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BANK MERGERS,
INTEREST RATES AND
THE FRAGILITY OF
LOAN MARKETS***

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ABSTRACT: We model the interaction between the concentration of the banking sector and the investment strategies of imperfectly competitive firms in the product market to address the question of whether competition makes loan markets more fragile. It is shown how a merger between two duopoly banks would typically decrease the interest rate and increase the investment volume of imperfectly competitive firms in the product market if investments were strategic complements and some other fairly mild conditions held. Under highly plausible conditions this implies that a merger will decrease the fragility of loan markets in the sense of decreasing equilibrium bankruptcy risk.

Keywords: Bank Competition, Bankruptcy Risk, Mergers, Credit Market Fragility.

JEL Classification: G21, G33, G34.


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I Introduction

In the 1980s the wave of financial deregulation has swept swiftly over many European countries which until quite recently had highly regulated financial markets. The deregulation process has been reinforced by one of the main goals of the financial integration within the framework of the European Union, namely to encourage competition in banking. Historically domestic banks have been protected from competition by stringent regulatory requirements and the special place they have been afforded in the operation of the monetary system. However, according to the prevailing "home country doctrine" of the European Union, a bank which is registered in any European country is eligible to open branches and to offer financial services in any other member country (see, for example, Mayer and Vives (1993)). It is claimed that financial integration will increase competition by removing sources of restrictive practices. Intensified competition is expected in turn to reduce margins between borrowing and lending rates and thus to improve the performance of the banking industry.

While increased competition undoubtedly will reduce profit margins in the banking industry, there are strong reasons to warn against assertions that competition in banking would automatically be beneficial. The banking industry is very different from most other industries. Due to the presence of asymmetric information causing adverse selection and moral hazard problems (see e.g. Stiglitz and Weiss (1981), but also Milde and Riley (1988)) the consequences of increased competition are more difficult to characterize. Broecker (1990) has studied the consequences of adverse selection due to the unobserved quality of borrowers. He has established how increased competition might make the adverse selection problems more severe by showing how average credit-worthiness decreases as the number of banks increases via negative externalities caused by the rejection decisions of other banks. In a similar vein Riordan (1993) has applied auction theory to the bank loan market and
demonstrated how fiercer competition can affect the screening decisions in the banking market by substantially increasing the proportion of bad loans. Competition for depositors under deregulation and protection of depositors against bank failure can also lead to instability problems such as runs and excessive risk-taking (see e.g. Diamond and Dybvig (1983) and Matutes and Vives (1995)). While these analyses have considered competition for either borrowers or depositors, Yanelle (1989) has studied aspects of "double competition", i.e. simultaneous competition for deposits as well as loan contracts. Yanelle argues that competition in intermediation does not automatically lead to efficient outcomes.

Despite these contributions, delineating the relationship between competition and risk-taking (project choice) in credit markets still seems to be a largely unexplored topic. The objective of this paper is to find out how a particular form (seemingly relevant, for example, in the Scandinavian countries) of competition in the loan market affects the lending rate charged, the investment decisions of firms in the product market and, in consequence, the default risk of loans and thereby the fragility of banks. For this purpose we focus on a simple economy with two interdependent firms with access to investment projects in the product market and we compare the cases of a banking industry with one and two banks, respectively. By identifying competition in the loan market with the number of banks in the banking industry we follow an approach with long traditions in industrial economics. In accordance with such an approach the effects of a banking merger are identified with the consequences for interest rates, investment volumes and bankruptcy risks of a reduction in the number of banks from two to one.

At the same time as the financial integration within the framework of the European Union has encouraged competition in banking, the ongoing banking crisis in many countries in the aftermath of financial deregulation has triggered a wave of substantial bank mergers. These have been considered as a way for governments to deal with troubled banks and the governments have actively tried to encourage mergers of sick banks into healthy ones (see, for example, The Economist (1995)).
The case of merging The Savings Bank of Finland (Suomen Säästöpankki) into its competitors in 1993 is a good example of such a policy. The present paper can be seen as contributing to our knowledge regarding the consequences of policies where the regulators solve the problems of a troubled bank by actively supporting mergers with healthy banks. On the other hand, a substantial part of the existing empirical research on the consequences of bank mergers has focused on potential economies of scale and scope in banking as a motive for mergers (see e.g. Berger, Hanweck and Humphrey (1987)). As a complement to this traditional approach our analysis centers around the consequences of mergers for lending rates, the volume of industry investments and bankruptcy risk.

Brander and Lewis (1986) have analyzed the linkages between imperfectly competitive product markets and the debt-equity positions of firms. In the presence of limited liability debt was proved to precommit the firm to an aggressive output strategy.¹ Poitevin (1989) made use of such a strategic relationship to demonstrate how firms belonging to the same imperfectly competitive industry can noncooperatively sustain some degree of collusion by borrowing from the same bank. In Poitevin's model firms borrow in order to finance production under circumstances where the production decisions are strategic substitutes and where the firms face firm-specific uncertainty. In contrast to Poitevin we present a model of the interaction between the concentration of the financial sector and the investment strategies of imperfectly competitive firms having access to interrelated investment projects. We compare the situation where firms in a duopoly with interrelated projects facing common uncertainty borrow from the same bank with the configuration where firms in a duopoly borrow from two different banks competing in a duopolistic banking market. Our analysis differs from that of Poitevin also in another highly important

¹ In relationship to Brander and Lewis (1986) Glazer (1994) has shown that the maturity of debt may matter as well in the sense that the behavior of firms in the product market is usually more aggressive with short-maturity than long-maturity debt. Thus by issuing long-term debt, rival firms may precipitate collusive behavior over some length of time.
respect. In addition to our characterization of how a common lender might be able to internalize externalities between interrelated investment projects, we delineate the relationship between the fragility in the sense of equilibrium bankruptcy risks and the concentration of the financial sector.

Even though it might be rare to find banking markets with such a high degree of concentration as exhibited in our analysis, it seems clear that the comparison we carry out should be able to capture crucial qualitative implications of bank mergers. And indeed, there are recent examples which would fit our model almost quite literally. The recent merger between Union Bank of Finland (UBF) and Kansallis-Osake-Pankki (KOP) has created a banking giant commanding as much as 45% of Finnish deposits and up to 60% of new loans to Finnish small and mid-size corporations. In our analysis we find that a merger between two bank duopolists would typically decrease the interest rate and increase the investments of a downstream industry if investments are strategic complements and some other fairly mild conditions hold. We also provide plausible conditions under which a bank merger would decrease the fragility of the loan market in the sense of decreasing its bankruptcy risk.

In presenting our study we first introduce the product market interaction between firms investing in risky projects in section II. In particular, we distinguish the impact of lending rate changes on the investments made in a banking duopoly from that prevailing in a banking monopoly. Section III starts by exploring the implications of a bank merger on lending rates as well as on total industry investment. In the latter part of section III the implications of a bank merger on the stability of the banking industry are developed by investigating how such a merger will affect the bankruptcy risk. The final section briefly sums up and discusses the findings of our analysis.
II Debt-Financed Investments in Risky Projects

II.1 Product Market Interaction

Consider two identical firms competing in the product market, which both have access to an investment project with uncertain return. If firm $i$ ($i = 1, 2$) invests $x_i$ while its competitor $j$ invests $x_j$ the project yields $\Theta \pi^i(x_i, x_j)$ for firm $i$ provided that the state of nature turns out to be $\Theta$. We assume a continuum of possible states of nature $\Theta$ distributed over the interval $[\theta_L, \theta_H]$ according to a cumulative distribution function $F(\Theta)$. The corresponding density function is denoted by $f(\Theta)$. Thus the firms face common uncertainty, which enters the profit realization in a multiplicative way.\(^2\) The investment technology is characterized in the following assumption.\(^3\)

**Assumption A** The revenues of firm $i$ from the project satisfy

\[(A1) \quad \pi^i_i(x_i, x_j) = 0, \quad (A2) \quad \pi^i_{ii}(x_i, x_j) < 0, \quad (A3) \quad \pi^i_i(x_i, x_j) > |\pi^i_j(x_j, x_i)|.\]

Assumptions (A1) and (A2) state that the revenue of firm $i$'s project is an increasing and strictly concave function of its own investment. Assumption (A3) formalizes the idea that "own effects" dominate over "cross-effects".

\(^2\) Showalter (1995) has recently demonstrated that the consequences of the strategic use of debt might depend on the type of uncertainty facing the product market industry.

\(^3\) Partial derivatives are denoted according to

\[\pi^i_i(x_i, x_j) = \frac{\partial \pi^i_i(x_i, x_j)}{\partial x_i} \text{ and } \pi^i_{ij}(x_i, x_j) = \frac{\partial^2 \pi^i_i(x_i, x_j)}{\partial x_i \partial x_j} \text{ and so on.}\]
The intention of the present paper is to analyze how competition between banks will impact on interest rates and investment decisions and thereby also on the risk exposure of a representative industry in the product market. For that reason we find it justified to assume that the firms have no capital of their own so that the investment projects have to be financed with debt from a bank. We will compare two separate configurations regarding the banking regime. In the first situation there are two competing banks and the two firms in the product market borrow from different banks. In the alternative scenario we focus on the case with a bank monopoly offering loans to both the duopolists in the product market.

II.2 Investments with a Banking Duopoly

We focus on two competing banks maximizing expected return in anticipation of the investment decisions of the firms in the product market. Since the banks commit themselves to lending rates at which they will satisfy the demand generated by the investment programs of the firms in the product market, the banks are engaged in two-stage competition. In choosing their lending rates, banks take into account how the lending rates will affect the investment decisions made by the project holders and thereby the value of the loan contract.

In this section we will restrict ourselves to an analysis of the investment decisions of the duopolists in the product market. Because the investment is financed with debt, there will be a surplus for the investing firm only when the state of nature is sufficiently good to cover the debt.\(^4\) In a standard debt contract the "breakeven" state of nature \(\eta_i\), in which firm \(i\) is just able to remain solvent, is defined by

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\(^4\) Gale and Hellwig (1985) have shown that in a one-period setting as here the standard debt contract with state-independent interest is the optimal incentive-compatible form of finance when lenders cannot observe an entrepreneur's return realizations without costly monitoring. See also Townsend (1979).
\[ \eta_i \pi^i(x_i, x_j) - R_i x_i = 0, \]  

where \( R_i = 1 + r_i \) is the lending rate factor.

Clearly, \( \eta_i = \eta_i(x_i, x_j, R_i) \), which states that the "breakeven" state of nature depends on the interest rate as well as on the investment level of both the firm itself and of its rival. The firm remains solvent for states of nature satisfying \( \theta > \eta_i \), while there is bankruptcy when \( \theta < \eta_i \). Consequently, the probability of bankruptcy is given by \( F(\eta_i) \).

If bank \( i \) commits itself to a lending rate \( R_i \) to firm \( i \), this firm will make its investment decision in order to maximize

\[ V^i(x_i, x_j) = \int_{\eta_i}^{\theta_i} (\theta \pi^i(x_i, x_j) - R_i x_i) dF(\theta). \]  

Maximization of (2) (for \( i=1,2 \)) yields the reaction functions

\[ V^i_i(x_i, x_j) = \int_{\eta_i}^{\theta_i} (\theta \pi^i_i(x_i, x_j) - R_i) dF(\theta) = 0. \]

Equation (3) can be rewritten according to

\[ \int_{\eta_i}^{\theta_i} \theta \pi^i_i(x_i, x_j) dF(\theta) = R_i [1 - F(\eta_i)]. \]

From this formulation we can see that the left hand side denotes the marginal revenue increase from an additional unit of investment adjusted to the average among those states of nature in which the firm is solvent. The right hand side in turn denotes
the marginal cost increase in debt from an additional unit of investment for those states of nature, where the firm can afford to pay back its debt in full.

Differentiation of (1) with respect to $x_i$ shows that

$$\frac{\partial \eta_i}{\partial x_i} = \frac{R_i}{\pi^i}(1 - e^{ui}) > 0,$$

where the scale elasticity of investment, $e^{ui}(x_i, x_j)$, satisfies

$$0 < e^{ui}(x_i, x_j) = \frac{x_i \pi^i(x_i, x_j)}{\pi^i(x_i, x_j)} < 1$$

because of (A2).

Thus, the "breakeven" state of nature is an increasing function of firm i's investment. Analogously, it can easily be established that

$$\frac{\partial \eta_i}{\partial R_i} > 0 \quad \text{and} \quad \text{sign}\left(\frac{\partial \eta_i}{\partial x_j}\right) = -\text{sign}(\pi^j)$$

so that firm i's "breakeven" state of nature increases with the interest rate charged while the effect of its rival's investment depends on the sign of $\pi^j$.

Even though we have provided an intuitive interpretation for FOC (3) it is hard to understand precisely when the underlying sufficient conditions for the first-order approach hold. The same is also true with respect to the impact of the lending rate on the expected marginal return on investment. In the light of these difficulties we reformulate the FOC (3). Following Clemenz (1986, p. 132) we can rewrite FOC (3) according to
\[ \pi_i(x_i, x_j) \left[ \eta_i + \frac{\epsilon_i}{\int (1 - F(\theta))d\theta} \right] - R_i = 0. \] (3')

Defining
\[ H(\eta_i) = \frac{\eta_i}{1 - F(\eta_i)}. \]

makes it possible to simplify (3') according to
\[ R_i(e^{\eta_i}(x_i, x_j) - 1) + \pi_i(x_i, x_j)H(\eta_i) = 0. \] (4)

This formulation turns out to have the benefit of being able to demonstrate how fairly mild restrictions on the profit functions and on the distribution function generate unambiguous results.

**Lemma 1** FOC (4) is a sufficient condition for optimal investment behavior if

\[ (B1) \quad \frac{\partial e^{\eta_i}(x_i, x_j)}{\partial x_i} \leq 0 \]

and

\[ (B2) \quad \text{the hazard rate of } F, \quad \frac{f(\eta_i)}{1 - F(\eta_i)}, \text{ increases with } \eta_i. \]

**Proof:** Differentiation of (4) yields
\[ V_{ii}^{i}(x_i, x_j) = R_{i} \frac{\partial e^{ii}}{\partial x_i} + \pi_{ii}^{i}(x_i, x_j)H(\eta_i) + \pi_{i}^{i}(x_i, x_j)H'(\eta_i) \frac{\partial \eta_i}{\partial x_i}. \]

As Milde and Riley (1988) have shown, an increasing hazard rate of \( F \) is sufficient for the property \( H'(\eta_i) < 0 \). Since \( \pi_{ii}^{i} < 0 \), \( \frac{\partial \eta_i}{\partial x_i} > 0 \) and \( \pi_{i}^{i} > 0 \) it follows that (B1) and (B2) are sufficient to justify the first-order approach.

QED

Assumption (B1) means that the scale elasticity is a non-decreasing function of the firm's own investment, which can be regarded as a reasonable assumption. Assumption (B2) is commonly used, for example in the principal-agent literature (as the monotone likelihood-ratio condition, see Rogerson (1985)), and, as Jorgensen, McCall and Radner (1967) show, it is satisfied for a wide range of distribution functions.

**Corollary 1** Under conditions (A2) and (B2) an increase in the lending rate will decrease the expected marginal return on investment.

Proof:
Differentiating (4) with respect to the interest rate gives

\[ V_{IR}^{i} = (e^{ii}(x_i, x_j) - 1) + \pi_{i}^{i}(x_i, x_j)H'(\eta_i) \frac{\partial \eta_i}{\partial R_i} < 0. \]  

(5)

The sign of (5) follows from the concavity of the profit function \( e^{ii} < 1 \), from (B2) and from the fact that \( \frac{\partial \eta_i}{\partial R_i} > 0 \).

QED
In what follows our analysis of the industry equilibrium is restricted to situations which satisfy the standard conditions for stability. As is well known, these conditions are \( \Delta = V_{ii}^j V_{jj}^i - V_{ij}^i V_{ji}^j > 0 \).

Provided that conditions (B1) and (B2) hold, the investment equilibrium \((x_1^*, x_2^*)\) is characterized by the system of equations

\[
V_i(x_i^*, x_j^*) = 0 \tag{6}
\]

\[
V_j(x_j^*, x_i^*) = 0. \tag{7}
\]

By totally differentiating the system of equations defined by (6) and (7) with respect to \( R_i \) we find that

\[
\frac{\partial x_i^*}{\partial R_i} = -\frac{V_j^i V_{ii}^j}{\Delta} \tag{8}
\]

and

\[
\frac{\partial x_j^*}{\partial R_i} = -\frac{V_j^i}{V_{jj}^j} \frac{\partial x_i^*}{\partial R_i}. \tag{9}
\]

For the analysis of (8) and (9) we differentiate the first order condition (4) with respect to \( x_j \) to find that

\[
V_{ij}^i = R_i e_{ij} + \pi_{ij}^i H(\eta_i) - \pi_{ij}^i H'(\eta_i) \frac{R_i x_i \pi_j^i}{(\pi_i)^2}. \tag{10}
\]

We make the following
Assumption C  \[ e_j^i = \frac{x_i^j(\pi_i^j \pi_j^i - \pi_i^j \pi_j^i)}{(\pi_j^i)^2} \geq 0. \] (C1)

Assumption (C1) states that the scale elasticity of firm i is a non-decreasing function of the investment level of firm j. We can directly see that a combination of conditions such that \( \pi_j^i < 0 \) and \( \pi_i^j > 0 \) would be sufficient for (C1) to hold. However, it might not be completely trivial to find examples where such a combination would hold and therefore we find it important to express our assumption in terms of the more general condition (C1). For example, with a particular revenue function

\[ \pi^i(x_i, x_j) = x_i^\alpha x_j^{\lambda \alpha}, \quad 0 < \lambda, \quad \alpha < 1, \]

of the Cobb-Douglas type, we find that \( \pi_j^i > 0, \pi_i^j > 0 \) and \( e_j^i = 0 \).

**Lemma 2** If the investments are strategic complements (\( \pi_i^j > 0 \)) and if conditions (B2) and (C1) hold, firm i’s expected marginal return on investment is an increasing function of its rival’s investment.

Proof:
The proof of lemma 2, i.e. \( V_{ij}^i > 0 \), follows directly from combining (10) and Lemma 1.

QED

The standard duopoly R&D race discussed in Harris and Vickers (1987) is a good example of a model where the investment decisions are strategic complements. Other characterizations of situations where this applies can be found in Bagwell and Staiger (1994).

We are now able to evaluate (8) and (9). Based on Lemma 2 and because of stability it must hold that
\[
\frac{\partial x_i^*}{\partial R_i} < \frac{\partial x_j^*}{\partial R_i} < 0.
\] (11)

Consequently, an increase in the lending rate levied on firm i will reduce the investment of both firm i and its rival because of strategic complementarity between their investments. But, as is quite natural, such an increase in the lending rate for firm i will reduce its investment to a larger extent than that of its rival as a result of the stability condition.

We can immediately see that if the investments are strategic substitutes and if \( c_{ij} \leq 0 \), firm i's expected marginal revenue from investment is a decreasing function of its rival's investment ( \( V_{ij} < 0 \) ), which would imply that
\[
\frac{\partial x_i^*}{\partial R_i} < 0 < \frac{\partial x_j^*}{\partial R_i}.
\]

Thus, an increase in the lending rate levied on firm i would increase, but to a lesser extent, the investment of firm j, because of strategic substitutability. A change in the lending rate would consequently affect the rival in a different direction. This feature is present also in Poitevin's (1989) analysis of the impact of lending rates on production decisions which are strategic substitutes.

The next subsection will focus on a bank monopoly granting loans to a symmetric duopoly with the intention of comparing the investment sensitivity to lending rate changes between the configurations of banking monopoly and duopoly.

II.3 Investments with a Banking Monopoly

In contrast to the previous section we here direct our attention to a monopoly in the banking market. The banking monopoly charges the identical duopolists in the
product market a common lending rate $R$. Keeping the notation otherwise unchanged, firm $i$ will then decide on its investment level in order to maximize

$$W^i(x_i, x_j) = \int_{\Theta} \left( \theta \pi^i(x_i, x_j) - R x_i \right) dF(\theta).$$  \hspace{1cm} (12)

Following an approach which is completely analogous to that of the previous section we can now characterize the impact of the lending rate on the equilibrium investment decisions $x^*_i(R) = x^*_j(R)$. Total differentiation of the first order conditions for the investments reveals that

$$\frac{\partial x^*_i}{\partial R} = - \frac{W^i_{IR}(W^i_{ii} - W^i_{ij})}{\Delta} < 0,$$  \hspace{1cm} (13)

where we have made use of (5), Lemma 1 and Lemma 2.

Under the assumptions prevailing, comparison of (8), (9) and (13) makes it possible to formulate the relationship

$$\frac{\partial x^*_i}{\partial R} < \frac{\partial x^*_i}{\partial R_i} < \frac{\partial x^*_j}{\partial R_i} < 0.$$  \hspace{1cm} (14)

From (14) we see that an increase in the monopoly lending rate will reduce industry investment to a greater extent than a corresponding increase in the lending rate charged by the banks in a duopolistic industry. The lending rate charged by a monopoly bank affects investment behavior of the firms directly and symmetrically. When a bank in a duopolistic industry increases its lending rate it will also affect the investment behavior of its customer’s rival in the product market. However, with duopoly banking an increase in the interest rate confronting firm $i$ generates a smaller investment contraction for firm $j$ than for firm $i$. Because the investment decisions are strategic complements the equilibrium investment with duopoly banking must therefore be reduced less than that with monopoly banking.
### III Interest Rate Decisions

Having analyzed the investment decisions of firms with a banking duopoly as well as with a banking monopoly, we turn to consider the lending rate decisions of banks. We start by looking at a banking duopoly and then move on to the case with a bank monopoly in order to be able to explore the implications of a bank merger on lending rates as well as on total industry investment. Finally, we examine the implications of a bank merger on the fragility of the banking industry by investigating how such a merger will affect the equilibrium bankruptcy risk.

#### III.1 Interest Rate Decisions in a Banking Duopoly and Monopoly

We now assume that the banks commit themselves to lending rate decisions according to which they finance the investments of the firms. The banks make these lending rate commitments taking into account how the interest rates will affect the investments of the firms in the product market. In the previous section we have already delineated the investment equilibrium \((x^*, y^*)\) resulting from the strategic interaction between the firms in the product market. Given this equilibrium, bank \(i\) commits itself to lend to firm \(i\) at an interest rate \(R_i\) which maximizes the expected value of the debt contract from bank \(i\)
\[ \Gamma_i(R_i, R_j) = \int_{\delta_i}^{\eta_i} \theta \pi^i(x_i^*, x_j^*) dF(\theta) + x_i^*[(1 - F(\eta_i))R_i - R_0], \] (15)

where \( R_0 = 1 + r_0 \) denotes the opportunity cost factor of granting loans, which is assumed to be constant. This means that loanable funds are supplied in a perfectly elastic fashion to the banks. The first term on the right hand side of (15) describes the bank’s profit in those states of nature where firm \( i \) goes into bankruptcy. The second term expresses the bank’s profits net of the opportunity cost of granting loans in those states of nature where the firm remains solvent.

Maximization of (15) yields the reaction functions

\[ \Gamma_i^*(R_i, R_j) = \frac{\partial \eta_i}{\partial R_i} \eta_i \pi^i(x_i^*, x_j^*)f(\eta_i) + x_i^* \left[ 1 - F(\eta_i) - R_i f(\eta_i) \frac{\partial \eta_i}{\partial R_i} \right] 
+ \frac{\partial x_i^*}{\partial R_i} [(1 - F(\eta_i))R_i - R_0] \]
\[ + \eta_i \int_{\delta_i}^{\eta_i} \left[ \frac{\partial \pi^i(x_i^*, x_j^*)}{\partial x_i^*} \frac{\partial x_i^*}{\partial R_i} + \frac{\partial \pi^i(x_i^*, x_j^*)}{\partial x_j^*} \frac{\partial x_j^*}{\partial R_i} \right] dF(\theta) = 0. \] (16)

The first two terms in (16) describe the direct interest rate effects, while the third and fourth terms express the indirect effects induced by changes in investments. The intersection between the reaction functions defined by (16) will constitute the subgame perfect interest rate equilibrium in duopoly competition between the banks.

In order to find out how bank competition affects the fragility of loan markets one has first to distinguish the interest rate equilibrium in duopoly competition from the optimal lending rate of a monopoly bank. A monopoly bank will choose an interest rate \( R \) in order to maximize
\[
\psi(R) = \int_{\theta_k}^{\eta_i} \pi^i(x^*_i, x^*_j) d\theta + x^*_i[(1 - F(\eta_i))R - R_0].
\]

An increase in the interest rate of such a bank will have a symmetric effect on the investments of the firms. For that reason the first-order condition can be written as

\[
\begin{align*}
\frac{\partial \eta_i}{\partial R} \pi^i(x^*_i, x^*_j) f(\eta_i) + x^*_i & \left[ 1 - F(\eta_i) - R_i f(\eta_i) \frac{\partial \eta_i}{\partial R} \right] \\
+ \frac{\partial x^*_i}{\partial R} [(1 - F(\eta_i))R - R_0] \\
+ \int_{\eta_i}^{\theta_k} & \left[ \frac{\partial \pi^i(x^*_i, x^*_j)}{\partial x_i} + \frac{\partial \pi^i(x^*_i, x^*_j)}{\partial x_j} \right] \frac{\partial x^*_i}{\partial R} = 0.
\end{align*}
\]

(17)

First-order conditions (16) and (17) allow us to compare the optimal lending rate of a bank monopoly with the interest rate equilibrium prevailing in a banking duopoly. Now one can establish

**Proposition 1** A merger of bank duopolists into a monopoly bank will generate a decrease in the interest rate.

**Proof:** Let us define the function \(g^i(R_i, R_j)\) according to

\[
g^i(R_i, R_j) = \frac{\partial \eta_i}{\partial R_i} \pi^i(x^*_i, x^*_j) f(\eta_i) + \frac{\partial x^*_i}{\partial R_i} [(1 - F(\eta_i))R_i - R_0] \\
+ x_i \left[ 1 - F(\eta_i) - R_i f(\eta_i) \frac{\partial \eta_i}{\partial R_i} \right]
\]

Consider the system of equations
\[ g^i(R_i, R_j) + \alpha \int_{\theta}^{\eta} \left[ \frac{\partial \pi^i(x^*_i, x^*_j)}{\partial x_i} + \frac{\partial \pi^i(x^*_i, x^*_j)}{\partial x_j} \right] dF(\theta) = 0. \] (18)

From the relationships (14) we know that a bank merger corresponds to a decrease in \( \alpha \). In order to find out the impact of a bank merger on the interest rate we totally differentiate the system of equations (18) with respect to \( \alpha \). Leaving out the arguments from the function \( g^i \), total differentiation yields

\[ g^i_{ii} \frac{\partial R_i}{\partial \alpha} + g^i_{ij} \frac{\partial R_j}{\partial \alpha} + \int_{\theta}^{\eta} \theta \left[ \frac{\partial \pi^i(x^*_i, x^*_j)}{\partial x_i} + \frac{\partial \pi^i(x^*_i, x^*_j)}{\partial x_j} \right] dF(\theta) = 0 \]

and, similarly with respect to firm \( j \),

\[ g^j_{jj} \frac{\partial R_j}{\partial \alpha} + g^j_{ij} \frac{\partial R_i}{\partial \alpha} + \int_{\theta}^{\eta} \theta \left[ \frac{\partial \pi^j(x^*_j, x^*_i)}{\partial x_j} + \frac{\partial \pi^j(x^*_j, x^*_i)}{\partial x_i} \right] dF(\theta) = 0. \]

Solution of this system of equations shows that

\[ \frac{\partial R_i}{\partial \alpha} = - \frac{g^i_{jj} - g^i_{ij}}{g^i_{ii} g^j_{jj} - g^i_{ij} g^j_{ij}} \int_{\theta}^{\eta} \theta \left[ \frac{\partial \pi^i(x^*_i, x^*_j)}{\partial x_i} + \frac{\partial \pi^i(x^*_i, x^*_j)}{\partial x_j} \right] dF(\theta) > 0. \]

This conclusion is based on a combination of ordinary sufficient second order conditions and on the assumption that "own effects" dominate over "cross-effects" (A3). Consequently, we have proved that \( \partial R_i/\partial \alpha > 0 \), from which the conclusion of the proposition follows.

QED

An intuitive explanation for why a merger leads to a lower lending rate is that with a banking duopoly a rise in the lending rate facing firm \( i \) also affects the investment...
behavior of firm i's rival in the product market because of the strategic complementarity between their investments. This "cross-effect" is, however, smaller than the "own effect". Hence, when firms in the product market compete with investment strategies which are strategic complements, the lending rate has to be raised more in a banking duopoly than in a banking monopoly. The strategic complementarity creates an externality which will be internalized by the single lender when a monopoly bank lends to both firms. Such an internalization of externalities does not take place in the case of competing bank duopolists when each project is financed by a different bank.

Proposition 1 has an immediate corollary as one can see from equation (14).

**Corollary 2** A bank merger will imply an expansion of the investment programs for an imperfectly competitive product market.

Proposition 1 and Corollary 2 show the potential advantages for interdependent firms of borrowing from a single monopoly bank. Of course, as a characterization of the impact of competition in the banking industry this conclusion is restricted to identifying competition with the number of banks. Such an identification could be justified by arguing that more competition is plausibly related to a higher degree of fragmentation of the banking industry and hence it could conceivably happen that interdependent firms are financed by different lenders with a higher probability when the degree of competition is high. A careful analysis of such aspects would, however, require microfoundations characterizing the process according to which the projectholders choose among different lenders. The rationale for the borrowers' choice of bank is outside the scope of the present paper.
III.2 Bank Competition and Bankruptcy Risk in Equilibrium

In the case of Scandinavian banking in the early 90's it has often been pointed out that the crisis was preceded by deregulation of the banking market. For that reason many observers have used the Scandinavian experience as evidence of how increased competition in the bank loan market will generate higher instability in the financial sector. In this section this issue is investigated within the framework of our model.

In the previous section we have established that a bank merger would decrease the interest rate and increase the levels of investments under the conditions delineated in our model. This suggests that the effect of a bank merger on the fragility of loan markets is not a priori clear. While a fall in the interest rate decreases the default risk, the resulting increase in investment does the reverse. In what follows we will analyze the relative magnitudes of those contrary effects more closely.

Let us consider the equation

\[ \eta_i^* \pi^i(x_i^*, x_j^*) - R_i x_i^* = 0, \]  \hspace{1cm} (19)

which has to hold according to the definition of \( \eta_i^* \). In equation (19) \( \eta_i^* \) denotes the "breakeven" state of nature generated by equilibrium investments and interest rates.

Differentiating \( \eta_i^* \) as defined by (19) we find that

\[ \frac{\partial \eta_i^*}{\partial R_i} = \frac{x_i^* + R_i \frac{\partial x_i^*}{\partial R_i} - R_i x_i^*}{\pi_i^*} \left( \frac{\pi_i^*}{\partial R_i} + \frac{\pi_j^*}{\partial R_i} \right) \]  \hspace{1cm} (20)

Defining the scale elasticity with respect to the rival's investment according to
\[ e^{ij} = \frac{\pi_j x_j}{\pi^i} \]  

(21)

and making use of symmetry \((x_i^* = x_j^*)\) we can reformulate (20) as

\[
\frac{\partial \eta_i^*}{\partial R_i} = \frac{x_i^*}{\pi^i} \left[ 1 + \frac{R_i}{x_i^*} \frac{\partial x_i^*}{\partial R_i} - \frac{R_i}{x_i^*} \left( \frac{\partial x_i^*}{\partial R_j} e^{ii} + \frac{\partial x_j^*}{\partial R_i} e^{ij} \right) \right]
\]

(22)

Making use of the interest rate elasticity of investment,

\[
\epsilon^* = - \frac{\partial x_i^*}{\partial R_i} > 0,
\]

\[
\frac{R_i}{x_i^*}
\]

and using (9) for the conclusion that

\[ 0 < \lambda = \frac{\partial x_j^*}{\partial R_i} = - \frac{V_{jj}^i}{V_{ij}^j} < 1, \]

we can express (22) according to

\[
\frac{\partial \eta_i^*}{\partial R_i} = \frac{x_i^*}{\pi^i} \left[ 1 - \epsilon^* (1 - (e^{ii} + \lambda e^{ij})) \right].
\]

(23)

We can thus conclude that
\[
\frac{\partial \eta_i^*}{\partial R_i} > 0 \text{ for all } \epsilon^* \text{ when } e^{ii} + \lambda e^{ij} \geq 1
\]  
(24a)

and that
\[
\frac{\partial \eta_i^*}{\partial R_i} > 0 \text{ iff } \epsilon^* < \frac{1}{1 - (e^{ii} + \lambda e^{ij})} \text{ when } e^{ii} + \lambda e^{ij} < 1,
\]  
(24b)

Consequently, (24a) and (24b) delineate the conditions in which the equilibrium "breakeven" state of nature depends positively on the interest rate. Under such conditions an interest rate increase will raise the equilibrium probability of default.

We can make the interpretation that \( e^{ii} + \lambda e^{ij} \) denotes the adjusted scale elasticity with respect to industry investments. The adjustment factor \( \lambda \) accounts for the fact that an increase in the lending rate for firm i will reduce its investment to a larger extent than that of its rival.

As Proposition 1 shows that a merger between bank duopolists will generate a decrease in the equilibrium interest rate, we can formulate

**Proposition 2**  A merger between bank duopolists will lessen the fragility of loan markets in the sense of reducing the probability of bankruptcy in equilibrium (a) always when \( e^{ii} + \lambda e^{ij} \geq 1 \), (b) iff
\[\epsilon^* < \frac{1}{1 - (e^{ii} + \lambda e^{ij})} \text{ when } e^{ii} + \lambda e^{ij} < 1.\]

Although there is no consensus about the response of firms' investment decisions to changes in interest rates, the literature offers some guidance regarding the size of this effect. With a production function with capital and labor of the Cobb-Douglas type, the interest rate elasticity \( \epsilon \) would be equal to one. This is regarded as an upper bound for the interest rate elasticity and it is argued that for various reasons this will most likely overestimate the response of the investment decisions to changes in the
interest rate (see Abel (1990), p. 762-63, for more details). Moreover, the huge empirical literature on investment functions strongly suggests that $\varepsilon$ is well below one (see Chirinko (1993) for a comprehensive survey). Thus we can safely expect the condition in Proposition 2 (b), $\varepsilon^* < \frac{1}{1 - (e^{ii} + \lambda e^{ij})}$, to hold when $e^{ii} + \lambda e^{ij} < 1$.

In the presence of bankruptcy risk the lending rate sensitivity of investments tends to be low because firms are interested only in those states of nature in which they remain solvent. Combined with the arguments delineated above we therefore have very strong reasons to expect that the sufficient condition for competition to make the loan market more fragile holds. In our framework bank competition has a destabilizing effect on the loan market because it results in higher lending rates and lower investments. In the likely case of the interest rate effect dominating the induced investment effect on the "breakeven" state of nature the bankruptcy risk increases with competition. (One can see from (24a) and (24b) that an increase in $e^{ij}$ increases the likelihood of a bank merger stabilizing loan markets.)

The fragility of financial markets and its relationship to competition has been analyzed to some extent in some earlier writings. Without exploring the implications of bank competition, Mankiw (1987) has stressed the possibility of financial collapse in credit markets; a small increase in the riskiness of some of the potential borrowers can cause credit markets for all of them to collapse, even though there may be no change in the expected returns on the investment projects. Broecker (1990) has studied the consequences of adverse selection due to the unobserved quality of borrowers on competition among the banks. He assumes that before a bank decides on granting a loan, it conducts a credit-worthiness test on the borrower. In the case of many banks borrowers that have been rejected at one bank can apply for loans at other banks so that the pool of applications that any bank gets has lower average quality than the population as a whole. As the "winner's curse" problem becomes more severe with more banks, banks become more conservative and charge a higher risk premium. In a similar vein Riordan (1993) has analyzed the bank loan market's
screening decisions by applying auction theory to the bank loan market with two types of borrowers who are indistinguishable ex ante. Each lender observes a signal about the loan quality and a loan is supplied by the lender observing the best signal. If the signal – the estimated probability of repayment – is above (below) some threshold level, a loan is offered (rejected). Riordan also shows how an increase in the number of competitors causes each bank to become more conservative in the sense that the threshold signal required to provide a loan is an increasing function of the number of competitors. Neither Broecker (1990) nor Riordan (1993), however, characterize precisely the relationships between the banking market structure, the lending rates, the volume of loans and investments and the resulting bankruptcy risk.

IV Concluding Discussion

We have modelled the interaction between the concentration of the banking sector and the investment strategies of imperfectly competitive firms in the product market to address the question of whether a competition makes loan markets more fragile. It has been shown how a merger between two duopoly banks would typically decrease the interest rate and increase the investment volume of imperfectly competitive firms in the product market if investments are strategic complements and some other fairly mild conditions hold. Under quite plausible conditions this implies that a merger would increase the stability of loan markets in the sense of decreasing bankruptcy risks. Thus an imperfectly competitive product market industry would face a lower probability of bankruptcy under monopoly than under duopoly banking.

This paper has delineated the advantages for interdependent firms of borrowing from the same monopoly bank rather than from competing oligopolists in the banking market. These advantages could be compared with those of a common agent as identified by Bernheim and Whinston (1985). However, our results are different from theirs, because a common monopoly bank will not be able to dictate the investment
decisions of the borrowing firms and this weakens the collusive power of a common lender in the banking market.

In general, it is reasonable to assume that there are industry-specific economies of scale in the processing of information as well as in the monitoring of loan applications. Such economies of scale might lead some banks to concentrate their lending activities in particular industries. On the other hand, arguments related to diversification should prevent banks from concentrating too large a proportion of their assets in one particular industry to the extent that risks borne by banks are not "macroeconomic" in nature and therefore diversifiable. In its selection of a banking strategy the bank engages in trading off the advantages of specialization against those of diversification. Our analysis has added one more important dimension to this tradeoff by delineating the link between the organization of the financial sector and the performance of an imperfectly competitive product market industry.

Throughout the analysis we have assumed that the banks commit themselves to lending rates at which they will satisfy the demand generated by the investment programs of the imperfectly competitive firms in the product market (horizontal supply function). In a world with risky investments the banks may not, however, want to commit themselves to such a simple strategy. Instead, as has been stressed in much of the banking discussions, the banks may choose as their strategy a "supply function" which specifies the volume of loans it provides as a function of the interest rate as e.g. in Milde and Riley (1988), who have analyzed this issue under perfect competition. Klemperer and Meyer (1989) have developed a richer model of competition under oligopoly along these lines. It is an important area of research to study the effect of competition on the performance of the bank loan markets under this more general strategy where the banks commit themselves to the volume of loans as a function of the interest rates.

A central conflict of interests between shareholders and bondholders is a typical feature of debt contracts. The shareholders place emphasis only on states of nature that are solvent, while in bankrupt states the shareholders' losses are truncated at zero.
due to limited liability. Typically this would lead the product market firms to engage in investment programs which are too aggressive relative to investment programs which are first-best from the point of view of the credit market. Debtholders, on the other hand, place emphasis only on bankrupt states, which would distort them to favor investment strategies which are too conservative relative to the first-best level. These observations led Stiglitz (1985) to suggest debtholder representation in the boards of borrowing firms as a mechanism of implementing first-best investment levels. Later on Brander and Poitevin (1992) have presented a much more detailed model of the agency costs of debt. In particular, they showed how the terms of the compensation contract offered to outside management by shareholders can reduce these agency costs substantially. It is an immediate implication of our results that the agency costs of debt are dependent on the market structure in the banking industry. Thus, implementation of first-best investment programs would require bank representation on the boards of the product market firms which is different in a banking monopoly from that in a banking duopoly. Alternatively, efficient (first-best) investment programs might also require compensation contracts offered to outside management which responded systematically to changes in the market structure of the banking industry.\(^5\)

\(^5\) It should be emphasized that the structure of the financial market is assumed not to affect the nature of competition in the product market. Of course, with bank representation on the boards of the firms the nature of competition in the product market could very well change.
REFERENCES


