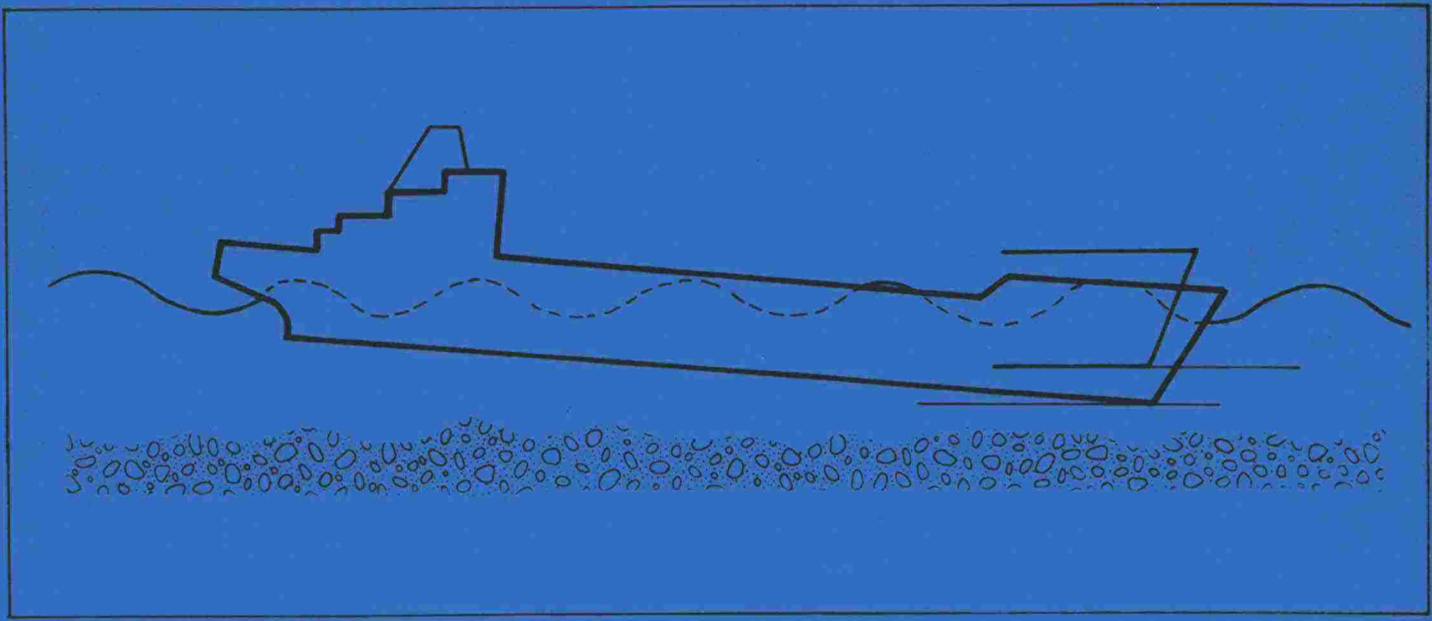
ADEQUACY OF COASAL NAVIGATION CHANNELS DURING STORMY SEAS

(With reference to Raahe approach channel)



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1. Introduction:

The present work aims at illucidating a recently developed method for estimating the probability for a certain vessel to hit the bottom of a certain channel, while passing through it with a certain speed during a certain sea condition. The probability nature of the outcome of such deterministic scheme arises from the involvement of the specified sea conditions' wave spectrum in the calculation.

Employing this method for either evaluating navigation safety of existing channels or designing the dredged depth of a new or modified channel, would help in rationalizing the planning processes. Illucidating it Leads to identifying the role of each of the parameters involved, and hence its influence on the result. Consequently, it would be easily possible to define the restricting measures on any or some of the involved parameters in order to ensure a certain desired safety level; or conversely to estimate the navigation safety level in a certain expected or investigated situation. The practical value of adopting such procedure is, thus, too obvious to ignore. The case of Raahe Approach channel is taken as an example to illustrate this point.

2. The parameters involved

These are the input data needed and can be grouped in 3 groups as follows (see fig. 2).

(a) Vessel's characteristics.
namely:

L_{pp} = Length between prependiculars

B = Beam

T = draugth

C_B = Block coefficient (taken as 0.7)

V = Speed

l = Distance between vessel´s ℄ and the toe of channel´s side slope. This is a measure of the vessel´s deviation from a course along the channel´s ℄ [$l = \frac{1}{2}$ channel´s bottom width in case of a ℄ course]

(b) Channel´s characteristics:
namely:

Orientation: Relative to georaphic directions and represented by the channel´s ℄ layout on sea charts [only straight reaches are considered]

b_o = Bottom width

L_{Gr} = Average estimate of natural undredged sea bottom level along side boundaries of channel. This is a measure of the degree of openness of the channel´s c.s. (an averge along both sides for a length not less than one ship length is to be considered at a time)

L_{dr} = dredged bottom level

(c) Sea dharacteristics
namely:

L_{wl} = Water level

U = Wind speed

t = Wind duration

Direc.= Wind geographic direction

S.B. = Sea bed level as averaged along the considered wind direction

3. The calculating procedure

(refere to fig^{s} (1), (2), and appendix (A) for mathematical details)

Starting from the input parameters as enlisted in item 2. above, the procedure for calculating the probability of hitting the bottom is as follows:

(a) From the characteristics of both the vessel and the channel, the value of the squat ΔT_1 is to be calculated from which the new water depth h and the under-keel clearence "a" are derived.

As could be seen from the schematic diagram in fig. (2), all the parameters defining both the vessel´s and the channel´s characteristics, except the channel´s orientation, are involved in this step; i.e any variation in the value of any of them would affect the resulting values of ΔT_1, h, and a

(b) Knowing the values of Froude NO. $F_n = V\ (gh)^{-0.5}$ and the ratio of $\frac{h}{T}$, the values of the vessel´s bow motion response operator $\frac{Z_B}{\zeta_A}\left(\frac{L_{pp}}{L_w}\right)$ as function of $\frac{L_{pp}}{L_w}$ (where $L_w \equiv$ sea wave length) are interpolated from the available tabulated results presented in appendix "B". These values of $\frac{Z_B}{\zeta_A}\left(\frac{L_{pp}}{L_w}\right)$ are valid for heading or following waves i.e $\mu = 180^{\circ}$ or 0° respectively; and thus need to be modified according to the actual value of the angle μ between the channel´s orientation and the considered wind "or wave" direction.

Modification is carried out by stretching the value of the constant step of $\frac{L_{pp}}{L_w}$ by the factor $\frac{1}{\cos\ (180 - \mu)}$ keeping the corresponding interpolated values of $\frac{Z_B}{\zeta_A}$ unchanged. The resul- values of $\frac{Z_B}{\zeta_A}\left(\frac{L_{pp}}{L_w}\right)_m$ are the actual Bow motion response operator values over the covered range of $\frac{L_{pp}}{L_w}$, that corresponds to the calculated squat and to the considered wind or wave direction relative to the channel´s orientation. As the available lists (actually listed curves) of $\frac{Z_B}{\zeta_A}\left(\frac{L_{pp}}{L_w}\right)$ are for $C_B = 0.7$ (appendix B), we had to presume that C_B for the considered vessel is 0.7. Moreover the value of μ to be considered in the modification factor has to be $> 90^{\circ}$ and $< 180^{\circ}$ while being aware of the fact that the most reliable range of it is $135^{\circ} \leqslant \mu \leqslant 180^{\circ}$; as for $\mu < 135^{\circ}$ the whole nature of the vessels response to wave action is changed due to rolling which was not considered in the derivation of the available $\frac{Z_B}{\zeta_A}\left(\frac{L_{pp}}{L_w}\right)$.

(c) Employing the sea characteristics and its boundaries along the considered wind direction, the expected corresponding wave spectrum S_{ζ} is calculated as function of $\frac{L_{pp}}{L_w}$ i.e $S_{\zeta}\left(\frac{L_{pp}}{L_w}\right)_{sh}$. The subscript "sh" stands for "shallow" and indicates that the influence of shallow water effect on S_{ζ} is also taken into account.

(d) The values of the vessel´s bow motion spectrum $S_B\left(\frac{L_{pp}}{L_w}\right)$ are calculated by multiplying the square of the ordinate $\frac{Z_B}{\zeta_A}$ as calculated in 3.b above by the ordinate S_{ζ} as calculated in 3.c above, for each corresponding value of $\frac{L_{pp}}{L_w}$

i.e $S_B\left(\frac{L_{pp}}{L_W}\right) = \left|\frac{Z_B}{\zeta_A}\left(\frac{L_{pp}}{L_W}\right)_{m}\right|^2 \cdot S_\zeta\left(\frac{L_{pp}}{L_W}\right)_{sh}$

(e) The first moment m_o of the vessels bow motion spectrum $S_B\left(\frac{L_{pp}}{L_W}\right)$ is calculated by numerically integrating S_B over the covered range of $\frac{L_{pp}}{L_W}$

as $m_o = \int S_B\left(\frac{L_{pp}}{L_W}\right) d\left(\frac{L_{pp}}{L_W}\right)$

(f) The probability density function of the vessel's bow vertical motion ΔT_2 is expressed by:

$$f(\Delta T_2) = \frac{\Delta T_2}{m_o} \cdot \exp.\left(-\frac{1}{2}\frac{\Delta T_2^2}{m_o}\right)$$

The probability that ΔT_2 exceeds the underkeel clearance in calm sea "a" is the probability that the vessel hits the bottom and is expressed by:

$$P(\Delta T_2 > a) = 1 - \int_o^a f(\Delta T_2)\, d\Delta T_2$$

$$= 1 - \left[1 - \exp\left(-\frac{1}{2}\frac{a^2}{m_o}\right)\right]$$

$$= \exp.\left(-\frac{1}{2}\frac{a^2}{m_o}\right)$$

This last equation governs the relation between the probability of agrounding $P(\Delta T_2 > a)$, the under-keel clearance in calm sea "a" and the first moment m_o of the vessel's bow vertical motion distribution, due to sea waves. This relation is plotted in fig (3) for $P(\Delta T_2 > a) \geqslant 1 \times 10^{-3}$, $0.1 \text{ m} \leqslant a \leqslant 10.0 \text{ m}$ and $0.001 \text{ m}^2 \leqslant m_o \leqslant 10.0 \text{ m}^2$.

It is to be noted here that the thus calculated propability is a conditional probability that holds if all other involved parameters are certain. To get an estimate of the absolute probability, the above conditional probability is to be multiplied by the probabilities of occurence of the adopted values of each of the other variable parameters, assuming the latter to be independent. If high values of the input parameters were employed for calculating $P(\Delta T_2 > a)$, then they would have very small probabilities of occurence and the product of them together with the calculated conditional probability would be much smaller indeed. This would explain the seemingly high value (1×10^{-3}) of the choosed lower limit of $P(\Delta T_2 > a)$ in fig. (3).

4. Employment of the method

(a) Evaluation of navigation safety of an existing channel in a certain situation determined by certain vessel´s and sea characteristics. This leads to whether the considered vessel is to be allowed to pass through during the considered sea condition or not, when an acceptable value of $P(\Delta T_2 > a)$ is predetermined. It could also be revealed what the appropriate speed of the considered vessel should be or the extremest sea conditions during which the vessel would be allowed to pass without violating the agreed upon safety limit as represented by the predetermined value of $P(\Delta T_2 > A)$. Ultimately, the conditions that implies the closure of the channel for navigation, or the restrictions on passing vessel´s speeds and sizes would be realistically predicted by employing the method.

In these cases, we have to calculate both "a", and m_o and check the value of $P(\Delta T_2 > a)$ from fig (3) from which it is clear that the increase of "a" and/or the decrease of m_o leads to decrease in the probability of hitting the bottom.

(d) Design of the dredged depth of either a new or to be improved channel. Actually this case would finally end to the results reached by the previous one, namely the prediction of the limits to be imposed on navigation to ensure a certain level of safety. However, in this case we have more freedom in determining these limits according to our expectations of the passing vessels´ characteristics. Here the value of m_o corresponding to a certain design vessel and the prevailing sea conditions is to be calculated, and together with an agreed upon P $(\Delta T_2 > a)$, the corresponding value of "a" is determined from fig. (3) from which the appropriate dredged depth could be determined together with any restrictions on the passing vessel´s speed. After that the previous case in item 4.a above is to be considered to determine the possible conditions that implies closure of navigation and the extremest conditions that allow vessel´s bigger than the design vessel to pass.

As different conditions are to be investigated in each case, in addition to the need to employ iteration method for calculating "a", a computing program designed to be executed on a handy desk calculator with proper capacity is the best solution for fascilitating the employment of this method by the office engineer. Even a central computer could be employed for evaluation of all navigation channels in the country after preparing the needed local input data for each. The value of such work cannot be denied as not only present day safety of navigation but also associated regulations and future improvement schemes of both alignments and waterways cross-sections would be rationally evident to all authorities concerned to appreciate.

5. Example of Raahe Approach channel

In the present context, Raahe Approach channel would serve as an example to illustrate the practical application of the above explained method. The channel is composed of two parts with different orientations (hence different values of μ for any wind direction), and each of them has to be treated separately. However one c.s. is suggested for both of the channel´s component parts (see fig (4)); whose navigation safety, in different possible weather conditions, has to be investigated during the passage of any of 3predetermined ship sizes of known dimensions. The results of the investigation would show the limiting weather conditions and/or ship speed for each ship size as against the calculated probability of hitting the bottom.

An inspection of the sea boundaries at the channel´s location along different possible wind directions showed that the site is exposed to wind action from: N-NW-W and SW directions. The values of the fetches and average water depths (S.B) were determined for each direction, together with the corresponding values of μ for both of the channel´s reaches. These data, together with wind speed U and duration t were employed to calculate the wave spectrum $S_{\zeta}\left(\frac{L_{pp}}{L_w}\right)_{sh}$. (see item 3.c above and appendix A).

It is to be noted here that the value $h` = (L_{GL} - L_{dr})$ is choosed to be 2.0 m, a value that might be different along any of the channel´s two reaches, pending upon the sea bed topography; however, the choosed value is thought to be representative while the influence of this parameter was included in the investigation objectives.

(a) Influence of sea condition, as represented by the value of m_o on the probability of hitting the bottom:

For ship size (1) and ship speed V = 3 m/sec. (a = 1.82 m) the values of m_o are plotted against μ for U = 20 m/sec. and 15 m/sec., and for two durations of t = 6 Hrs and 9 Hrs. (fig. 5). The plotted values are those got for all considered wind directions relative to the 2 orientations of the channel's 2 reaches.

To illustrate, the considered vessel if passes through the first reach of the channel (i.e osuus 1 of fig 4) at the time that 15 m/sec. wind from NW (μ = 132.5°) has been blowing for 6 Hrs would correspond to m_o = 0.0041 m^2 (fig. 5). Setting this value of m_o against the value of a = 1.82 m in fig (3) shows that the probability of its hitting the bottom is far below the lower limit of the graph of 1×10^{-3} (actually it is $< 1 \times 10^{-99}$). If the ship passes 3 hours later, while the wind is still blowing from the same NW direction (i.e t = 9 Hr), fig 5 shows that the value of m_o would then be = 0.0915 m^2, and this together with a = 1.82 m are seen to correspond to $P(\Delta T_2 > a)$ that is still below 1×10^{-3} but higher than the first case (actually 1×10^{-8}). However, and for the first case of t = 6 Hr, when the ship passes reach 2 of the channel (i.e osuus 2 of fig. 4) whose μ relative to NW wind is only 105.5°, the value of m_o is 0.3501 m^2 and the probability of hitting the bottom is 0.0088. For the 2nd case of t = 9 Hr. the corresponding values of m_o and $P(\Delta T_2 > a)$ are 0.8552 m^2 and 0.1442, respectively.

This example, where the water depth is supposed to be 11.0 m (its average value) and the ship's course is along the channel's ℄ , shows that a channel that changes oirentation might be safe for navigation during certain weather condition in its seaward part while not so in its landward part. Whether to allow a ship to pass or not depends upon the allowable safety level on the worst part of the channel (the one of less μ). However, and as stated before, values of $\mu < 135^{o}$ involves a greater portion of rolling according to how less it is from 135^{o}, and as rolling is not included in the evaluation of m_o, the obtained results are doubtfull, as the nature of the problem would not be the same and the pilot reaction to rolling motion is basically different than to pitching and heaving.

For wind speed of 20 m/sec., duration t of 6 Hrs and 9 Hrs, the values of m_o are plotted against μ for each of the 3 ship sizes in fig. (6). All ships are assumed to have the same speed of 3 m/sec. and to keep to ℄ course while the water depth is 11.0 m. The values of "a" for ships of sizes 1, 2, and 3 are 1.82 m, 1.31 m, and 0.81 m respectively. Fig (6) shows the influence of the ship size on the value of m_o. It is very clear that m_o decreases with the increase of ship size, which is to be expected as bigger ships are less responsive to the disturbing wave action; but this does not lead to safer passages for bigger ships due to the decrease in the values of "a".

To illustrate, passage of the considered ships through reach 1 of the channel during a storm of 20 m/sec. from SW (i.e $\mu = 137.5^{o}$) that lasted for 6 Hrs would result in $m_o = 0.0672\ m^2$, $0.0350\ m^2$, and $0.0177\ m^2$ for ship sizes 1, 2, and 3 respectively. These together with the values of "a" for each ship size would result in probabilities of hitting the bottom amounting to: 0.1235, 0.1948, and 0.3876 for sizes 1, 2, and 3 respectively, fig (3). For passage of the same ships through reach 2 of the channel during the same SW storm wind, we have to take into account the new value of $\mu = 164.5^{o}$ whose influence on reducing the values of m_o is considerable (being greater than μ_{sw} for reach 1). To illustrate, and for t = 9 Hrs, the values of m_o are: $1.1675\ m^2$ $1.10860\ m^2$, and $0.0427\ m^2$ for sizes 1, 2, and 3 respectively. These values, together with the values of "a" for each ship size vould lead to $P\ (\Delta T_2 > a) = 5 \times 10^{-5}$, 4×10^{-5}, and 0.0005 for sizes 1, 2, and 3 respectively which are all lower than the 1×10^{-3} boundary in fig. (3).

The critical influence of the wind direction relative to the channel´s orientation is clear from the above illustration which shows that navigation safety is more hampered in reach 1 of the channel during SW wind storm, while reach 2 is the one most hampered during NW wind storm. This fact together with an estimation of the influence of wind direction, intensity, and duration at the channel´s location would lead to evaluating the need for reorienting any or both of the channel´s reaches. After all, preknowledge of the sea condition (U, direc., and t) at the channel´s location would determine whether a ship is to be allowed to pass, even if it exceeds in size the design vessel of the channel

(b) Influence of under-keel clearance "a", and the parameters influencing it on the probability of hitting the bottom

It is clear from fig (3) that, for the same m_o, the P ($\Delta T_2 > a$) increases with the decrease of the underkeel clearance "a". It has been also demonstrated, in the above, how the influence of the decrease in "a" exceeds that of the decrease in m_o as a result of increase in ship size. Moreover, and as the influence of "a" on navigation safety holds even in calm weather, with its ultimate value being equal to 0.00, it is of utmost importance to investigate whatever factors that might influence it. The case of Raahe approach channel cross section (fig 4) would serve to illustrate the following discussion.

(b.1) Influence of variation in water level L_{wl}

For a certain dredged level L_{dr} and surrounding sea bed level L_{Gr}, the value of the height of the channel c.s.^{s} side boundary h`is constant (= 2.0 m); while any variation in water level $L_{wl}$ would be reflected on the water depth $h_o = L_{wl} - L_{dr}$. The values of under-keel clearance "a" are plotted in fig. (7) against water depth $h_o$, while h`is kept constant (2.0 m), for each of the considered ship sizes and speeds along the channel's ℄ of 2.5 m/sec., 3.0 m/sec and 3.5 m/sec. It is very clear that both ship size and water level play a great roll on the value of under-keel clearance while an increase of 40 % in the ship speed decreases the value of "a" by 9.5 %, 16.1 % and 28.9 % for ship sizes 1, 2, and 3 respectively.

(b.2) Influence of variation in dredged level L_{dr}

For a certain water level L_{wl} and surrounding sea bed level L_{Gr}, variation in dredged level L_{dr} leads to variation in both the water depth h_o and the height of the channel c.s^{s} side boundary h'. The values of under-keel clearance "a" are plotted in fig (8) against water depth h_o and the corresponding h' for the 3 considered ship sizes and speeds. While the variation in "a", here, includes the influence of both h_o and h', it is noted that there is no practical difference between the plotting in fig (8) and that for constant h' in fig (7). This is due to the small range of variation in h' as adopted in fig (8); however for the whole range of h', between 0.0 (i.e unbounded c.s.) and h_o (i.e closed c.s) the influence of h' is clearly seen in fig (9) which shows the variation in "a" for h_o = 11.0 m. A decrease in "a" of 6.6 %, 11.5 %, and 17.7 %, for ship size "2" and speeds of 2.5 m/sec., 3.0 m/sec., and 3.5 m/sec. respectively, exists due to increase in h' from 0.0 till h_o. The % age decrease in "a" is also seen to increase with ship size such that for example its max value "corresponding to V = 3.5 m/sec. is 10.79 %, 17.7 %, and 36.98 % for ship size "1", "2", and "3" respectively.

(b.3) Influence of deviation from ℄ course

For h_o = 11.0 m, h' = 2.0 m, and the 3 considered ship sizes and speeds the values of under-keel clearance "a" are plotted against the distance between the ship's ℄ and the toe of the channel's side slope l. The value of l = 55.0 (i.e half the channel's bottom breadth) corresponds to the ℄ course, while l = 35.0 m corresponds to a 20.0 m deviation from it. A decrease in under-keel clearance "a" of 1.45 %, 1.52 %, and 3.22 %, for ship of size 2 and speeds of 2.5 m/sec., 3.0 m/sec., and 3.5 m/sec. respectively; exists due to the 20 m deviation from the ℄ course. However, the % age decrease in "a" is also seen to increase with ship size such that its max value "corresponding to $V = 3.5$ m/sec.) is 1.71 %, 3.22 %, and 6.84 % for ship size "1", "2" and "3" respectively.

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6. References

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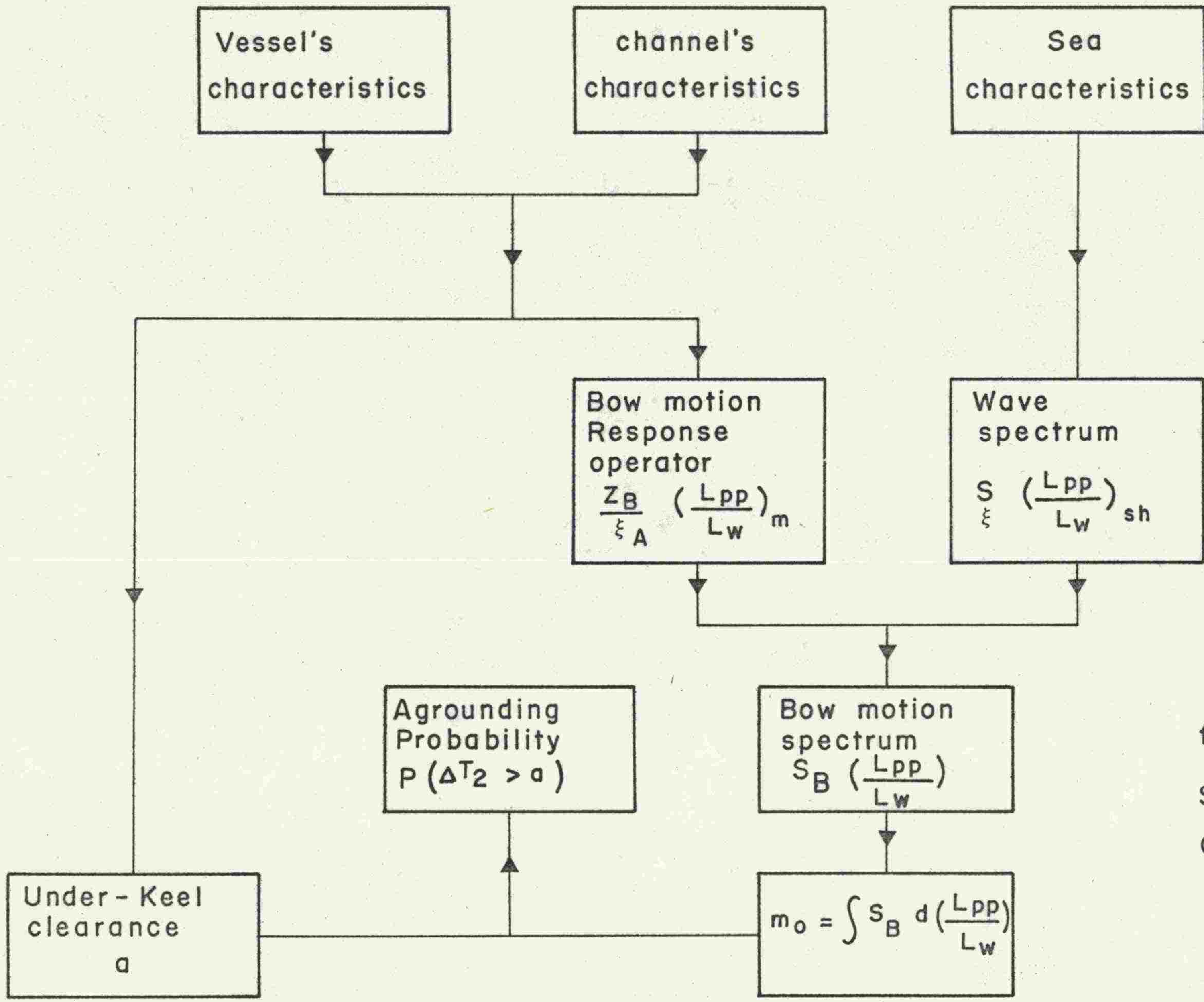

fig (1)

Schematic Procedure for Calculating agrounding Probability

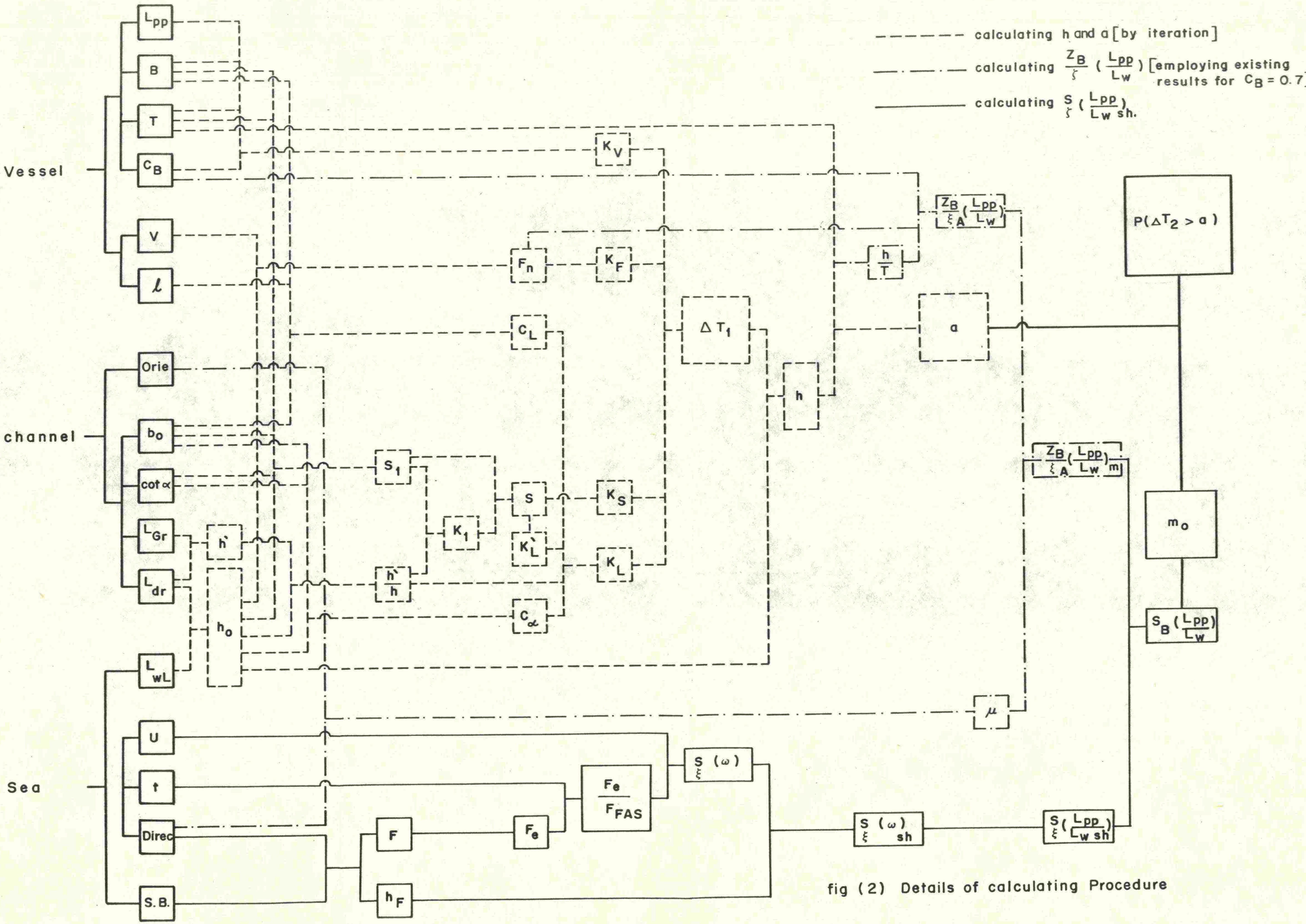

fig (2) Details of calculating Procedure

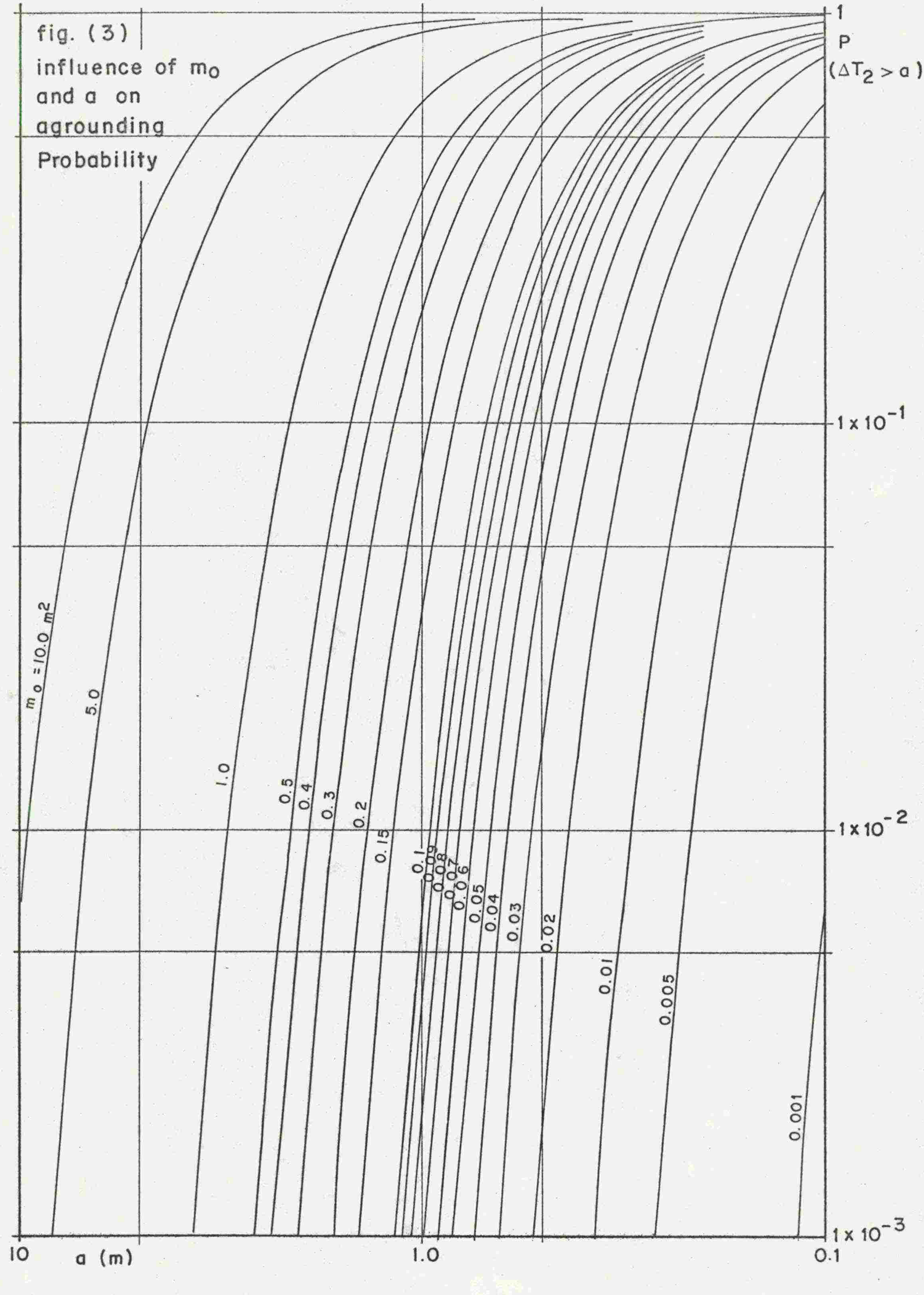

fig. (3)
influence of m_o and a on agrounding Probability
P
$(\Delta T_2 > a)$
1
1×10^{-1}
1×10^{-2}
1×10^{-3}
10
a (m)
1.0
0.1
$m_o = 10.0\ m^2$
5.0
1.0
0.5
0.4
0.3
0.2
0.15
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.005
0.001

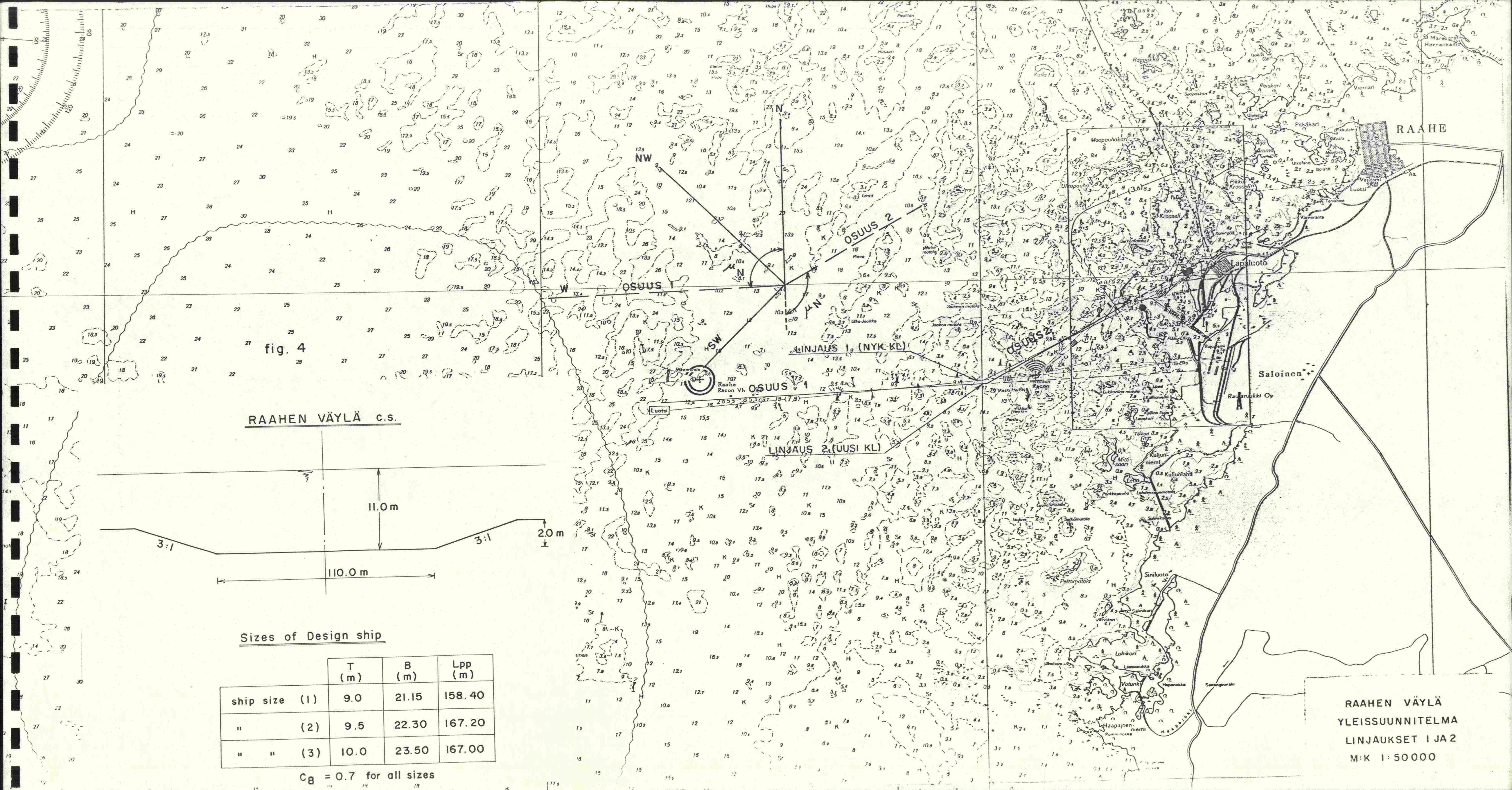

fig. 4

RAAHEN VÄYLÄ c.s.

Sizes of Design ship

	T (m)	B (m)	Lpp (m)
ship size (1)	9.0	21.15	158.40
" (2)	9.5	22.30	167.20
" " (3)	10.0	23.50	167.00

$C_B = 0.7$ for all sizes

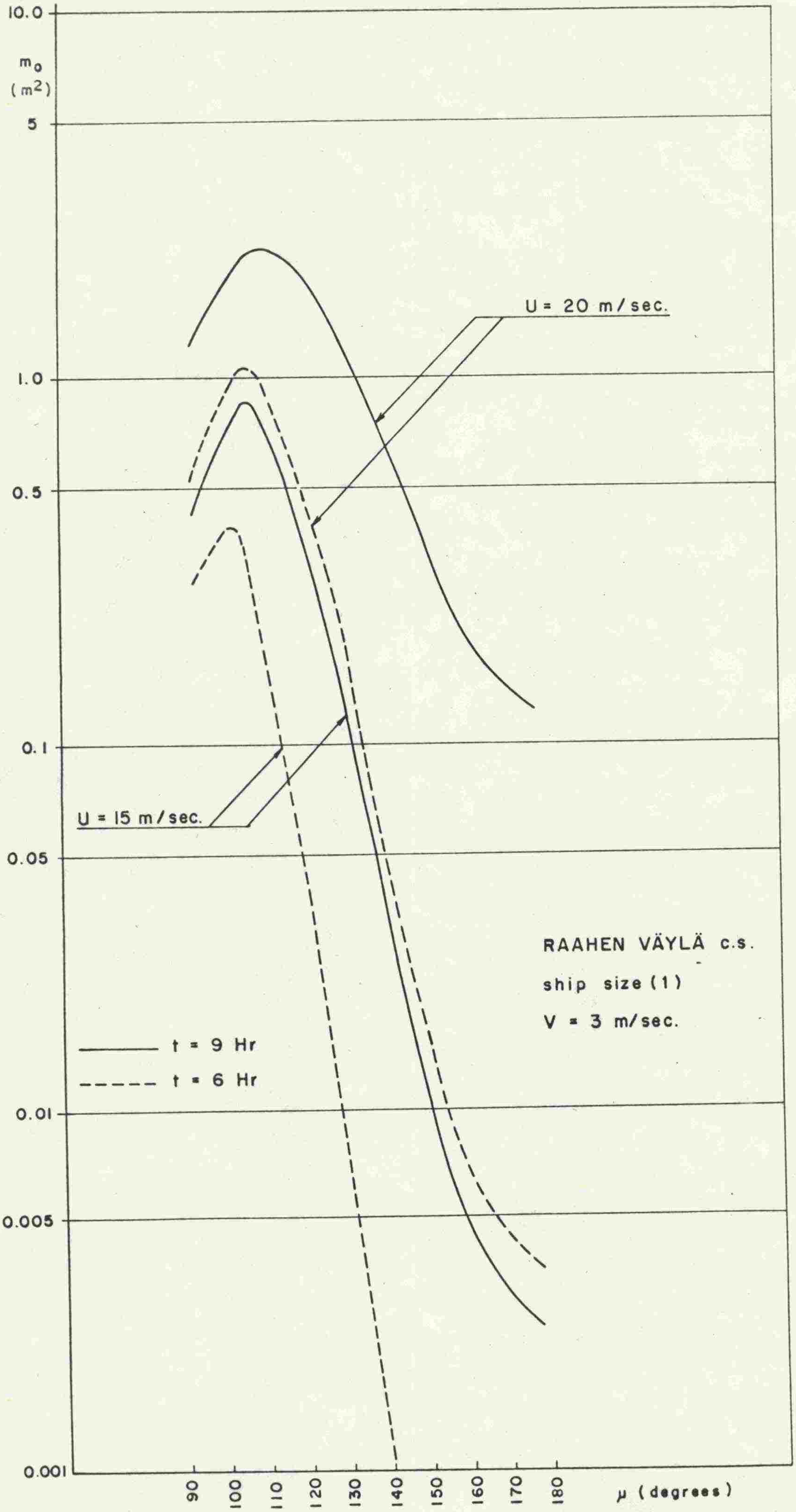

fig (5) influence of U, t, and μ on m_o

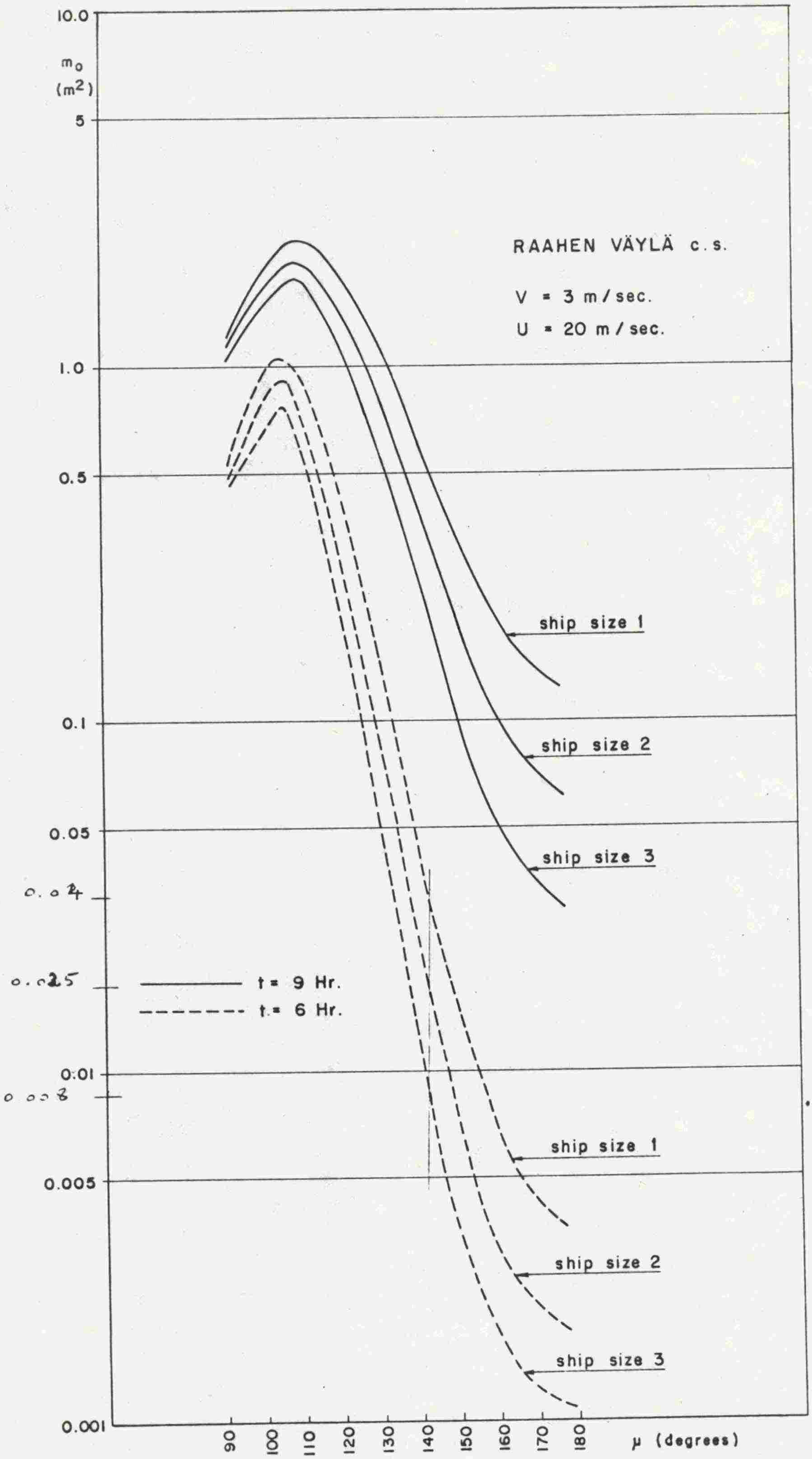

fig (6) influense of ship size on m_0

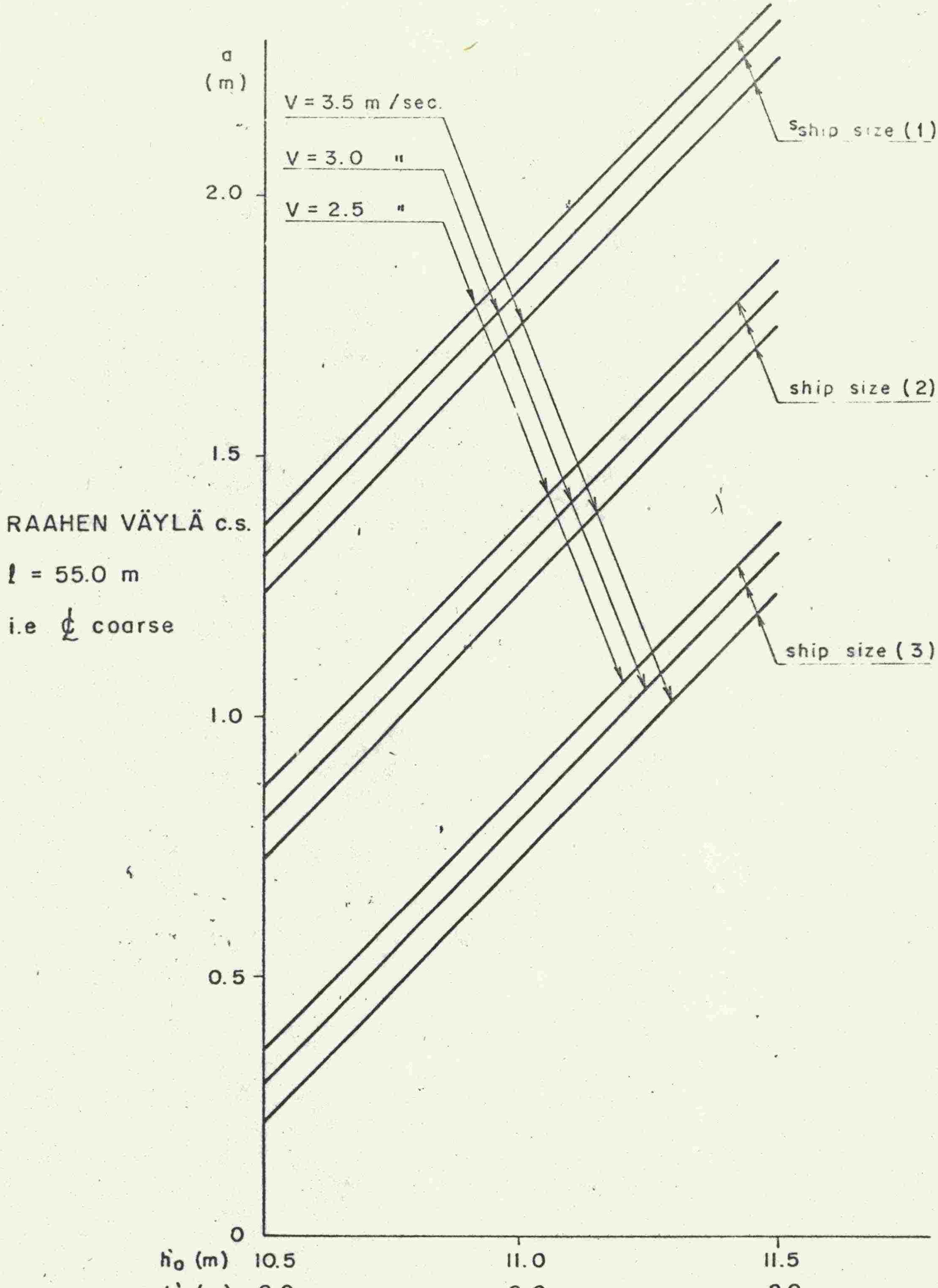

fig (7) influence of water Level L_{WL} on under-Keel clearance a "Calm Sea"

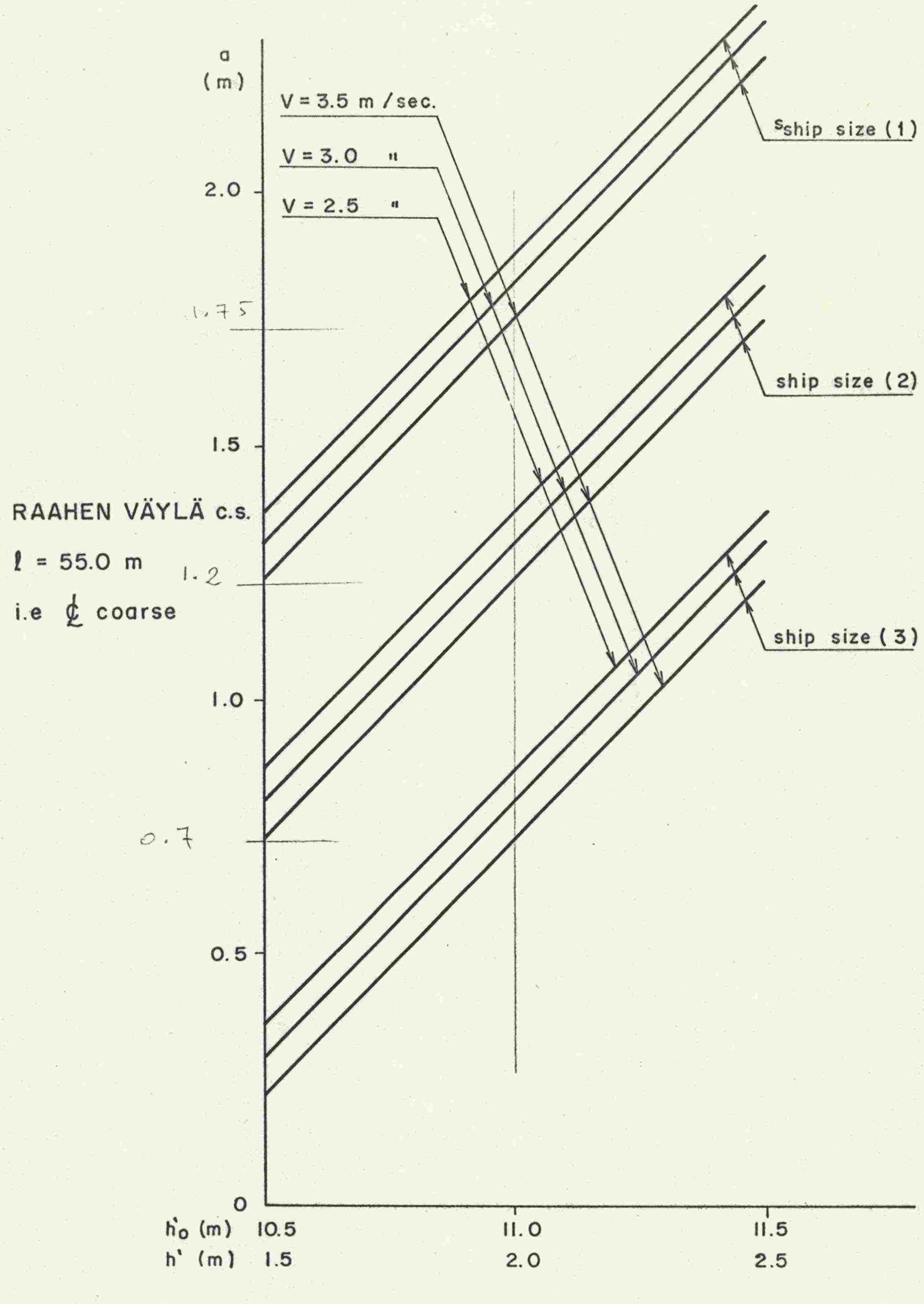

fig (8) influence of channel's dredged bottom Level "L_{dr}" on under-Keel clearance "a"

Calm Sea

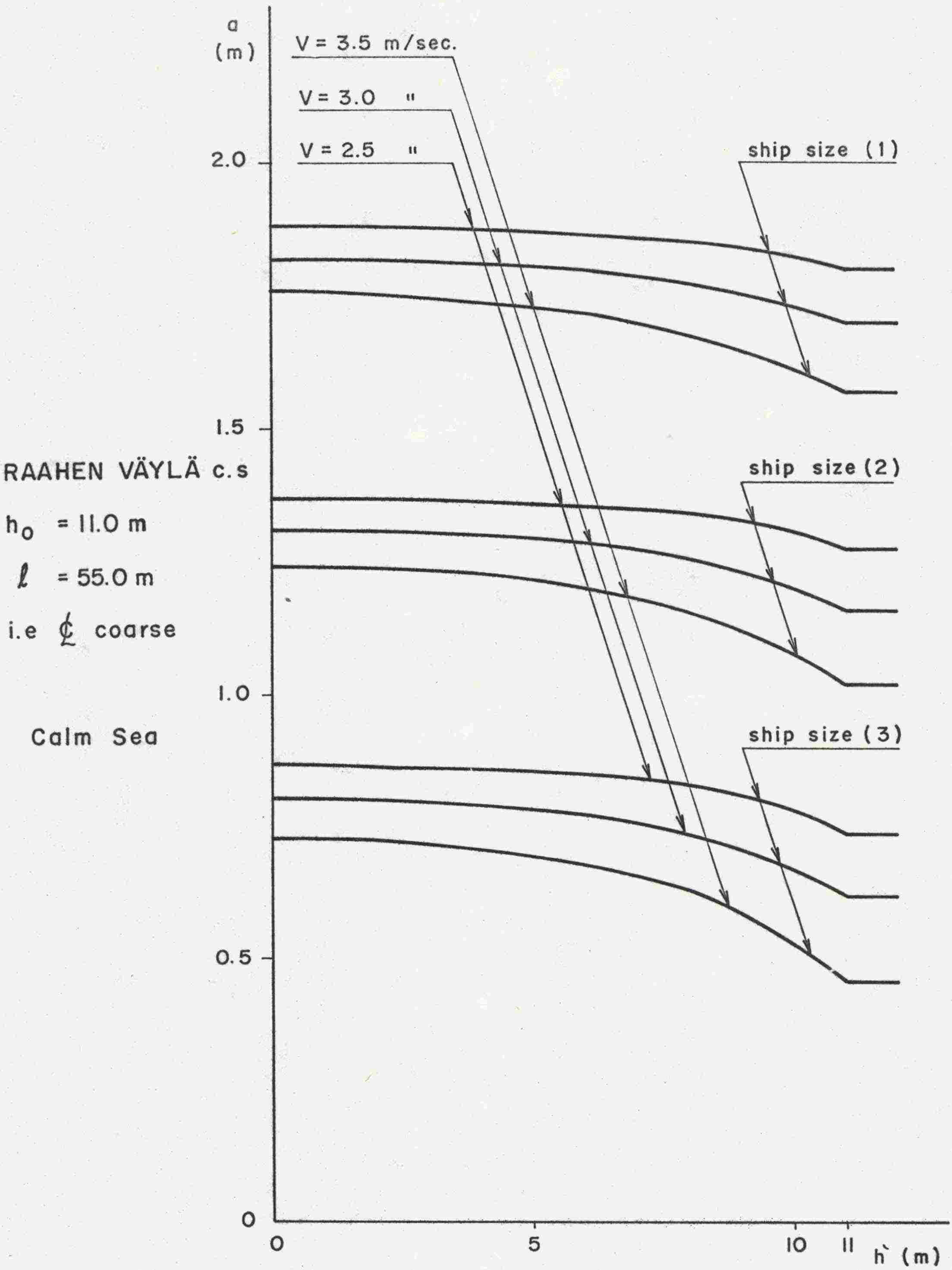

fig (9) influence of Ground Level along the side boundaries of the channel's c.s., L_{Gr}, on under-Keel clearance a

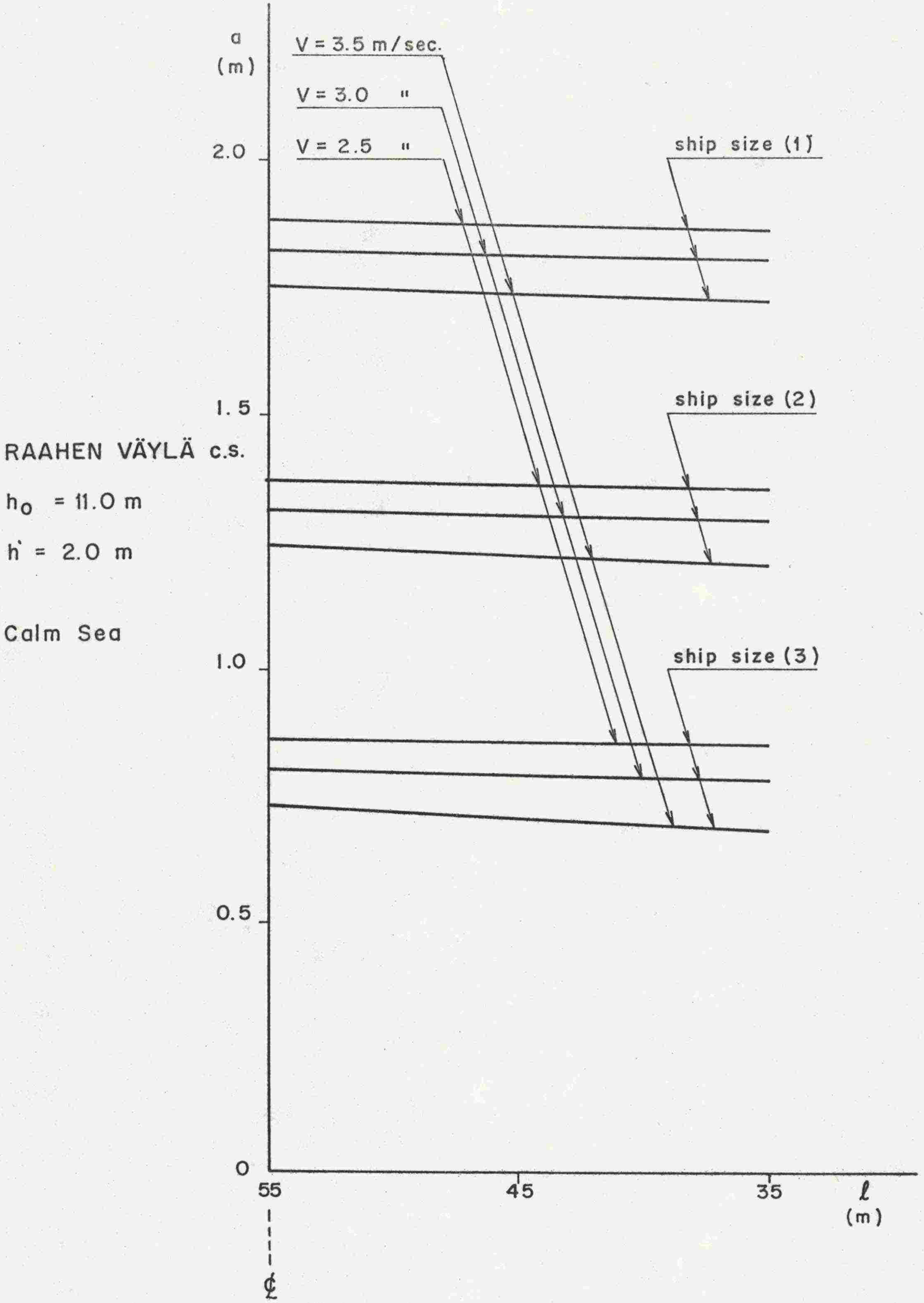

fig (10) influence of deviation from channel's ℄ coarse on under-Keel clearance a

Appendix "A"
Formulas and calculating details

Refere to fig.^{s} 1, and 2

1. Calculate the squat ΔT_1 and the under-keel clearance $a = h_o - (T + \Delta T_1)$

$\Delta T_1 = 2.4\ K_V\ K_F\ K_S\ K_L$

where the K^s are different parameters to be evaluated as follows:

$K_V \equiv$ the vessel´s characteristics parameter

$$= \frac{TB\ C_B}{L_{pp}}$$

note that C_B is taken = 0.7 to suit the only available data of the ship response operator. "Appendix B"

$K_F \equiv$ Froude Number´s parameter

$$= F_n^2\ (1-F_n{}^2)^{-0.5}$$

where $F_n = V\ (gh)^{-0.5}$

and g = acceleration of gravity

$K_S \equiv$ The area ratio parameter

$= 7.45\ S + 0.76$ for $S \geqslant 0.03$

$= 1.0$ for $S < 0.03$

where:

$$S = S_1 \times \frac{1}{K_1}$$

$$S_1 = \frac{A_x}{A_{ch_1}}$$

A_x = area of main frame of the vessel $\simeq$ 0.98 BT

A_{ch_1} = cross section area of the channel, as bounded by the extended side slopes of its dredged c.s up till water level

$= h\ (b_o + h \cot\alpha)$

$b_o = b_o \qquad\qquad$ if $b_o < 10\ B$

$\quad = 10B \qquad\qquad$ if $b_o \geqslant 10\ B$

$K_1 \equiv$ correction factor for A_{ch_1}, to get the effective area of the channel c.s.

$$= 2\ a\ S_1^{\ b}\ e^{cS_1}\ (1 - \frac{h'}{h}) + 1$$

where:

$e = 2.718...$

$a = 3282.205$

$b = 3.027662$

$c = -10.76032$

$K_L \equiv$ ship location parameter

$$= C_L \left[40\ (S_1 - 0.25)^2\right] \frac{h'}{h} \cdot \frac{1}{C_\alpha} + 1$$

where:

$C_L = 0 \qquad\qquad$ if $l > 5\ B$

$C_L = \dfrac{b_o - 2\ l}{b_o - B} \qquad$ for $b_o \leqslant 10\ B$

$C_L = \dfrac{10\ B - 2\ l}{9\ B} \qquad$ for $b_o < 10\ B$

$C_\alpha = \dfrac{b_o + h \cot\alpha}{b_o + h}$

It is to be noted that the squat calculation is to be carried out by iteration, starting with a value of the water depth $h = h_o$, calculating corresponding ΔT_1, then $h = h_o - \Delta T_1$ which is to be taken as the new value of the water depth, and new values of ΔT_1 and h are calculated. The operation is repeated untill the difference between two successive calculated values of ΔT_1 is as insignificant as the desired accuracy.

2. Interpolating the values of the vessel´s response operator $\frac{Z_B}{\zeta_A}$ as function of $\left(\frac{L_{pp}}{L_w}\right)_m$. The tables in Appendix "B" include the available values of $\frac{Z_B}{\zeta_A}$ against $\frac{L_{pp}}{L_w}$ for the range (0.00 - 3.00) in steps of 0.0625; F_n = 0.0, 0.1, 0.2, and $\frac{h}{T}$ = 1.00 to 2.5 in steps of 0.25. These tables had been derived from graphs of $\frac{Z_B}{\zeta_A}$ against $\frac{L_{pp}}{L_w}$ for C_B = 0.7 and $\mu = 180^o$. referebce "1". The value of μ would determine the stretched value of the $\frac{L_{pp}}{L_w}$ step, being"

$$\delta\left(\frac{L_{pp}}{L_w}\right)_m = \frac{0.0625}{\cos\,(180 - \mu)}$$

which would determine the modified values of $\frac{L_{pp}}{L_w}$ that correspond to the tables values. Then, and for each $\frac{L_{pp}}{L_w}$, as in the tables, the value of $\frac{Z_B}{\zeta_A}$ corresponding to F_n and $\frac{h}{T}$, as got from the squat calculation, is linearly interpolated. The result would be a numerical tabulation of $\frac{Z_B}{\zeta_A}$ against $\left(\frac{L_{pp}}{L_w}\right)_m$ that corresponds to the actual values of μ, F_n, and $\frac{h}{T}$; while C_B is 0.7.

3. Calculation of the shallow water wave spectrum $S_{\zeta}\left(\frac{L_{pp}}{L_w}\right)_{sh}$.

(3.1) First the spectral density function of the developing sea in deep water conditions $S_{\zeta}(w)$, as function of the angular frequency $w = \frac{2\pi}{T}$

($T \equiv$ wave period), is expressed by (PM results): reference "3"

$$S_{\zeta}(w) = \frac{\alpha_1}{w^5} \cdot \exp\left(-\frac{\beta_1}{w^4}\right)$$

where:

$$\alpha_1 = 0.0081\, g^2 \left(\frac{F_e}{F_{FAS}}\right)^{-0.194}$$

$$\beta_1 = 0.1 \frac{g^4}{U^4} \cdot \exp. \left[(\ln 7.4) \left(\frac{F_e}{F_{FAS}}\right)^{-0.284}\right]$$

$U = U_{19.5}$

$\equiv$ Wind speed at a height of 19.5 m above Mean Sea Level in m/sec.

F_e The effective fetch F, taking into account the influence of the irregular boundaries of the considered sea area and wind direction. Using a map and employing a simplified method due to Saville - refer. "2" - the adjustment factor $\frac{F_e}{F}$ is to be estimated, where F is the straight fetch along the considered wind direction

F_{FAS} The fetch required for the sea to reach the state of Fully Arisen Sea (FAS), when subjected to a certain $U_{19.5}$ for a time period $\geqslant t_{FAS}$. Here t_{FAS} is the wind duration required for the sea to reach the FAS state when subjected to a certain $U_{19.5}$ along an effective fetch $F_e \geqslant F_{FAS}$.

Both F_{FAS} and t_{FAS} are essentially important parameters that would determine, together with the considered effective fetch F_e and wind duration t; whether the sea state has reached the FAS state or still in what is called Developing State. This has direct influence on the value of $\frac{F_e}{F_{FAS}}$ to be employed in the S_ℓ (w) formulas above, and the procedure is as follows

(a) calculate both F_{FAS} and t_{FAS} as follows (according to Pierson, Moskowitz results as analysed by Inoue. refr. 3)

$$F_{FAS} = 16.030 \ U_{19.5}^{1.5} \quad \text{Km}$$

$$t_{FAS} = 11.096 \ U_{19.5}^{0.5} \quad \text{hr}^{s}$$

($U_{19.5}$ is in m/sec.)

Hence, calculate the ratios: $\frac{F_e}{F_{FAS}}$ and $\frac{t}{t_{FAS}}$

(b) calculate the $\frac{t}{t_{FAS}}$\`s equivilant $\frac{F}{F_{FAS}}$

from: (refer. "3")

$$\frac{t}{t_{FAS}} = \frac{(F/F_{FAS})^2}{\int_o^{(F/F_{FAS})} \left\{ \frac{3}{4 \times 0.1 \exp. \left[(\ln 7.4) \ (F/F_{FAS})^{-0.284} \right]} \right\}^{1/4} d \ (F/F_{FAS})}$$

(c) If the value of $\frac{t}{t_{FAS}}$\`s equivilant $\frac{F}{F_{FAS}}$ as calculated in (b) above, is less than the actual $\frac{F_e}{F_{FAS}}$, then it is the value to be used in the S_ζ (w) formula.

Otherwise, i.e if the $\frac{t}{t_{FAS}}$´s equivilant $\frac{F}{F_{FAS}}$ is more or equal to the actual $\frac{F_e}{F_{FAS}}$ as calculated in (a) above, this actual value is the one to be used in the S (w) formula.

(3.2) The spectral density function of the developing sea in shallow water $S_\zeta (w)_{sh}$ is to be obtained from that of deep water by:

$$S_\zeta (w)_{sh} = S_\zeta (w) \cdot K^2 (w)$$

where:

$$\frac{1}{K^2 (w)} = \frac{1}{\left(\frac{H}{H_o}\right)^2} = \tanh \frac{2\pi h_o}{L_w} \left[1 + \frac{\frac{4\pi h_o}{L_w}}{\sinh \frac{4\pi h_o}{L_w}} \right]$$

where:

H = wave height in water depth h_o

H_o = deep water wave height

L_w = wave length in water depth h_o

h_o = average water depth along the considered fetch

It is to be noted here that both S_ζ (w) and $S_\zeta (w)_{sh}$ are Rayleigh distributions.

(3.3) As the ship´s response operator $\frac{Z_B}{\zeta_A} \left(\frac{L_{pp}}{L_w}\right)_m$ is evaluated as function of the independent variable $\frac{L_{pp}}{L_w}$, and as the bow motion spectrum S_B is expressed by:

$$S_B = \frac{Z_B}{A}^2 \cdot S$$

assuming a linear system; then it is needed to express the wave spectrum S_ζ as function of the same independent variable $\frac{L_{pp}}{L_w}$ instead of w. This implies multiplication with

$$\frac{d \quad w\ (\frac{L_{pp}}{L_w})}{d \quad \left(\frac{L_{pp}}{L_w}\right)}$$ after expressing w in terms of $\left(\frac{L_{pp}}{L_w}\right)$, i.e

$$w\ (\frac{L_{pp}}{L_w}).$$

Now:

$$w\ (\frac{L_{pp}}{L_w}) = \left[(\frac{2\pi g}{L_{pp}} \cdot \frac{L_{pp}}{L_w}) \cdot \tanh\ (\frac{2\pi h_o}{L_{pp}} \cdot \frac{L_{pp}}{L_w})\ \right]^{0.5}$$

and

$$\frac{dw}{d\ (\frac{L_{pp}}{L_w})} = \frac{(\frac{\pi g}{L_{pp}})}{w\ (\frac{L_{pp}}{L_w})} \cdot \frac{1}{K^2\ (\frac{L_{pp}}{L_w})}$$

$$\therefore\ S_\zeta\ (\frac{L_{pp}}{L_w})\, sh = S_\zeta\ (w\ (\frac{L_{pp}}{L_w}))\ \cdot \frac{(\frac{\pi g}{L_{pp}})}{w\ (\frac{L_{pp}}{L_w})}$$

$$= \frac{\alpha_1\ (\frac{\pi g}{L_{pp}})}{w^6 (\frac{L_{pp}}{L_w})} \cdot \exp.\ (\frac{-\beta_1}{w^4 (\frac{L_{pp}}{L_w})})$$

which is the spectral density function of the developing sea in shallow water, expressed as function of $\frac{L_{pp}}{L_w}$ as independent variable instead of w.

This formula, together with that of $w\left(\frac{L_{pp}}{L_w}\right)$ above, enables the calculation of any ordinate $S_{\zeta}\left(\frac{L_{pp}}{L_w}\right)_{sh}$ corresponding to a certain $\frac{L_{pp}}{L_w}$, after the suitable value of $\frac{F_e}{F_{FAS}}$ has been determined as in (3.1) above.

4. Calculation of the Bow Motion Spectrum $S_B\left(\frac{L_{pp}}{L_w}\right)$, and the value of m_o.

Assuming a linear system, the spectral density function of the ship's bow vertical motion due to the wave spectrum as determined in the previous item (item "3"), is:

$$S_B\left(\frac{L_{pp}}{L_w}\right) = \left|\frac{Z_B}{\zeta_A}\left(\frac{L_{pp}}{L_w}\right)_m\right|^2 \cdot S_{\zeta}\left(\frac{L_{pp}}{L_w}\right)_{sh}$$

For each of the modified value of $\frac{L_{pp}}{L_w}$ as tabulated in item "2" above the corresponding value of $S_{\zeta}\left(\frac{L_{pp}}{L_w}\right)_{sh}$ is calculated and multiplied by the square of the corresponding interpolated value of $\frac{Z_B}{\zeta_A}$.

The resulting tabulation constitutes the Bow motion spectrum $S_B\left(\frac{L_{pp}}{L_w}\right)$ which is still a Rayleigh distribution.

This distribution is the one to be numerically integrated to get m_o

$$m_o = \int_0^{\infty} S_B\left(\frac{L_{pp}}{L_w}\right) d\left(\frac{L_{pp}}{L_w}\right)$$

Integration is to be carried over the available range of $\frac{L_{pp}}{L_w}$,

which is so chosen to cover the most significant range for the present purpose; however, and especially due to the stretching of $\frac{L_{pp}}{L_w}$ step by the μ factor, the accuracy here is much affected and it should be investigated whether there is need to reduce the numodified step of $\frac{L_{pp}}{L_w}$, namely the 0.0625 step in the response operator´s tables.

The value of the ship´s bow vertical motion ΔT_2, as expressed by S_B $(\frac{L_{pp}}{L_w})$ is defined by its probability density function:

$$f\ (\Delta T_2) = \frac{\Delta T_2}{m_o} \cdot \exp.\ (-\frac{1}{2}\ \frac{\Delta T_2{}^2}{m_o})$$

5. The probability of hitting the bottom or that $\Delta T_2 >$ the under-keel clearance a.

$$P\ (\Delta T_2 > a) = 1 - \int_0^a f\ (\Delta T_2)\ d\,\Delta T_2$$

$$= 1 - \left[1 - \exp.\ (-\frac{1}{2}\ \frac{a^2}{m_o})\right]$$

$$= \exp.\ (-\frac{1}{2}\ \frac{a^2}{m_o})$$

Appendix "B"

Interpolated Values of the Bow

Motion Amplitude response operator

$$\frac{Z_B}{\zeta_A}\left(\frac{L_{pp}}{L_w}\right)$$

Bow Motion Amplitude Response Operator: $\frac{Z_b}{\zeta_A}$

$F_n = 0, \mu = 180^{\circ}, C_B = 0.7$

$$\frac{L_{pp}}{L_w} = x = \left(\frac{L_{ppe}}{L_w}\right) / [\cos(180 - \mu)]$$

Serial No.	L_{ppe}/L_w	(h_s/T) 1.00	1.25	1.50	1.75	2.00	2.25	2.50
1	0.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	0.0625	1.006	1.034	1.063	1.058	1.053	1.050	1.047
3	0.1250	1.031	1.078	1.119	1.122	1.125	1.110	1.094
4	0.1875	1.059	1.125	1.191	1.190	1.188	1.188	1.188
5	0.2500	1.071	1.238	1.333	1.323	1.313	1.297	1.281
6	0.3125	1.141	1.281	1.406	1.406	1.406	1.391	1.375
7	0.3750	1.197	1.338	1.519	1.525	1.531	1.500	1.469
8	0.4375	1.275	1.422	1.666	1.675	1.684	1.636	1.588
9	0.5000	1.371	1.571	1.905	1.869	1.833	1.776	1.719
10	0.5625	1.434	1.625	2.003	2.009	2.016	1.930	1.844
11	0.6250	1.516	1.716	2.172	2.164	2.156	2.070	1.984
12	0,6875	1.625	1.813	2.319	2.327	2.334	2.253	2.172
13	0,7500	1.714	1.976	2.467	2.455	2.444	2.386	2.382
14	0.8125	1.750	2.016	2.500	2.528	2.556	2.544	2.531
15	0.8750	1.797	2.063	2.563	2.586	2.609	2.631	2.653
16	0.9375	1.806	2.078	2.563	2.596	2.628	2.673	2.719
17	1.0000	1.762	2.031	2.581	2.603	2.625	2.692	2.759
18	1.0625	1.600	1.984	2.453	2.508	2.563	2.625	2.688
19	1.1250	1.484	1.881	2.344	2.394	2.444	2.441	2.438
20	1.1875	1.344	1.750	2.250	2.263	2.275	2.232	2.188
21	1.2500	1.286	1.609	2.119	2.099	2.078	1.922	1.766
22	1.3125	1.094	1.484	1.891	1.871	1.850	1.652	1.453
23	1.3750	1.000	1.366	1.719	1.667	1.616	1.363	1.109
24	1.4375	0.853	1.203	1.594	1.485	1.375	1.105	0.834
25	1.5000	0.762	1.071	1.476	..311	1.147	0.894	0.641
26	1.5625	0.625	0.906	1.313	1.125	0.938	0.711	0.484
27	1.6250	0.509	0.781	1.156	0.958	0.759	0.551	0.344
28	1.6875	0.438	0.719	1.100	0.842	0.584	0.423	0.263
29	1.7500	0.405	0.705	1.105	0.819	0.533	0.376	0.219
30	1.8125	0.406	0.703	1.141	0.828	0.516	0.366	0.216
31	1.8750	0.453	0.747	1.181	0.858	0.534	0.377	0.219
32	1.9375	0.500	0.781	1.188	0.899	0.609	0.419	0.228
33	2.000	0.548	0.809	1.190	0.923	0.656	0.445	0.234

$F_n = 0$
(cont.)

Serial No	$\frac{L_{ppe}}{L_W}$	(h_s/T) 1.00	1.25	1.50	1.75	2.00	2.25	2.50
34	2.0625	0.563	0.778	1.116	0.902	0.688	0.455	0.222
35	2.1250	0.563	0.744	1.031	0.860	0.688	0.453	0.219
36	2.1875	0.541	0.694	0.922	0.791	0.659	0.431	0.203
37	2.250	0.523	0.666	0.809	0.717	0.625	0.407	0.188
38	2.3125	0.466	0.609	0.709	0.644	0.578	0.375	0.172
39	2.3750	0.434	0.547	0.594	0.560	0.525	0.341	0.156
40	2.4375	0.397	0.472	0.528	0.736	0.472	0.303	0.134
41	2.5000	0.371	0.429	0.453	0.431	0.409	0.267	0.125
42	2.5625	0.319	0.375	0.375	0.370	0.366	0.240	0.113
43	2.6250	0.288	0.319	0.334	0.167	0.313	0.203	0.094
44	2.6875	0.250	0.281	0.291	0.285	0.278	0.183	0.088
45	2.7500	0.216	0.244	0.250	0.238	0.225	0.148	0.072
46	2.8125	0.181	0.197	0.219	0.203	0.188	0.122	0.056
47	2.8750	0.141	0.156	0.206	0.174	0.141	0.094	0.047
48	2.9375	0.113	0.125	0.188	0.149	0.109	0.074	0.038
49	3.0000	0.084	0.097	0.172	0.132	0.091	0.061	0.031

[illegible] Amplitude Response Operator: $\frac{Z_b}{\zeta_A}$

$F_n = 0.1$, $\mu = 180^\circ$, $C_B = 0.7$

$$\frac{L_{pp}}{L_w} = x = \left(\frac{L_{ppe}}{L_w}\right) / \left[\cos\,(180 - \mu)\right]$$

Serial No.	$\frac{L_{ppe}}{L_w}$	(h_s/T) 1.00	1.25	1.50	1.75	2.00	2.25	2.50
1	0.000	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2	0.0625	1.006	1.0312	1.0781	1.0656	1.0531	1.0578	1.0625
3	0.1250	1.031	1.0781	1.1562	1.1562	1.1562	1.1406	1.1250
4	0.1875	1.069	1.1562	1.2500	1.2500	1.2500	1.2500	1.2500
5	0.2500	1.071	1.2562	1.3437	1.3749	1.4062	1.3906	1.3750
6	0.3125	1.141	1.3437	1.4687	1.5077	1.5468	1.5234	1.5000
7	0.3750	1.197	1.4125	1.6000	1.6828	1.7656	1.7109	1.6562
8	0.4375	1.275	1.5000	1.8125	1.8984	1.9843	1.8984	1.8125
9	0.5000	1.371	1.6406	2.0937	2.1406	2.1875	2.0781	1.9687
10	0.5625	1.4375	1.7187	2.3125	2.3671	2.4218	2.2890	2.1562
11	0.6250	1.5062	1.8125	2.4843	2.5546	2.6250	2.4765	2.3281
12	0.6875	1.6250	1.9187	2.6312	2.7218	2.8125	2.6875	2.5625
13	0.7500	1.7187	2.0312	2.7500	2.8437	2.9375	2.8671	2.7968
14	0.8125	1.7750	2.1250	2.7750	2.8750	2.9750	2.9953	3.0156
15	0.8750	1.8125	2.1593	2.6875	2.7968	2.9062	3.0077	3.1093
16	0.9375	1.7656	2.1406	2.5312	2.6406	2.7500	2.9062	3.0625
17	1.0000	1.6718	2.1031	2.4531	2.5703	2.6875	2.8203	2.9531
18	1.0625	1.4687	2.0156	2.1875	2.3046	2.4218	2.5234	2.6250
19	1.1250	1.2500	1.7500	1.9531	2.0468	2.1406	2.2031	2.2656
20	1.1875	1.0937	1.5156	1.7968	1.8359	1.9375	1.9531	1.9687
21	1.2500	0.9375	1.2968	1.5781	1.6328	1.6875	1.5312	1.3750
22	1.3125	0.7500	1.000	1.3281	1.3906	1.4531	1.2187	0.9843
23	1.37500	0.6093	0.8375	1.1250	1.1718	1.2187	0.9062	0.5937
24	1.4375	0.4687	0.6562	0.9687	0.9687	0.9687	0.6640	0.3593
25	1.5000	0.3906	0.5312	0.7500	0.7500	0.7500	0.5156	0.2812
26	1.5625	0.3281	0.4750	0.5781	0.5859	0.5937	0.4093	0.2250
27	1.6250	0.2812	0.4437	0.4062	0.4249	0.4437	0.3165	0.1875
28	.,6875	0.2562	0.4343	0.3125	0.3359	0.3593	0.2687	0.1781
29	1.7500	0.2500	0.4218	0.2812	0.3046	0.3281	0.2499	0.1718
30	1.8125	0.2500	0.4343	0.2781	0.2859	0.3000	0.2140	0.1781
31	1.8750	0.2656	0.4375	0.2781	0.2828	0.2875	0.2312	0.1750
32	1.9375	0.2812	0.4437	0.2812	0.2827	0.2843	0.2280	0.1718
33	2.000	0.2875	0.4375	0.2812	0.2812	0.2812	0.2218	0.1625

$F_n = 0.1$
(cont.)

Serial No	$\frac{L_{ppe}}{L_w}$	(h_s/T) 1.00	1.25	1.50	1.75	2.00	2.25	2.50
34	2.0625	0.2968	0.4343	0.2781	0.2718	0.2656	0.2109	0.1562
35	2.2150	0.2875	0.4187	0.2625	0.2531	0.2437	0.1999	0.1562
36	2.1875	0.2812	0.4031	0.2406	0.2249	0.2093	0.1780	0.1468
37	2.2500	0.2656	0.3750	0.2187	0.1984	0.1781	0.1593	0.1406
38	2.3125	0.2500	0.3468	0.2031	0.1796	0.1562	0.1406	0.1250
39	2.3750	0.2343	0.3125	0.1843	0.1562	0.1281	0.1187	0.1093
40	2.4375	0.2187	0.2812	0.1656	0.1421	0.1187	0.1062	0.0937
41	2.5000	0.1875	0.2593	0.1406	0.1171	0.0937	0.0906	0.0875
42	2.5625	0.1718	0.2343	0.1281	0.1046	0.0812	0.1749	0.0687
43	2.6250	0.1562	0.2031	0.1093	0.0859	0.0625	0.0625	0.0625
44	2.6875	0.1281	0.1812	0.0937	0.0702	0.0468	0.0468	0.0468
45	2.7500	0.1218	0.1562	0.0781	0.0546	0.0312	0.0312	0.0312
46	2.2125	0.0781	0.1250	0.0656	0.0406	0.0156	0.0156	0.0156
47	2.87500	0.0687	0.1000	0.0468	0.0234	0.000	0.000	0.000
48	2.9375	0.0625	0.0718	0.0312	0.0156	0.000	0.000	0.000
49	3.000	0.0468	0.0625	0.0218	0.0109	0.000	0.000	0.000

Bow Motion Amplitude Response Operator: $\frac{Z_b}{\zeta_A}$

$F_n = 0.2$, $\mu = 180^\circ$, $C_B = 0.7$

$$\frac{L_{pp}}{L_w} = x = \left(\frac{L_{ppe}}{L_w}\right) / \left[\cos(180 - \mu)\right]$$

Serial No.	L_{ppe}/L_w	(h_s/T) 1.00	1.25	1.50	1.75	2.00	2.25	2.50
1	0.000	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2	0.0625	1.006 0.1006	1.0343	1.0843	1.0739	1.0625	1.0765	1.0906
3	0.1250	1.031	1.0875	1.1718	1.1718	1.1718	1.1796	1.1875
4	0.1875	1.059	1.1625	1.2812	1.2968	1.3125	1.3125	1.3125
5	0.2500	1.071	1.2718	1.3906	1.4453	1.5000	1.4843	1.4687
6	0.3125	1.141	1.3718	1.5343	1.6030	1.6718	1.6562	1.6406
7	0.3750	1.197	1.4593	1.7031	1.8046	1.9062	1.8593	1.8125
8	0.4375	1.275	1.5562	1.9843	2.0702	2.1562	2.0937	2.0312
9	0.500	1.371	1.7031	2.3125	2.3593	2.4062	2.3281	2.2500
10	0.5625	1.4062	1.8062	2.4843	2.6015	2.7187	2.6249	2.5312
11	0.6250	1.4781	1.9062	2.7812	2.8671	2.9531	2.8671	2.7812
12	0.6875	1.5875	2.0281	2.9687	3.0781	3.1875	3.1406	3.0937
13	0.7500	1.6562	2.1562	2.9750	3.1515	3.3281	3.3359	3.3437
14	0.8125	1.7187	2.2343	2.9062	3.1268	3.3375	3.3953	3.4531
15	0.8750	1.7375	2.2443	2.7187	2.9374	3.1562	3.2812	3.4062
16	0.9375	1.7031	2.1875	2.4843	2.6952	2.9062	3.0624	3.2187
17	1.000	1.6093	2.1031	2.3593	2.5702	2.7812	2.8671	2.9531
18	1.0625	1.3125	1.8281	2.0625	2.2421	2.4218	2.3984	2.3750
19	1.1250	1.0625	1.6093	1.7968	1.9687	2.1406	2.0468	1.9531
20	1.1875	0.8906	1.3593	1.6093	1.7265	1.8437	1.7187	1.5937
21	1.2500	0.7187	1.0781	1.3906	1.4609	1.5312	1.2577	0.9843
22	1.3125	0.5312	0.7968	1.0781	1.1718	1.2656	0.9453	0.6250
23	1.3750	0.3906	0.6250	0.8750	0.9218	0.9687	0.6874	0.4062
24	1.4375	0.2812	0.4062	0.6875	0.7031	0.7187	0.4999	0.2812
25	1.5000	0.2343	0.3281	0.4687	0.4843	0.5000	0.3593	0.2187
26	1.5625	0.1968	0.2812	0.3125	0.3281	0.3437	0.2577	0.1718
27	1.6250	0.1843	0.2531	0.1875	0.2234	0.2593	0.1952	0.1312
28	1.6875	0.1875	0.2500	0.1468	0.1749	0.2031	0.1656	0.1281
29	1.7500	0.1875	0.2500	0.1343	0.2484	0.1812	0.1531	0.1250
30	1.8125	0.1875	0.2687	0.1343	0.1468	0.1593	0.1437	0.1281
31	1.8750	0.1843	0.2750	0.1375	0.1468	0.1562	0.1406	0.1250
32	1.9375	0.1781	0.2781	0.1312	0.1437	0.1562	0.1406	0.1250
33	2.000	0.1875	0.2781	0.1312	0.1437	0.1562	0.1390	0.1218

$F_n = 0.2$ 2
(cont.)

Serial No	$\frac{L_{ppe}}{L_w}$	(h_s/T) 1.00	1.25	1.50	1.75	2.00	2.25	2.50
34	2.0625	0.1843	0.2656	0.1250	0.13828	0.1406	0.1281	0.1156
35	2.1250	0.1843	0.2531	0.1250	0.1281	0.1312	0.1202	0.1093
36	2.1875	0.1750	0.2468	0.1218	0.1234	0.1250	0.1125	0.1000
37	2.2500	0.1562	0.2218	0.1156	0.1171	0.1187	0.1077	0.0968
38	2.3125	0.1531	0.2156	0.1093	0.1062	0.1031	0.0968	0.0906
39	2.3750	0.1406	0.1937	0.0968	0.0952	0.0937	0.0853	0.0781
40	2.4375	0.1312	0.1781	0.0875	0.0828	0.0781	0.0737	0.0687
41	2.500	0.1250	0.1593	0.0656	0.0640	0.0625	0.0625	0.0625
42	2.5625	0.1093	0.1500	0.0625	0.0500	0.0375	0.0453	0.0531
43	2.6250	0.1000	0.1250	0.0468	0.0390	0.0312	0.0359	0.0406
44	2.6875	0.0906	0.1093	0.0375	0.0312	0.0250	0.0281	0.0312
45	2.700	0.0781	0.0937	0.0312	0.0202	0.0093	0.0124	0.0156
46	2.8125	0.0671	0.0843	0.0218	0.0109	0.000	0.000	0.000
47	2.8750	0.0375	0.0687	0.0062	0.0036	0.000	0.000	0.000
48	2.9375	0.0312	0.0593	0.0000	0.000	0.000	0.000	0.000
49	3.000	0.0218	0.0343	0.0000	0.000	0.000	0.000	0.000