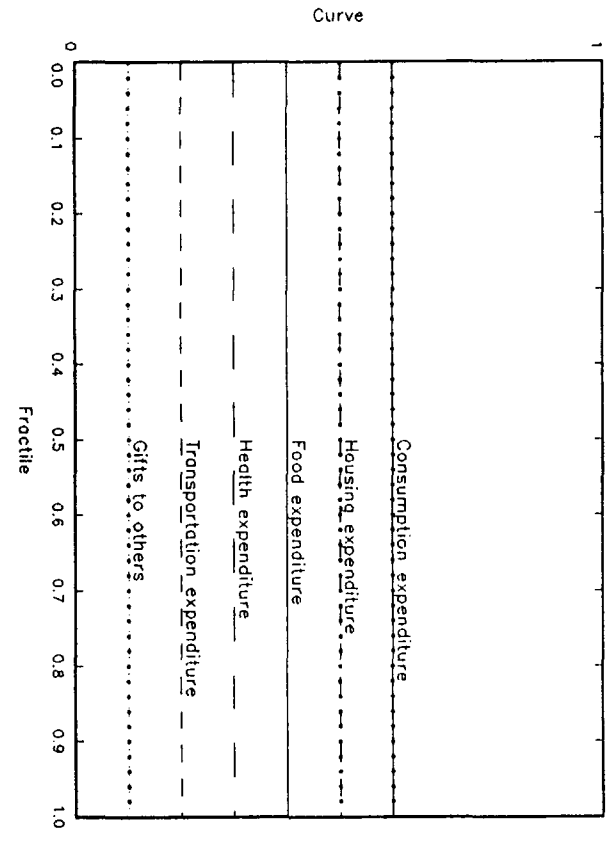



**Figure 2. Lorenz curves and concentration curves w.r.t. consumption in 1985; all households**

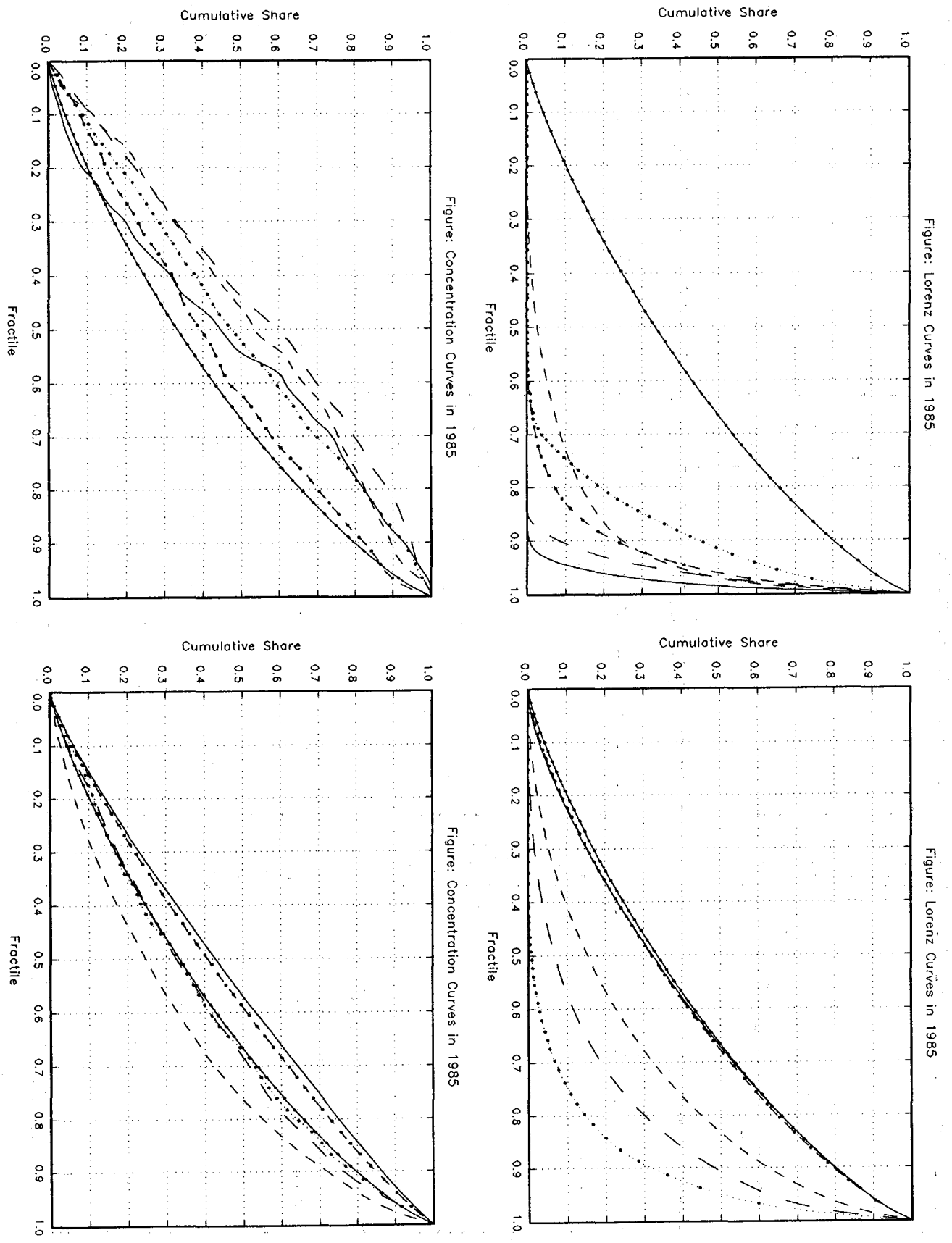


**Figure 3. Difference of concentration curves relative to social services in 1985; all households.**

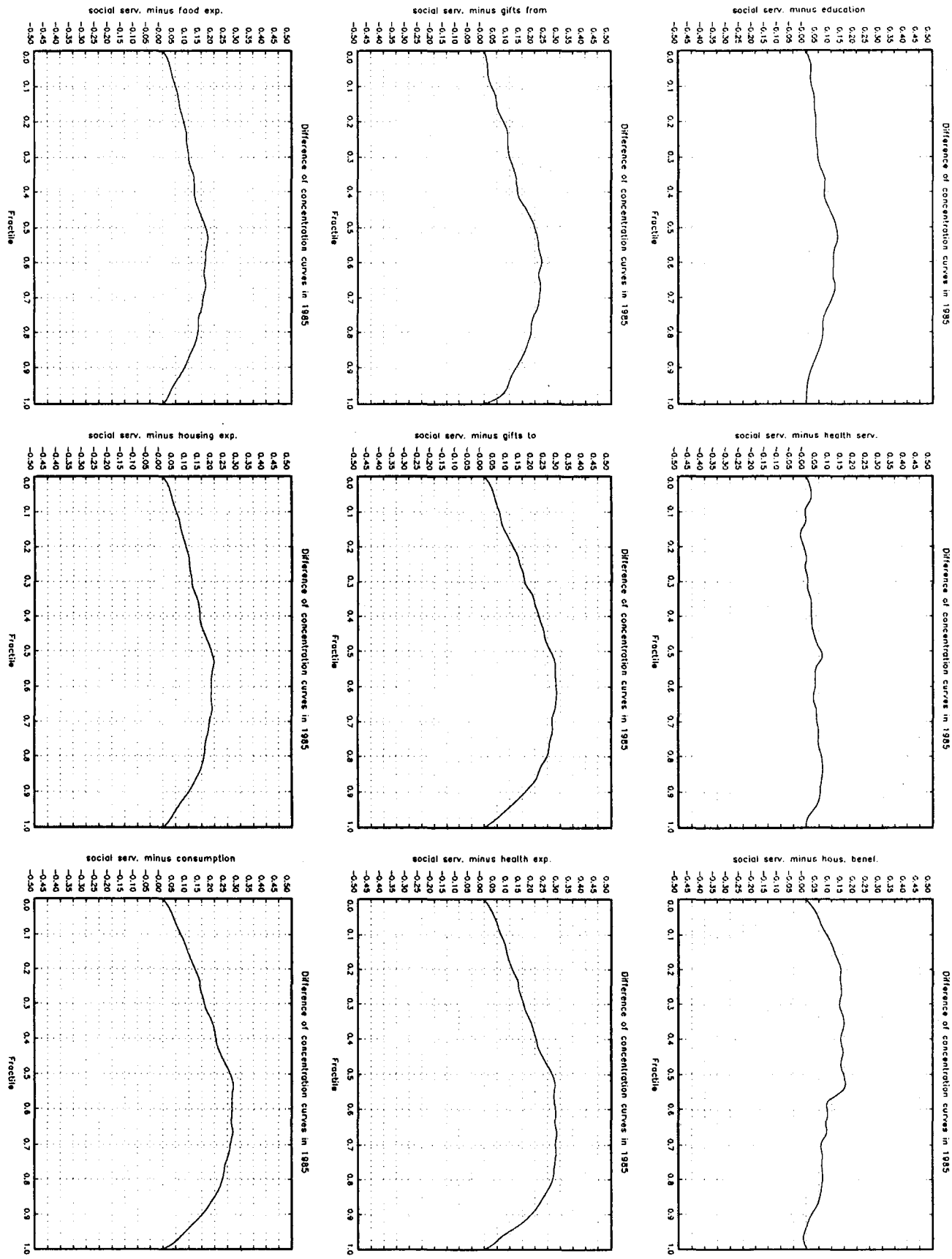


**Figure 4. Difference of concentration curves relative to transportation expenditure in 1985; all households**

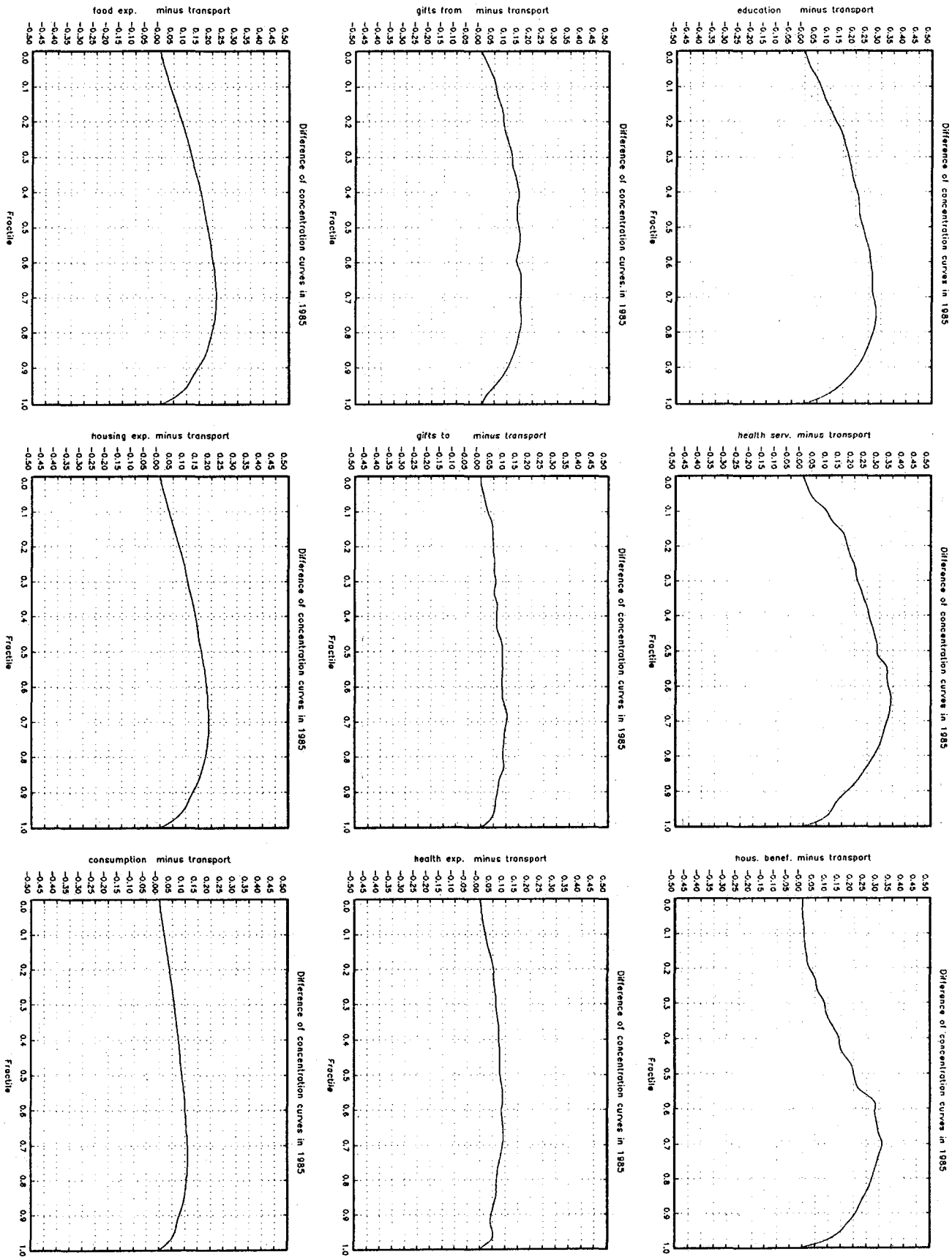


**Figure 5. Pairwise differences of concentration curves in 1985; all households**

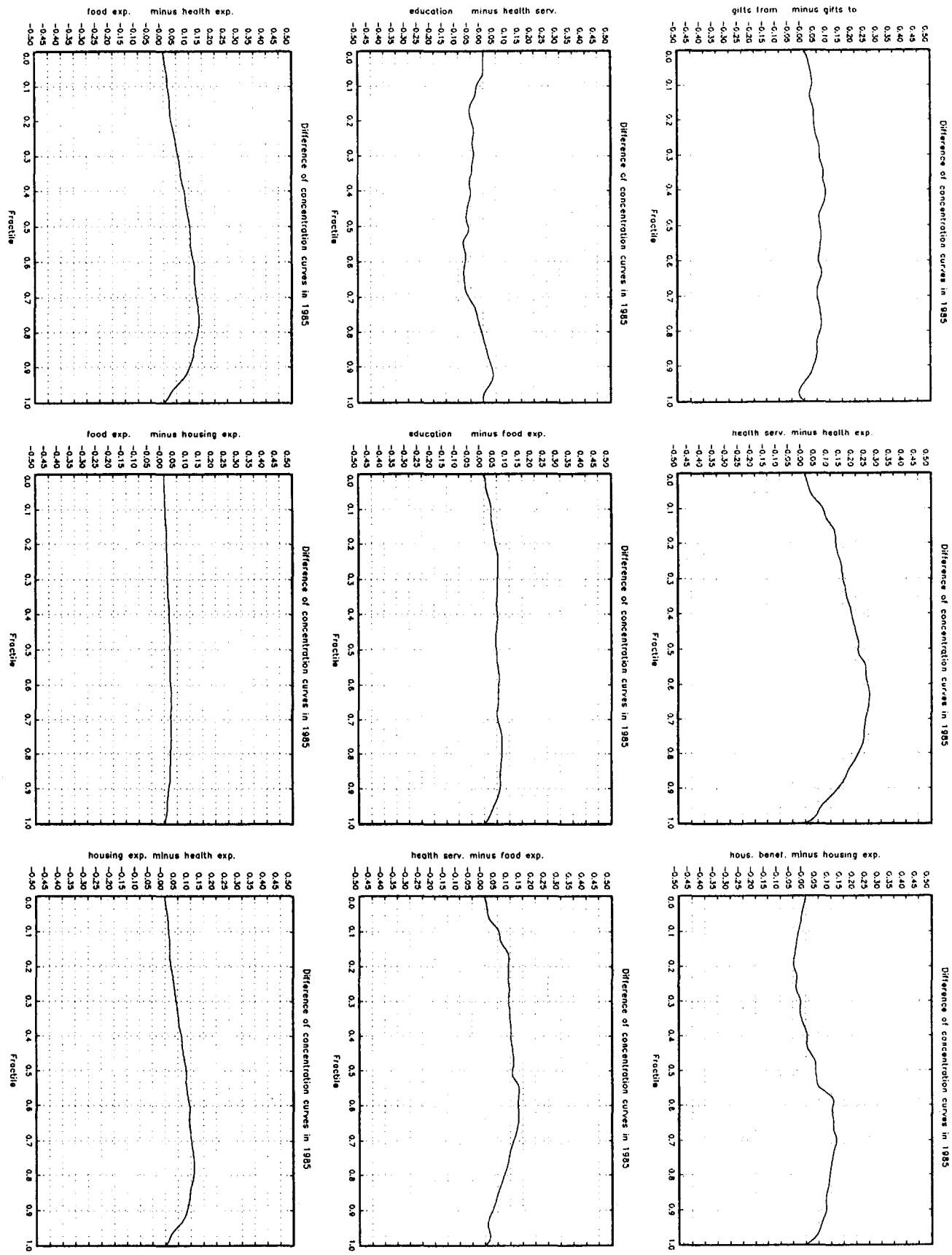


**Figure 6 a. Lorenz curves and concentration curves w.r.t. consumption; single adult**

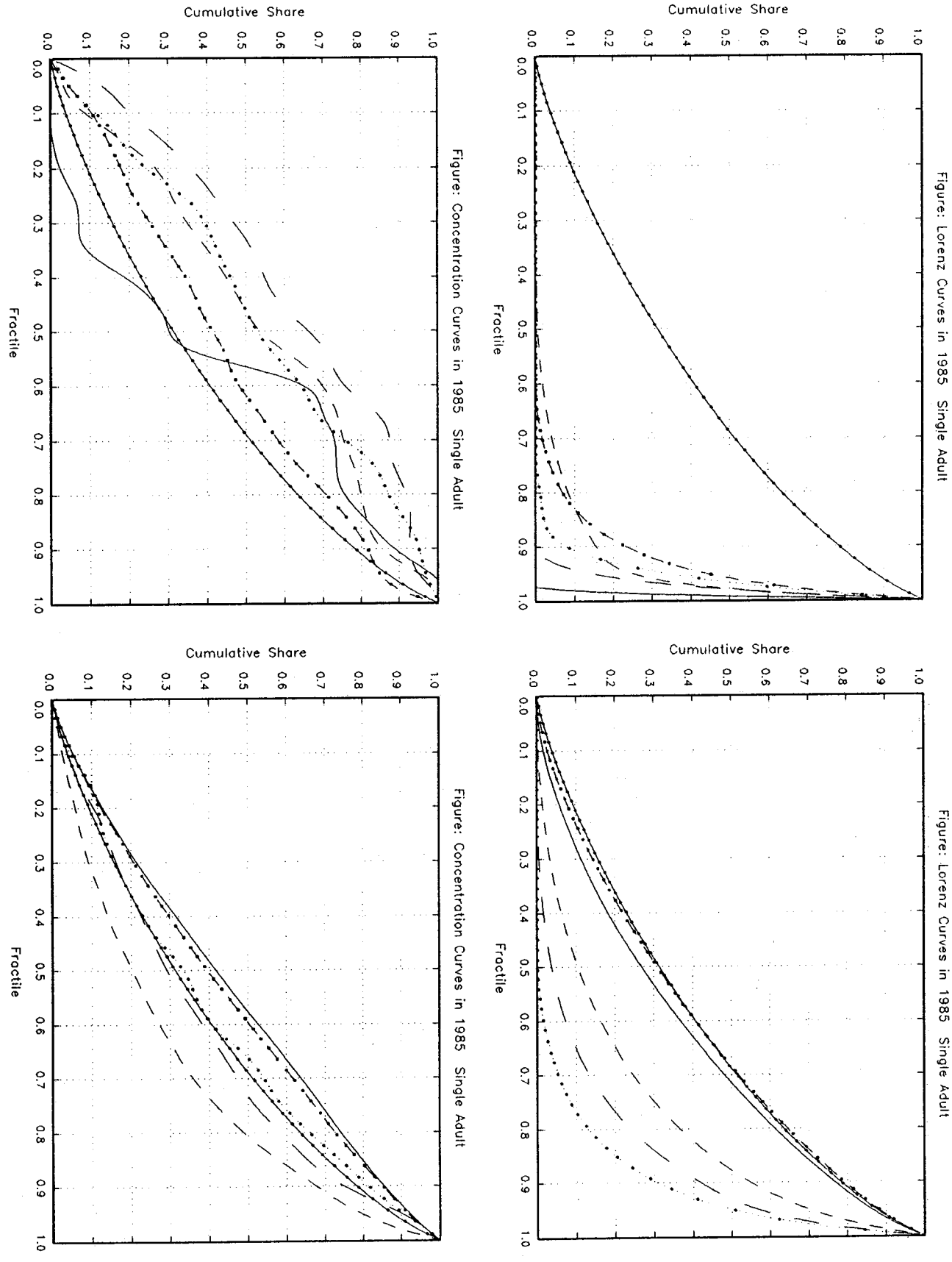


**Figure 6 b. Lorenz curves and concentration curves w.r.t. consumption; couple with no children**

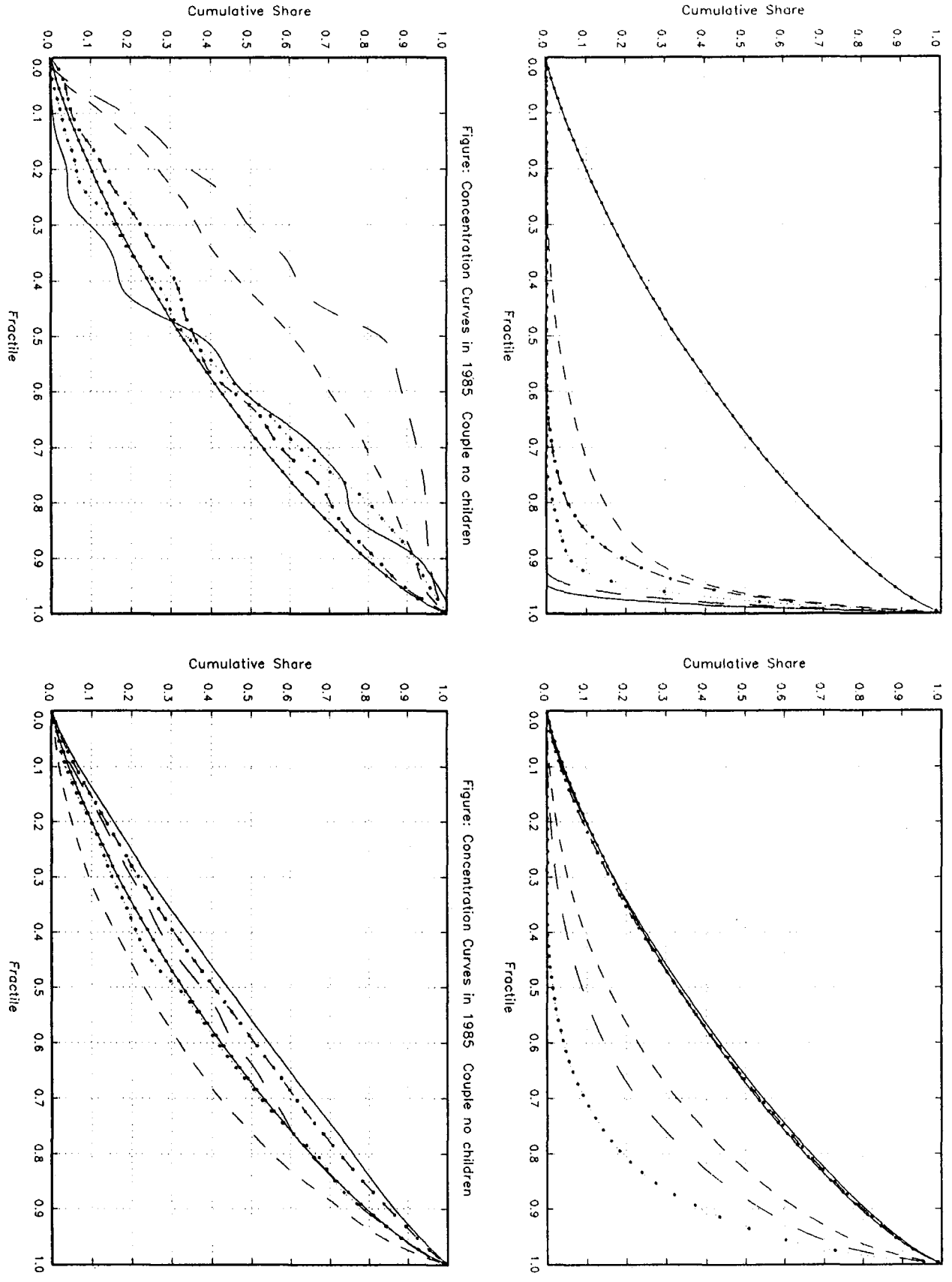


**Figure 6 c. Lorenz curves and concentration curves w.r.t. consumption; single provider**

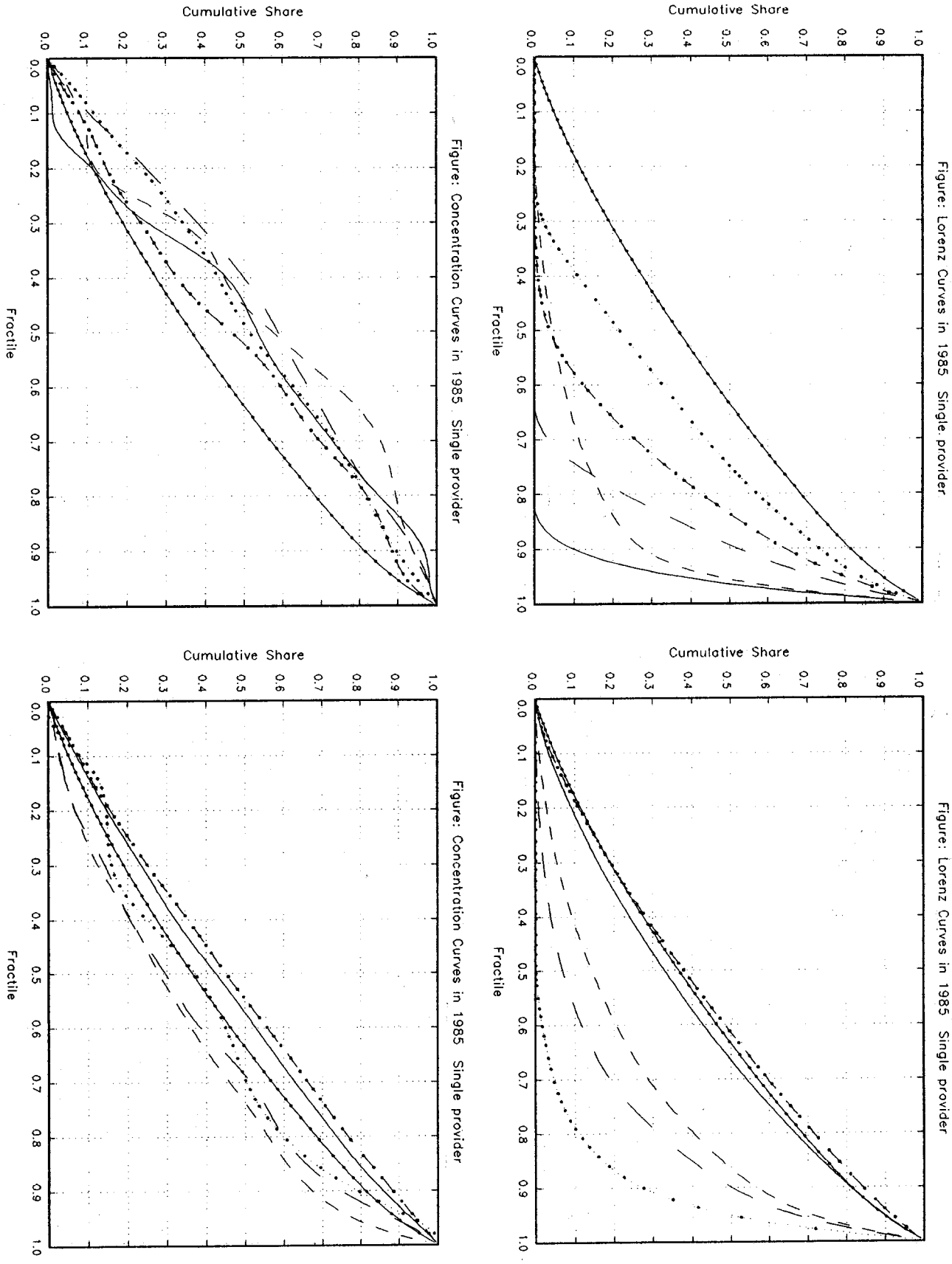


**Figure 6 d. Lorenz curves and concentration curves w.r.t. consumption; couple with children**

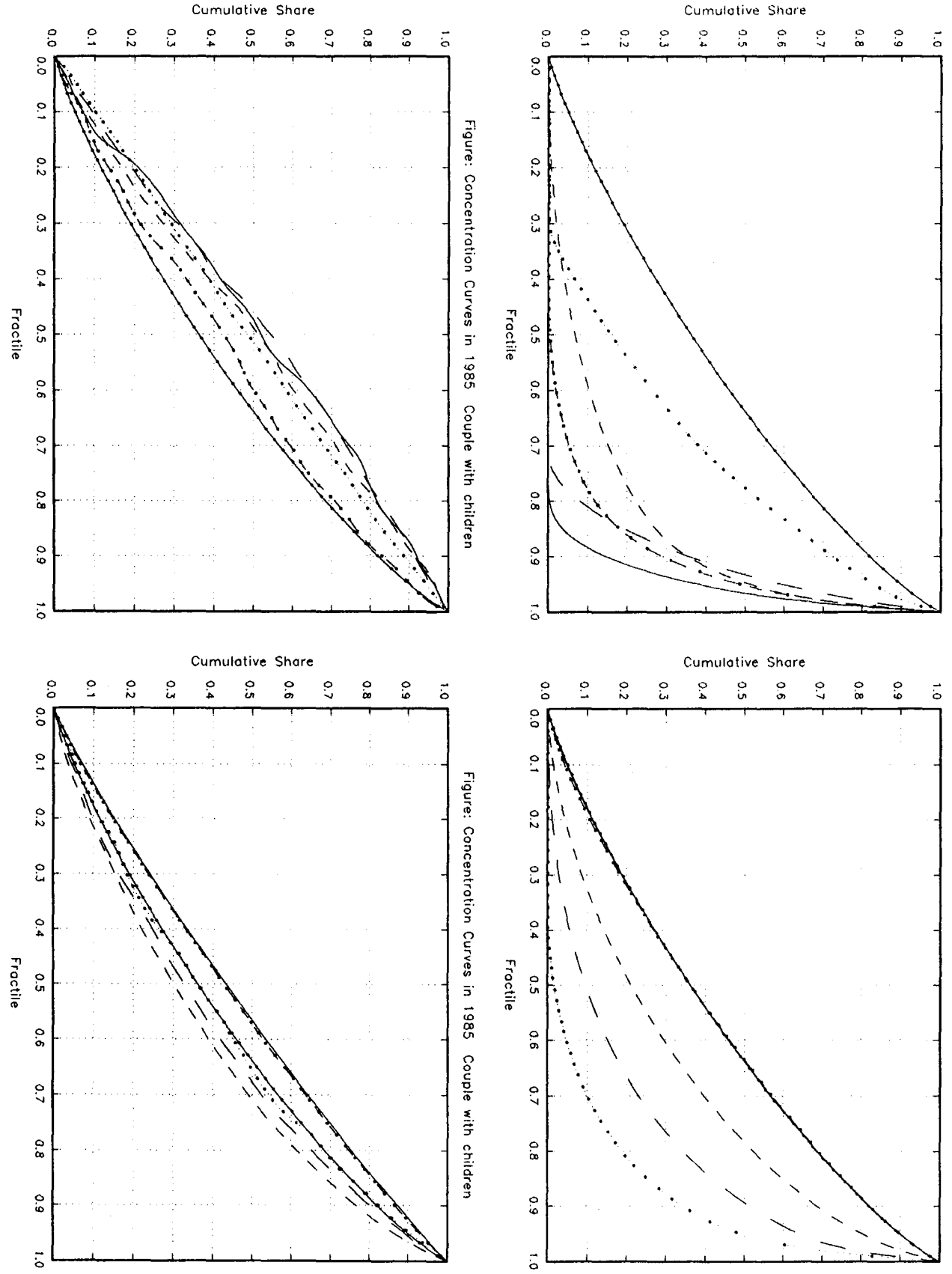
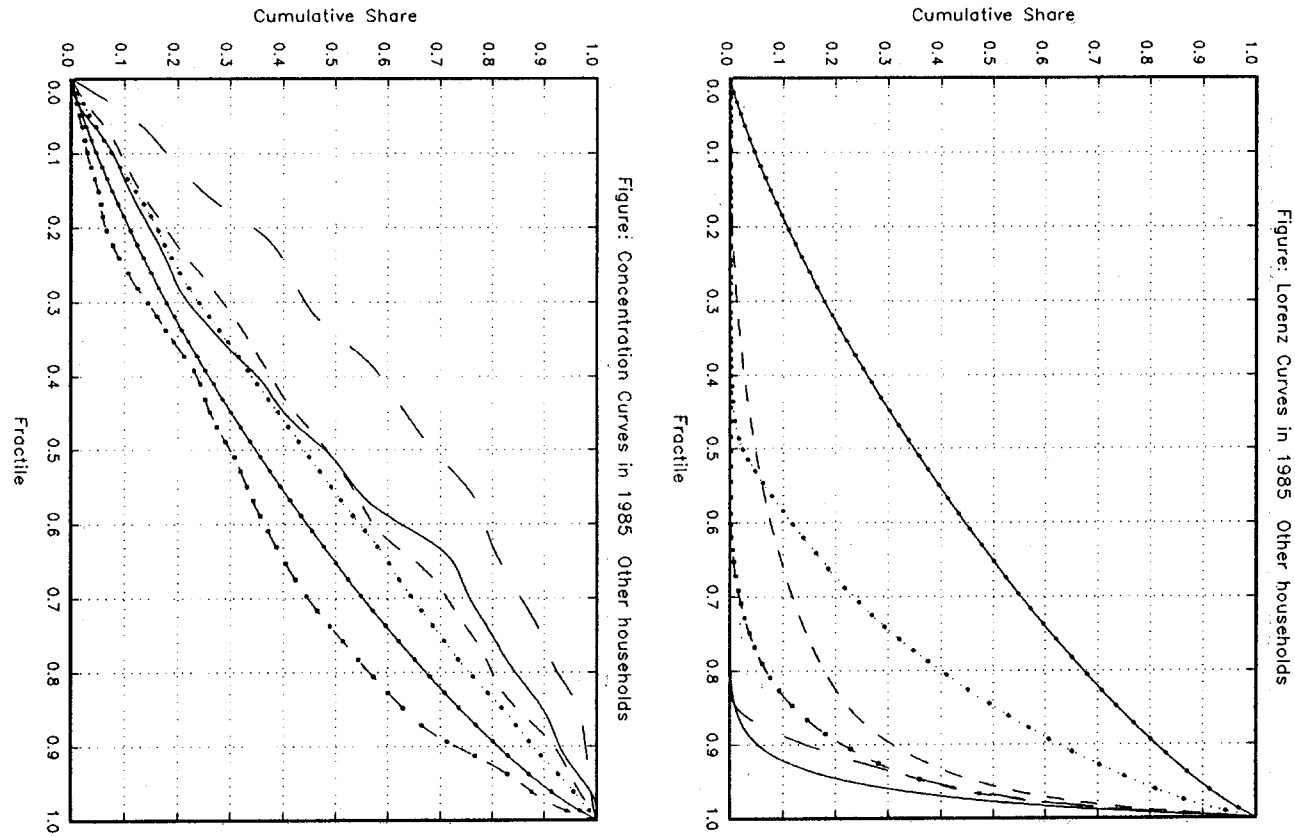


**Figure 6 e. Lorenz curves and concentration curves w.r.t. consumption; other households**







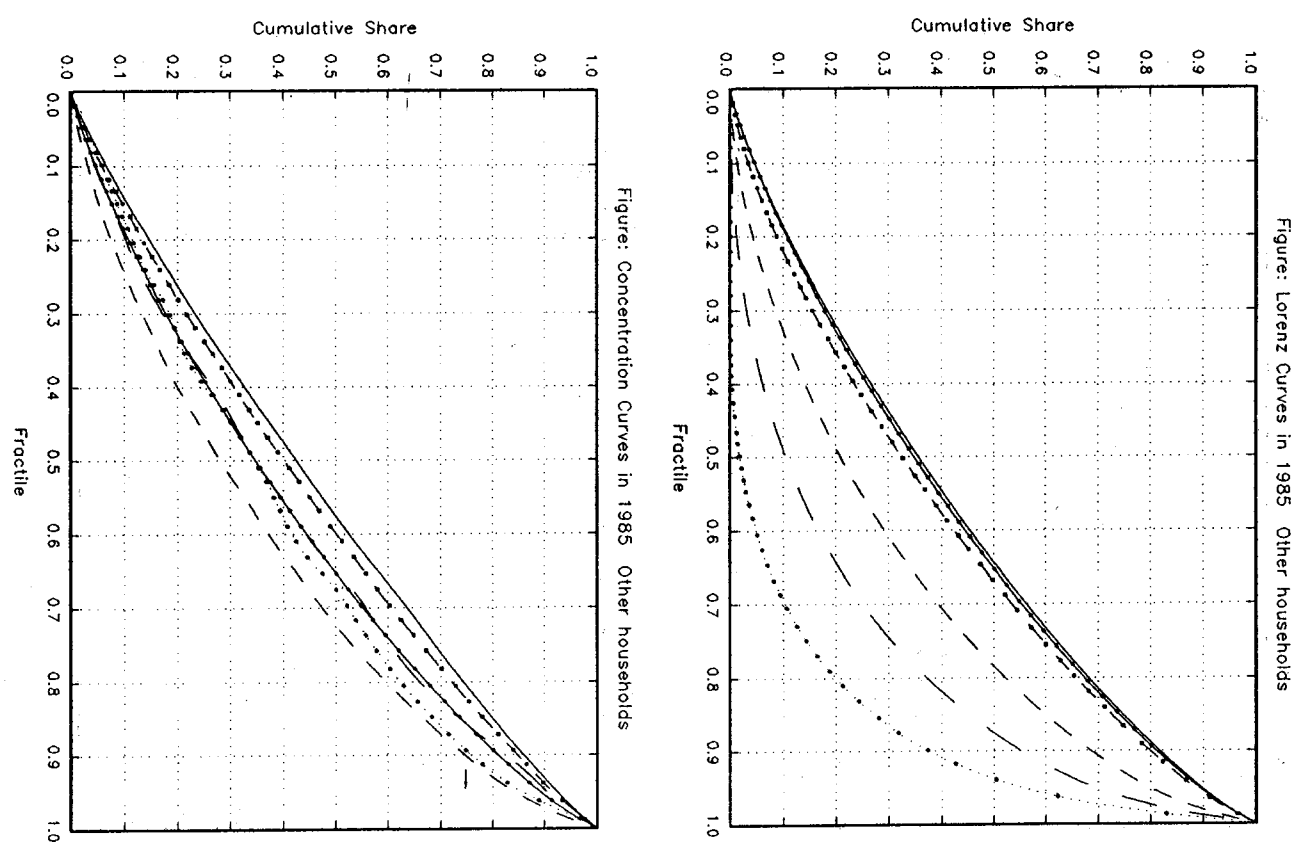


**Figure 6 f. Lorenz curves and concentration curves w.r.t. consumption; elderly household**

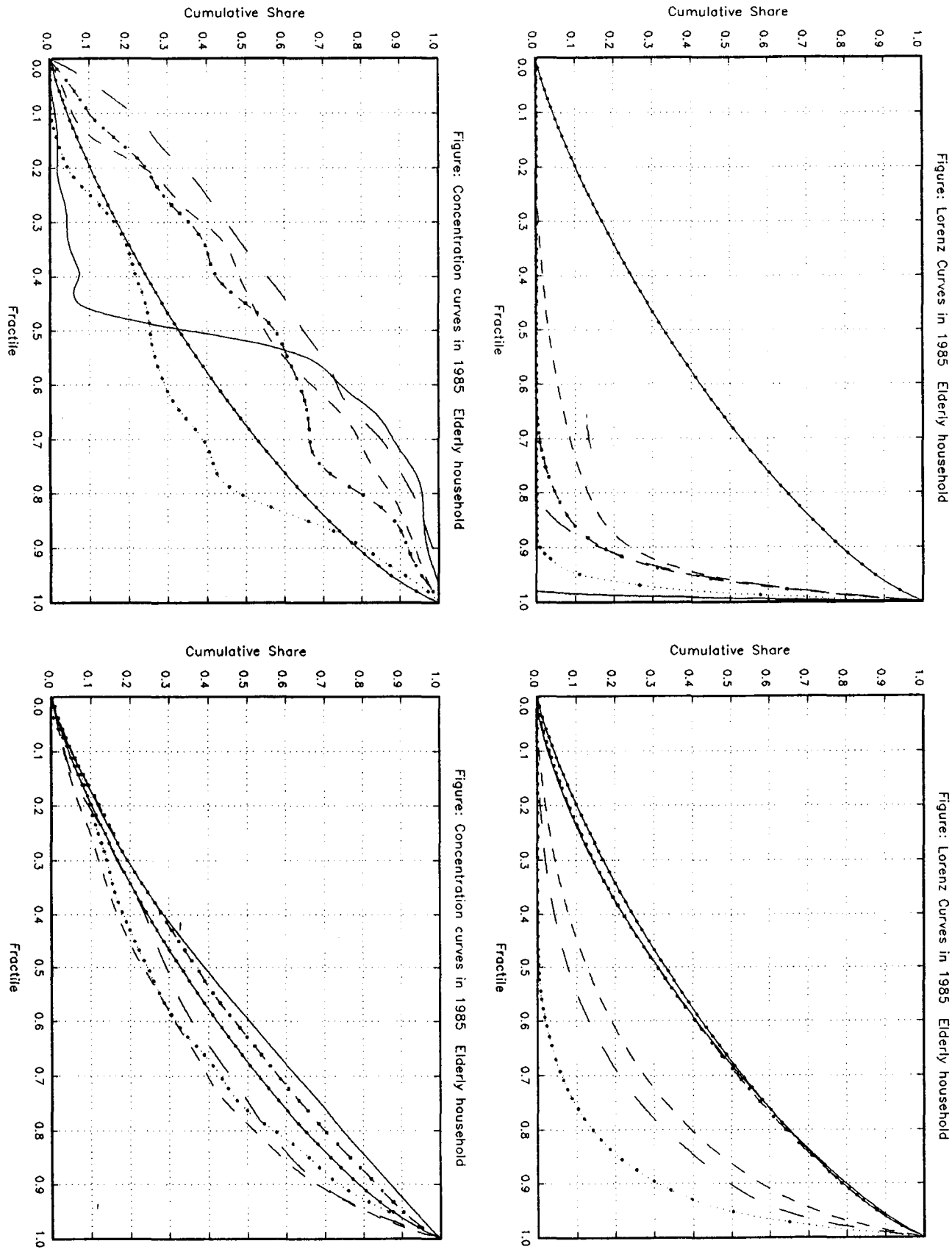


**Figure 7 a. Difference of concentration curves relative to social services; single adult**

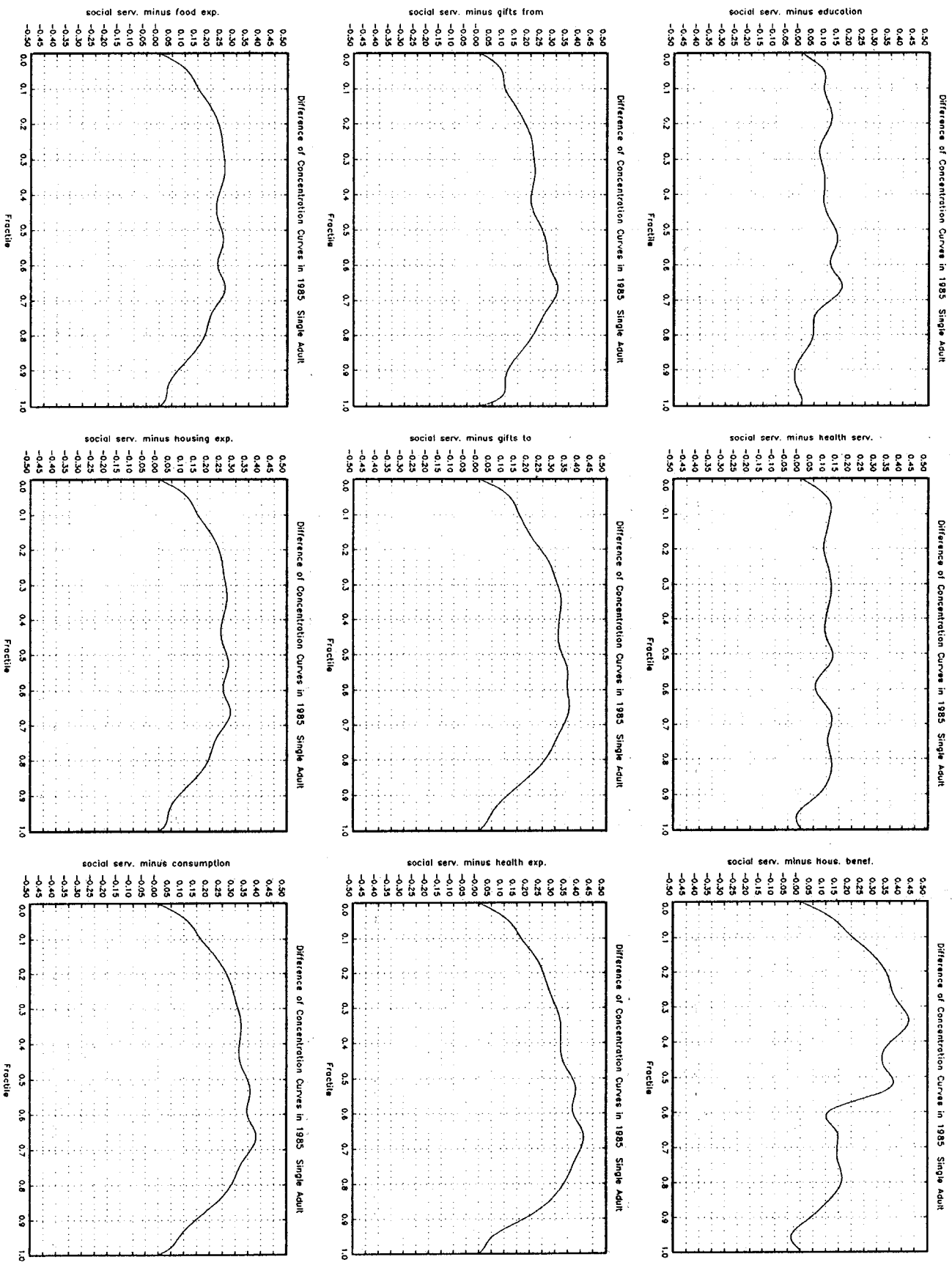


**Figure 7 b. Difference of concentration curves relative to social services; couple with no children**

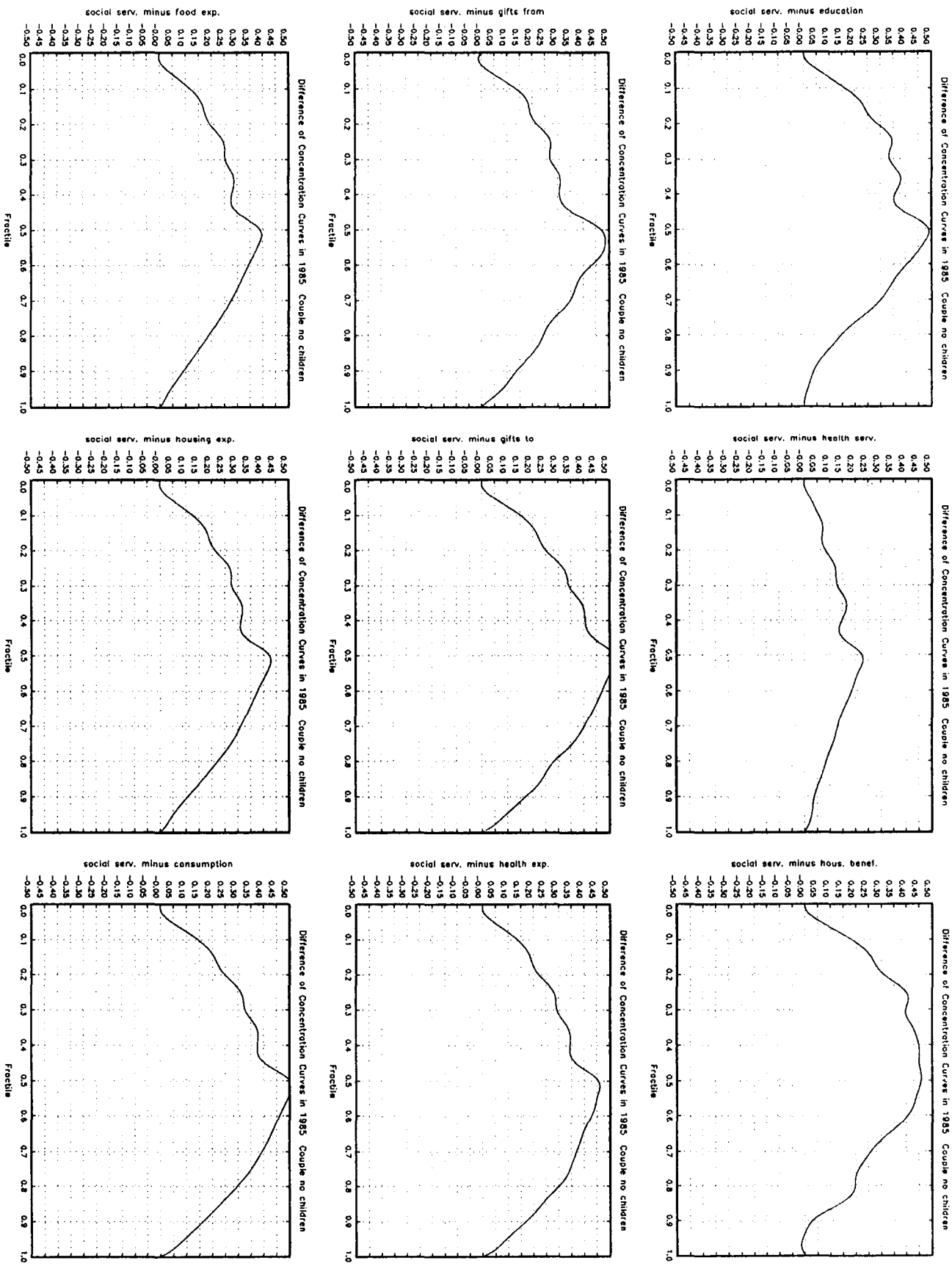
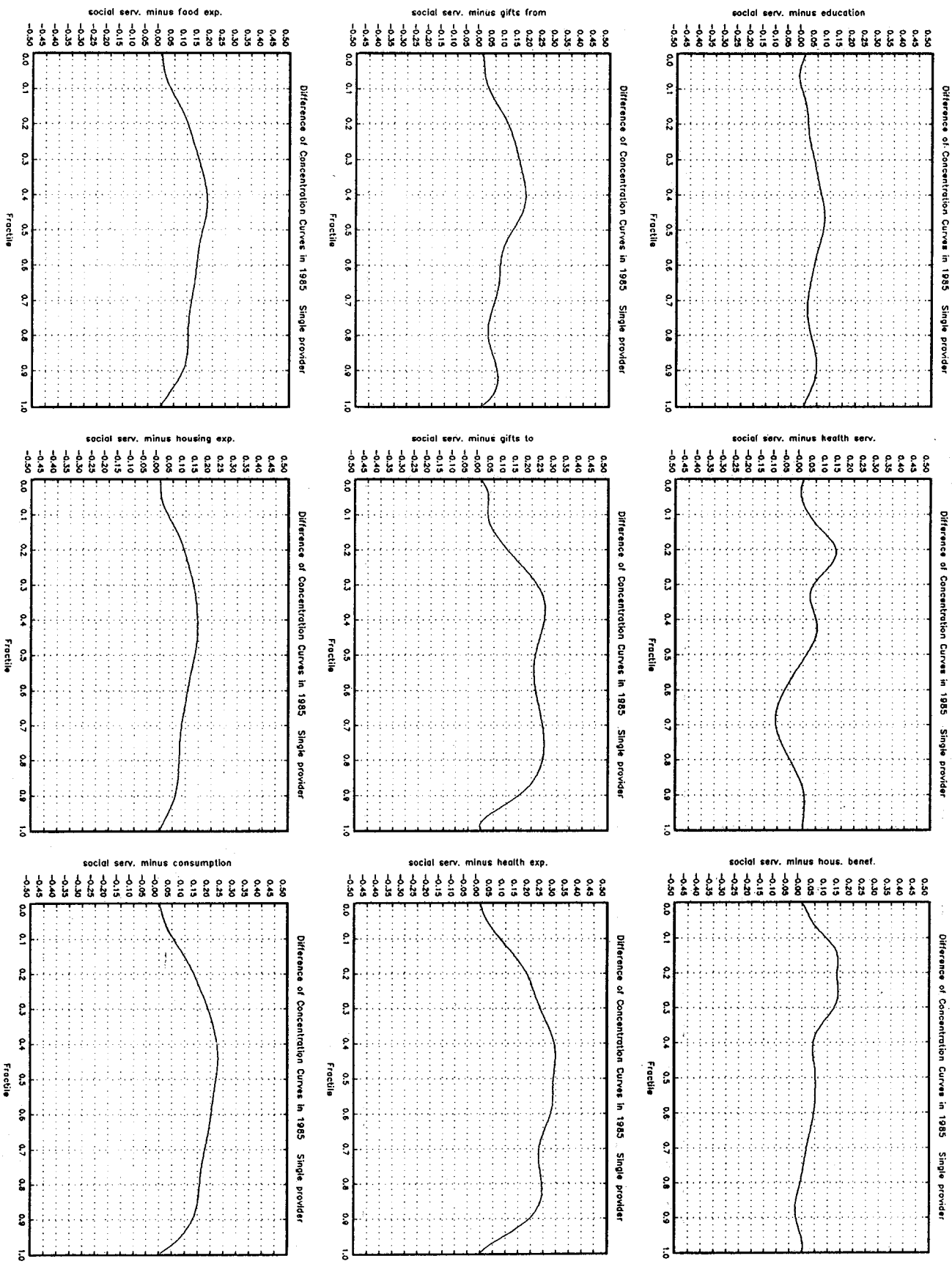
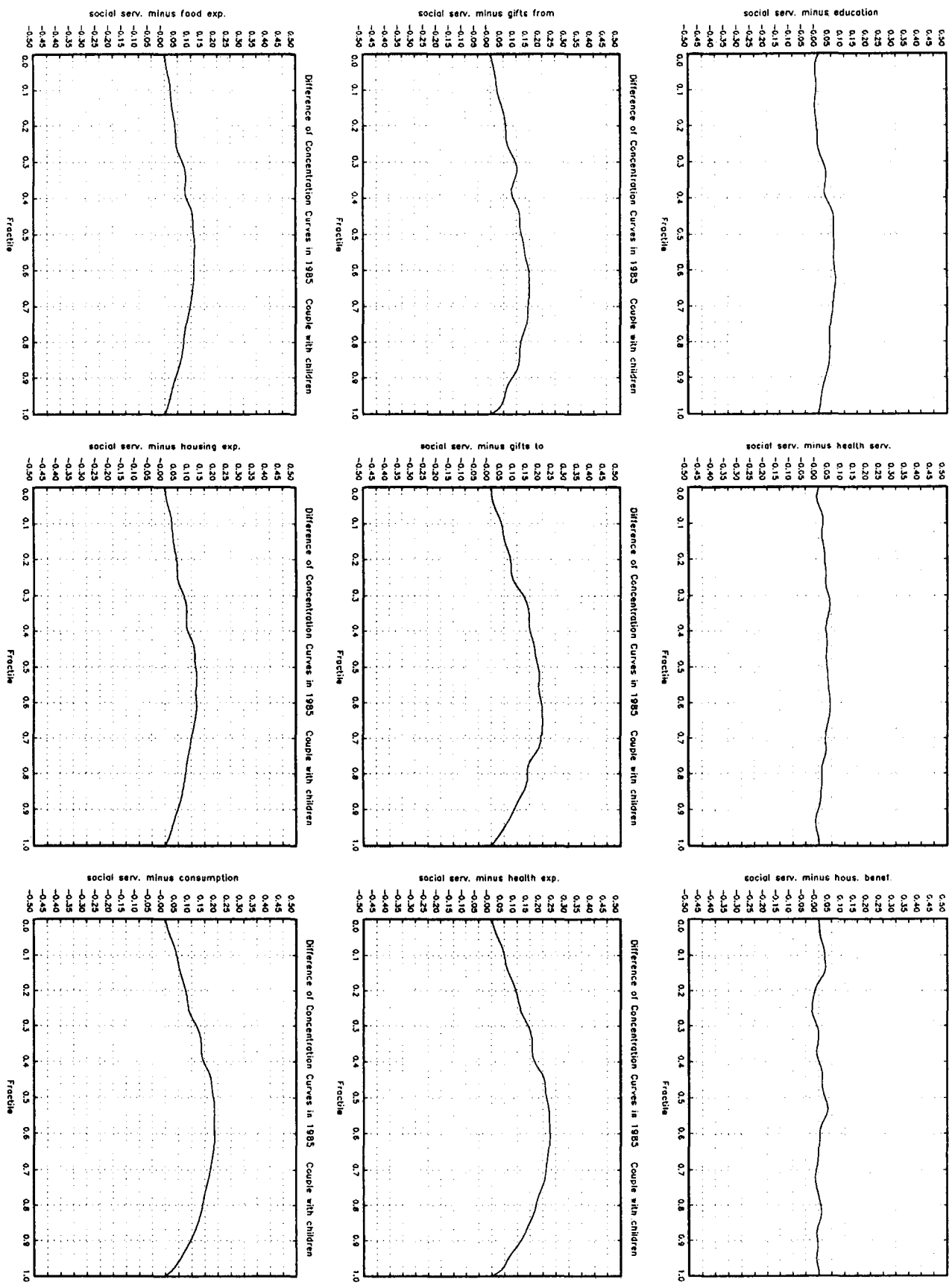


**Figure 7 c. Difference of concentration curves relative to social services; single provider**



# Figure 7 d. Difference of concentration curves relative to social services; couple with children



# Figure 7 e. Difference of concentration curves relative to social services; other households

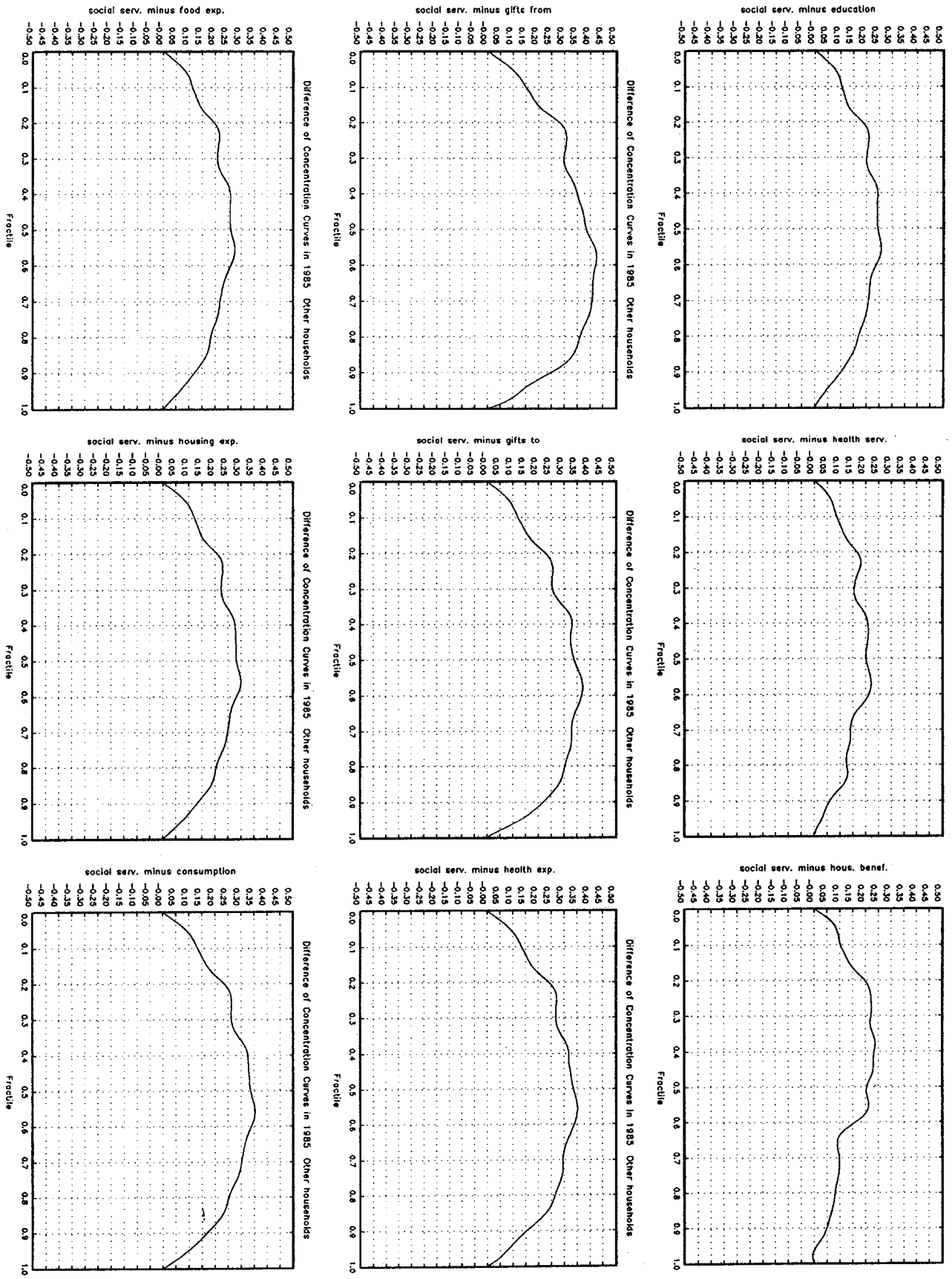


**Figure 7 f. Difference of concentration curves relative to social services; elderly household**

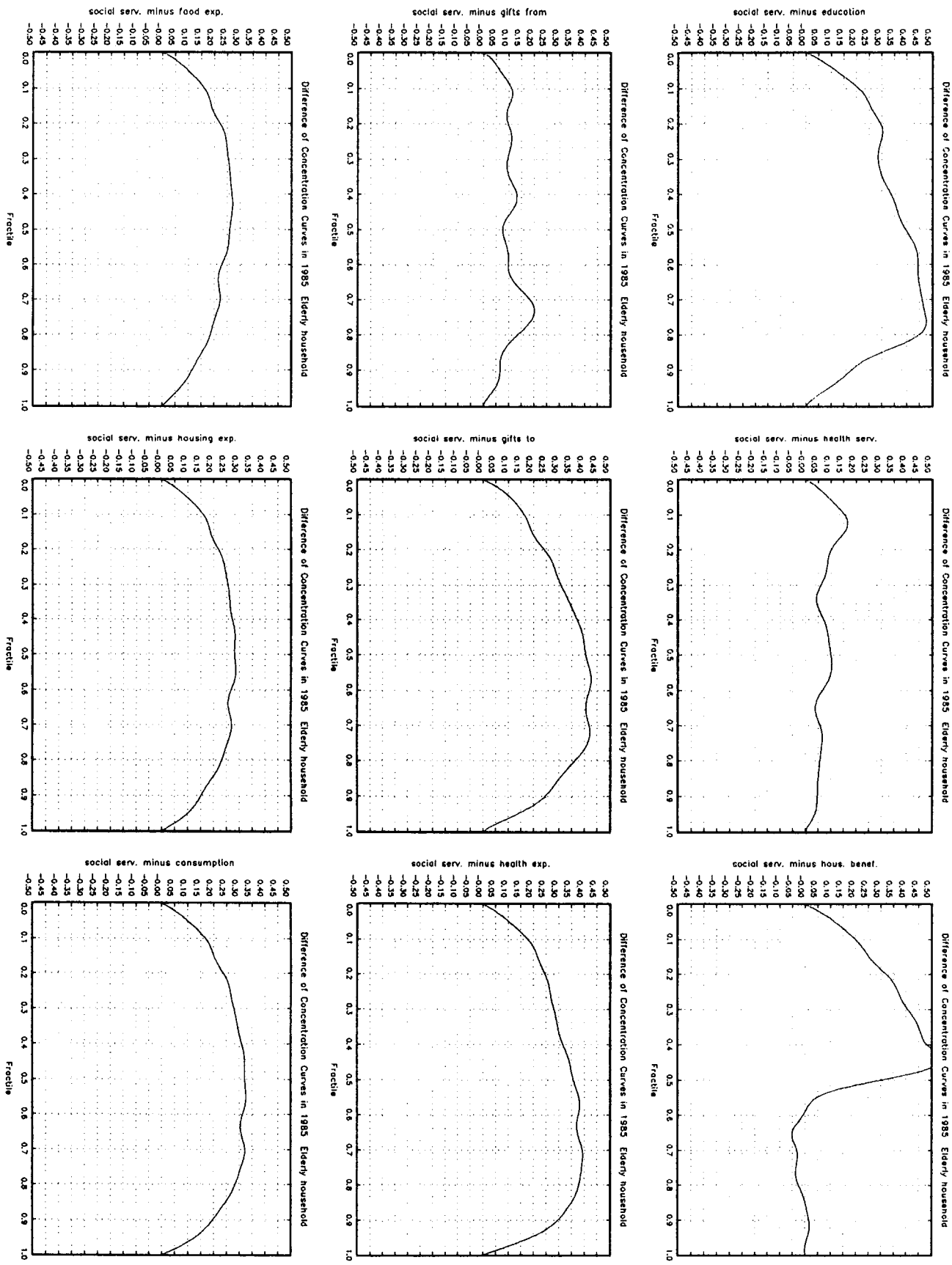


**Figure 8 a. Difference of concentration curves relative to transportation expenditure; single adult**

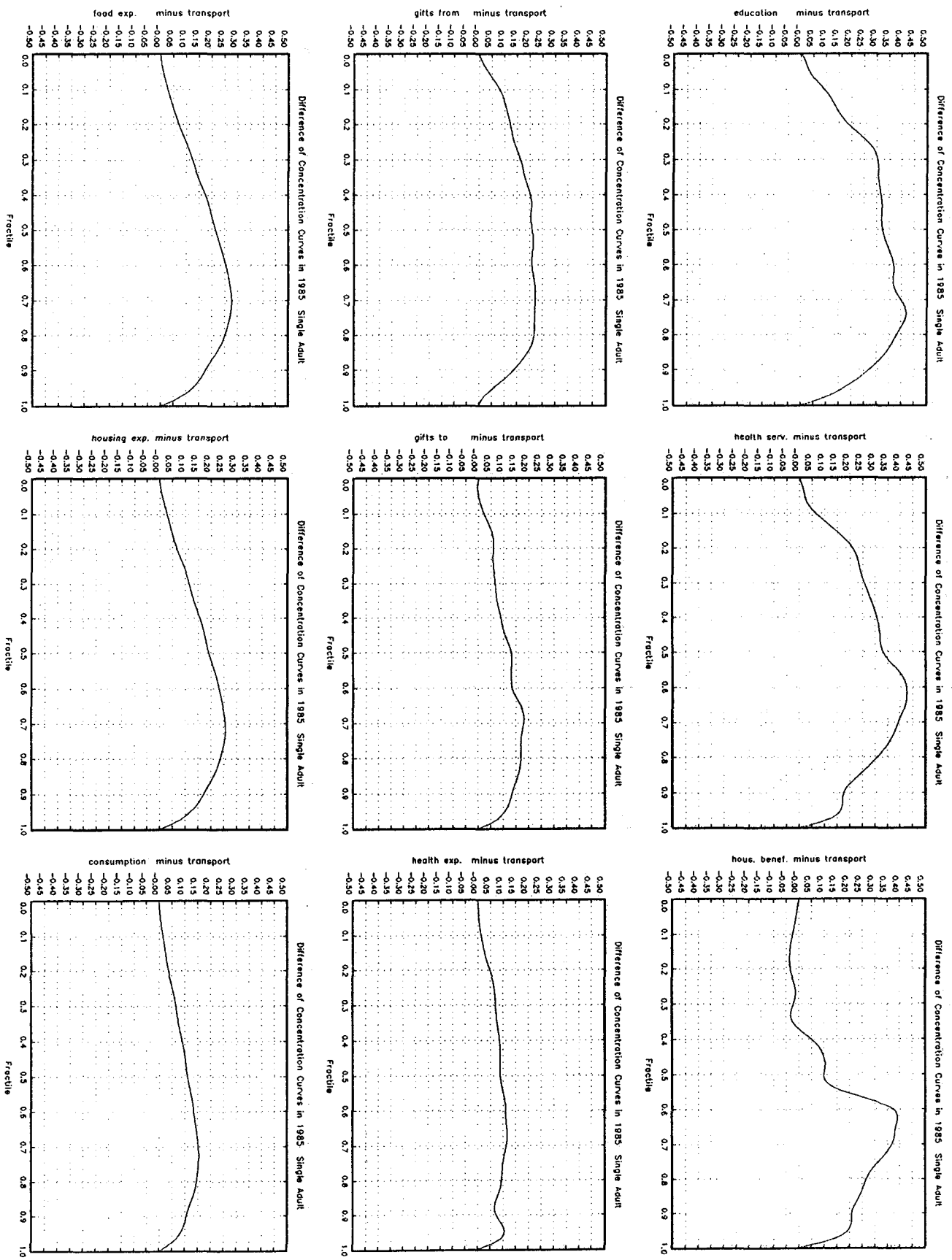
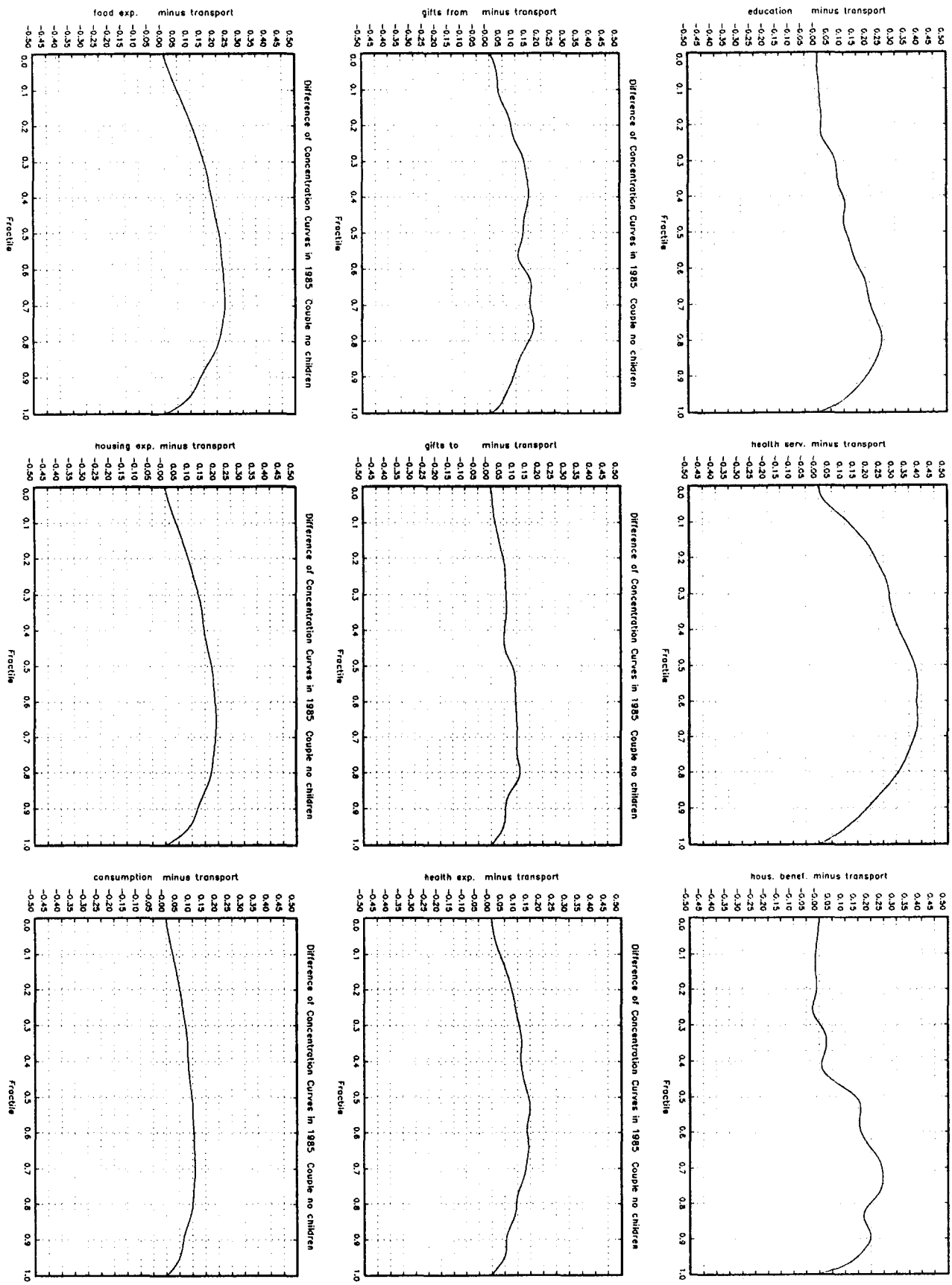


**Figure 8 b. Difference of concentration curves relative to transportation expenditure; couple with no children**



# Figure 8 c. Difference of concentration curves relative to transportation expenditure; single provider

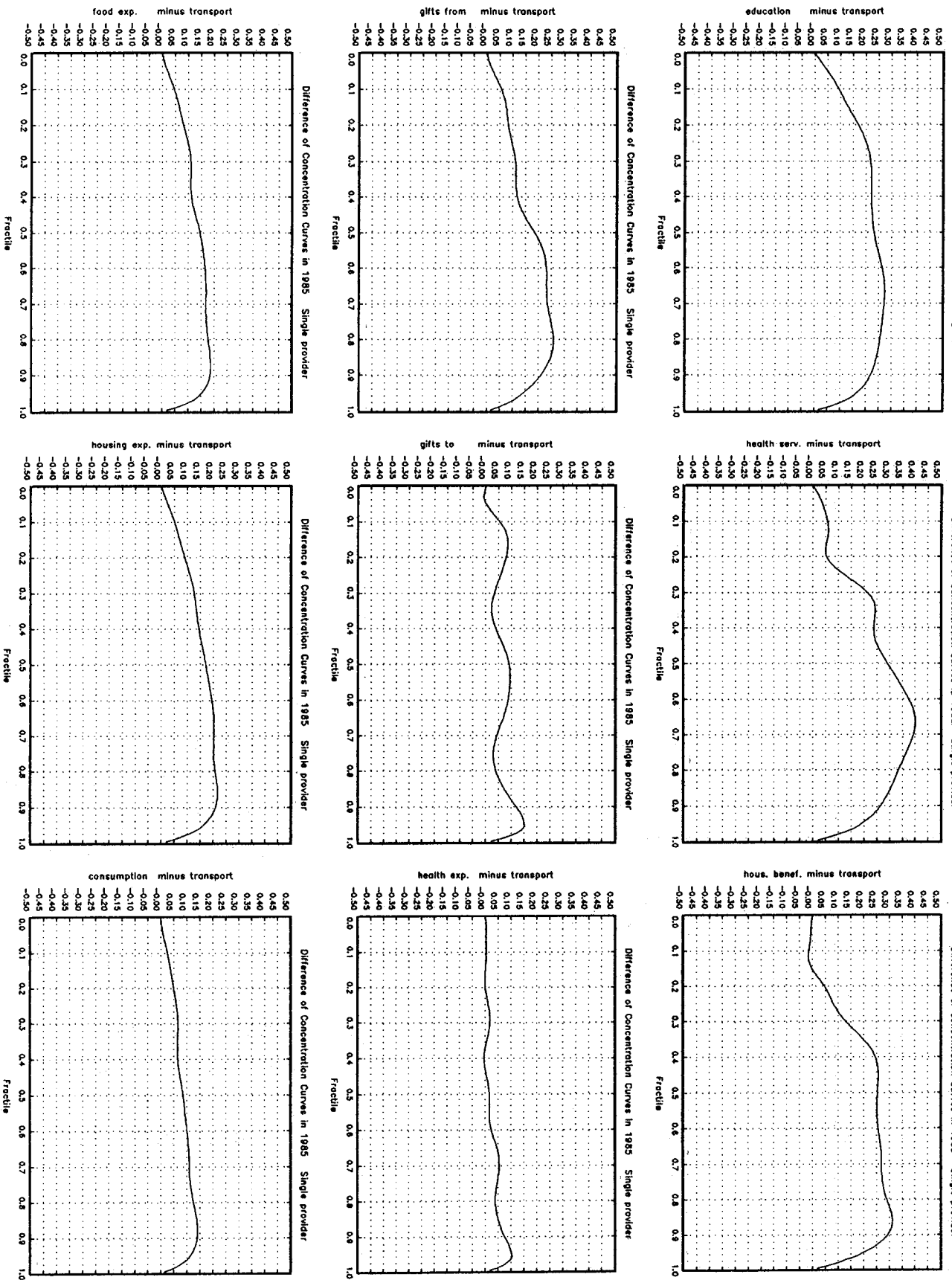


**Figure 8 d. Difference of concentration curves relative to transportation expenditure; couple with children**

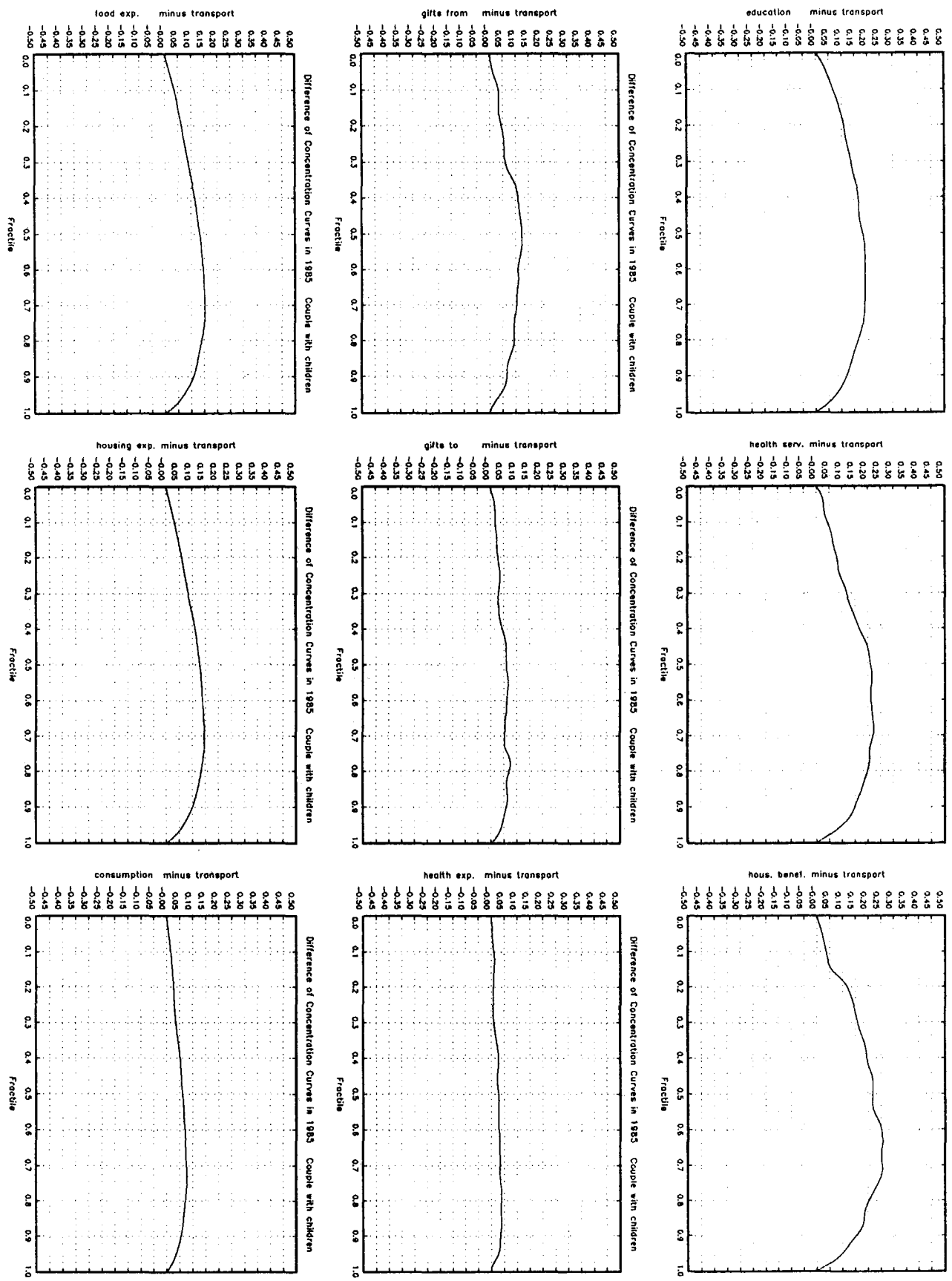


**Figure 8 e. Difference of concentration curves relative to transportation expenditure; other households**

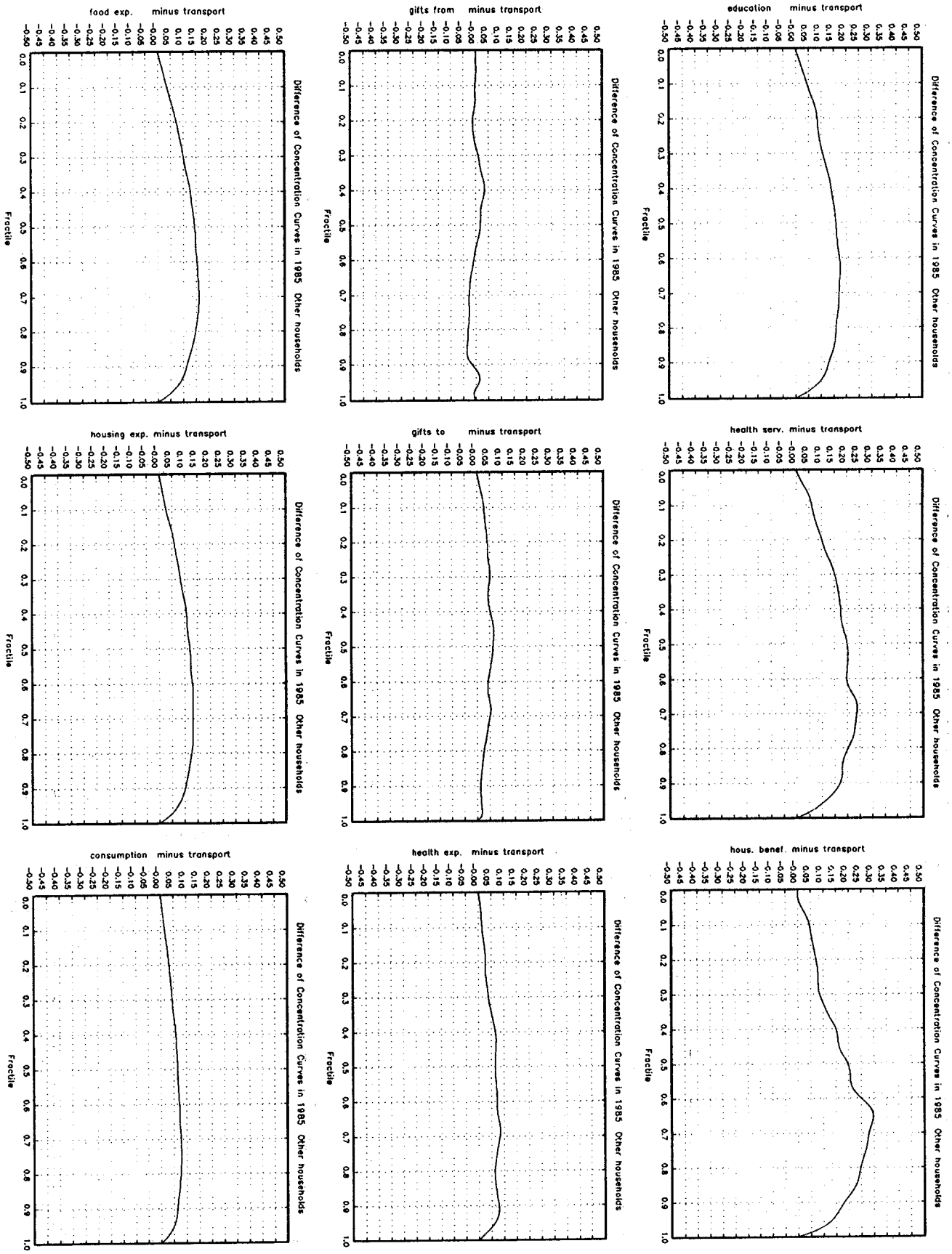


**Figure 8 f. Difference of concentration curves relative to transportation expenditure; elderly household**

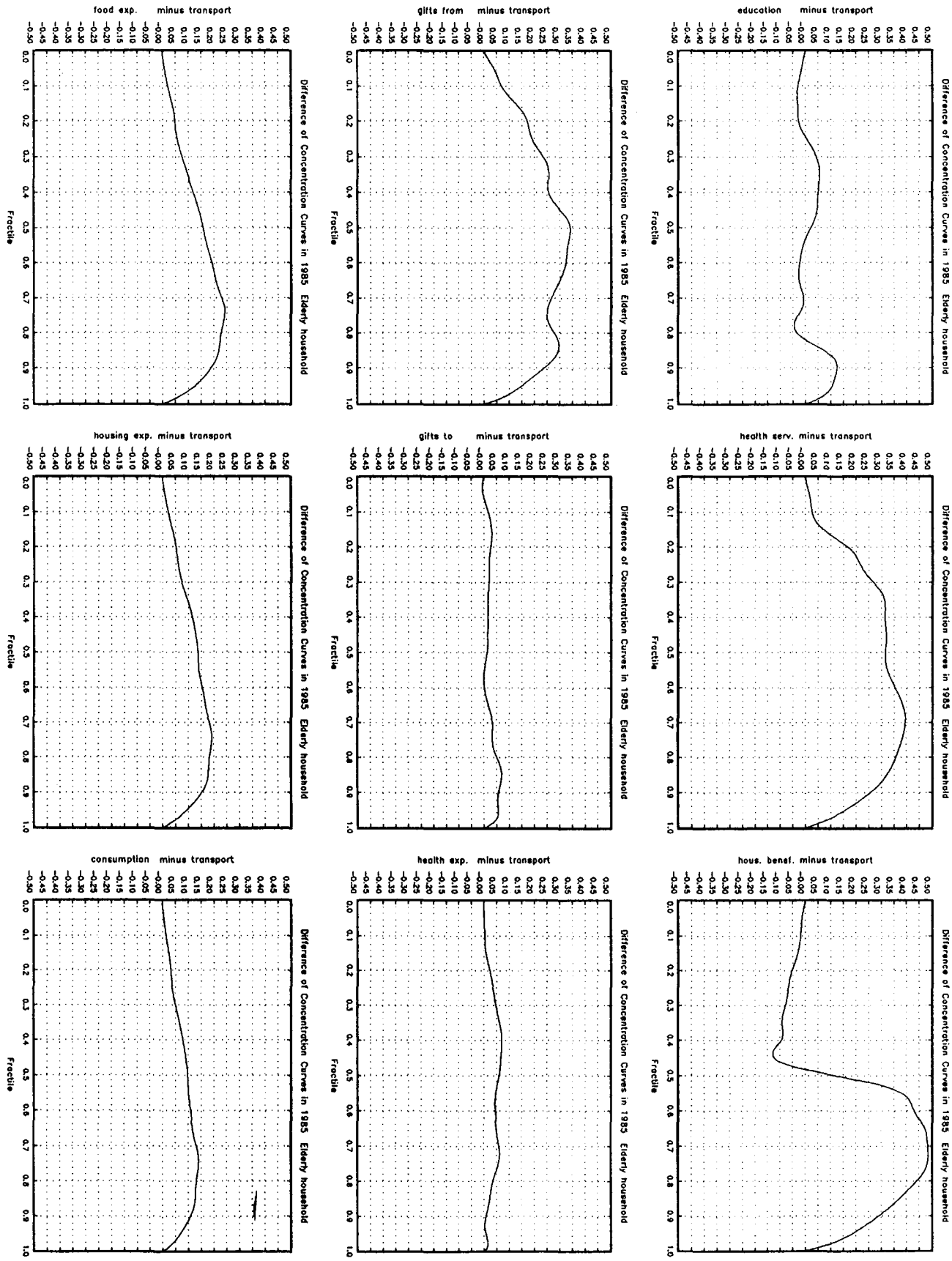


**Figure 9 a. Pairwise differences of concentration curves; single adult**

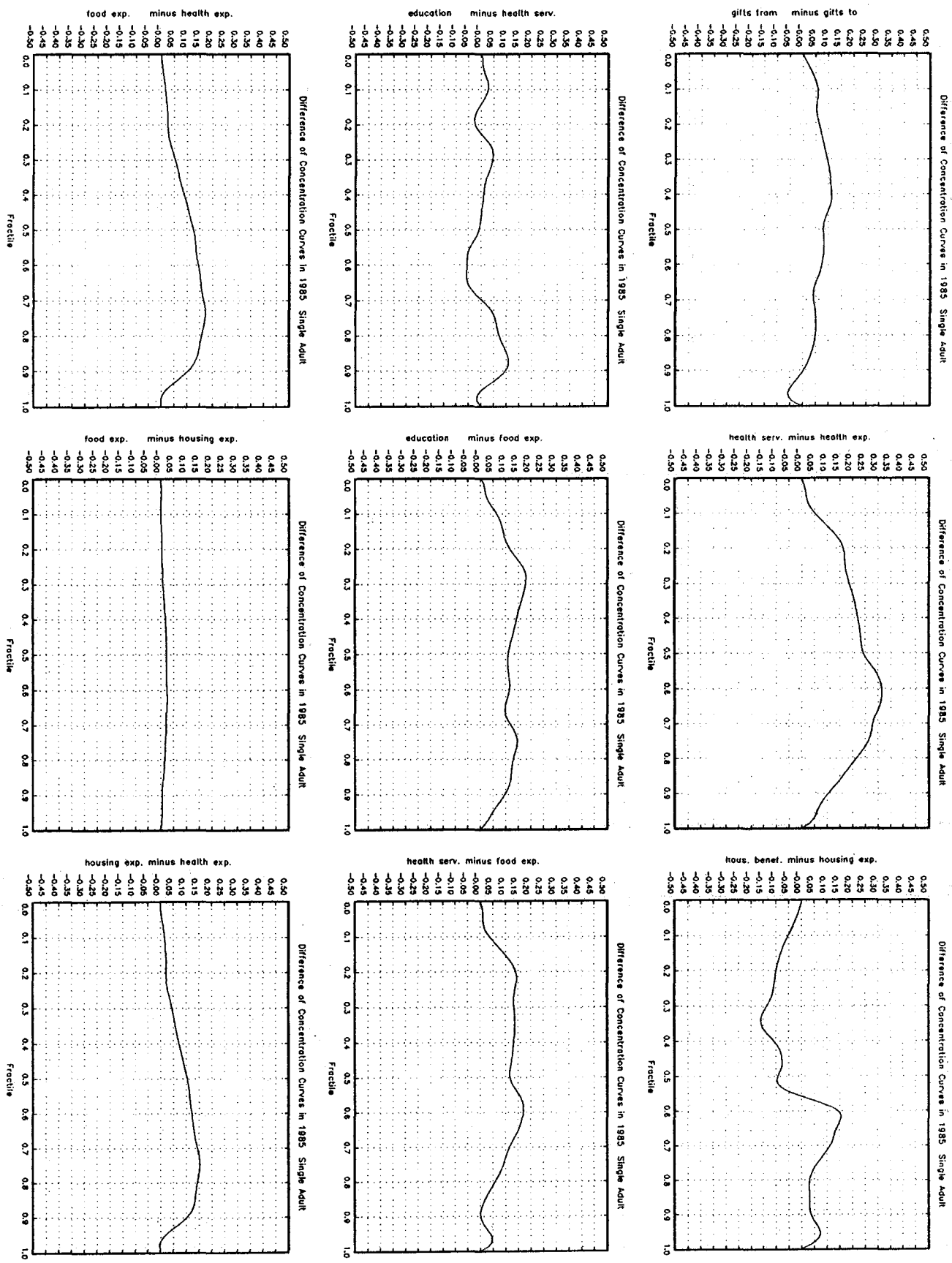
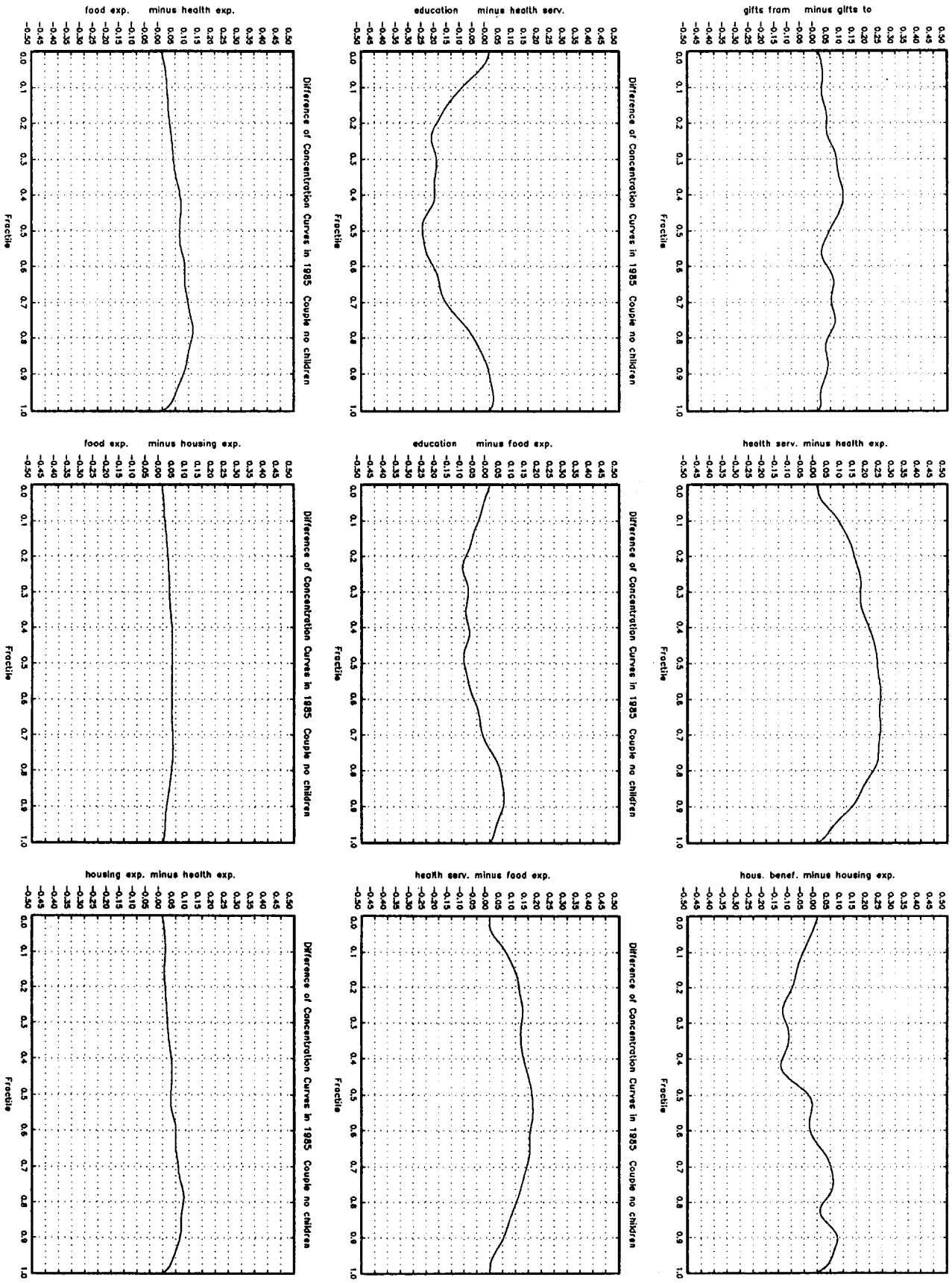
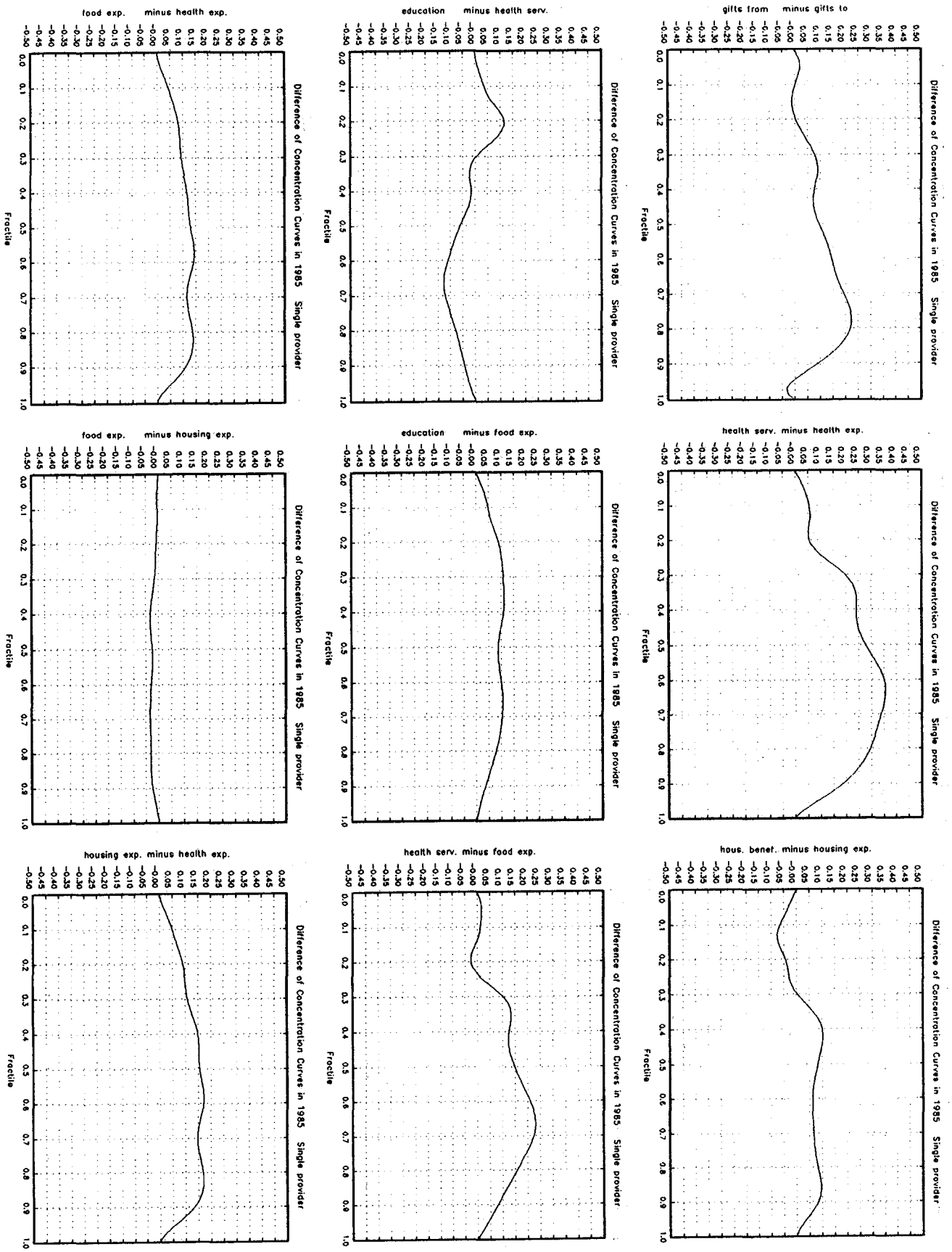


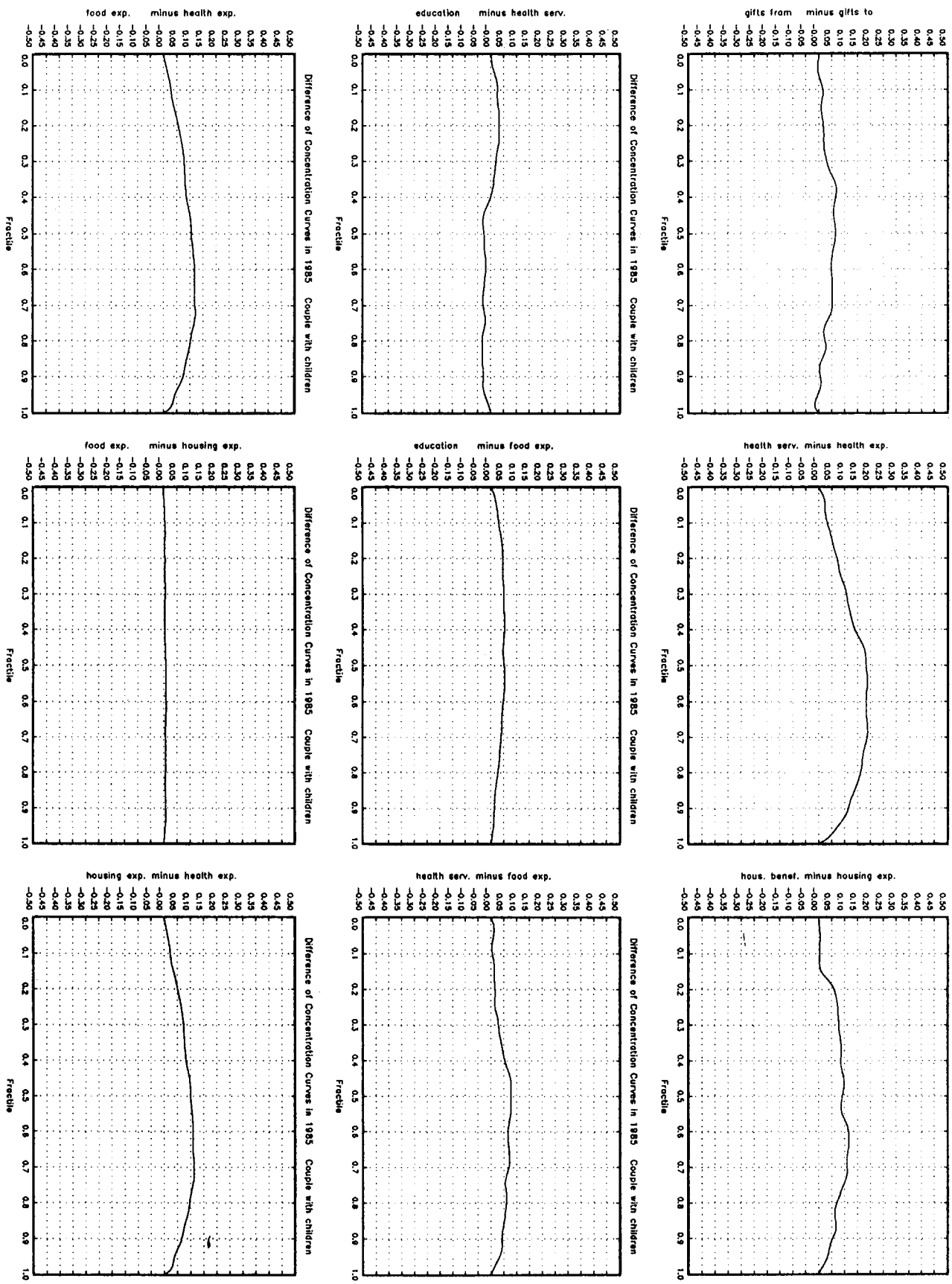
**Figure 9 b. Pairwise differences of concentration curves; couple with no children**



# Figure 9 c. Pairwise differences of concentration curves; single provider



# Figure 9 d. Pairwise differences of concentration curves; couple with children



# Figure 9 e. Pairwise differences of concentration curves; other households

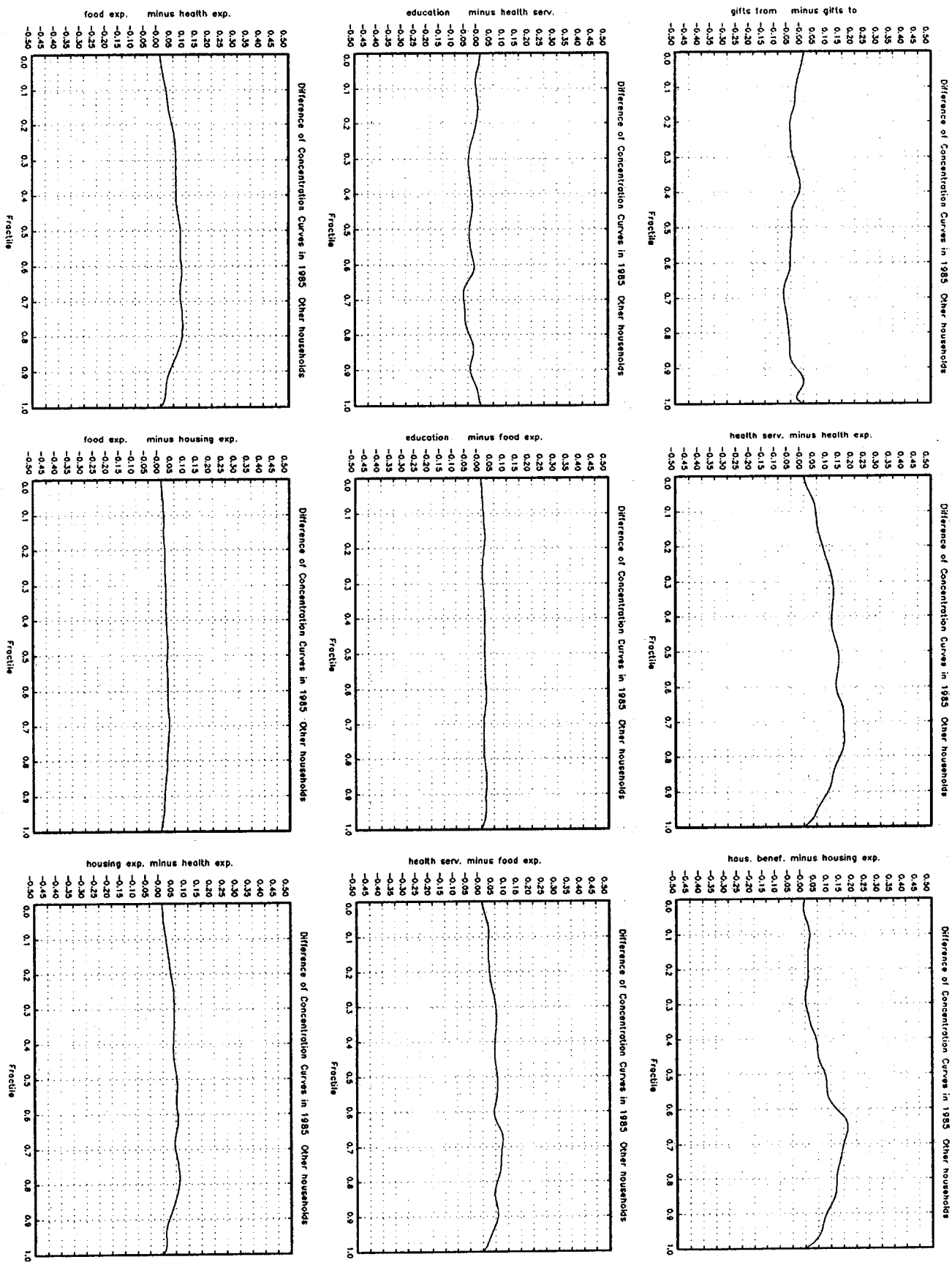


**Figure 9 f. Pairwise differences of concentration curves; elderly household**