Q. F. F. Q. S.

PARTEM PRIOREM
SPECIMINIS ACADEMICI,
QUO
RESOLVUNTUR NONNULLA

PROBLEMATA,

POSITA

FIGURA TELLURIS ELLIPSOIDICA,

Consens. Ampliss. Senat. Philos. in Reg. Acad. Aboënsi,

PRÆSIDE

MARTINO JOHANNE WALLENIO,

MATHES. PROFESSORE REG. & ORDIN.
Acad. h. t. RECTORE,
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THOMAS MATTHEISZEN,

NYLANDUS.

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Högaktade

Herr MATTH. MATTHEISZEN,

Och

Herr CARL MATTHEISZEN.

Mine Käraste Bröder.

Så onekeligit det är at emot undfångna välgärningar et tacksamt sinne bör svara; så oundvikelig anser jag äfven min skyldighet, at vid detta tilfälle offenteligen betyga, huru mycket jag är Eder, Mine Käraste Bröder, förbunden. Då I genom mångfaldiga mig bevista kärleks - prof ökt den förbindelse, hvarmed natur och blods - band oss förenat: så borde jag ju anses för den otacksammaste, om ingen erkänsla för sådant hos mig skulle finnas. Til et litet men dock upriktigt vedermäle af den tacksamhet, jag innom mig hyser, varder fördenskuld detta mitt Academiska arbete Eder tilägnadt, under hjertelig önskan, at den Högsta Försynen täcktes uppehålla Eder, jämte Eder kära Omvårdnad, vid all sielfönskelig sällhet.

Jag är med beständig tilgifvenhet och ömmaste vänskap

Mine käreste Bröders

trognaste bror

THOMAS MATTHEISZEN.

I. N. J.

§. 1.

Qvantum intersit cum Astronomiæ tum scientiarum ipsi adgnatarum & conjunctissimarum, Geographiæ atqve Artis Navigandi, Figuram Telluris cognosci, hoc loco dicere nostrum non est. Hanc a Sphærica nonnihil abludere, & qvidem ad polos compressiorem esse, ut prælucente ratione atqve institutis adcuratius observationibus & mensuris compertum fuit, Ellipticam assumere convenientissimum videbatur. Qvæ hypothesis, licet eam non modo dubiam sed & minus veram esse, postmodum graves omnino rationes non tam arguerint qvam evicerint: nec dum tamen prorsus derelicta a Mathematicis aut neglecta jacet; forte qvod aliam vel veriorem vel faciliorem nondum extare, vel illam ipsam penitiori adhuc examini subjiciendam judicent. Qvidqvid sit, hanc Figuræ Ellipticæ hypothesin, qvæ singulari elegantia se commendat & egregia calculi adminicula præstat, nos qvoqve nunc supponimus.

 §. 2.

§. 2.

Esto igitur POAD Ellipsis repræsentans meridianum terrestrem, cujus seu ipsius Telluris centrum C, polus P, A punctum Æqvatoris, semidiameter æqvatoris C A $= a$, semiaxis terræ C P $= na$, OLUZ recta normalis ad Ellipsin seu linea verticalis in loco observatoris O, cujus loci Latitudo scilicet angulus ALO dicatur L. Ad OZ, qvæ axibus Ellipseos occurrat in L & Z, perpendicularis esto CU. Centro C radio CA descriptus sit circulus AH, cui in H occurrat recta per O ad CA ducta perpendicularis OK. Sit etjam ad CP perpendicularis OR. Circulum in H tangens recta HT occurrat productæ CA in T. His ita positis, in antecessum monemus:

I. Per naturam Ellipseos esse HK : OK :: CA : CP :: $1 : n$, seu OK $= n$. HK.

II. Junctam rectam TO tangere Ellipsin in O (per Elem. Sect. Con.). Et qvemadmodum (Eucl. El. III. 17. 18. VI. 8) proportionales sunt CK. (CH vel) CA, CT; ita idem valere constat circa axem alterum, scilicet si recta Ellipsin in O tangens occurrat ipsi CP in S, esse ∺ CR, CP, CS, seu CR × CS $=$ CP².

Cor. Qvia rectus est angulus LOT æqve ac CHT, ideoqve KL × KT $=$ OK² & CK × KT $=$ HK², erit KL × KT : CK × KT vel Subnormalis KL : CK :: (OK² : HK² :: per I.) CP² : CA² :: $nn : 1$.

 III.

III. $OL \times OU = CP^2 = n^2a^2$. Nam (Eucl. VI. 2. 8.) OL: CR:: (LZ: CZ:: CZ: ZU::) CS: OU, ideoqve $OL \times OU = CR \times CS =$ (II) CP^2.

IV. In Ellipsi (ut & reliqvis Coni Sectionibus) Radium Curvaturæ in puncto qvolibet O esse proportionalem cubo normalis OL, & qvidem (*) = huic cubo applicato ad qvadratum e semiparametro Axis principalis, seu $= \frac{CA.^2\ OL^3}{CP^4} = \frac{OL^3}{n^4a^2}$.

Schol. 1. Similiter & ex dictis facile demonstratur, pari ratione esse Subnormalem RZ: CR adeoqve & normales OZ: OL:: CA^2: CP^2:: 1: nn; $OZ \times OU = CA^2 = aa$; atqve radium Curvaturæ in $O = \frac{CP.^2\ OZ^3}{CA^4} = \frac{nn.}{aa} OZ^3$. Seqvuntur etjam hæc omnia ex valoribus mox (§ 3) inveniendis.

Schol. 2. Si CU producta occurrat Ellipsi in D: erit CD, qvippe (constr.) parallela tangenti OT, semidiameter Conjugata ipsi CO; qvamobrem (doctr. Sect. Con) rectang. $OU \times CD = CA \times CP$. Qva propositione, (qvod in transcursu observamus) collata cum istis: $OU \times OL = CP^2$ & $OU \times OZ = CA^2$, seqvitur esse CP: CA:: OL: CD, nec non CA: CP:: OZ: CD, adeoqve $CD = \frac{OL}{n} = n. OZ$ semper proportionalem normali OL vel OZ.

(*) Kongl. Vet. Acad. Handl. 1744 p. 158. DE LA CAILLE Lec. Elem. de Mathem. § 887. Cfr. sis Rob. SIMSON Sect. Con. Lib. V. prop. 40. aliosve passim.

§. 3.

Datis CA & CP, ſaltem proportione: pro loco O, latitudine dato, potiſſimum invenire oportet angulum COZ & ſemidiametrum CO, qvibus ſpeciatim ad determinandas Parallaxes Aſtronomicas opus eſt. Qværantur inſuper etjam rectæ OL, OZ, OU, CL, CZ, CU, LU, UZ, LZ, & Radius Curvaturæ Meridiani.

Eo fine ante omnia qværo angulos ACH & ACO, qvi dicantur M & N reſpective. Eſt autem Tang M: Tang N:: KH: KO:: (§ 2. I) 1 : n, & Tang N: Tang L:: $\frac{OK}{CK} : \frac{OK}{KL}$ :: KL : CK :: (§ 2. Cor.) nn: 1. Ergo Tang M = n. Tang L & Tang N = nn. Tang L. Cognito ſic vel utroqve vel, qvod plerumqve ſufficit, alterutro angulorum M & N, non tantum mox innoteſcit (ſcilicet ſolo angulo N invento) angulus qvæſitus COZ = L − N (Eucl. I. 32) qvi aberratio Latitudinis vocari poterit; ſed & reliqva inveniri poterunt, vario qvidem modo, fortasſis autem optime ut ſeqvitur. Poſito Sinu toto = 1, ſunt HK = a. Sin M, CK = a. Coſ M, OK = (n. HK =) na. Sin M, KL = (§ 2. Cor. nn. CK =) nna. Coſ M. Jam in Δ:lo COH eſt CO: CH:: Coſ. M: Coſ N; in Δ:lo OKL, OL : OK :: 1 : Sin L; in Δ:lo ORZ, OZ: OR vel CK:: 1: Coſ L; OU = (§ 2. III) $\frac{nnaa}{OL}$; CL = CK − KL, vel in Δ:lo CLO, CL: CO:: Sin $\overline{L - N}$: Sin L; in Δ:lo CLZ, CZ : CL:: Tang L: 1, & LZ: CL:: 1: Coſ L; in Δ:lo CLU,

CU:

CU : CL : LU :: Sin L : 1 : Cos L; deniqve in Δ:lo CUZ, UZ : CZ :: Sin L : 1. Sic igitur obtinentur $CO = \frac{a.\,\text{Cos}\,M}{\text{Cos}\,N} = (*)\,\frac{na.\,\text{Sin}\,M}{\text{Sin}\,N}$; $OL = \frac{na.\,\text{Sin}\,M}{\text{Sin}\,L}$; $OZ = \frac{a.\,\text{Cos}\,M}{\text{Cos}\,L} = (*)\,\frac{a.\,\text{Sin}\,M}{n.\,\text{Sin}\,L}$; $OU = \frac{na.\,\text{Sin}\,L}{\text{Sin}\,M} = (*)\,\frac{a.\,\text{Cos}\,L}{\text{Cos}\,M}$; $CL = \overline{1-nn}.\,a.\,\text{Cos}\,M$; $CZ = \overline{1-nn}.\,a.\,\text{Tang}\,L.\,\text{Cos}\,M = (*)\,\frac{\overline{1-nn}.\,a.}{n}\,\text{Sin}\,M$; $CU = \overline{1-nn}.\,a.\,\text{Sin}\,L.\,\text{Cos}\,M = (*)\,\frac{\overline{1-nn}.\,a.}{n}\,\text{Cos}\,L.\,\text{Sin}\,M$; $LU = \overline{1-nn}.\,a.\,\text{Cos}\,L.\,\text{Cos}\,M$; $UZ = \frac{\overline{1-nn}.\,a.}{n}\,\text{Sin}\,L.\,\text{Sin}\,M$; $LZ = \overline{1-nn}.\,a.\,\frac{\text{Cos}\,M}{\text{Cos}\,L} = (*)\,\frac{\overline{1-nn}.\,a.}{n}\,\frac{\text{Sin}\,M.}{\text{Sin}\,L}$ Vel si, ad inveniendas CL &c., seqventibus, nostro tamen judicio non æqve commodis, uti mavis formulis: $CL = \frac{a.\,\text{Cos}\,M.\,\text{Sin}\,(L-N)}{\text{Cos}\,N.\,\text{Sin}\,L} = \frac{na.\,\text{Sin}\,M.\,\text{Sin}\,\overline{L-N}}{\text{Sin}\,L.\,\text{Sin}\,N}$;

(*) Qvia semper Cos : Sin :: 1 : Tang. atqve (dem) angulorum L, M, N, Tangentes sunt ut 1, n, nn.

Cor. Qvia, si jungatur recta HL, est Tang ALH : Tang ALO :: (HK : OK ::) 1 : n; erunt Tangentes qvatvor horum angulor. ALH, ALO, ACH, ACO continue proportionales & qvidem secundum rationem 1 : n.

$CZ = a. \text{Cos} M. \frac{\text{Sin}\,\overline{L-N}}{\text{Cos} N. \text{Cos} L} = na. \frac{\text{Sin} M. \text{Sin}\,\overline{L-N}}{\text{Cos} L. \text{Sin} N};$

$CU = a. \text{Cos} M. \frac{\text{Sin}\,(L-N)}{\text{Cos} N} = na. \frac{\text{Sin}\,\overline{L-N}. \text{Sin} M}{\text{Sin} N};$

$LU = a. \frac{\text{Cos} M. \text{Sin}\,\overline{L-N}}{\text{Cos} N. \text{Tang} L} = na. \frac{\text{Sin} M. \text{Sin}\,\overline{L-N}}{\text{Tang} L. \text{Sin} N};$

$UZ = a. \text{Tang} L. \frac{\text{Cos} M. \text{Sin}\,\overline{L-N}}{\text{Cos} N} = na. \frac{\text{Tang} L. \text{Sin} M. \text{Sin}\,\overline{L-N}}{\text{Sin} N}; LZ = a. \frac{\text{Cos} M. \text{Sin}\,\overline{L-N}}{\text{Sin} L. \text{Cos} L. \text{Cos} N} = na. \frac{\text{Sin} M. \text{Sin}\,\overline{L-N}}{\text{Sin} L. \text{Cos} L. \text{Sin} N}.$ Deniqve (§ 2. IV) Radius Curvaturæ $= \frac{a}{n} \left(\frac{\text{Sin} M}{\text{Sin} L}\right)^3 =$ (*) $nna \left(\frac{\text{Cos} M}{\text{Cos} L}\right)^3$.

§. 4.

Exhibuimus (§ 3) formulas, qvæ ad calculum numericum, & qvidem ad singula qvæsitorum independenter a reliqvis invenienda, sint aptissimæ, seu qvæ commodas regulas practicas, in usus Astronomicos & Geographicos, contineant. Poterunt autem, qvod nonnunqvam utile erit, in alias converti, speciatim tales, qvas non nisi CA, CP, atqve Latitudinis vel Tangens vel Sinus vel Cosinus ingrediantur. Sic posito Tang $L = t$, sunt Cos $L = \frac{1}{\sqrt{1+tt}}$, Sin $L = \frac{t}{\sqrt{1+tt}}$, Tang $M = nt$, Cos $M = \frac{1}{\sqrt{1+nntt}}$,

$\frac{1}{\sqrt{1+nntt}}$, Sin M $= \frac{nt}{\sqrt{1+nntt}}$, Tang N $= nnt$, Cos N $= \frac{nnt}{\sqrt{1+n^4tt}}$; [qvorum valorum substitutione in § 3, prodeunt formulæ, qvæ non nisi *n*, *a* & *t*, continebunt. Juvat etiam notare, qvod Tang COL = Tang $\overline{L-N}$ = $\left(\text{*} \frac{\text{Tang L} - \text{Tang N}}{1 + \text{Tang L. Tang N}}\right) = \frac{(1-nn)t}{1+nntt} = \frac{1-nn}{n}$. Tang L. Cos M² = ($\frac{1-nn}{n}$. Tang M. Cos M² =) $\frac{1-nn}{n}$. Sin M. Cos M $= \frac{1-nn}{2n}$. Sin 2 M. Seqvitur idem ex inventis (§ 3) valoribus ipsarum CU, OU, qvatenus Tang COL $= \frac{CU}{OU}$. Sic etiam Sin COL $= \frac{CU}{CO} = \overline{1-nn}$. Sin L. Cos N.

Schol. Aliis usibus inservire poterunt formulæ, qvas ingrediatur non *t* sed v. g. abscissa CK = *x*. No-

(*) Hoc Theorema passim (ut MACLAUR. Tr. of Flux. §. 922) absqve demonstratione adhibitum, facile probatur; vid. sis, F. C. MAYER Comm. Petrop. T. II. p. 15; DE LA LANDE Astron. §. 2932; Exerc. Misc. Math. Phys. Fasc. II. §. 13. Aboæ 1758.

Notandum igitur quod $t = \left[\frac{OK}{KL} = (\S\ 2.\ I.\ \&\ II.\ Cor.)\ \frac{n.\ HK}{m.\ CK} = \frac{HK}{n.\ CK} = \right] \frac{\sqrt{aa - xx}}{nx}$, qvi valor loco ipsius t adhibeatur.

§. 5.

Definiatur sub qva Latitudine loci & qvantus sit angulus COZ maximus. Qvia (§ 3) $COZ = L - N$, oportet iam esse fluxionem $dL = dN$, hoc est (ob Tang $L = t$, & Tang $N = nnt$,) $\frac{dt}{1+tt} = \frac{nndt}{1+n^4tt}$, unde $tt = \left(\frac{1-nn}{nn-n^4} = \right) \frac{1}{nn}$, adeoqve t seu Tang $L = \frac{1}{n} = \frac{CA}{CP}$, & nnt seu Tang $N = n = \frac{CP}{CA}$; junctâ igitur AP, est qvæsita Latitudo $L =$ ang CPA, $N =$ CAP, angulus maximus $COZ = CPA - CAP = 2 CPA - 90° = 90° - 2 CAP$, ideoqve (Elem. Trig.) Tang $\frac{1}{2} COZ = \frac{CA - CP}{CA + CP} = \frac{1-n}{1+n}$.

Vel hoc modo: maximus erit Ang. COZ qvando ejus Tangens vel (§ 4) huic proportionalis qvantitas $\frac{t}{1+nntt}$ maxima fuerit, ideoqve evanuerit hujus sumenda fluxio $\frac{(1-nn\,tt)\,dt}{(1+nntt)^2}$, h.e. ubi $1 = nntt$,

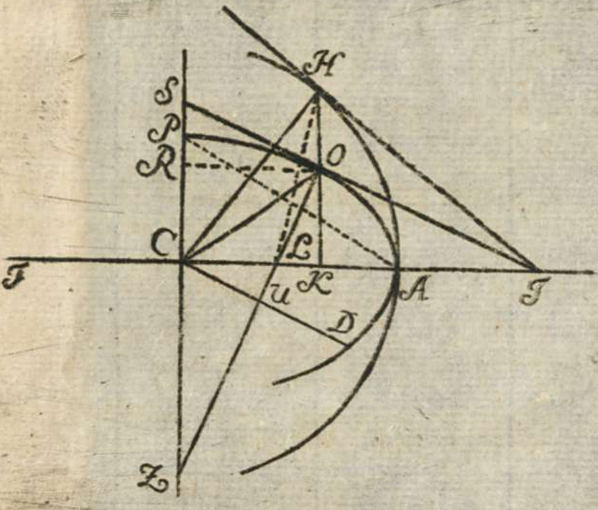
ac

ac proinde $t = \frac{1}{n}$ ut antea. Vel, non adhibendo methodum fluxionum, sic: Tang COZ est (§ 4) proportionalis ipsi Sin 2 M, at Sin 2 M est maximus qvando M $= 45°$ adeoqve Tang M seu $nt = 1$. Hunc ipsius t valorem $\frac{1}{n}$ substituendo in generali Tangentis COZ valore (§ 4), in casu maximi fit Tang COZ $= \frac{1 - nn}{2n} = \frac{1 + n. 1 - n}{2n} = \frac{(CA + CP)(CA - CP)}{2 CA. CP} = \frac{CA^2 - CP^2}{2 CA. CP} =$ (si F fuerit Focus Ellipseos) $\frac{CF^2}{2 CA. CP}$.

Schol. Cum sit semper COZ complementum ipsius COS (= OCU) vel COT *i. e.* anguli ordinatarum diametro CO: patet COZ maximum esse, qvando angulus, qvo CO & diameter ipsi conjugata (cfr § 2 Schol. 2) ad se inclinantur, fuerit maxime obliqvus, adeoqve maximus (ab una parte) vel minimus (ab altera). De hoc casu & de inveniendis constructione punctis O ita positis in Ellipsi, vid. sis SIMSON Sect. Con. L. IV. prop. 6. Cas. 1. coll. prop. 4. Corr. vel potius HAUSEN Elem. Sect. Con prop. 13. ejusqve Schol.

§. 6.

Similiter qværere lubet, qvo casu & qvanta sit maxima CU, distantia scilicet lineæ verticalis



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OZ a Centro C. Qvia tunc etjam maximum erit CU^2 ideoqve (§. §. 3. 4). Sin L. Cof M^2 seu $\frac{tt}{(1+tt).(1+nntt)}$; sumenda hujus fluxio $\frac{2t(1-nnt^4)dt}{(1+tt.\ 1+nntt)^2}$ statuatur $=0$, h. e. $nnt^4=1$; unde fit t seu Tang $L=\sqrt{\frac{1}{n}}=\sqrt{\frac{CA}{CP}}$; & hunc ipsius t valorem inferendo generali expressioni $\frac{(1-nn)at}{\sqrt{(1+tt)(1+nntt)}}$ ipsius CU, obtinemus distantiam maximam $CU = \overline{1-n}.a = CA - CP$.

Cor. 1. Hæc Latitudo minor est Latitudine illa (§ 5) cui respondet ang. COZ maximus, utraqve autem $> 45°$. Nam ob $n<1$, erit $\sqrt{\frac{1}{n}}<\frac{1}{n}$ ($=t$ in § 5) at utraqve >1 seu Tang $45°$.

Cor. 2. Et quidem Latitudinis hujus, cui competit maxima CU, Tangens est medius proportionalis inter tangentes anguli semirecti atqve Latitudinis habentis maximam aberrationem COZ. Etenim $\div 1, \sqrt{\frac{1}{n}}, \frac{1}{n}$.

Cor. 3. Ubi CU est maxima, fit (§ 4) Tang $COZ = \frac{1-n}{\sqrt{n}} = \frac{CA-CP}{\sqrt{CA.\ CP}} = \frac{CU}{\sqrt{CA.CP}}$ ideoqve $OU = \sqrt{CA.\ CP} = CD$ (§. 2. Schol. 2.)